
<https://doi.org/10.15407/ujpe69.8.600>

M. BULAKHOV,^{1,2} A.S. PELETMINSKII,^{1,2} YU.V. SLYUSARENKO^{1,2,3}

¹O.I. Akhiezer Institute of Theoretical Physics,
National Scientific Center “Kharkiv Physical and Technical Institute”,
Nat. Acad. of Sci. of Ukraine
(1, Akademichna Str, Kharkiv 61108, Ukraine)

²V.N. Karazin National University of Kharkiv
(4, Svobody Sq., Kharkiv 61022, Ukraine)

³National University “Lviv Polytechnic”
(12, S. Bandery Str., Lviv 79013, Ukraine)

INFLUENCE OF PHOTON SUBSYSTEM ON THE MAGNETIC PROPERTIES OF QUANTUM GASES

The possibility for the photon component to affect the magnetic properties of a system of quantum gases of two-level atoms staying in thermodynamic equilibrium with radiation (photons) has been studied. A corresponding simple model has been proposed, which enabled the general equations describing the thermodynamic equilibrium in this system to be derived. The resulting equations are solved in the temperature interval far from the degeneracy temperatures of all three system components. The analysis of the solutions testified to a non-trivial behavior of the system’s magnetic state as a response to changes in the photon density and the intensity of the external magnetic field. It is shown that the growth of the photon density induced in the system by external sources can increase both system’s magnetization and the density of excited atoms. Such a conclusion is not trivial a priori given the fact that photons in the vacuum have no magnetic moment.

Keywords: quantum gases, two-level atoms, photons, external magnetic field, thermodynamic equilibrium, non-degenerate state, inverse population, magnetic properties of the medium.

1. Introduction

The very concept of “quantum gases”, i.e., quantum-mechanical systems with many identical particles in the gaseous aggregate state, where the overlapping effect of the wave functions of individual particles

cannot be neglected, has become commonly accepted over the almost century-long history of their research, which was started in the works by S. Bose, A. Einstein, E. Fermi, and P. Dirac [1–4]. However, objects of this kind still continue to surprise their researchers by manifesting more and more new properties, which can serve not only as a demonstration of bright examples of quantum-mechanical effects at the microscopic level, but they also provide reasonable hopes for their use in practical applications; see, for example, work [5].

Some of the mentioned new phenomena and effects in quantum gases can be related to the inter-

Citation: Bulakhov M., Peletminskii A.S., Slyusarenko Yu.V. Influence of photon subsystem on the magnetic properties of quantum gases. *Ukr. J. Phys.* **69**, No. 8, 600 (2024). <https://doi.org/10.15407/ujpe69.8.600>.

© Publisher PH “Akademperiodyka” of the NAS of Ukraine, 2024. This is an open access article under the CC BY-NC-ND license (<https://creativecommons.org/licenses/by-nc-nd/4.0/>)

action of atoms in such gases with radiation (photons). For example, in works [6, 7], the study of quantum gases staying in equilibrium with radiation at ultralow temperatures made it possible to predict some “exotic” effects in those systems, which can be used to prove the possibility of realization of the photon Bose–Einstein condensation (BEC). Recall that, in the vacuum, the existence of BEC of photons is impossible, first of all, due to the lack of their mass.

However, in the environment, the realization of the photon BEC is possible, which was first demonstrated in work [8] by experimenting with a liquid dye in equilibrium with radiation at near-room temperatures. In the experiments, the availability of photon mass was associated with the emergence of a standing wave along one of the Cartesian coordinates; as a result, the system became as if it were two-dimensional. The preservation of the number of photons in the system (which is one of the necessary conditions for the BEC to exist) was provided by a system of mirrors. In works [6, 7], the mass of photons was considered to be a consequence of the photon interaction with atoms of ultracold gases. The effect of photon mass acquisition due to the interaction with the environment (quantum gases) was later proved in work [9].

However, it should be noted that, when studying weakly interacting gases in equilibrium with radiation (see works [6, 7]), the issues concerning the magnetic properties of the mentioned systems were neglected. Namely, of the greatest interest is the influence of the photon component on the magnetization of a system that is in equilibrium with ultracold atomic gases. This interest is primarily caused by the fact that photons do not possess a magnetic moment. Therefore, the very formulation of the problem concerning the influence of the photon component on the magnetic properties of the system may seem paradoxical. This work is devoted to clarifying the posed problem. In the framework of a simple model, whose basis was formulated in works [6, 7], with a certain modification to take into account the external magnetic field, we intend to show that the influence of the photon component that is in equilibrium with atoms in quantum gases on the magnetization of the system can be quite substantial if the system parameters acquire certain values.

2. Basic Equations of Thermodynamic Equilibrium of Photons with Quantum Gases

The conditions of thermodynamic equilibrium between radiation and quantum gases will be formulated in a form close to similar conditions presented in works [6, 7]. Namely, let us consider an ideal gas of Fermi or Bose atoms that are in thermodynamic equilibrium with photons in the presence of an external constant and uniform magnetic field \mathbf{H} . The atoms of this gas can be in only two quantum states: ground and excited ones (two-level atoms). In other words, the transition from one atomic state to another occurs through the absorption or emission of a photon. Thus, an excited atom can be considered as a “bound state” formed by a photon and an atom in the ground state. It should be emphasized that the atomic classification using the terms “ground state” and “excited state” is conditional and introduced for convenience. Hence, we may talk about the commonly accepted notions of the ground and excited states, as well as about other states connected through a one-photon transition.

The possibility of considering the system as an ideal gas of atoms and photons can be “justified” exactly by the fact that we describe the system in a state of thermodynamic equilibrium between its components. That is, the state of thermodynamic equilibrium is achieved by means of all possible interactions in the system, of course. However, we will assume that, after establishing such an equilibrium state, interaction effects can be neglected when describing this state. Taking this assumption into account, we note that, from a formal viewpoint, the system can be considered as consisting of atoms of two types (denoted by the subscripts $i = 1$ and 2), which are in thermodynamic equilibrium with the photon gas. In other words, one atomic component is composed of atoms in the ground state, and the other component is formed by the atoms in the excited state.

Let us also assume that the atoms of two types are characterized by a set of quantum numbers $\alpha_i \equiv i$, spins $S_i \equiv (S_1, S_2)$, and the function $f_{S_{iz}}(\mathbf{p}; \beta, \zeta_i)$ of atomic distribution over the momentum \mathbf{p} ,

$$f_{S_{iz}}(\mathbf{p}; \beta, \zeta_i) = \{\exp[\beta(\varepsilon_i(\mathbf{p}) - \zeta_i - \mu H S_{iz})] \pm 1\}^{-1}, \quad (1)$$

where

$$\varepsilon_i(\mathbf{p}) = \varepsilon_i + \frac{\mathbf{p}^2}{2M}, \quad \varepsilon_i < 0 \quad (2)$$

is the energy of a moving atom with the momentum \mathbf{p} , in the i -th state, and with a set of quantum-mechanical indices $\alpha_i \equiv i$ in the absence of an external magnetic field, $\varepsilon_i \equiv (\varepsilon_1, \varepsilon_2)$ is the energy of a stationary atom in the corresponding quantum-mechanical state in the absence of magnetic field, $\beta \equiv \frac{1}{T}$ is the inverse temperature T of the system (expressed in energy units), H is the external magnetic field directed along the axis z , m is the mass of an atom (the mass of an atom is assumed to be identical for different atomic states). Formula (1) also introduces such parameters as the magnetic moments of the atoms of the i -th type, $\mu\mathbf{S}_i \equiv (\mu\mathbf{S}_1, \mu\mathbf{S}_2)$. Their projections on the z -axis (or, equivalently, on the direction of the magnetic field H ; see above) enter this formula. In so doing, we may assume that the proportionality coefficient μ , which determines the value of the magnetic moment of an atom, is given by the expression $\mu = g\mu_B$, where μ_B is the Bohr magneton, and g is the so-called atomic g-factor (see, for example, work [10]). In addition, the notation ζ_i in Eq. (1) denotes the chemical potential of atoms in the i -th state. For definiteness, we also assume that the set of quantum numbers α_1 (the quantities with subscript “1”) corresponds to the ground atomic state, and its counterpart α_2 (the set of the quantities with subscript “2”) corresponds to the excited atomic state; see also Eq. (2). The sign “+” in formula (1) corresponds to the case of fermion atoms, and the sign “-” to bosons. It should also be noted that the spins of atoms in the ground and excited states are considered to be different, $\mathbf{S}_1 \neq \mathbf{S}_2$. This is a reflection of the circumstance that the spin of an atom is the vector sum of the spin of its nucleus (which in turn is the vector sum of the spins of the nucleons and their orbital moments) and the sum of all orbital moments and spins of the electrons in its electron shell. Since the ground state of an atom and its excited state are connected by the absorption or emission of a photon, the spin of the atom must also change. The issue of a specific (quantitative) change in the spin of atoms during the transition from one state to another is quite difficult and cannot be solved in the framework of the approach used in this article.

Let us now turn to the characteristics of the photon component, which is in the state of thermodynamic equilibrium with the quantum two-component gas whose characteristics are described by formulas (1) and (2). We assume that the photon component

is characterized by the distribution function

$$f_{\text{ph}}(\mathbf{k}; \beta, \zeta_{\text{ph}}) = \{\exp[\beta(\hbar\omega(\mathbf{k}) - \zeta_{\text{ph}})] - 1\}^{-1}, \quad (3)$$

where $\omega(\mathbf{k})$ is the dispersion law of photons, and ζ_{ph} is the chemical potential of photons. The very fact of introducing the chemical potential of photons into consideration means that the preservation of their total number in a given state of thermodynamic equilibrium is assumed, i.e., the total number of free photons in the medium and the photons formed by excited atoms is constant; for more details, see works [5, 6]. It should be specially emphasized that since photons are in equilibrium with the medium, they should be considered as quasi-particles with the effective mass m_{ph} , which is determined by the so-called “cutoff frequency” $\omega_0 = \omega(\mathbf{k} = 0)$,

$$\omega_0 = \omega(\mathbf{k} = 0), \quad m_{\text{ph}} = \frac{\hbar\omega_0}{v^2}, \quad (4)$$

where v is the velocity of light in the medium. Since the medium is composed of dilute quantum gases, the velocity of light in it can be considered close to the velocity of light in vacuum, c .

Some more remarks have to be added. In this article, we ignore the issue of the possibility of acquiring a spin (a magnetic moment) by a photon in the medium. This issue is not simple, requires separate studies, and remains controversial; see, for example, works [11–13]. Therefore, formula (3) does not contain terms that characterize the interaction of a photon (even in the medium) with an external magnetic field.

Regarding the explicit form of the photon dispersion law, the following remark can be made. In works [5, 6, 8], it was assumed, in essence, that the dispersion law of photons could be accepted in the form that is usual for relativistic particles,

$$\omega(\mathbf{k}) = \sqrt{\omega_0^2 + v^2k^2}. \quad (5)$$

It is easy to see that, in the interval of small wave vectors, this dispersion law acquires the following simplified form:

$$\hbar\omega(\mathbf{k}) \approx \varepsilon_{\text{ph}} + \frac{p^2}{2m_{\text{ph}}}, \quad \varepsilon_{\text{ph}} = \hbar\omega_0, \quad \mathbf{p} = \hbar\mathbf{k}, \quad (6)$$

where the photon mass m_{ph} is determined by formula (4). It was already mentioned above that the effect of

mass acquisition by photons due to their interaction with quantum gases was substantiated in work [9]. (In the cited work, explicit expressions for possible photon dispersion laws in quantum gases, including those of types (5) and (6), were also obtained.)

Formula (6) attracts attention to the extremely important role of the interval of small wave vectors in the dispersion law of photons in a medium. In paper [6], the application of approximation (6) was justified by the fact that the main task of that article was to demonstrate the possibility of Bose–Einstein condensation of photons in ultracold gases. At the same time, it was pointed out that, actually, as we will see below, the intervals of large wave vectors do not play a significant role in the problem under consideration, because their contribution to effects and phenomena in the relevant temperature interval is exponentially small.

As was mentioned above, the system is in the state of thermodynamic equilibrium with respect to the photon absorption (emission) process. At a given temperature, this condition of “chemical reaction” can be presented in the following form (see work [14]):

$$\left(\frac{\partial\Phi_1}{\partial N_1}\right) + \left(\frac{\partial\Phi_{\text{ph}}}{\partial \tilde{N}_{\text{ph}}}\right) = \left(\frac{\partial\Phi_2}{\partial N_2}\right), \quad (7)$$

where Φ_1 , Φ_2 , and Φ_{ph} are partial Gibbs potentials, which correspond to the components of atoms of the first and second kinds and the photon component, respectively; $N_1(H, T)$ and $N_2(H, T)$ are the numbers of particles in the atomic components of the first and second kinds, respectively; and $\tilde{N}_{\text{ph}}(H, T)$ is the number of free photons (not absorbed by the atoms). Recall that the Gibbs energy can be found using the formula

$$\Phi = E - TS - \Omega, \quad (8)$$

where $E = \sum_{\mathbf{p}} \varepsilon(\mathbf{p})f(\mathbf{p})$ is the internal energy, $\Omega = \mp T \sum_{\mathbf{p}} \ln(1 \pm f(\mathbf{p}))$ is the grand thermodynamic potential [the upper (lower) sign corresponds to the Bose–Einstein (Fermi–Dirac) statistics], $S = -(\partial\Omega/\partial T)_{V, \zeta}$ is the entropy, and $f(\mathbf{p})$ is the distribution function of the corresponding component in the system.

Combining condition (7) with the preservation conditions for the total number of the atoms of both types, N , and the total number of photons, N_{ph} , in-

cluding those contained in the excited atom and forming, with it, the “bound state”, we obtain the following system of equations:

$$\begin{aligned} n &= n_1(H, T) + n_2(H, T), \\ n_{\text{ph}} &= n_2(H, T) + n_{\text{ph}}(H, T), \\ \left(\frac{\partial\Phi_1}{\partial N_1}\right) + \left(\frac{\partial\Phi_{\text{ph}}}{\partial \tilde{N}_{\text{ph}}}\right) &= \left(\frac{\partial\Phi_2}{\partial N_2}\right), \end{aligned} \quad (9)$$

Here, $n = N/V$ is the total density of atoms in the system, and $n_{\text{ph}} = \tilde{N}_{\text{ph}}/V$ is the total density of photons in the system. In accordance with formulas (1)–(3), the density of the atoms of the i -th kind ($i = 1, 2$), $n_i(H, T) = N_i(H, T)/V$, and the density of photons in the free (unbound) state, $n_{\text{ph}}(H, T) = \tilde{N}_{\text{ph}}(H, T)/V$, are given by the formulas

$$\begin{aligned} n_1(H, T) &= \frac{1}{V} \sum_{S_{1z}=-S_1}^{S_1} \sum_{\mathbf{p}} \times \\ &\times \{\exp[\beta(\varepsilon(\mathbf{p}) + \varepsilon_1 - \zeta_1 - \mu HS_{1z})] \pm 1\}^{-1}, \end{aligned} \quad (10)$$

$$\begin{aligned} n_2(H, T) &= \frac{1}{V} \sum_{S_{2z}=-S_2}^{S_2} \sum_{\mathbf{p}} \times \\ &\times \{\exp[\beta(\varepsilon(\mathbf{p}) + \varepsilon_2 - \zeta_2 - \mu HS_{2z})] \pm 1\}^{-1}, \end{aligned} \quad (11)$$

$$\begin{aligned} n_{\text{ph}}(H, T) &= \frac{1}{V} \sum_{\mathbf{p}} \times \\ &\times \{\exp[\beta(\varepsilon_{\text{ph}}(p) + \varepsilon_{\text{ph}} - \zeta_{\text{ph}})] - 1\}^{-1}, \end{aligned} \quad (12)$$

where V is the volume of the system.

Recall that the main task of this work is to demonstrate the influence of the photon subsystem on the magnetic properties of quantum gases in equilibrium with radiation. In this regard, it is necessary to introduce the magnetic characteristics of the system into consideration. Namely, these are the magnetizations of atomic components, $M_{1z}(H, T)$ and $M_{2z}(H, T)$. The magnetization density of the component of unexcited atoms, in accordance with Eq. (1) [see also Eqs. (10) and (11)], is given by the expression

$$\begin{aligned} m_1(H, T) &= \frac{M_1(H, T)}{V} = \frac{\mu}{V} \sum_{S_{1z}=-S_1}^{S_1} S_{1z} \sum_{\mathbf{p}} \times \\ &\times \{\exp[\beta(\varepsilon(\mathbf{p}) + \varepsilon_1 - \zeta_1 - \mu HS_{1z})] \pm 1\}^{-1}, \end{aligned} \quad (13)$$

The magnetization density of the component with the excited atoms in the system is determined by the similar formula

$$m_2(H, T) = \frac{M_2(H, T)}{V} = \frac{\mu}{V} \sum_{S_{2z}=-S_2}^{S_2} S_{2z} \sum_{\mathbf{p}} \times \{ \exp [\beta (\varepsilon(\mathbf{p}) + \varepsilon_2 - \zeta_2 - \mu H S_{2z})] \pm 1 \}^{-1}. \quad (14)$$

As a result, we arrive at the following expression for the magnetization density of the entire system:

$$m(H, T) = m_1(H, T) + m_2(H, T). \quad (15)$$

We recall that, in this work, as was mentioned above, we neglect the possibility for a photon in the medium to acquire a magnetic moment. Therefore, free (not bound in the excited state of atoms) photons make no direct contribution to the total magnetization of the system, which is reflected in formulas (13)–(15).

Thus, formulas (1) and (9)–(15) contain, in principle, all the necessary information for studying the thermodynamics and magnetic properties of a system that consists of quantum gases in excited and ground states and is in thermodynamic equilibrium with photons. In particular, using formula (8), and formulas (1), (3), (13), and (14), Eq. (7) can be presented in the form

$$\begin{aligned} \zeta_1 - \varepsilon_1 + H \left(\frac{\partial m_1}{\partial n_1} \right) + \zeta_{\text{ph}} - \varepsilon_{\text{ph}} = \\ = \zeta_2 - \varepsilon_2 + H \left(\frac{\partial m_2}{\partial n_2} \right). \end{aligned} \quad (16)$$

So, now we have to supplement the first two equations in the system of equations (9) with the newly obtained equation (16) and, in view of Eqs. (13) and (14), consider the result as a closed system of equations describing the state of thermodynamic equilibrium.

The research algorithm for the formulated equations is clear in principle. At the first stage, it consists in finding the chemical potentials of the system components by solving the related equations (9). Afterwards (again in the principle sense), it is possible to calculate the magnetization densities of the system, Eqs. (13)–(15). It is the obtained expression for the total magnetization density of the whole system, Eq. (15), that has to be analyzed from the viewpoint

of the influence of the parameters of the photon component on the magnetic properties of the system. Recall that, as was announced above, just this effect is of interest, because the photon itself (at least in vacuum; see some analysis of this issue in works [11–13]) does not possess a magnetic moment. So, as it could seem, the photonic component cannot affect the magnetic properties.

The analytic solution of the system of equations (9) is a difficult task, which is possible in a limited number of cases. Even finding a numerical solution to the problem in the general case is also a difficult task. Therefore, in what follows, we confine ourselves to the analytic study of the solution of the indicated system of equations in the case where all three gas components of the system are non-degenerate.

3. A System of Non-Degenerate Gases of Atoms in Equilibrium with a Non-Degenerate Gas of Free Photons

In the case where all components of the researched system are non-degenerate, expressions (1) and (9)–(15) can be substantially simplified because of the necessity to satisfy the relationships

$$\begin{aligned} \exp \left(\frac{\varepsilon_1 - \zeta_1 - \mu H S_{1z}}{T} \right) \gg 1, \\ \exp \left(\frac{\varepsilon_2 - \zeta_2 - \mu H S_{2z}}{T} \right) \gg 1, \\ \exp \left(\frac{\varepsilon_{\text{ph}} - \zeta_{\text{ph}}}{T} \right) \gg 1. \end{aligned} \quad (17)$$

By taking them into account, the expressions for the distribution functions of atoms in the excited and ground states [Eq. (1)] and for the distribution function of free photons [Eq. (3)] acquire the following simple forms in the main approximations:

$$\begin{aligned} f_{S_{1z}}(\mathbf{p}; \beta, \zeta_1) \approx e^{\frac{\zeta_1 - \varepsilon_1 + \mu H S_{1z}}{T}} e^{-\frac{p^2}{2mT}}, \\ f_{S_{2z}}(\mathbf{p}; \beta, \zeta_2) \approx e^{\frac{\zeta_2 - \varepsilon_2 + \mu H S_{2z}}{T}} e^{-\frac{p^2}{2mT}}, \\ f_{\text{ph}}(\mathbf{p}; \beta, \zeta_{\text{ph}}) \approx e^{\frac{\zeta_{\text{ph}} - \varepsilon_{\text{ph}}}{T}} e^{-\frac{p^2}{2m_{\text{ph}}T}}, \end{aligned} \quad (18)$$

As a result, expressions (10)–(12) for the particle densities in all three components of the system can also

be simplified in the same approximation:

$$\begin{aligned} n_1(H, T) &\approx e^{\frac{\zeta_1 - \epsilon_1}{T}} \left(\frac{mT}{2\pi\hbar^2} \right)^{3/2} \sum_{S_{1z} = -S_1}^{S_1} e^{\frac{\mu H S_{1z}}{T}}, \\ n_2(H, T) &\approx e^{\frac{\zeta_2 - \epsilon_2}{T}} \left(\frac{mT}{2\pi\hbar^2} \right)^{3/2} \sum_{S_{2z} = -S_2}^{S_2} e^{\frac{\mu H S_{2z}}{T}}, \\ n_{\text{ph}}(H, T) &\approx g_{\text{ph}} e^{\frac{\zeta_{\text{ph}} - \epsilon_{\text{ph}}}{T}} \left(\frac{m_{\text{ph}} T}{2\pi\hbar^2} \right)^{3/2}, \end{aligned} \quad (19)$$

where the factor g_{ph} makes allowance for the possible degeneracy of photon states.

Next, let us introduce the following functions into consideration:

$$\begin{aligned} g_1(H, T) &\equiv \left(\frac{mT}{2\pi\hbar^2} \right)^{3/2} \sum_{S_{1z} = -S_1}^{S_1} e^{\frac{\mu H S_{1z}}{T}}, \\ g_2(H, T) &\equiv \left(\frac{mT}{2\pi\hbar^2} \right)^{3/2} \sum_{S_{2z} = -S_2}^{S_2} e^{\frac{\mu H S_{2z}}{T}}, \\ g_{\text{ph}}(T) &\equiv g_{\text{ph}} \left(\frac{m_{\text{ph}} T}{2\pi\hbar^2} \right)^{3/2}. \end{aligned} \quad (20)$$

Taking them into account, expressions (19) for the particle densities in all three components of the system can be presented in the following form:

$$\begin{aligned} n_1(H, T) &= e^{\frac{\zeta_1 - \epsilon_1}{T}} g_1(H, T), \\ n_2(H, T) &= e^{\frac{\zeta_2 - \epsilon_2}{T}} g_2(H, T), \\ n_{\text{ph}}(H, T) &= e^{\frac{\zeta_{\text{ph}} - \epsilon_{\text{ph}}}{T}} g_{\text{ph}}(T). \end{aligned} \quad (21)$$

Note that the dependence of the quantity n_{ph} on the magnetic field H in expression (21) is not obvious. At this point, we may assume that this notation was introduced only for the sake of symmetry with the expressions for the density of atomic particles. The justification of this notation will become clear only after finding the quantities $e^{\frac{\zeta_1 - \epsilon_1}{T}}$, $e^{\frac{\zeta_2 - \epsilon_2}{T}}$, $e^{\frac{\zeta_{\text{ph}} - \epsilon_{\text{ph}}}{T}}$, which, as will be shown, contain the dependence on the magnetic field H . Expressions for these quantities can be obtained by solving the system of equations (9). With regard for formulas (17)–(21), this system looks like

$$\begin{aligned} n &= e^{\frac{\zeta_1 - \epsilon_1}{T}} g_1(H, T) + e^{\frac{\zeta_2 - \epsilon_2}{T}} g_2(H, T), \\ n_{\text{ph}} &= e^{\frac{\zeta_{\text{ph}} - \epsilon_{\text{ph}}}{T}} g_{\text{ph}}(T) + e^{\frac{\zeta_2 - \epsilon_2}{T}} g_2(H, T), \\ e^{\frac{\zeta_1 - \epsilon_1}{T}} e^{\frac{\zeta_{\text{ph}} - \epsilon_{\text{ph}}}{T}} &= e^{\frac{\zeta_{\text{ph}} - \epsilon_{\text{ph}}}{T}} \psi(H, T), \end{aligned} \quad (22)$$

where the third equation is the rewritten equation (16) with the introduced function

$$\psi(H, T) = \exp \left\{ \left(\frac{\partial m_2}{\partial n_2} - \frac{\partial m_1}{\partial n_1} \right) \frac{H}{T} \right\}, \quad (23)$$

to make the calculations more convenient. The function $\psi(H, T)$ is assumed to be known, and it will be determined later.

Thus, (22) is a system of algebraic equations for finding the quantities $e^{\frac{\zeta_1 - \epsilon_1}{T}}$, $e^{\frac{\zeta_2 - \epsilon_2}{T}}$, and $e^{\frac{\zeta_{\text{ph}} - \epsilon_{\text{ph}}}{T}}$. Its solving does not cause principal difficulties, but the final result is rather cumbersome:

$$\begin{aligned} e^{\frac{\zeta_{\text{ph}} - \epsilon_{\text{ph}}}{T}} &= \frac{1}{2g_{\text{ph}}(T)} \left\{ -(\kappa(H, T) + n - n_{\text{ph}}) + \right. \\ &\quad \left. + \sqrt{(\kappa(H, T) + n - n_{\text{ph}})^2 + 4n_{\text{ph}}\kappa(H, T)} \right\}, \\ e^{\frac{\zeta_1 - \epsilon_1}{T}} &= \frac{1}{2g_1(H, T)} \left\{ -(\kappa(H, T) + n_{\text{ph}} - n) + \right. \\ &\quad \left. + \sqrt{(\kappa(H, T) + n - n_{\text{ph}})^2 + 4n_{\text{ph}}\kappa(H, T)} \right\}, \\ e^{\frac{\zeta_2 - \epsilon_2}{T}} &= \frac{1}{2g_2(H, T)} \left\{ (\kappa(H, T) + n + n_{\text{ph}}) - \right. \\ &\quad \left. - \sqrt{(\kappa(H, T) + n - n_{\text{ph}})^2 + 4n_{\text{ph}}\kappa(H, T)} \right\}, \end{aligned} \quad (24)$$

Here, we introduced the function

$$\begin{aligned} \kappa(H, T) &\equiv \frac{g_1(H, T)g_{\text{ph}}(T)}{g_2(H, T)} \psi(H, T) = \kappa_0(T) \times \\ &\times \sum_{S_{1z} = -S_1}^{S_1} e^{\frac{\mu H S_{1z}}{T}} \left(\sum_{S_{2z} = -S_2}^{S_2} e^{\frac{\mu H S_{2z}}{T}} \right)^{-1}, \\ \kappa_0(T) &\equiv g_{\text{ph}} \left(\frac{m_{\text{ph}} T}{2\pi\hbar^2} \right)^{3/2}, \end{aligned} \quad (25)$$

dependent on the temperature and the magnetic field. Recall that the function $\psi(H, T)$ still needs to be defined.

Formulas (24)–(25) obtained for the quantities $e^{\frac{\zeta_1 - \epsilon_1}{T}}$, $e^{\frac{\zeta_2 - \epsilon_2}{T}}$, and $e^{\frac{\zeta_{\text{ph}} - \epsilon_{\text{ph}}}{T}}$ make it possible to write down expressions for particle densities of all system components in terms of system parameters, which are

considered, as before, to be already known:

$$\begin{aligned}
n_{\text{ph}}(H, T) &= \frac{1}{2} \left\{ -(\kappa(H, T) + n - n_{\text{ph}}) + \right. \\
&\quad \left. + \sqrt{(\kappa(H, T) + n - n_{\text{ph}})^2 + 4n_{\text{ph}}\kappa(H, T)} \right\}, \\
n_1(H, T) &= \frac{1}{2} \left\{ -(\kappa(H, T) + n_{\text{ph}} - n) + \right. \\
&\quad \left. + \sqrt{(\kappa(H, T) + n - n_{\text{ph}})^2 + 4n_{\text{ph}}\kappa(H, T)} \right\}, \\
n_2(H, T) &= \frac{1}{2} \left\{ (\kappa(H, T) + n_{\text{ph}} + n) - \right. \\
&\quad \left. - \sqrt{(\kappa(H, T) + n - n_{\text{ph}})^2 + 4n_{\text{ph}}\kappa(H, T)} \right\}.
\end{aligned} \tag{26}$$

These expressions, in turn, make it possible to write down rather simple formulas for the particle distribution functions of all three components:

$$\begin{aligned}
f_{S_{1z}}(\mathbf{p}; \beta, \zeta_1) &= e^{-\frac{p^2}{2mT} + \frac{\mu HS_{1z}}{T}} \left(\frac{2\pi\hbar^2}{mT} \right)^{3/2} \times \\
&\quad \times \left(\sum_{S'_{1z}=-S_1}^{S_1} e^{\frac{\mu HS'_{1z}}{T}} \right)^{-1} n_1(H, T),
\end{aligned} \tag{27}$$

$$\begin{aligned}
f_{S_{2z}}(\mathbf{p}; \beta, \zeta_2) &= e^{-\frac{p^2}{2mT} + \frac{\mu HS_{2z}}{T}} \left(\frac{2\pi\hbar^2}{mT} \right)^{3/2} \times \\
&\quad \times \left(\sum_{S'_{2z}=-S_2}^{S_2} e^{\frac{\mu HS'_{2z}}{T}} \right)^{-1} n_2(H, T),
\end{aligned} \tag{28}$$

$$\begin{aligned}
f_{\text{ph}}(\mathbf{p}; \beta, \zeta_{\text{ph}}) &= \\
&= \frac{1}{g_{\text{ph}}} e^{-\frac{p^2}{2m_{\text{ph}}T}} \left(\frac{2\pi\hbar^2}{m_{\text{ph}}T} \right)^{3/2} n_{\text{ph}}(H, T).
\end{aligned} \tag{29}$$

Now, in view of Eqs. (13)–(15) and (27)–(29), we arrive at the following expressions determining the magnetization of the system:

$$m_1(H, T) = \mu \frac{\left(\sum_{S_{1z}=-S_1}^{S_1} S_{1z} e^{\frac{\mu HS_{1z}}{T}} \right)}{\left(\sum_{S'_{1z}=-S_1}^{S_1} e^{\frac{\mu HS'_{1z}}{T}} \right)} n_1(H, T), \tag{30}$$

$$m_2(H, T) = \mu \frac{\left(\sum_{S_{2z}=-S_2}^{S_2} S_{2z} e^{\frac{\mu HS_{2z}}{T}} \right)}{\left(\sum_{S'_{2z}=-S_2}^{S_2} e^{\frac{\mu HS'_{2z}}{T}} \right)} n_2(H, T). \tag{31}$$

Let us pay attention to the fact that the sums over the spin projections, which enter expressions (25) and (30)–(31), can be calculated following the rules for the calculation of the geometric progression sum; in particular,

$$\begin{aligned}
\sum_{S'_{1z}=-S_1}^{S_1} e^{\frac{\mu HS'_{1z}}{T}} &= \frac{e^{-\frac{\mu HS_1}{T}} \left(e^{\frac{\mu H(2S_1+1)}{T}} - 1 \right)}{e^{\frac{\mu H}{T}} - 1} = \\
&= \frac{\sinh \frac{\mu H}{T} (S_1 + \frac{1}{2})}{\sinh \frac{\mu H}{2T}}.
\end{aligned} \tag{32}$$

Then the sum ratio in the determination of the quantity κ [see Eq. (25)] can be represented in the form

$$\begin{aligned}
\sum_{S_{1z}=-S_1}^{S_1} e^{\frac{\mu HS_{1z}}{T}} \left(\sum_{S_{2z}=-S_2}^{S_2} e^{\frac{\mu HS_{2z}}{T}} \right)^{-1} &= \\
&= \frac{\sinh \frac{\mu H}{T} (S_1 + \frac{1}{2})}{\sinh \frac{\mu H}{T} (S_2 + \frac{1}{2})}.
\end{aligned} \tag{33}$$

On the other hand, for the sum ratios in Eqs. (30) and (31), we arrive at the following expressions:

$$\begin{aligned}
\sum_{S_{1z}=-S_1}^{S_1} S_{1z} e^{\frac{\mu HS_{1z}}{T}} \left(\sum_{S'_{1z}=-S_1}^{S_1} e^{\frac{\mu HS'_{1z}}{T}} \right)^{-1} &= \\
&= \left(S_1 + \frac{1}{2} \right) \coth \frac{\mu H}{T} \left(S_1 + \frac{1}{2} \right) - \frac{1}{2} \coth \frac{\mu H}{2T},
\end{aligned} \tag{34}$$

$$\begin{aligned}
\sum_{S_{2z}=-S_2}^{S_2} S_{2z} e^{\frac{\mu HS_{2z}}{T}} \left(\sum_{S'_{2z}=-S_2}^{S_2} e^{\frac{\mu HS'_{2z}}{T}} \right)^{-1} &= \\
&= \left(S_2 + \frac{1}{2} \right) \coth \frac{\mu H}{T} \left(S_2 + \frac{1}{2} \right) - \frac{1}{2} \coth \frac{\mu H}{2T}.
\end{aligned} \tag{35}$$

Now, we can return to the definition of the function $\psi(H, T)$. In accordance with Eq. (23) and accounting for Eqs. (30), (31), (34), and (35),

$$\begin{aligned}
\psi(H, T) &= \\
&= e^{\left\{ (S_2 + \frac{1}{2}) \coth \frac{\mu H}{T} (S_2 + \frac{1}{2}) - (S_1 + \frac{1}{2}) \coth \frac{\mu H}{T} (S_1 + \frac{1}{2}) \right\} \frac{\mu H}{T}}.
\end{aligned} \tag{36}$$

With the known expression (36), the function $\kappa(H, T)$ [see Eq. (25)] also becomes completely defined,

$$\kappa(H, T) = \kappa_0(T) \frac{\sinh \frac{\mu H}{T} (S_1 + \frac{1}{2})}{\sinh \frac{\mu H}{T} (S_2 + \frac{1}{2})} \times$$

$$\times e^{\left\{ \left(S_2 + \frac{1}{2} \right) \coth \frac{\mu H}{T} \left(S_2 + \frac{1}{2} \right) - \left(S_1 + \frac{1}{2} \right) \coth \frac{\mu H}{T} \left(S_1 + \frac{1}{2} \right) \right\} \frac{\mu H}{T}} \quad (37)$$

Therefore, in expressions (24) and (26)–(31), the quantity $\kappa(H, T)$ has now to be taken in form (37).

For the sake of a qualitative rather than a quantitative analysis of the obtained results, let us consider the cases of low $\left(\frac{\mu H}{T} \ll 1 \right)$ and high $\left(\frac{\mu H}{T} \gg 1 \right)$ magnetic fields. It can be easily verified that, in the case of low magnetic fields, formula (33) takes the form

$$\sum_{S_{1z}=-S_1}^{S_1} e^{\frac{\mu H S_{1z}}{T}} \left(\sum_{S_{2z}=-S_2}^{S_2} e^{\frac{\mu H S_{2z}}{T}} \right)^{-1} \approx \frac{2S_1 + 1}{2S_2 + 1} \left(1 + \frac{1}{6} (S_1 - S_2)(S_1 + S_2 + 1) \left(\frac{\mu H}{T} \right)^2 \right). \quad (38)$$

At the same time, in the case of high magnetic fields (in the above sense), the same formula shows an exponential field dependence,

$$\sum_{S_{1z}=-S_1}^{S_1} e^{\frac{\mu H S_{1z}}{T}} \left(\sum_{S_{2z}=-S_2}^{S_2} e^{\frac{\mu H S_{2z}}{T}} \right)^{-1} \approx e^{(S_1 - S_2) \frac{\mu H}{T}}. \quad (39)$$

The dependence of the magnetization density of the system on the magnetic field is also determined, as one can see, from Eqs. (30) and (31), by quantities (34) and (35), which enter the definition of $\kappa(H, T)$ as well. In the two limiting approximations proposed for consideration, these quantities are given by the following expressions:

(i) if $\frac{\mu H}{T} \ll 1$,

$$\sum_{S_{1z}=-S_1}^{S_1} S_{1z} e^{\frac{\mu H S_{1z}}{T}} \left(\sum_{S'_{1z}=-S_1}^{S_1} e^{\frac{\mu H S'_{1z}}{T}} \right)^{-1} \approx \frac{1}{3} S_1 (S_1 + 1) \frac{\mu H}{T}, \quad (40)$$

$$\sum_{S_{2z}=-S_2}^{S_2} S_{2z} e^{\frac{\mu H S_{2z}}{T}} \left(\sum_{S'_{2z}=-S_2}^{S_2} e^{\frac{\mu H S'_{2z}}{T}} \right)^{-1} \approx \frac{1}{3} S_2 (S_2 + 1) \frac{\mu H}{T}, \quad (41)$$

(ii) and if $\frac{\mu H}{T} \gg 1$,

$$\sum_{S_{1z}=-S_1}^{S_1} S_{1z} e^{\frac{\mu H S_{1z}}{T}} \left(\sum_{S'_{1z}=-S_1}^{S_1} e^{\frac{\mu H S'_{1z}}{T}} \right)^{-1} \approx S_1, \quad (42)$$

$$\sum_{S_{2z}=-S_2}^{S_2} S_{2z} e^{\frac{\mu H S_{2z}}{T}} \left(\sum_{S'_{2z}=-S_2}^{S_2} e^{\frac{\mu H S'_{2z}}{T}} \right)^{-1} \approx S_2. \quad (43)$$

Thus, taking into account formulas (38) and (39)–(43), expression (37) for the system parameter $\kappa(H, T)$ takes the form

$$\kappa(H, T) \approx \kappa_0(T) \frac{2S_1 + 1}{2S_2 + 1} \times \left(1 + \frac{1}{6} (S_2 - S_1)(S_1 + S_2 + 1) \left(\frac{\mu H}{T} \right)^2 \right) \quad (44)$$

for low magnetic fields $\left(\frac{\mu H}{T} \ll 1 \right)$ and transforms into the simple expression

$$\kappa(H, T) \approx \kappa_0(T) \quad (45)$$

for high magnetic fields $\left(\frac{\mu H}{T} \gg 1 \right)$. As one can see from expression (44), in the interval of low magnetic fields, $\kappa(H, T)$ increases quadratically with the increasing field if $S_2 > S_1$. In the case of high magnetic fields, as follows from Eq. (45), the quantity $\kappa(H, T)$ in the main approximation ceases to be affected by the magnetic field at all as the magnetic field increases. Its dependence on the field intensity manifests itself in the following summands of the type $\exp\left(-\frac{\mu H}{T}\right)$, which are exponentially decaying at large parameter values. As one can see by comparing formulas (44) and (45), within the whole interval of the magnetic field change from $\frac{\mu H}{T} \ll 1$ to $\frac{\mu H}{T} \gg 1$, the value of $\kappa(H, T)$ with $S_2 > S_1$ increases from $\kappa(H, T) \approx \kappa_0(T) \frac{2S_1 + 1}{2S_2 + 1}$ to $\kappa(H, T) \approx \kappa_0(T)$. Recall that this quantity characterizes the dependence of the particle densities of all three components on the magnetic field in two considered limiting cases. Thus, as one can see, by changing the magnetic field, we can affect both the density of free photons in the system, $n_{\text{ph}}(H, T)$, and the degree of the inverse population of atomic levels characterized by the density $n_2(H, T)$.

Now, with regard to Eq. (26), let us analyze formulas (26), (30), (31), and (15) and find how the photonic component of the system affects its magnetization. Again, since the problem is complicated due to its multiparametric character and the complexity of the relevant transcendental equations, we will follow the influence of photons on system's magnetization in the framework of a certain perturbation

theory. Namely, consider the case of low photon component density in the system, $n_{\text{ph}} \ll n$, and the case $n_{\text{ph}} \sim n$. Note at once that the case $n_{\text{ph}} \gg n$ cannot be correctly considered in the framework of the model discussed in this article because it contradicts the two-level atom approximation.

In the approximation of low photon density, expressions (26) become simpler:

$$\begin{aligned} n_{\text{ph}}(H, T) &\approx \frac{n_{\text{ph}}\kappa(H, T)}{\kappa(H, T) + n}, \\ n_1(H, T) &\approx n - \frac{n_{\text{ph}}n}{\kappa(H, T) + n}, \\ n_2(H, T) &\approx \frac{n_{\text{ph}}n}{\kappa(H, T) + n}, \end{aligned} \quad (46)$$

which makes it possible, using Eqs. (44), (40), and (41), to immediately write down the formula for the magnetic moment density of the system in the case of a low external magnetic field (see Eqs. (30) and (31)),

$$\begin{aligned} m(H, T) &= \frac{\mu^2 H}{3T} \left([S_2(S_2 + 1) - S_1(S_1 + 1)] \times \right. \\ &\times \left. \frac{n_{\text{ph}}n}{\kappa_0(T) \frac{2S_1+1}{2S_2+1} + n} + S_1(S_1 + 1)n \right), \end{aligned} \quad (47)$$

where the quantity $\kappa_0(T)$ is determined by formula (25).

From formula (31), it is easy to see that, in this approximation (a low magnetic field, $\frac{\mu H}{T} \ll 1$, and a low photon component density), the system magnetization at $S_1 < S_2$ increases not only with the growth of the external field magnitude, but also with the growth of the photon density. That is, provided the availability of a low magnetic field affecting the system, the ‘‘pumping’’ of additional photons into the system from outside contributes to the increase of its magnetic moment, even if the field itself does not increase.

In the case of high magnetic fields, $\frac{\mu H}{T} \gg 1$, and a low photon component density, $n_{\text{ph}} \ll n$, and assuming $S_1 < S_2$ again, we obtain the following expression for the magnetic moment density of the whole system:

$$m(H, T) \approx \mu n S_1 + \mu (S_2 - S_1) \frac{n_{\text{ph}}n}{\kappa_0(T) + n}, \quad (48)$$

where the quantity $\kappa_0(T)$, as above, is determined by formula (25). Again, we can see that, provided

the magnetic field is constant and large by strength, the increase of the total photon density induces the growth of the system magnetization. We would like to emphasize that we are talking – in both cases, (47) and (48) – only about the tendency of such a magnetization growth precisely because there is a necessity to keep the consideration within the approximation $n_{\text{ph}} \ll n$.

Finally, let us consider the case $n_{\text{ph}} \sim n$. Recall that the case $n_{\text{ph}} \gg n$ is incorrect in the framework of the applied model, because it violates the assumption about the two-level atom. Therefore, if the relationship $n_{\text{ph}} \sim n$ is valid, expressions (26) for the component densities can be given in the form

$$\begin{aligned} n_{\text{ph}}(H, T) &\approx \frac{1}{2} \left\{ \sqrt{\kappa^2(H, T) + 4n\kappa(H, T)} - \kappa(H, T) - \right. \\ &- \left. \left(1 + \frac{\kappa(H, T)}{\sqrt{\kappa^2(H, T) + 4n\kappa(H, T)}} \right) \delta n \right\}, \\ n_1(H, T) &\approx \frac{1}{2} \left\{ \sqrt{\kappa^2(H, T) + 4n\kappa(H, T)} - \kappa(H, T) + \right. \\ &+ \left. \left(1 - \frac{\kappa(H, T)}{\sqrt{\kappa^2(H, T) + 4n\kappa(H, T)}} \right) \delta n \right\}, \\ n_2(H, T) &\approx \\ &\approx n - \frac{1}{2} \left\{ \sqrt{\kappa^2(H, T) + 4n\kappa(H, T)} - \kappa(H, T) - \right. \\ &- \left. \left(1 - \frac{\kappa(H, T)}{\sqrt{\kappa^2(H, T) + 4n\kappa(H, T)}} \right) \delta n \right\}, \end{aligned} \quad (49)$$

where $\delta n \equiv n - n_{\text{ph}}$, and the quantity $\kappa(H, T)$ is determined by formula (37).

Below, we present formulas for system’s magnetization, which were obtained proceeding from Eq. (49) in the main approximation $\delta n = 0$, in the cases of high and low magnetic fields, as was discussed above. For low magnetic fields, $\frac{\mu H}{T} \ll 1$, the expression for the medium magnetization reads

$$\begin{aligned} m(H, T) &\approx \frac{\mu^2 H}{3T} \left(-\frac{1}{2} \left(\sqrt{\kappa^2(T) + 4n\kappa(T)} - \kappa(T) \right) \times \right. \\ &\times \left. [S_2(S_2 + 1) - S_1(S_1 + 1)] + S_2(S_2 + 1)n \right), \end{aligned} \quad (50)$$

where $\kappa(T)$ is given by expression (44) in the main approximation at $\frac{\mu H}{T} \ll 1$. In the same limiting approximation for densities, $n_{\text{ph}} = n$, the formula for the system magnetization in the case $\frac{\mu H}{T} \gg 1$ looks

like

$$m(H, T) \approx \mu S_2 n - \frac{1}{2} \mu (S_2 - S_1) \times \left(\sqrt{\kappa_0^2(T) + 4n\kappa_0(T)} - \kappa_0(T) \right), \quad (51)$$

where $\kappa_0(T)$ is given by expression (45) in the main approximation at $\frac{\mu H}{T} \gg 1$.

From a comparative analysis of expressions (46) and (49), as well as expressions (47) and (48) with expressions (50) and (51), it is possible to understand that the tendency for the component density to grow with the increasing inverse population of energy levels, as well as for the magnetization density of the system to grow with the increase of the total photon density in the system, n_{ph} , is preserved within the whole interval $0 < n_{\text{ph}} \leq n$ allowed by the model of this parameter change (see above).

The presented analysis testifies that the inverse level population can be controlled and the magnetization of the medium can be changed by injecting photons into the medium from the outside, i.e., by increasing the value of n_{ph} . However, it is also worth noting that the behavior of the inverse population and the magnetic moment density of the medium can be substantially determined by the quantity $\kappa(H, T)$, i.e., the variation of the external magnetic field. In this regard, it should be noted that the quantity $\kappa(H, T)$ determined by the expression (25) also contains the temperature and the parameters of the photon energy spectrum (the cutoff frequency or the photon mass; see Eqs. (5) and (6) in formulas (24), (26), (46), and (49)–(51). That is, the value of $\kappa(H, T)$ can be changed by varying both the magnetic field and the temperature [not violating the limits of the high-temperature approximation; see Eq. (19)].

4. Conclusions

To summarize, the possibility has been proved that the photon component can affect the magnetic properties of a system of quantum gases that are in thermodynamic equilibrium with radiation (photons) and under the influence of an external magnetic field. Such a conclusion is not trivial *a priori* given the fact that photons in the vacuum do not possess a magnetic moment. The statement about the substantial influence of the photon subsystem on the medium

magnetization follows from the analysis of the solutions of the general equations describing the thermodynamic equilibrium state in the system, far from the degeneracy temperatures of all three of its components. The possibility of controlling the inverse population of the levels of atomic components, as well as the magnetic properties of the medium, by means of additional pumping of photons into the system and changing the intensity of the external magnetic field and the temperature has been illustrated.

The work was sponsored by the National Academy of Sciences of Ukraine (project No. 0121U108722), the Ministry of Education and Science of Ukraine (project No. 0122U001575), and the Science and Technology Center in Ukraine (project No. 9918).

1. A. Einstein. Quantentheorie des einatomigen idealen Gases. *Sitzber. Kgl. Preuss. Akad. Wiss.* **1**, 3 (1925).
2. S.N. Bose. Plancks Gesetz und Lichtquantenhypothese. *Z. Phys.* **26**, 178 (1924).
3. E. Fermi. Sulla quantizzazione del gas perfetto monoatomico. *Rend. Lincei* **3**, 145 (1926).
4. P.A.M. Dirac. On the theory of quantum mechanics. *Proc. R. Soc. Lond. A* **112**, 661 (1926).
5. A.S. Peletniskii, Yu.V. Slyusarenko, A.G. Sotnikov. *Theory of Exotic States in Quantum Fermi and Bose Systems* (Naukova dumka, 2023) (in Ukrainian).
6. A. Kruchkov, Y. Slyusarenko. Bose–Einstein condensation of photons in an ideal atomic gas. *Phys. Rev. A* **88**, 013615 (2013).
7. N. Boichenko, Y. Slyusarenko. Coexistence of photonic and atomic Bose-Einstein condensates in ideal atomic gases. *Condens. Matter Phys.* **18**, 43002 (2015).
8. J. Klaers, J. Schmitt, F. Vewinger, M. Weitz. Bose–Einstein condensation of photons in an optical microcavity. *Nature* **468**, 545 (2010).
9. Y.V. Slyusarenko, O.Y. Sliusarenko. Kinetic theory of weakly ionized dilute gas of hydrogen-like atoms of the first principles of quantum statistics and dispersion laws of eigenwaves. *J. Math. Phys.* **58**, 1133021 (2017).
10. Y. Kawaguchi, M. Ueda. Spinor Bose–Einstein condensates. *Phys.Rept.* **520**, 253 (2012).
11. B. Altschul. Astrophysical bounds on the photon charge and magnetic moment. *Astropart. Phys.* **29**, 290 (2008).
12. H. Pérez Rojas, E. Rodríguez Querts. The photon magnetic moment problem revisited. *Eur. Phys. J. C* **74**, 2899 (2014).
13. Z. Saglam, G. Sahin. Magnetic moment of photon. *J. Mod. Phys.* **6**, 937 (2015).
14. L.D. Landau, E.M. Lifshitz. *Statistical Physics. Part 1* (Pergamon Press, 1980).

Received 16.08.24.

Translated from Ukrainian by O.I. Voitenko

*М. Булахов, О.С. Пелетминський,
Ю.В. Слюсаренко*

ВПЛИВ ФОТОННОЇ
ПІДСИСТЕМИ НА МАГНІТНІ
ВЛАСТИВОСТІ КВАНТОВИХ ГАЗІВ

Вивчено можливість впливу фотонного компонента на магнітні властивості системи із квантових газів, що перебувають у термодинамічній рівновазі з випромінюванням (фотонами). Запропоновано просту модель системи, в рамках якої здобуто загальні рівняння, що описують термодинамічну рівновагу квантових газів дворівневих атомів із фотонами. Отримані рівняння розв'язано в області температур, далеких від температури виродження всіх трьох ком-

понентів системи. Аналіз розв'язків свідчить про нетривіальну поведінку магнітного стану системи як відповідь на зміну густини фотонів та інтенсивності зовнішнього магнітного поля. Показано, що збільшення густини фотонів в системі за рахунок зовнішніх джерел може приводити до збільшення як намагніченості системи, так і густини збуджених атомів. Такий висновок не є *a priori* тривіальним з огляду на те, що фотони у вакуумі не мають магнітного моменту.

Ключові слова: квантові гази, дворівневі атоми, фотони, зовнішнє магнітне поле, термодинамічна рівновага, невироджений стан, інверсна заселеність, магнітні властивості середовища.