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DISSIPATIVE RAYLEIGH–TAYLOR INSTABILITY AND ITS CONTRIBUTION TO THE FORMATION OF AN INTERFACE BETWEEN BIOMATERIALS AT THEIR ELECTRIC WELDING

The dissipative Rayleigh–Taylor instability, which can arise at the interface between biomaterials under the action of high-frequency currents, has been considered. A mechanism describing the formation of a corrugated boundary of the mesomorphic phase is proposed. The corrugation period and depth are evaluated.

Keywords: dissipative Rayleigh–Taylor instability, biological materials, instability increment, bipolar electric welding.

1. Introduction

The electric coagulator (electrosurgical coagulator, medical coagulator, cauterodyne) is a device that is most commonly used in surgical operations in modern medical practice. This is a high-frequency surgical apparatus for dissecting tissues and simultaneously stopping the bleeding. The concept of electrocoagulation as a method of treatment was introduced by E.L. Doyen in 1909. He reported about a bipolar method of electrocoagulation of malignant tumors [1]. In 1910, Czerny introduced the notion of "passive electrode" and, using a needle as an "active electrode", described the dissection of tissues by a high-frequency current [2].

The modern electrocoagulator includes a generator of high-frequency electric current, an active needle-like electrode, and a passive electrode. The active electrode is supplied with a high-frequency electric current, which induces the heating of the intracellular fluid and an incision at the place of electrode contact with the tissue. The device is used for the surgical treatment of patients in many medical areas, such as general surgery, gynecology, urology, neurosurgery, endoscopy, oncology, vascular surgery, orthopedics, and otolaryngology.

The application of an electrocoagulator for the tissue dissection is inseparably linked with the problem of tissue connection. Conventional methods are not always efficient. Moreover, they can invoke the following serious consequences of the suture material (brackets, glue) application:

• an inevitable development of the inflammatory reaction in response to the presence of such materials in the wound;

• the danger of the infection spread from hollow organs (intestines, stomach) through the suture material, which could result in the development of severe postoperative complications;

• the risk of anastomotic stenosis owing to the development of the coarse scarring in the long-term postoperative period; and others.

To solve the problems indicated above, the technology of bipolar electric welding making use of high-frequency currents and the corresponding equipment were developed at the E.O. Paton Institute of Electric Welding of the National Academy of Sciences of Ukraine [3]. The practical application of this technology in surgical operations makes it possible to shorten the operation time by 20–40 min on average, to reduce the blood loss by about 200–250 ml (sometimes, this amount is a few times larger), to obtain the economical effect due to a reduction of the application of expensive devices and staplers, to save the thread suture material – e.g., catgut, silk threads, nylon, capron (polyamide), lavsan (polyester) – or reject mechanical connectors such as titanium clips from the application.

The further development and improvement of the technology of bipolar electric welding by means of high-frequency currents are associated with the research of mechanisms governing the process of electric welding of biological tissues at the microscopic level. A specific feature of the electric welding process consists in that the connection of biological tissues occurs without the participation of mechanical connectors: threads, glues, metal brackets, and so forth. A contact between the tissues is achieved and supported due the formation of a certain mesomorphic phase, which arises in both connected tissues. Proceeding from a hypothetical model, this phase is created owing to the interaction between collagen structures [4]. A molecular model of structural changes that occur in soft biological tissues at their electric welding, which was proposed in work [4], provides information about the time characteristics of the electric field: the pulse duration, frequency, and time evolution of the current strength at all stages of the process. However, this model does not answer the question about the physical model of biological tissue connection and does not determine the geometry of the interface between the tissues.

For the first time, the interface shape at the welding of two different metals in the solid phase was discussed in work [5]. The cited authors experimentally showed that, when different metals are rolled together in vacuum, there arise unstable modes at certain rolling speeds, which are characterized by the appearance of a wave-like structure of the metal interface along the rolling direction. To describe the interface shape, a physical model was proposed, which was based on the Kelvin–Helmholtz instability phenomenon and involved the viscosity and surface tension of metals, the so-called dissipative Kelvin–Helmholtz instability. In the framework of this model, conditions of the instability emergence were considered, and the characteristic length and time scales of unstable perturbations were determined. The qualitative and quantitative agreement of the proposed model with experimental results was demonstrated.

In this work, the dissipative Rayleigh–Taylor instability, which can arise at the interface between biomaterials under the influence of high-frequency currents, is considered. The mechanism of formation of corrugated boundary of the mesomorphic phase is proposed, and the period and depth of interface corrugation are evaluated.

2. Rayleigh–Taylor Instability in Viscous Media

Let two incompressible and viscous liquids be in equilibrium in the field of a gravity force that is directed oppositely to the z-axis direction (the Cartesian reference frame). The liquids are characterized by the densities ρ_1 and ρ_2, and their dynamic viscosity coefficients are μ_1 and μ_2. The unperturbed interface between the media coincides with the plane $z = 0$. Fluid 1 occupies the half-space $z > 0$, i.e. it is located over fluid 2.

Let us consider small deviations of the generalized system parameters from the equilibrium values, by assuming the density to be equal to $\rho + \delta\rho$ ($|\delta\rho| \ll \rho$), the pressure to $p + \delta p$ ($|\delta p| \ll p$), and the perturbed velocity components to u_l, where the subscripts $l = 1, 2, 3$ correspond to the coordinates x, y, and z, respectively. The perturbed velocity components have the same order of magnitude as the density and pressure.

The system of equations that describe the dependence of small deviations of perturbed quantities on the coordinates and time includes [6, 7] a linearized equation of motion, which takes the viscous and surface tension forces into account,

$$\rho\frac{\partial u_l}{\partial t}=-\frac{\partial\delta p}{\partial x_l}+\frac{\partial p_{lk}}{\partial x_k}+\left(-g\delta\rho+\delta(z)\sigma\Delta_\perp\eta\right)e_l,\quad (1)$$

and a linearized continuity equation

$$\frac{\partial\left(\rho+\delta\rho\right)}{\partial t}+\frac{\partial\left(\rho+\delta\rho\right)u_l}{\partial x_l}=0,\quad (2)$$

where $e_l=(0,0,1)$, $p_{lk}=\mu\left(z\right)\left(\frac{\partial u_l}{\partial x_k}+\frac{\partial u_k}{\partial x_l}\right)$ is the viscous stress tensor, $\mu\left(z\right)$ the dynamic viscosity coefficient dependent on the coordinate z, σ the surface tension coefficient, $\delta\left(z\right)$ the Dirac delta-function, $\eta\left(x,y,t\right)$ a small deviation of the interface from the equilibrium position $z=0$, and $\Delta_\perp=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$ the two-dimensional Laplacian. Equation (1) was obtained proceeding from the condition of the force balance in the equilibrium state

$$\frac{\partial p}{\partial x_l}=-g\rho e_l.\quad (3)$$

Together with Eq. (2), the equality

$$\frac{\partial\delta\rho}{\partial t}+u_l\frac{\partial\rho}{\partial x_l}=0\quad (4)$$

has to be obeyed, which gives rise to the condition of fluid incompressibility.

Let us transform Eqs. (1) and (2) with regard for Eqs. (3) and (4). Then the initial system of equations reads

$$\rho\frac{\partial u_l}{\partial t}=-\frac{\partial\delta p}{\partial x_l}+\mu\Delta u_l+\frac{\partial\mu}{\partial z}\left(\frac{\partial u_l}{\partial z}+\frac{\partial u_z}{\partial x_l}\right)+$$
$$+\left(-g\delta\rho+\delta(z)\sigma\Delta_\perp\eta\right)e_l,\quad (5)$$

$$\frac{\partial u_l}{\partial x_l}=0,\quad (6)$$

$$\frac{\partial\delta\rho}{\partial t}+u_z\frac{\partial\rho}{\partial z}=0,\quad (7)$$

where $\Delta=\Delta_\perp+\frac{\partial^2}{\partial z^2}=\Delta_\perp+D^2$ is the three-dimensional Laplacian. Equations (5)–(7) have solutions that are proportional to the normal modes $NM\left(x,y,t\right)=\exp\left(ik_xx+ik_yy+\lambda t\right)$:

$$\delta p=p_0(z)NM(x,y,t),\quad (8)$$

$$\delta\rho=\rho_0(z)NM(x,y,t),\quad (9)$$

$$u_l=u_{l0}(z)NM(x,y,t),\quad (10)$$

where $p_0\left(z\right)$, $\rho_0\left(z\right)$, and $u_{l0}\left(z\right)$ are z-dependent quantities; k_x and k_y are the projections of the perturbation wave vector on the axes x and y, respectively; λ is a constant; and i the imaginary unit.

The substitution of Eqs. (8)–(10) into the system of equations (5)–(7) brings about the following system of differential equations for the quantity u:

$$\lambda\rho u_x=-ik_x\delta p+\mu(D^2-k^2)u_x+D\mu(ik_xu_z+Du_x),\quad (11)$$

$$\lambda\rho u_y=-ik_y\delta p+\mu(D^2-k^2)u_y+D\mu(ik_yu_z+Du_y),\quad (12)$$

$$\lambda\rho u_z=-D\delta p+\mu(D^2-k^2)u_z+$$
$$+2D\mu Du_z-g\delta\rho-k^2\delta(z)\sigma\eta,\quad (13)$$

$$ik_xu_x+ik_yu_y=-Du_z\quad (14)$$

$$\delta\rho=-\frac{u_z(D\rho)}{\lambda},\quad (15)$$

where $k^2=\sqrt{k_x^2+k_y^2}$. Equations (11), (12), and (14) give rise to the equation

$$k^2\delta p=(-\lambda\rho+\mu(D^2-k^2))du_z+(D\mu)(D^2+k^2)u_z.\quad (16)$$

From Eqs. (13) and (15), it follows that

$$D\delta p=-\lambda\rho u_z+\mu(D^2-k^2)u_z+$$
$$+2(D\mu)Du_z+g\frac{(D\rho)}{\lambda}u_z-\frac{k^2}{\lambda}\delta(\eta)\sigma u_z.\quad (17)$$

Expression (17) contains the delta-function. This means that the contribution of the surface tension is substantial near $z=\eta(x,y,t)$. This contribution can be taken into consideration by integrating Eq. (17) over the coordinate z within a narrow ε-vicinity of the surface $\eta\left(x,y,t\right)$, i.e., we integrate Eq. (17) over a thin transient layer:

$$\langle A(x,y,z,t)\rangle=\lim_{\varepsilon\to0}\left(\int\limits_{\eta-\varepsilon}^{\eta+\varepsilon}A(x,y,z,t)dz\right).\quad (18)$$

By applying procedure (18) to equality (17), we obtain

$$\delta p\Big|_{\eta-0}^{\eta+0}=2\langle(Du_z)(D\mu)\rangle+\frac{g}{\lambda}u_z\rho\Big|_{\eta-0}^{\eta+0}-\frac{k^2}{\lambda}\sigma u_z(z=\eta).\quad (19)$$

In this equation, the notation $a(z)|_{\eta-0}^{\eta+0} = a(\eta+0) - a(\eta-0)$ is used, where $a(\eta \pm 0)$ are the values of the parameter a in the upper, $a(\eta+0)$, and the lower, $a(\eta-0)$, media with respect to their interface. Since the unperturbed interface coincides with the plane $z = 0$, the parameter η in Eq. (19) has to be put equal to zero.

On the other hand, equality (16) makes it possible to calculate the difference between the perturbation pressure values on the both sides of the boundary $z = \eta(x, y, t)$:

$$\delta p\Big|_{\eta-0}^{\eta+0} = k^{-2}[(-\lambda\rho + \mu(D^2 - k^2))Du_z + + (D\mu)(D^2 + k^2)u_z]\Big|_{\eta-0}^{\eta+0}. \tag{20}$$

By excluding the pressure difference $\delta p|_{\eta-0}^{\eta+0}$ from Eqs. (19) and (20), we obtain a relation between the perturbed quantities at the interface:

$$k^{-2}[(-\lambda\rho+\mu(D^2-k^2))Du_z+(D\mu)(D^2+k^2)u_z]\Big|_{\eta-0}^{\eta+0} = = 2\langle(Du_z)(D\mu)\rangle + \frac{g}{\lambda}u_z\rho\Big|_{\eta-0}^{\eta+0} - \frac{k^2}{\lambda}\sigma u_z(z=\eta). \tag{21}$$

Expression (21), which describes the relationship of perturbed quantities at the interface, should be appended by an equation that describes this relationship far from the interface. For this purpose, we exclude the perturbed pressure described by expression (16) from Eq. (17). As a result, we obtain

$$D\left[\left(\rho - \frac{\mu}{\lambda}(D^2-k^2)\right)Du_z - \frac{1}{\lambda}D\mu(D^2+k^2)u_z\right] = = k^2\left[\left(-g\frac{D\rho}{\lambda^2} + \frac{k^2}{\lambda^2}\delta(\eta)\sigma\right)u_z + + \left(\rho - \frac{\mu}{\lambda}(D^2-k^2)\right)u_z - \frac{2}{\lambda}D\mu Du_z\right]. \tag{22}$$

Provided that the density and viscosity of the media are constant, expression (22) is simplified:

$$\left(1 - \frac{\nu}{\lambda}(D^2-k^2)\right)(D^2-k^2)u_z = 0, \tag{23}$$

where $\nu = \mu/\rho$ is the kinematic viscosity coefficient.

In work [7], the general solution of Eq. (23) was proposed as a linear combination of partial solutions $\exp(\pm kz)$ and $\exp(\pm qz)$, where $q = \sqrt{k^2 + \lambda/\nu}$ is the perturbation penetration depth. However, this is not proper. One can easily be convinced of this by expressing Eq. (2) in terms of the speed potential φ ($\mathbf{u} = -\boldsymbol{\nabla}\varphi$, where $\boldsymbol{\nabla} = \mathbf{e}_l\frac{\partial}{\partial x_l}$ is the gradient operator). Then Eq. (2) transforms into the Laplace equation $\Delta\varphi = 0$. The expression for the potential φ, e.g., in the lower half-space can be found from the definition of the vertical velocity component:

$$\varphi(x,y,z,t) = \left(\frac{A}{k}e^{kz} + \frac{B}{q}e^{qz}\right)NM(x,y,t). \tag{24}$$

Potential (24) does not satisfy the Laplace equation. Therefore, the solution of Eq. (23) is sought in the form

$$\varphi_{1,2}(x,y,z,t) = A_{1,2}e^{\pm kz}NM(x,y,t), \tag{25}$$

where subscripts 1 and 2 correspond to the signs + and −, respectively.

Proceeding from the expression for the solution describing the perturbed vertical velocity, let us calculate the quantity $\langle(Du_z)(D\mu)\rangle$ in Eq. (21):

$$\langle(Du_z)(D\mu)\rangle = \lim_{\varepsilon\to 0}\left(\int\limits_{\eta-\varepsilon}^{\eta+\varepsilon}(Du_z)(D\mu)dz\right) = = \lim_{\varepsilon\to 0}\left(\int\limits_{\eta-\varepsilon}^{\eta}(Du_z)(D\mu)dz\right) + + \lim_{\varepsilon\to 0}\left(\int\limits_{\eta}^{\eta+\varepsilon}(Du_z)(D\mu)dz\right) = = -ku_z(\mu(\eta-\varepsilon) + \mu(\eta+\varepsilon)). \tag{26}$$

According to Eq. (25), the vertical velocity looks like

$$\begin{aligned} u_{z1}(x,y,z,t) &= A_1e^{-kz}NM(x,y,t) \quad (z>0), \\ u_{z2}(x,y,z,t) &= A_2\,e^{kz}NM(x,y,t) \quad (z<0). \end{aligned} \tag{27}$$

At the interface $z = \eta(x,y,t) = 0$, the continuity conditions (21) for the velocity u_z component and the pressure must be satisfied. From whence, we obtain two equations for the constants A_1 and A_2:

$$\begin{aligned} &A_1 = A_2, \\ &-\frac{\lambda(-k\rho_1A_1 - k\rho_2A_2)}{k^2} = \\ &= -2kA_1(\mu_1+\mu_2) + \frac{g}{\lambda}(\rho_1-\rho_2)A_1 - \frac{k^2}{\lambda}\sigma A_1. \end{aligned} \tag{28}$$

From Eqs. (27), it is easy to obtain a dispersion equation that describes the dependence of the λ-value on the parameters of the media:

$$\lambda^2 + 2k^2\lambda\frac{(\mu_1+\mu_2)}{(\rho_1+\rho_2)} - gk\frac{(\rho_1-\rho_2)}{(\rho_1+\rho_2)} + \frac{k^3\sigma}{(\rho_1+\rho_2)} = 0. \tag{29}$$

It should be noted that Eq. (29) can be obtained from the dispersion equation for the dissipative Kelvin–Helmholtz instability provided that the motion velocities in the both media equal zero [8].

In terms of the dimensionless variables $\Lambda = \lambda t_0$, $q = kl_0$, and

$$\mu^* = \frac{(\mu_1+\mu_2)\,t_0}{(\rho_2+\rho_1)\,l_0^2},$$

where

$$l_0 = \sqrt{\frac{\sigma}{g\,|\rho_1-\rho_2|}}, \quad t_0 = \sqrt{\frac{(\rho_1+\rho_2)\,l_0}{g\,|\rho_1-\rho_2|}},$$

Eq. (29) looks like

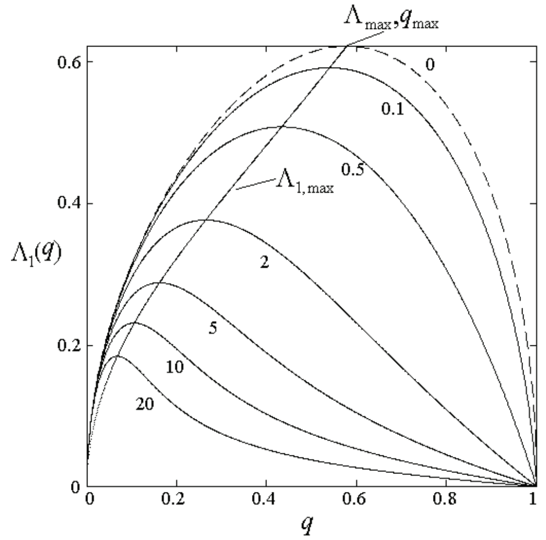
$$\Lambda^2 + 2\Lambda q^2\mu^* - \operatorname{sign}(\rho_1-\rho_2)\,q + q^3 = 0. \tag{30}$$

In the absence of viscosity ($\mu_1, \mu_2 \to 0$), Eq. (30) describes a state of unstable (the Rayleigh–Taylor instability) or stable equilibrium of the plane interface between two fluids with different densities, depending on whether the denser fluid is at the top or at the bottom. In the former case, the instability increment is determined from the equation

$$\Lambda^2 = \operatorname{sign}(\rho_1-\rho_2)\,q - q^3. \tag{31}$$

From whence, we get that, if $\rho_1 < \rho_2$, then $\Lambda_{1,2} = \pm\sqrt{-q-q^3}$ are purely imaginary values, and the interface perturbations do not increase by amplitude, but oscillate. If $\rho_1 > \rho_2$, the increment Λ is positive at $0 < q < 1 = q_c$. The instability increment maximum is attained at $q_{\max} = 1/\sqrt{3}$ and equals $\Lambda_{\max} = \sqrt{2\times 3^{-3/2}}$.

Hence, in the absence of viscosity, the Rayleigh–Taylor instability has a threshold character with respect to the wave number. In other words, it takes place only at wave numbers smaller than q_c. The rate of interface destruction is maximum for the wave number $q_{\max} = 1/\sqrt{3}$.



Dispersion dependences of the dimensionless increment of instability of the interface between two fluids calculated for various dimensionless viscosities of the media (indicated by the numbers near the curves)

Now, let us consider the influence of the medium viscosity on the interface instability. In this case, the instability increment is determined by the expression

$$\Lambda_{1,2} = \pm\sqrt{q^4\mu^{*2} + \operatorname{sign}(\rho_1-\rho_2)\,q - q^3} - q^2\mu^*. \tag{32}$$

Let us analyze the dependence of this expression on the wave number q. If $\rho_1 < \rho_2$, the perturbations are always stable, because $\Lambda_{1,2} < 0$. If $\rho_1 > \rho_2$, the instability increment is determined by the expression

$$\Lambda_1(q) = \sqrt{q^4\mu^{*2} + q - q^3} - q^2\mu^*,$$

which implies that the interface perturbations have wavelengths in the interval $0 < q < 1 = q_c$, analogously to the case of viscosity absence.

Figure illustrates the dependences $\Lambda_1(q)$ of the instability increment for the medium interface on the dimensionless wave number for various viscosity values μ^*'s. The figure also demonstrates the dependence of the maximum increment $\Lambda_{1,\max}$ on the wave number q, where

$$\Lambda_{1,\max} = \sqrt{q^4\mu^{*2} + q - q^3} - q^2\mu^*$$

at

$$\mu^* = \sqrt{\frac{9q^4 - 6q^2 + 1}{8q^3\,(q^2+1)}}.$$

From this figure, it follows that the magnitude of maximum increment decreases with the growth of viscosity in the media, and the maximum itself shifts toward the long-wavelength section of the perturbation spectrum.

3. Interface between Biopolymers at Their Bipolar Electric Welding Using a High-Frequency Current

Now, let us consider the formation of an interface between two identical biopolymers at their bipolar electric welding making use of a high-frequency current. Let the biopolymers have the same dynamic viscosity and density. At the initial stage, they are in contact with each other owing to an external mechanical influence [9]. Then a high-frequency current with the corresponding strength and frequency controllably flows through their interface. As a result of the current action, the protein coagulates and forms an interface.

Our research concerns the shape of the interface between the electrically welded biological tissues. For this purpose, let us use the results obtained in the previous section for the Rayleigh–Taylor instability in a gravitational field with regard for the surface tension and viscosity. Since the densities of contacting media are equal, $\rho_1 = \rho_2 = \rho$, the gravitational term is absent. However, in this process, there exists a non-gravitational force of attraction between the dielectric half-spaces separated by a thin dielectric layer with the thickness d and the dielectric constant ε_0. This force arises in the dielectric half-spaces embedded into an electric field with the strength $\mathbf{E}$ [10]. In the plane geometry, the field strength $\mathbf{E}$ is evaluated from the output voltage of a welding apparatus,

$$E = \frac{U_0 \cos(\Omega t)}{d}, \tag{33}$$

where $U_0 = 10 \div 200$ V and $\Omega \simeq 10 \div 60$ kHz are the amplitude and frequency, respectively, of the output voltage of a soft tissue welding apparatus EK-300M "SVARMED" [11], and $d = 1 \div 10$ μm is the gap width between the tissues at the welding beginning.

The magnitude of the attraction force is about

$$F = \varepsilon_0 \frac{\overline{E^2}}{8\pi} \Delta S = \varepsilon_0 \frac{U_0^2}{16\pi d^2} \Delta S, \tag{34}$$

where $\overline{E^2}$ is the squared field strength averaged over the period of high-frequency oscillations, and ΔS is the area of the high-frequency voltage influence. The magnitude of the weld seam depth h is about a few millimeters [11]. Therefore, the effective acceleration corresponding to force (34) is determined by the expression

$$g^* = g^*(\rho) = \frac{\varepsilon_0 U_0^2 \Delta S}{16\pi d^2 \rho V}, \tag{35}$$

where $V = h\Delta S$ is the volume of a welded material. From Eq. (35), we obtain that the acceleration equals

$$g^*(\rho) = \frac{\varepsilon_0 U_0^2}{16\pi d^2 \rho h}.$$

For $\varepsilon_0 = 100$, $U_0 = 100$ V, $\rho = 1$ g/cm^3, and $h = 0.1$ cm, it amounts to $g^* \simeq 2 \times 10^8$ cm/s^2. This value should be substituted into the dispersion equation (29), provided that $g^*(\rho_1) = g^*$ and $g^*(\rho_2) = -g^*$. Furthermore, since the Antonov rule is not applicable to the (bio)polymer–(bio)polymer interphase layer [12], we assume that the surface tension preserves its initial value.

Under those conditions, the dispersion equation that characterizes the increment dependence on the wave number reads

$$\lambda^2 + 2k^2\lambda\frac{\mu}{\rho} - g^*k + \frac{k^3\sigma}{2\rho} = 0. \tag{36}$$

In terms of new dimensionless variables $\Lambda_b = \lambda t_b$, $q_b = kl_b$, and $\mu_b = \mu\, t_b / \left(\rho l_b^2\right)$, Eq. (36) acquires the universal form

$$\Lambda_b^2 + 2\Lambda_b q_b^2 \mu - q_b + q_b^3 = 0, \tag{37}$$

where $l_b = \sqrt{\sigma/(2\rho g^*)}$ and $t_b = \sqrt{l_b/g^*}$.

The dependence of the increment Λ_b on the wave number q_b is determined by the solution of Eq. (37). This solution coincides with solution (32) for fluids with different densities and viscosities, which was discussed in the previous section. The curves in Figure completely correspond to the dispersion of perturbations in a biopolymer medium, provided that the substitutions $\Lambda \to \Lambda_b$, $q \to q_b$, and $\mu^* \to \mu_b$ are made.

In order to evaluate the parameters of the weld seam between biotissues, we took the values of dynamic viscosity, $\mu \approx 10 \div 90$ Pa s, and density, $\rho \approx 1.1$ g/cm^3, that are typical of a human organism [13]. The surface tension of biotissues is determined by the interfacial tension at the surfaces of cell membranes, which does not exceed 5 dyn/cm [14]. On the basis of the typical values for biotissue parameters quoted above, we obtain

$$l_b = 1.067\times10^{-4}\,\mathrm{cm},\ t_b = 0.73\times10^{-6}\,\mathrm{s},\ \mu_b = 100\div900. \tag{38}$$

From Figure, it follows that, at $\mu_b = 100$, the wave number of the most unstable mode is small, $q_b \ll 1$. Then the corrugation period of the biomaterial interface, $L = 2\pi/k$, can be determined from the equality

$$\mu_b = \sqrt{\frac{9q_b^4 - 6q_b^2 + 1}{8q_b^3\,(q_b^2 + 1)}}.$$

As a result, we obtain

$$\mu_b = 10^4, \quad L = 0.029 \text{ cm}. \tag{39}$$

The corrugation depth a is evaluated from the condition that, before the moment of tissue welding, the amplitude of perturbations is small, i.e. $ka \ll 1$. From whence in view of values (39), we obtain the upper limit for the corrugation amplitude $a \ll L/(2\pi) = 4 \times 10^{-3}$ cm.

4. Conclusions

To summarize, the Rayleigh–Taylor instability has been considered with regard for the viscosities of contacting media. The analysis of the equations describing the dependence of perturbations at the interface on the vertical coordinate showed that their solution is not the sum of two exponential functions, but a single exponential function that satisfies the Laplace equation for the velocity potential. The other exponential function is a fault solution and cannot be taken into consideration in the framework of the given problem. It is shown that, as was in the case of viscosity absence, the instability can be realized in the case of viscous fluids, if the denser fluid is on the top. However, the instability character changes to the dissipative one, if the growth of the viscosity in the media is accompanied by a reduction of the instability increment and a shift of its maximum into the region of long-wave perturbations. On the basis of the dissipative Rayleigh–Taylor instability, the process of electric welding of biological materials is described. The resulting interface between the biomaterials is shown to be a corrugated surface. The period and depth of the corresponding interface corrugation are evaluated.

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ДИСИПАТИВНА НЕСТІЙКІСТЬ
РЕЛЕЯ–ТЕЙЛОРА ТА ЇЇ ВНЕСОК У ФОРМУВАННЯ
МЕЖІ З'ЄДНАННЯ ПРИ ЕЛЕКТРИЧНОМУ
ЗВАРЮВАННІ БІОМАТЕРІАЛІВ

Р е з ю м е

Розглянута дисипативна нестійкість Релея–Тейлора, яка може виникати на межі з'єднання біоматеріалів при впливі струмів високої частоти, та запропонований механізм формування гофрованої межі мезоморфної фази з оцінкою періоду та глибини її гофрування.