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IMPACTS OF VISCOUS MATTER AND RADIATION ON THE TEMPORAL EVOLUTION OF THE UNIVERSE

We propose that the cosmic background is simply characterized by an equation of state of a relativistic fluid consisting of homogeneously and isotropically distributed single particle and single photon. Other components such as dark matter and dark energy (cosmological constant) are not taken into consideration. We assume that the expansion of the Universe is isotropic. Accordingly, the shear viscosity coefficient vanishes. We have derived expressions for the energy density and the bulk viscosity and found that both quantities largely increase with the inclusion of the photon. As for their temporal evolution, there is a rapid decrease in the early stages of the Universe. This seems to saturate at large co-moving time. We also found that the scale factor in non-viscous and viscous cosmic backgrounds is almost non-distinguishable in the early Universe. The latter suggest going beyond the standard cosmological model (SCM). The non-viscous cosmic background seems to have a scale factor larger than that of the viscous one. The result obtained for the temporal evolution of the Hubble parameter confirms the scale factor results. Namely, the distinguishability between non-viscous and viscous cosmic backgrounds is the largest in the early Universe.

Keywords: classical general relativity, classical statistical mechanics, early Universe.

1. Introduction

The temporal evolution of the Universe can be determined by reliable equations of state (EoS), which characterize the matter and radiation fillings in the cosmological geometry. Such a description for the early Universe is known as "solid description," as it draws a qualitative, if not even quantitative picture, although no single observational evidence for that is practically available [1–3]. The present work is an extension of previous studies [4, 5], where the cosmic background matter was conjectured to be consisted of two parts: the single particle with a mass m_p and the second part integrating radiation (single photon). Such a background could be characterized by Newton's theory [6]. It was assumed that other components such as the dark energy and dark matter would not matter much during the early stages of the Universe. For the seek of simplicity, we set aside these additional ingredients. Accordingly, the cosmic background could then be determined by an EoS for a particle with mass m_p and a photon with frequency ν and energy $h\nu$, where h is Planck's constant.

Recent lattice QCD simulations [7–11] and high-energy experiments [12] have taught us that the matter under extreme conditions surely does not behave itself like an ideal gas, but rather as a relativistic fluid. This fluid is a strongly correlated medium with specific properties such as the non-vanishing thermal conductivity and finite viscosity coefficients (bulk ξ and shear η) [13, 14]. The related EoS could be taken from Ref. [15].

In the present work, we introduce a toy model based on a thermodynamical approach in order to propose a framework for a finite bulk viscosity for the cosmic background, i.e., beyond the standard cosmological model (SCM), with which we aim at deriving the Friedmann equations using Newtonian gravity. Finally, we apply thermodynamics in order to drive expressions for various cosmological quantities such as the scale factor $a(t)$ and the Hubble parameter $H(t)$. Throughout this work, we assume natural units for all physical quantities. We also assume that the different forms of matter and radiation are homogeneously and isotropically distributed.

The present paper is organized as follows. A toy model for the cosmic background is studied in Section 2. Expressions for the time evolution of the scale factor, the Hubble parameter, and the energy density for non-viscous and viscous cosmic bacrgrounds are also presented in Section 2. Using the proposed expressions, our calculations for the bulk viscosity of the cosmic background are elaborated in Section 3. Section 4 is devoted to the final conclusions.

2. The Approach

In a toy model, we firstly assume that the only type of energy in the very early stages of the Universe is the heat Q. We then assume that there is just one particle occupying the entire cosmic geometry [4]. Thus, we can, for this model, formulate the first law of thermodynamics $dQ = 0$,

$$dQ = dU + pdV, \tag{1}$$

where p, V, and U are the pressure, the volume, and the internal energy, respectively. The volume $V \simeq \simeq a^3$ and the energy density ρ can be related to each other as $\rho = U/V$. We assume that, in co-moving coordinates, the mass of the particle can be estimated as $m_p = U$. Therefore, $d\rho$ can be given as

$$d\rho = \frac{1}{V^2}(VdU - UdV), \tag{2}$$

From Eq. (1), we obtain

$$d\rho = -3(p + \rho)\frac{da}{a}. \tag{3}$$

By multiplying both sides of Eq. (3) by $1/dt$, a time element, we get

$$\dot{\rho} = -3(p + \rho)\, H. \tag{4}$$

We notice from this equation that $\dot{\rho}$ depends on both ρ and p. Accordingly, Eq. (4) can be solved for a specific equation of state (EoS) of the system, which is dominated by matter (just one particle). Second, as the Universe expands, both ρ and p become time-depending quantities.

Now, if we want to integrate another component to constituents (heat and the mass of the particle), we might consciously think of the photon [4]. For seek of simplicity, we assume that this photon has a frequency $\nu = 1$,

$$\dot{\rho} = -3\left((p + \rho) + \frac{2\,\pi}{a^3}\right)H. \tag{5}$$

In light of this discussion, we also have to account for a radiation-dominated phase, $p = \rho/3$. Then, the Hubble parameter, for instance, when assuming a flat curvature and the vanishing cosmological constant, can be expressed as

$$H(t) = \frac{-\dot{\rho}}{3\left(p + \rho + \frac{2\,\pi}{a^3}\right)}, \tag{6}$$

Now, let us assume that two components of the cosmic background are positioned at a distance a from a specific point in the Universe. For the particle, the kinetic energy reads $m_p\dot{a}^2/2$. For the photon, the kinetic energy is given as $h\nu$. Of course, there is a gravitational force between the masses of both components and the entire mass of the absolute background, which can be determined as $M = (4\pi/3)a^3\rho$. Thus, the total energy can be given as [5]

$$E = \frac{1}{2}\,m_p\,\dot{a}^2 - \frac{M\,m_p}{a} + 2\,\pi - \frac{M\,m_\gamma}{a}, \tag{7}$$

where the second term represents the gravitational potential energy of the particle with the rest of the Universe, the third term represents photon's energy, and the fourth term gives photon's gravitational potential energy. All quantities are given in natural units.

This novel *toy* model (beyond SCM) is apparently based on the Newtonian cosmology, with which we shall try to derive the Friedmann equations. We assume now that both particle and photon are positioned in a viscous surrounding (beyond SCM). Then, the total energy, Eq. (7), gets an additional contribution from the viscosity work, which obviously affects the expansion of the Universe. Whether this indeed slows down or rather adds to the effective cosmic pressure is still controversial. Many of the current literature more likely support the second scenario, on one hand. Let us focus the discussion on the earlier case, which is more likely favored in Refs. [1–5] based on

Refs. [10, 11, 13, 14]

$$E = \frac{1}{2} m_p \, \dot{a}^2 - \frac{M \, m_p}{a} + 2\pi - \frac{M \, m_\gamma}{a} - \xi a^3 \, \frac{\ddot{a}}{\dot{a}}, \tag{8}$$

where $\xi = \xi_p + \xi_\gamma$ combines the bulk viscosity coefficients of the particle and the photon. For an isotropic expansion of the Universe, the shear viscosity coefficient is conjectured to vanish.

Multiplying Eq. (8) by $(2/m_p)$ and substitute the value of M give

$$\frac{2E}{m_p} = \dot{a}^2 - \frac{8\,\pi}{3} a^2 \rho + \frac{4\pi}{m_p} - 2\xi \frac{a^3 \, \ddot{a}}{m_p \, \dot{a}}, \tag{9}$$

compare the previous equation with the first equation of Fridemann solution $\dot{a}^2 + k = \frac{8\,\pi}{3} a^2 \rho$, the curvature parameter k can be obtained

$$k = -2 \frac{E}{m_p} - 2 \frac{\xi}{m_p} \frac{\ddot{a}}{\dot{a}} a^3 + \frac{4\pi}{m_p}. \tag{10}$$

• For a flat Universe, i.e., $k = 0$, m_p has to be very large. Otherwise, if m_p would remain finite, then the scale factor has to be given as

$$a(t) = \left(3 \, \frac{2\pi - E}{\xi}\right)^{1/3} t^{1/3}, \tag{11}$$

which apparently limits the total energy E to be less than $2\,\pi$, photon's energy.

• For a positive or negative curvature,

$$a(t) = \left(3 \, \frac{4\pi \mp m_p - 2E}{2\xi}\right)^{1/3} t^{1/3}, \tag{12}$$

respectively. The $-$ sign refers to a positive curvature, while $+$ sign stands for a negative curvature. In deriving Eqs. (11) and (12), the approximation $\ddot{a}/\dot{a} \approx \dot{a}/a$ has been applied [1]. Alternatively, we have to find solutions for

$$\text{at } k = 0 \quad a^3 \, \frac{\ddot{a}}{\dot{a}} = \frac{2\,\pi - E}{\xi}, \tag{13}$$

$$\text{at } k = \pm 1 \quad a^3 \, \frac{\ddot{a}}{\dot{a}} = \frac{2\,\pi - E \pm m_p/2}{\xi}. \tag{14}$$

[1] where $d/dt(H) = d/dt(\dot{a}/a) = \ddot{a}/\dot{a} - (\dot{a}/a)^2$ implying that $d/dt(H) \approx H - H^2$. This gives the solution $H = e^t/(e^t + e^{c_1})$, where c_1 is the integration constant.

To this end, we propose $\dot{a} = y$. Accordingly, $\ddot{a} = y\, y_a^{'}$. Then, the generic solution reads

$$a(t) = \left(-\frac{3}{2} \, \frac{2\,\pi - E \pm m_p/2}{\xi}\right)^{1/3} t^{1/3}, \tag{15}$$

which is nearly compatible with Eq. (12). Accordingly, the constraint on the total energy E assuring a positive or negative curvature is given as

$$E > 2\,\pi \mp \frac{m_p}{2}. \tag{16}$$

Once again, we conclude that the matter/energy content defines the Universe curvature. So far, we also conclude that the dependence of the scale factor $a(t)$ on the co-moving time t significantly changes, when accounting for the viscous properties for the background matter/radiation. Recalling the expressions for the dependence of the energy density ρ on the co-moving time t [5], we get

• For the particle only,

$$\rho(t) = \frac{t^{-2}}{36\,\pi}, \tag{17}$$

• For a particle and a photon in the non-viscous cosmic background,

$$\rho(t) = \frac{t^{-2}}{36\,\pi} + \frac{6\pi}{c_2^3} \, t^{-1}, \tag{18}$$

where c_2 is an integration constant.

• For a particle and a photon in the viscous cosmic background,

$$\rho(t) = \frac{1}{36\,\pi} \, t^{-2} + \frac{6\pi}{c_2^3} \, t^{-1} + \xi \, \frac{c_2^3}{9} \, t - 1, \tag{19}$$

Finally, from Eq. (12), we can deduce an expression for the evolution of the bulk viscosity

$$\xi(t) = \left(-\frac{3}{2} \, \frac{2\,\pi - E \pm m_p(t)/2}{a(t)^3}\right) t, \tag{20}$$

where $a(t)$ can be substituted from the previous equations, for example, Eqs. (13)–(14).

3. Results

In this section, Eq. (20) is used to determine the dependence of the bulk viscosity on the co-moving time t. We firstly propose to start with a statistical fit for the results depicted in Fig. 1 of Ref. [5]. We obtain

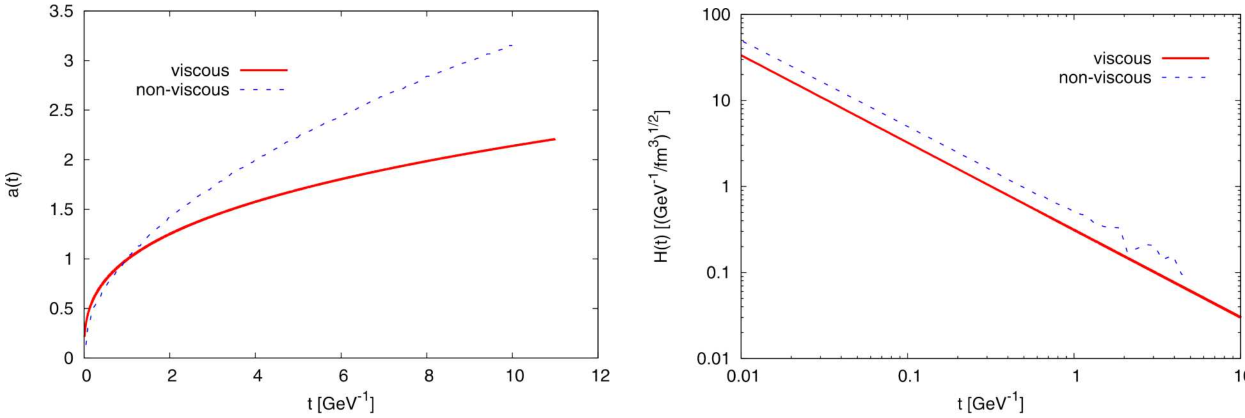

Fig. 1. Left-hand panel depicts the dependence of the scale factor $a(t)$ on the co-moving time t. The right-hand panel presents the Hubble parameter $H(t)$ in dependence on t. All physical quantities are given in neutral units. Taken from Ref. [5]

an expression for the time evolution of the scale factor $a(t)$

$$a(t) = c_3\, t^{1/3}, \tag{21}$$

where c_3 is the proportionality constant $0.993 \pm \pm 0.0012$. When substituting $a(t)$ from Eq. (21) into Eq. (20)

$$\xi(t) = \left(-\frac{3}{2}\,\frac{2\,\pi - E \pm m_p(t)/2}{c_3^3}\right). \tag{22}$$

Second, we need to figure out how the particle mass evolves with the co-moving time, $m_p(t)$. As discussed previously, we have that $m_p = U = \rho V$, where $V = = a^3$. When substituting ρ from Eq. (17), we get

$$m_p(t) = \frac{a(t)^3}{36\,\pi}\, t^{-2}. \tag{23}$$

The resulting $m_p(t)$ can then be substituted in Eq. (22),

$$\xi(t) = \left[-\frac{3}{2}\,\frac{2\,\pi - E \pm \frac{1}{2}\left[\frac{a(t)^3}{36\,\pi}\, t^{-2}\right]}{c_3^3}\right]. \tag{24}$$

In order to determine the dependence of the energy density of the viscous cosmic background filled out with a particle and a photon, we substitute $\xi(t)$ from Eq. (24) into Eq. (19)

$$\rho(t) = \frac{1}{36\,\pi}\, t^{-2} + \frac{6\pi}{c_3^3}\, t^{-1} - \\ -\frac{1}{6}\left[2\,\pi - E \pm \frac{1}{2}\left(\frac{a(t)^3}{36\,\pi}\, t^{-2}\right)\right] t - 1, \tag{25}$$

As reported in Ref. [5], Fig. 1 depicts the dependence of the scale factor $a(t)$ (left-hand panel) and the Hubble parameter $H(t)$ (right-hand panel) on the co-moving time t. The dashed and solid curves refer to the non-viscous and viscous backgrounds, respectively. As proposed in Ref. [5], the scale factor in non-viscous and viscous cosmic backgrounds can be related to t as follows

$$a(t)|_{\texttt{non-visc}} \propto t^{1/2}, \\ a(t)|_{\texttt{visc}} \propto t^{1/3}. \tag{26}$$

We notice in Fig. 1 that, at low t, both non-viscous and viscous scale factors are almost non-distinguishable, and their time-dependence is apparently large. With increasing t, the scale factor in the non-viscous cosmic background becomes larger than that in the viscous cosmic background. This behavior results from the finite bulk viscosity which seems to slow down the expansion of the Universe. Furthermore, we found that, at a small range of t, the expansion of the bulk viscous Universe becomes much more rapid than in the non-viscous one.

The time-dependence of the Hubble parameter, right-hand panel, apparently confirms the results obtained for the scale factor (left-hand panel). We found that the distinguishability between non-viscous ($H = = 1/2t$) and viscous cosmic backgrounds ($H = 1/3t$) [5] is the largest at low t. With increasing t, we observe the corresponding time evolution of the Hubble parameter in both types of the cosmic background comes close each other, on one hand. On the other hand, the viscous cosmology seems to have a faster Hubble parameter than the non-viscous one.

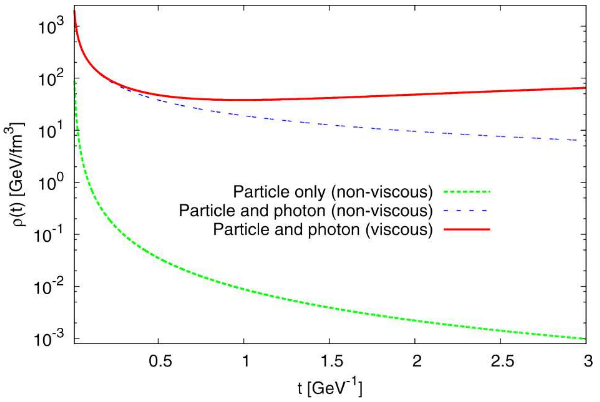

Fig. 2. The energy density $\rho(t)$ is given in dependence of the co-moving time t using Eqs. (17), (18), (25) for the particle only (non-viscous), one particle and one photon (in non-viscous), and one particle and one photon (in viscous cosmic background), depcited as dotted, dashed, and solid curves, respectively

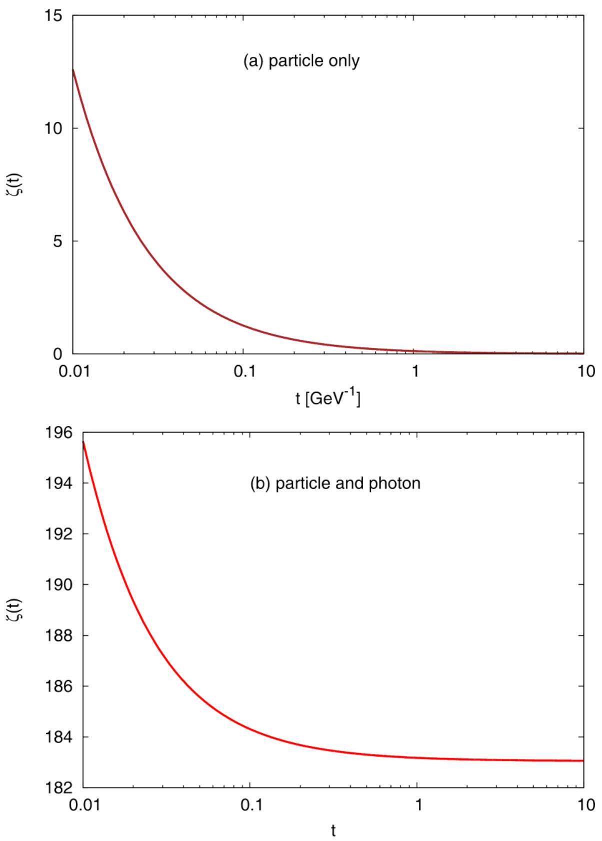

Fig. 3. The bulk viscosity $\xi(t)$ is given in dependence on the co-moving time t, Eq. (24). Panel (a) shows the temporal dependence of ξ for one particle. Panel (b) presents the same, but for one particle and one photon

Figure 2 illustrates the dependence of the energy density $\rho(t)$ on the co-moving time t for one particle filling in the non-viscous cosmic background (dotted curve), for one particle together with one photon in the non-viscous cosmic background (dashed curve), and for one particle and one photon in viscous cosmic background (solid curve) [5]. The corresponding expressions are given in Eqs. (17), (18), and (25), respectively. We notice a similar trend of $\rho(t)$ vs. t in both cases; a particle only and a particle with a photon in the non-viscous cosmic background. The latter corresponds to a larger $\rho(t)$ than the former. There is a difference of nearly 3–4 orders of magnitudes. For the viscous cosmic background, the resulting $\rho(t)$ is greater than that for the non-viscous cosmic background. We also note that $\rho(t)$ first decreases with increasing t, which is then followed by an increasing $\rho(t)$ with a further increase in t. This late increasing is stemming from the impacts of the propertied viscosity added to the background geometry. At small t, the effect of the viscosity seems being negligible so that the resulting curves for the cosmic viscous and non-viscous backgrounds are very similar to each other. Then with the increase in the co-moving time the effect of the viscosity seems to increase and accordingly enlarges the value of the energy density, especially for the viscous cosmic background relative to the non-viscous one. Such an enhancement can be understood from the positive sign in front of the viscosity term in Eq. (19).

Figure 3 presents the dependence of the bulk viscosity $\xi(t)$ on the co-moving time t as determined from Eq. (24). Panel (a) shows the dependence of ξ on t for one particle. Panel (b) shows the same dependence, but here an additional photon is included. A similar observation was found in the energy density, Fig. 2. It is obvious that, for both configurations, $\xi(t)$ rapidly decreases with increasing t. Afterwards, at large t, the value of the viscosity seems to saturate. We also observe that the temporal evolutions of ξ for both cases look very similar, despite the fact that the values of ξ, when including one photon, are much higher than when excluding this.

4. Conclusions

We have utilized the same toy model introduced in Refs. [4, 5]. This model proposes that the cosmic background could simply be characterized by an equation of state of a relativistic fluid consisting of one particle and one photon. In doing this, we have assumed that other components such as the dark energy and dark matter are not impacting the evolution of the Universe. We have also assumed the vanishing cosmological constant. The different forms of matter and radiation are distributed homogeneously and isotropically. Specifically, we assume that the particle and the photon are positioned at a distance a from a specific point in the Universe. Therefore, each component will have eigen kinetic energy. For the particle, this reads $m_p\dot{a}^2/2$, while for the photon, we have $h\nu$. There is a gravitational force between the masses of both components and the entire mass of the absolute background, which can be determined as $M = (4\pi/3)a^3\rho$. Finally, we assume that the expansion of the universe is isotropic, and, accordingly, the shear viscosity coefficient is conjectured to vanish. The main contribution of the present paper is that both particle and photon are in a viscous surrounding, i.e., going beyond the standard cosmological model. Thus, the total energy gets an additional contribution from the viscosity, which apparently would slow down the expansion of the Universe.

Based on the laws of thermodynamics, some basic cosmological parameters, namely, the scaling factor and Hubble parameter have been determined in dependence on the co-moving time t. We have obtained that the scale factors for the the non-viscous and viscous backgrounds are strongly time-dependent. At low t, the scale factors in non-viscous and viscous cosmic backgrounds are almost non-distinguishable. With increasing t, we note that the scale factor in the non-viscous cosmic background becomes larger than that in the viscous one. This result can be understood due to the finite bulk viscosity which seems to slow down the expansion of the Universe. Furthermore, we found that, at small range of t, early stages of the Universe, the expansion of the bulk viscous Universe becomes much more rapid than in the non-viscous one. The results obtained on the time-dependence of the Hubble parameter confirm the results obtained for the scale factor. The distinguishability among the non-viscous and viscous cosmic backgrounds is the largest at low t. With increasing t, the temporal evolution of the Hubble parameter for both types of a cosmic background comes close each other. We also observe that the viscous cosmology has a faster Hubble parameter than for the non-viscous one.

Furthermore, we have studied the effects of a finite bulk viscosity on these cosmological quantities and found noticeable changes as a result of the finite bulk viscosity. The dependence of the energy density ρ on the cosmic background, i.e., whether viscous or non-viscous one, has been studied, as well. In addition, we note that $\rho(t)$ first decreases and then increases with increasing t. This late increasing is due to the impacts of the viscosity added to the background geometry, while, at small t, the effect of the viscosity seems being negligible so that the resulting temporal evolutions for cosmic viscous and non-viscous backgrounds are very similar to each other. At large t, the effect of the viscosity increases, and, accordingly, the energy density increases as well.

Last but not least, we have deduced an expression for the bulk viscosity of the cosmic background in two cases – one particle in additional to one photon and one particle only – to observe the temporal evolution of the bulk viscosity. We note that ξ in both cases decreases rapidly with increasing t. Similar to the observation found in the energy density, the values of ξ, when including one photon, are much larger than that, when excluding this.

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ВПЛИВ В'ЯЗКОЇ РЕЧОВИНИ ТА ВИПРОМІНЮВАННЯ НА ЧАСОВУ ЕВОЛЮЦІЮ ВСЕСВІТУ

Ми припускаємо, що космічний фон характеризується рівнянням стану релятивістичної рідини, яка складається з однорідно та ізотропно розподілених однієї частинки та одного фотона. Інші компоненти, такі як темна речовина і темна енергія, не розглядаються. Ми вважаємо, що Всесвіт розширюється ізотропно. Відповідно, коефіцієнт в'язкості зсуву дорівнює нулю. Отримано вирази для густини енергії та об'ємної в'язкості і показано, що вони суттєво зростають із введенням до розгляду фотона. Що стосується їхньої часової еволюції, то на початковій стадії вони швидко зменшуються, але з часом виходять на насичення. Ми також виявили, що масштабний фактор майже однаковий для нев'язкого та в'язкого космічного фону у ранньому Всесвіті. Це передбачає вихід за межі стандартної космологічної моделі. Виглядає так, що у нев'язкого космічного фону масштабний коефіцієнт більший, ніж у в'язкого. Часова еволюція параметра Хаббла узгоджується з результатами для масштабного фактора. А саме, відмінність між нев'язким та в'язким космічним фоном є найбільшою у ранньому Всесвіті.

Ключові слова: класичний загальний релятивізм, класична статистична механіка, ранній Всесвіт.