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MINIMIZATION OF DOSE LOAD IN ALGORITHMS OF X-RAY COMPUTED TOMOGRAPHY

An algorithm has been developed for the reconstruction of an X-ray image obtained at the minimum dose load on the researched object and provided a given image accuracy. This algorithm combines approaches typical of the inverse projection and regularization methods. The image is formed by overlaying filtered projections, and the filtering parameters are determined from the minimum condition for the difference between the discrepancy and the experimental error.

Keywords: computed tomography, dose load, reconstruction methods, theory of ill-posed problems.

1. Introduction

X-rays are known to be used in computed tomography to obtain three-dimensional images of the internal structure of organism. When choosing the radiation intensity, two mutually contradictory tendencies are faced with. On the one hand, a desire to improve the image quality requires that the radiation intensity should be as high as possible. On the other hand, this intensity has to be reduced in view of the harmful effects of X-rays. Therefore, there arises a problem to determine the minimum level of dose load on the organism, which would provide a required image accuracy. This principle is also called ALARA (As Low As Reasonably Achievable). In this paper, one of possible ways to solve this problem has been proposed.

2. X-ray Damping and Its Physical Model Used in Computed Tomography

It is known [1] that any living organism considered as a physical system can be characterized by the presence of different structural levels. The latter are conventionally classified into two groups. The first group includes macroscopic levels. Among them, the following levels are distinguished: the structural level of organ systems, the organ level, the level of morphofunctional organ units, and the level of tissues, cellular systems, and non-cellular structures. The second group includes microscopic levels: cellular, subcellular, molecular, and atomic ones.

When studying a structure at any of indicated macroscopic levels, a continuum model is used, in which an organism is regarded as an inhomogeneous continuum. This is a model of organism that computed tomography is based on. In this model, the structure is characterized by the spatial dependence of the X-ray damping coefficient $\mu(\mathbf{x})$, where $\mathbf{x} = \{x, y, z\}$ is the radius vector of a point in the continuum. The determination of the function $\mu(\mathbf{x})$ for a given organism is the ultimate goal of computed tomography from the physical viewpoint.

The function $\mu(\mathbf{x})$ is determined experimentally. An object is scanned in a definite plane at various angles using X-ray irradiation, and the intensity of radiation passed through the object is measured with the help of a detector. Analogous measurements are repeated for neighbor planes.

Which is a relationship between the function $\mu(\mathbf{x})$ and the experimentally measured data? Let the plane $Z = 0$ be one of those planes. In this plane, together with the laboratory coordinate frame (x, y), a moving coordinate frame (x', y') is introduced, in which the y'-axis is directed along the X-ray propagation direction. Denoting the angle between the x- and x'-axes as ϕ, we have the obvious equality

$$x' = x\cos\phi + y\sin\phi. \tag{1}$$

The intensity of radiation passed through the object is determined by the known formula

$$I_\phi(x') = I_\phi^0(x')\exp\left[-\int \mu(x, y)dy\right], \tag{2}$$

where I_ϕ^0 is the radiation intensity in the absence of an object. In the framework of the problem concerned, a new quantity is introduced into consideration,

$$\lambda_\phi(x') = -\ln\frac{I_\phi(x')}{I_\phi^0(x')}, \tag{3}$$

which is called the object projection. Taking formulas (1) and (2) into account, the equation for the object projection looks like

$$\lambda_\phi(x') = \iint_{-\infty}^{\infty} \mu(x, y)\delta(x\cos\phi + y\sin\phi - x')dxdy, \tag{4}$$

where $\delta(x)$ is the delta function. If the function $\lambda_\phi(x')$ is known from experimental data, the problem consists in determining the function $\mu(x, y)$ on the basis of Eq. (4). In other words, the ultimate aim of computed tomography is to solve integral equation (4).

The function $\mu(x, y)$ is visually represented as a planar diagram. The x- and y-axes lie in the plane of this diagram, and the function values μ correspond to different color intensities of the diagram sections. A diagram of this kind is called the tomographic cross-section of the object. Accordingly, the procedure of calculation of $\mu(x, y)$ is called the reconstruction of the object cross-section on the basis of its projections.

3. Reconstruction Algorithm Based on Inverse Projection

This method for the solution of Eq. (4) forms the basis of reconstruction algorithms in the vast majority of modern tomographs [2–4]. Its input data include the two-dimensional inverse Fourier transform of the function $\mu(x, y)$ in polar coordinates,

$$\mu(x, y) = \int_0^\pi \int_{-\infty}^{\infty} \mu^*(r, \phi) \times \\ \times \exp\left[2\pi r(x\cos\phi + y\sin\phi)\right]|r|\,drd\phi, \tag{5}$$

where $\mu^*(r, \phi)$ is the two-dimensional direct Fourier transform of the function $\mu(x, y)$ in polar coordinates. Relation (5) can be written down as

$$\mu(x, y) = \int_0^\pi \lambda_\phi^*(x')d\phi, \tag{6}$$

where x' is determined by formula (1), and the function $\lambda_\phi^*(x')$ by the expression

$$\lambda_\phi^*(x') = \int_{-\infty}^{\infty} \mu^*(r, \phi)\exp\left[i2\pi r(x')\right]|r|\,dr. \tag{7}$$

According to the projection-slice theorem, the two-dimensional Fourier transform of $\mu(x, y)$ is equal to the one-dimensional Fourier transform of the projection $\lambda_\phi(x')$. Therefore, expression (7) is rewritten in the form

$$\lambda_\phi^*(x') = \int_{-\infty}^{\infty} \lambda_\phi(x)p(x' - x)dx, \tag{8}$$

in which the following notation was introduced:

$$p(x) = R^2\left\{\frac{\sin(2Rx)}{Rx} - \left[\frac{\sin(Rx)}{Rx}\right]^2\right\}. \tag{9}$$

Formula (8) means that the projection $\lambda_\phi(x)$ is subjected to filtering. Accordingly, the function $\lambda_\phi^*(x)$ is called the filtered projection, and the constant R the filtration coefficient.

However, one can see that formulas (5)–(9) do not contain quantities that would characterize the errors. At the same time, the latter inevitably arise at computations. Therefore, it is evident that the problem about the relation between the dose load and the image accuracy cannot be solved in the framework of an algorithm based on the inverse projection method.

4. Reconstruction Algorithm Based on the Tikhonov Regularization Method

Let us rewrite Eq. (4) in the operator form:

$$A\mu = \lambda. \tag{10}$$

The functions μ and λ are considered as elements of the sets M and Λ, i.e.

$$\mu \in M, \quad \lambda \in \Lambda. \tag{11}$$

By introducing the corresponding distances $\rho(\lambda_1, \lambda_2)$ and $\rho(\mu_1, \mu_2)$, the sets M and Λ are transformed into metric spaces.

As was already mentioned, the image reconstruction procedure is reduced to the solution of the integral equation (10) or, equivalently, Eq. (4). The specific feature of this equation consists in that the element λ is given with a certain error that is not taken into account. Such a problem belongs to the class of inverse problems in mathematical physics [5]. The procedure of finding an approximate solution for them is called regularization, and the approximate solution itself is called regularized.

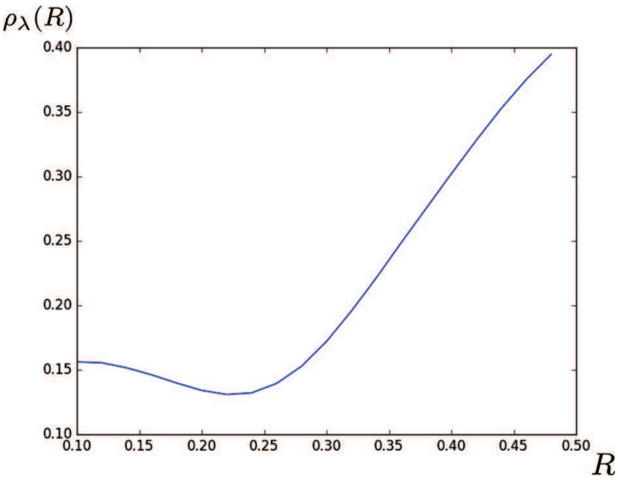

Fig. 1. Dependence $\rho_\lambda(R)$ at $\delta_\lambda = 0.03$

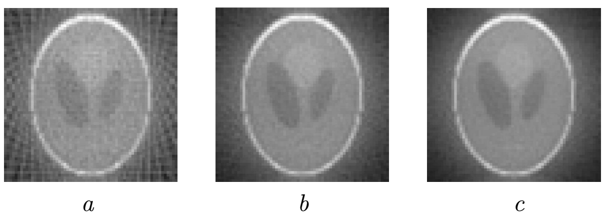

Fig. 2. Phantom reconstruction at $\delta_\lambda = 0.03$ and $N = 24$ (*a*), 60 (*b*), and 120 (*c*)

There are a number of methods for the solution of ill-posed problems. The most wide-spread among then is the Tikhonov regularization method. According to it, instead of Eq. (10), the equation

$$A^*A\mu + \alpha\mu = A^*\lambda \tag{12}$$

is dealt with. Here, A^* is an operator conjugate to the operator A, and α is the regularization coefficient. The latter can be determined from the equation

$$\rho^2\left(A\tilde{\mu}, \lambda\right) = \delta_\Lambda^2, \tag{13}$$

where $\tilde{\mu}$ is an approximate solution of the equation.

It is evident that, unlike the inverse projection method, the solution of Eq. (10) or, equivalently, Eq. (4) depends on the error. Nevertheless, it is unclear how this solution can be related to the radiation intensity.

5. Reconstruction Algorithm Making Allowance for the Dose Load

A comparison of both methods described above demonstrates that, mathematically, the regularization method is more rigorous than the inverse projection one. So, why is the latter preferable in practice? The matter is that its implementation requires much less amount of the computational time (by orders of magnitude). Let us try to combine them and to preserve the positive features of both methods. Let equality (13) be a basis for such a unification. It contains the basic idea of the regularization theory, namely, the computational and experimental errors have to be made identical. For this purpose, a regularization parameter α is introduced, and Eq. (12) is solved instead of Eq. (10), which results in a smoothed solution $\tilde{\mu}$. In this sense, the terms "smoothing" and "regularization" can be regarded as synonyms.

One of the manifestations that the inverse projection method is insufficiently consistent from the mathematical viewpoint is the fact that it does not substantiate the choice of the filtration coefficient R. The filtering procedure performed by means of Eq. (8) is nothing else but a smoothing of the projection $\lambda_\phi(x)$. Therefore, the coefficient R actually plays the same role as the regularization parameter α does. In the course of computations, there emerges

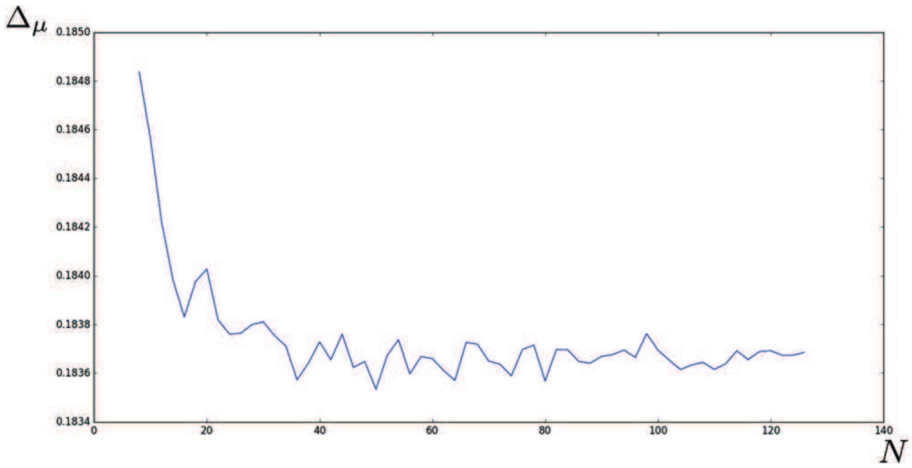

Fig. 3. Dependence $\Delta_\mu(N)$ at $\delta_\lambda = 0.03$

another factor that stimulates the smoothing. Really, if λ^*_ϕ is determined from expression (8), the integration procedure is substituted by the summation of discrete function values calculated with a certain step Δx of the argument. So, this is the interval Δx, over which the function $\lambda_\phi(x)$ is smoothed at the numerical integration. Therefore, both parameters responsible for the smoothing, R and Δx, have to be put in agreement. By its content, the quantity $1/R$ is also an interval, over which the smoothing takes place. Hence, the required agreement is reduced to the simple equality

$$R = \frac{1}{\Delta x}. \tag{14}$$

The idea of smoothing – one can see that it is inherent in both methods concerned – suggests us how the dose load can be introduced into consideration. In the calculation, integral (6) is substituted by the sum

$$\mu(x, y) = \sum_{i=1}^{N} \lambda^*_{\phi_i} \Delta\phi, \tag{15}$$

where $\Delta\phi = \frac{\pi}{N}$. Summation (15) is another smoothing. It can be characterized by the parameter N, which plays the role of another regularization parameter.

Thus, we arrive at a calculation scheme characterized by two regularization parameters, R and N. Therefore, let us assume that the function μ depends on them, i.e.

$$\mu = \mu(x, y; R, N). \tag{16}$$

As one can see from formula (15), N projections are used for the determination of the function $\mu(x, y)$. It is evident that larger N values correspond to higher dose loads. Therefore, the parameter N can be regarded as the measure of a dose load.

The function $\mu(x, y; R, N)$ can be determined as follows. By selecting arbitrary R- and N-values, we calculate this function using formulas (6), (8), and (9), as is required in the inverse projection method. The calculation is repeated for different (R, N) pairs. As an approximate solution, we select the function $\tilde{\mu}(x, y; R, N)$, for which the following expression is minimum:

$$Q = \rho^2 \left(A\tilde{\mu}, \lambda\right) - \delta^2_\Lambda. \tag{17}$$

Hence, the proposed reconstruction method combines the ideas taken from both inverse projection and regularization methods.

6. Numerical Experiment

The experimental procedure consists of the following steps.

A mathematical phantom with a known intensity distribution $\mu_0(x, y)$ is chosen.

Making use of expressions (2) and (3), the value $\lambda_\phi(x)$ of this phantom is calculated for a given number N of projections. Let us denote this value as $\lambda_{0\phi}(x)$.

A definite error δ_λ is added to the exact function $\lambda_{0\phi}(x)$.

With the help of formulas (6), (8), and (9), a minimum of difference (17) regarded as a function of R is determined.

The error Δ_μ of the reconstructed image $\mu(x, y)$ is determined as $\Delta_\mu = \rho(\mu_0, \mu)$.

Steps 2 to 5 are repeated for various values of the parameter N. Then the dependence of $\Delta_\mu = \rho(\mu_0, \mu)$ on N is plotted. This dependence is used to choose the optimal value for N.

In order to verify the proposed approach, an experiment was performed, by using the Shepp–Logan phantom [6]. The latter approximately simulates a cross-section of human head. It is used as a standard, when the efficiency of reconstruction methods is verified. When calculating the distance ρ between the functions, the latter were assumed to be elements of a Hilbert metric space.

A procedure for the determination of the parameter R is illustrated in Fig. 1, where the dependence $\rho_\lambda(R)$ is plotted for $\delta_\lambda = 0.03$. According to the figure, the parameter $R \approx 0.23$ in this case. Figure 2 illustrates the corresponding reconstruction results obtained at $N = 24$, 60, and 120. Finally, Fig. 3 demonstrates the obtained dependence of Δ_μ on N. From this figure, it is evident that, after a certain number of projections (in the case concerned, this is 60), the quality of the reconstructed image does not increase significantly. A conclusion can be made that the value $N = 60$ is the optimal number of projections for the given experimental error δ_λ.

A specific value of the patient dose load depends on a variety of parameters, such as the current through an X-ray tube and the voltage across it, the exposure time per one image, and the number of images [7]. In our research, we showed a method for the calculation of an optimal number of images required to obtain a tomogram with desired quality.

7. Conclusions

An importance of the problem of patient dose load in the course of treatment and diagnosis has never been doubted. We may talk about two approaches, when calculating the dose load value. One of them is aimed at determining the upper limit of the permissible dose load. This limit is determined by the biological features of the examined object. For the other approach to be applied, the lower load limit has to be determined. This approach is typical of diagnostic methods, when the diagnostic accuracy becomes lost, if the radiation intensity is not sufficiently high. The lower limit value obviously depends on the required accuracy. Unlike the upper limit, this value is not determined by object's biology, but by the physical and mathematical apparatus, which this diagnostic method is based on. The indicated dependence for computed tomography was obtained in this paper. The authors hope for that the application of the described approach to the choice of the number of images will substantially reduce the dose load on the patient, keeping the diagnostic quality of the obtained tomographic images at a high level.

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МІНІМІЗАЦІЯ ДОЗОВОГО
НАВАНТАЖЕННЯ В АЛГОРИТМАХ
РЕНТГЕНІВСЬКОЇ КОМП'ЮТЕРНОЇ ТОМОГРАФІЇ

Р е з ю м е

Розроблено алгоритм реконструкції рентгенівського зображення, для якого при заданій точності зображення дозове навантаження на об'єкт дослідження є мінімальним. В цьому алгоритмі поєднані підходи, характерні для методу оберненого проеціювання та методу регуляризації. Зображення формується в результаті накладання фільтрованих проекцій, а параметри фільтрації визначаються із умови мінімуму різниці між нев'язкою та похибкою експерименту.