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CHIRALITY PRODUCTION DURING AXION INFLATION

We study the generation of a chiral charge during the axion inflation, where the pseudoscalar inflaton field ϕ couples axially to the electromagnetic field via the term $(\beta/M_p)\phi\, \boldsymbol{E}\cdot\boldsymbol{B}$ with the dimensionless coupling constant β. To describe the evolution of the electromagnetic field and to determine $\langle\boldsymbol{E}\cdot\boldsymbol{B}\rangle$ sourcing the chiral asymmetry during the inflation due to the chiral anomaly, we employ the gradient-expansion formalism. It operates with a set of vacuum expectation values of the bilinear electromagnetic functions and allows us to consider the backreaction of generated fields on the inflaton evolution, as well as the Schwinger production of charged fermions. In addition, we assume that the produced fermions thermalize and include the chiral magnetic effect contribution to the electric current given by $\boldsymbol{j}_{\rm CME} = e^2/(2\pi^2)\mu_5\boldsymbol{B}$, where μ_5 is the chiral chemical potential which quantifies the produced chiral asymmetry. Solving a set of equations for the inflaton field, scale factor, quadratic functions of the electromagnetic field, and the chiral charge density (chiral chemical potential), we find that the chirality production is quite efficient leading to the generation of a large chiral chemical potential at the end of the axion inflation.

Keywords: axion inflation, gradient-expansion formalism, Schwinger effect, chiral anomaly, chiral asymmetry.

1. Introduction

The concept of inflation is a highly successful idea that effectively addresses numerous cosmological problems. Particularly, it provides the mechanism for the generation of primordial scalar and tensor perturbations leaving their imprint in the cosmic microwave background spectrum and leading to the large-scale structure formation [1–3] (see Refs. [4, 5] for a review). Additionally, there is a common assumption that the primordial magnetic field originates from the inflation, as discussed in seminal papers [6–9]. This magnetic field is often regarded as a "seed" for magnetic fields observed in astrophysical objects such as galaxies and galaxy clusters [10–18]. On the other

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hand, in cosmic voids containing vanishingly small amounts of matter, the magnetic field may persist in its original form unaffected by astrophysical processes. Consequently, it could contain a crucial information about earlier stages of the Universe's history. Indeed, there are evidences for the presence of magnetic fields in voids following from the gamma-ray observations of distant blazars [19–28] (for a recent review, see Ref. [29]).

One of the simplest and widely accepted models of the inflation involves a real scalar field ϕ, referred to as the inflaton. This inflaton slowly rolls along the slope of its effective potential and exhibits the vacuumlike equation of state $p = -\rho$. However, to achieve this behavior, one needs a sufficiently flat inflaton potential. On the other hand, the coupling of the inflaton to matter fields, necessary for the successful reheating of the Universe after the inflation, may introduce quantum corrections to the inflaton potential, potentially spoiling its flatness and disabling the inflation. Therefore, a mechanism is required to preserve the form of the inflaton potential at least in the range of the inflaton values relevant for the inflation. One example of such a mechanism is realized in the axion inflation, where the role of the inflaton is played by the pseudo-Nambu–Goldstone boson of the shift symmetry – the axion field. In the context of magnetogenesis, the axion inflation is particularly advantageous, as it gives rise to helical magnetic fields in the early Universe [30–50]. This property significantly enhances the chances of the survival for these magnetic fields compared to the case of nonhelical magnetic fields due to the inverse cascade of magnetic helicity [51–59].

In the axion inflation, the inflaton field ϕ couples to the Abelian gauge field (referred to as the electromagnetic field hereafter) by means of the $\propto I(\phi)\, F_{\mu\nu}F_{\alpha\beta}\epsilon^{\mu\nu\alpha\beta}$ interaction term. Here, $F_{\mu\nu}$ is the gauge-filed stress tensor, and $I(\phi)$ is the axial coupling function. To preserve the parity symmetry, $I(\phi)$ must be a pseudoscalar quantity. The simplest and most commonly used choice for $I(\phi)$ is a linear function of the inflaton field, i.e., $I(\phi) \propto \phi$. This coupling breaks the conformal invariance of the Maxwell action which is the necessary condition for the generation of electromagnetic fields from quantum fluctuations [60]. In the axion inflation model, only one circularly polarized mode of the electromagnetic field gets enhanced due to the interaction with the inflaton, which results in a nonzero helicity of a generated field.

In addition to the generation of a magnetic field, the inflation also gives rise to an electric field, leading to two primary consequences: (i) it produces pairs of charged particles due to the Schwinger effect [61–63] and (ii) nonzero scalar product $\boldsymbol{E} \cdot \boldsymbol{B}$ gives rise to a chiral asymmetry in the fermionic sector because of the chiral anomaly [64, 65]. The Schwinger pair production during the inflation has been extensively studied in various works [42, 46, 48, 66–89]. In particular, it was shown that this phenomenon may significantly affect the generation of gauge fields and make a substantial contribution to the Universe reheating. However, the chiral anomaly during the axion inflation has received a relatively little attention [42, 48]. To the best of our knowledge, the quantitative analysis of the chiral charge generation is still missing in the literature. In addition, the backreaction of this chiral asymmetry on the gauge-field evolution due to the chiral magnetic effect has not been investigated yet. This provides the main motivation for the study in this paper.

A recently proposed and efficiently applied method for studying the magnetogenesis during the axion inflation is the gradient-expansion formalism, introduced and utilized in recent works [46, 50]. This formalism involves a set of bilinear electromagnetic functions in the position space which are the vacuum expectation values of the scalar products of the electric and magnetic field vectors with an arbitrary number of spatial curls. Its advantage over conventional approaches, which deal with separate Fourier modes of the electromagnetic fields, is that it automatically accounts for all relevant modes simultaneously. Therefore, it can be applied even in the presence of complex nonlinear phenomena such as the backreaction and the Schwinger effect, which couple all electromagnetic modes to one another. In this study, we extend the gradient-expansion formalism by considering the presence of a chiral asymmetry in the fermionic sector (assuming that fermions attain the state of local equilibrium with temperature T and the chiral chemical potential μ_5) and the additional term in the electric current induced by this chiral asymmetry – the chiral magnetic effect current $\boldsymbol{j}_{\rm CME} = e^2/(2\pi^2)\mu_5\boldsymbol{B}$ (for a review, see Ref. [90] and references therein).

The paper is organized as follows. The axion inflation model and the gradient-expansion formalism for its description are discussed in Sec. 2. The chiral magnetic effect is introduced and incorporated into the gradient-expansion formalism in Sec. 3. Numerical results for the chirality production in the axion inflation are presented in Sec. 4 and summarized in Sec. 5.

2. Magnetogenesis in Axial-Coupling Model

In the axion inflation, the coupling of the electromagnetic field A_μ to the pseudoscalar inflaton field ϕ breaks the conformal invariance of the Maxwell action and, thus, is crucial for the electromagnetic field generation. The action of the axion inflation model is given by

$$S = \int d^4x\sqrt{-g}\Bigg[\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - V(\phi) - \\ - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}I(\phi)F_{\mu\nu}\tilde{F}^{\mu\nu} + \mathcal{L}_{\rm f}(\psi,\, A_\mu)\Bigg], \qquad (1)$$

where $g = \det g_{\mu\nu}$ is the determinant of the space-time metric, $V(\phi)$ is the inflaton potential, $I(\phi)$ is the axial-coupling function, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the gauge-field strength tensor, and its dual tensor is defined by

$$\tilde{F}^{\mu\nu} = \frac{1}{2\sqrt{-g}}\,\varepsilon^{\mu\nu\lambda\rho}F_{\lambda\rho}, \qquad (2)$$

where $\varepsilon^{\mu\nu\lambda\rho}$ is the absolutely antisymmetric Levi-Civita symbol with $\varepsilon^{0123} = +1$. The last term in Eq. (1) describes the matter fields charged under the $U(1)$ gauge group and, therefore, coupled to the electromagnetic four-potential A_μ. In this work, we consider a toy model in which only one massless charged fermionic field ψ is present. Moreover, we will not consider the dynamics of this matter field in detail and only describe its observable quantities such as the electric current, energy density, *etc.*

The following Euler–Lagrange equations for the inflaton and electromagnetic fields are easily obtained from action (1):

$$\frac{1}{\sqrt{-g}}\,\partial_\mu\left[\sqrt{-g}\,g^{\mu\nu}\,\partial_\nu\phi\right] + \frac{dV}{d\phi} + \frac{1}{4}\frac{dI}{d\phi}F_{\mu\nu}\tilde{F}^{\mu\nu} = 0, \quad (3)$$

$$\frac{1}{\sqrt{-g}}\,\partial_\mu\left[\sqrt{-g}\,F^{\mu\nu}\right] + \frac{dI}{d\phi}\,\tilde{F}^{\mu\nu}\partial_\mu\phi = j^\nu, \qquad (4)$$

where $j^\nu = -\partial\mathcal{L}_{\rm f}(\psi,\, A_\mu)/\partial A_\nu$ is the electric four-current of fermions induced by the electromagnetic field.

In addition, the dual gauge field strength tensor satisfies the Bianchi identity

$$\frac{1}{\sqrt{-g}}\,\partial_\mu\left[\sqrt{-g}\,\tilde{F}^{\mu\nu}\right] = 0. \qquad (5)$$

The energy–momentum tensor in our model is given by

$$T_{\mu\nu} = \partial_\mu\phi\,\partial_\nu\phi - g^{\lambda\rho}F_{\mu\lambda}F_{\nu\rho} - g_{\mu\nu}\Bigg[\frac{1}{2}\partial_\alpha\phi\,\partial^\alpha\phi - \\ - V(\phi) - \frac{1}{4}F_{\alpha\beta}F^{\alpha\beta}\Bigg] + T^{\rm f}_{\mu\nu}, \qquad (6)$$

where the last term defines the contribution of the charged fermionic field. The zero-zero component of $T_{\mu\nu}$ gives the energy density of the Universe.

Now, we restrict ourselves to the case of the spatially-flat FLRW metric and assume that the inflaton field ϕ is spatially homogeneous, i.e., depends only on time. It is convenient to use the Coulomb gauge for the electromagnetic four-potential $A_\mu = (0,\, -\boldsymbol{A})$. Then the three-vectors of electric $\boldsymbol{E}$ and magnetic $\boldsymbol{B}$ fields, as measured by the comoving observer, are defined in the conventional way as

$$\boldsymbol{E} = -\frac{1}{a}\,\partial_0\boldsymbol{A}, \quad \boldsymbol{B} = \frac{1}{a^2}\,{\rm rot}\,\boldsymbol{A}. \qquad (7)$$

The gauge-field stress tensor and its dual tensor are expressed in terms of the electric and magnetic fields as follows:

$$\begin{aligned} &F_{0i} = aE^i, \quad F_{ij} = -a^2\varepsilon_{ijk}B^k, \\ &\tilde{F}_{0i} = aB^i, \quad \tilde{F}_{ij} = a^2\varepsilon_{ijk}E^k. \end{aligned} \qquad (8)$$

Finally, we assume that the plasma of charged fermions is spatially homogeneous and quasineutral, i.e., $j^0 = 0$, while the electric current density is described by the Ohm's law

$$j^i = \frac{1}{a}\,\sigma_{\rm f}E^i. \qquad (9)$$

It was shown in the literature that this is, indeed, the case for fermions produced by the Schwinger effect during the inflation, if the electromagnetic field is constant [48, 71]. In such a case, the generalized conductivity $\sigma_{\rm f}$ depends only on the absolute values of the electric and magnetic fields. We will use the same form of the electric current assuming that the electromagnetic field changes adiabatically slowly. The applicability of this approximation was discussed in detail in Ref. [50].

Then the full system of equations in our model consists of the Friedmann equation for the Hubble parameter $H = \dot{a}/a$ determining the expansion rate of the Universe

$$H^2 = \frac{\rho}{3M_{\rm p}^2} = \frac{1}{3M_{\rm p}^2}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi) + \right.$$
$$\left. + \frac{1}{2}\left\langle \boldsymbol{E}^2 + \boldsymbol{B}^2 \right\rangle + \rho_{\rm f}\right], \tag{10}$$

the Klein–Gordon equation for the inflaton field

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = I'(\phi)\left\langle \boldsymbol{E}\cdot\boldsymbol{B}\right\rangle, \tag{11}$$

and Maxwell's equations for the electromagnetic field

$$\dot{\boldsymbol{E}} + 2H\boldsymbol{E} - \frac{1}{a}\,\mathrm{rot}\,\boldsymbol{B} + I'(\phi)\,\dot{\phi}\,\boldsymbol{B} + \sigma_{\rm f}\boldsymbol{E} = 0, \tag{12}$$

$$\dot{\boldsymbol{B}} + 2H\boldsymbol{B} + \frac{1}{a}\,\mathrm{rot}\,\boldsymbol{E} = 0, \tag{13}$$

$$\mathrm{div}\boldsymbol{E} = 0, \quad \mathrm{div}\boldsymbol{B} = 0. \tag{14}$$

In Eq. (10), $M_{\rm p} = (8\pi G)^{-1/2} = 2.43\times10^{18}$ GeV is the reduced Planck mass. Since we consider the electromagnetic field as a quantum field, the vacuum expectation value is taken, when it appears in the classical equations of motion (10) and (11). They are denoted by the angle brackets.

In order to close the system of equations, we need to specify the conductivity. In the case of one massless Dirac fermion species with charge e, the conductivity induced by the Schwinger effect is constant and, for collinear electric and magnetic fields in the de Sitter spacetime, has the form [48]

$$\sigma_{\rm f} = \frac{e^3}{6\pi^2}\frac{|B|}{H}\coth\left(\frac{\pi|B|}{|E|}\right), \tag{15}$$

where $|E| \equiv \sqrt{\langle \boldsymbol{E}^2\rangle}$ and $|B| \equiv \sqrt{\langle \boldsymbol{B}^2\rangle}$. During the axion inflation, the generated gauge fields are helical, and vectors $\boldsymbol{E}$ and $\boldsymbol{B}$ are, indeed, nearly collinear. Expression (15) was derived in the strong-field regime, $|eE| \gg H^2$, which is the most important one for physical applications.

The energy of the electric field which is dissipated due to a finite conductivity is transferred into charged fermions produced by the Schwinger effect. Then the energy density of fermions satisfies the following equation which can be derived using the energy balance in the system:

$$\dot{\rho}_{\rm f} + 4H\rho_{\rm f} = \sigma_{\rm f}\langle \boldsymbol{E}^2\rangle. \tag{16}$$

For massless fermions, apart from the ordinary $U(1)$ gauge symmetry, at the classical level, there is also the global $U(1)$ chiral symmetry which "rotates" the phases of the right- and left-handed components of the spinor ψ with opposite signs. According to Noether's theorem, it should correspond to a conserved current j_5^μ which is the difference of the currents of right- and left-handed particles. However, in quantum field theory, this symmetry is violated by the renormalization of ultraviolet divergences. As a result, the corresponding Noether current is not conserved, i.e. there is a chiral anomaly (see Refs. [64, 65]). In the expanding Universe, the chiral anomaly equation reads as

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}j_5^\mu) = -\frac{e^2}{8\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu}. \tag{17}$$

Assuming again the homogeneous fermion distribution, we rewrite this equation in the form

$$\dot{n}_5 + 3Hn_5 = \frac{e^2}{2\pi^2}\langle \boldsymbol{E}\cdot\boldsymbol{B}\rangle, \tag{18}$$

where $n_5 = j_5^0$ is the chiral charge density of fermions [1]. Since the nonzero value of $\langle \boldsymbol{E}\times\boldsymbol{B}\rangle$ is generated in the axion inflation, the fermion chirality is not conserved. At the beginning of the inflation, there were no gauge fields and fermions (thus, the initial value of chiral asymmetry was equal to zero). Then the nonconservation of the chiral charge implies that the nonzero n_5 will be generated during the axion inflation.

In order to study this process numerically, one needs to solve the coupled system of the Friedmann (10), Klein–Gordon (11), and Maxwell equations (12)–(14) together with the anomaly equation (18). However, the Maxwell equations (12)–(14) describe the evolution of quantum electric and magnetic fields; therefore, one has to deal with the mode functions of these fields in the Fourier space. In the presence of the backreaction or the Schwinger effect, all modes become coupled to one another that makes numerical calculations very demanding. To overcome this problem, we utilize the gradient expansion formalism developed in Ref. [50]. It operates with the

[1] We do not account for perturbative chirality-flipping processes which equilibrate the chiral asymmetry, since they are negligible at temperatures above 80 TeV [91, 92], which is true in our model, see Sec. 4.

vacuum expectation values of bilinear electromagnetic functions in the coordinate space that include all physically relevant modes at once. These functions are defined as follows:

$$\mathcal{E}^{(n)} = \frac{1}{a^n}\,\langle \boldsymbol{E}\cdot\mathrm{rot}^n\boldsymbol{E}\rangle, \tag{19}$$

$$\mathcal{G}^{(n)} = -\frac{1}{a^n}\,\langle \boldsymbol{E}\cdot\mathrm{rot}^n\boldsymbol{B}\rangle, \tag{20}$$

$$\mathcal{B}^{(n)} = \frac{1}{a^n}\,\langle \boldsymbol{B}\cdot\mathrm{rot}^n\boldsymbol{B}\rangle. \tag{21}$$

The equations of motion for these quantities can be derived from the Maxwell's equations (12) and (13) and have the form [46, 50]:

$$\dot{\mathcal{E}}^{(n)} + [(n+4)H + 2\sigma_{\mathrm{f}}]\,\mathcal{E}^{(n)} - 2I'(\phi)\dot{\phi}\,\mathcal{G}^{(n)} + 2\mathcal{G}^{(n+1)} = [\dot{\mathcal{E}}^{(n)}]_b, \tag{22}$$

$$\dot{\mathcal{G}}^{(n)} + [(n+4)H + \sigma_{\mathrm{f}}]\,\mathcal{G}^{(n)} - \mathcal{E}^{(n+1)} + \mathcal{B}^{(n+1)} - I'(\phi)\dot{\phi}\,\mathcal{B}^{(n)} = [\dot{\mathcal{G}}^{(n)}]_b, \tag{23}$$

$$\dot{\mathcal{B}}^{(n)} + (n+4)H\,\mathcal{B}^{(n)} - 2\mathcal{G}^{(n+1)} = [\dot{\mathcal{B}}^{(n)}]_b. \tag{24}$$

where $[\dot{\mathcal{E}}^{(n)}]_b$, $[\dot{\mathcal{G}}^{(n)}]_b$, and $[\dot{\mathcal{B}}^{(n)}]_b$ are the boundary terms.

The origin of these terms is the following. All possible modes of quantum gauge field fluctuations are always present in the physical vacuum. Due to a quasiexponential expansion of the Universe during the inflation, the wavelength of some modes of the electromagnetic field becomes larger than the Hubble horizon (more precisely, they become, at some point, tachyonically unstable and stop oscillating). Such long-wavelength modes can be treated as the Fourier modes of a classical electromagnetic field [93]. Modes deep inside the horizon oscillate in time without a significant change in their amplitude. Their total energy is infinite and its contribution to the electromagnetic energy density should be excluded. The number of modes that cross the horizon and become physically relevant constantly grows during the inflation. This leads to an additional time dependence of electromagnetic quantities which can be described by means of the boundary terms in the equations of motion.

The boundary terms for the system of equations (22)–(24) were derived for the first time in Ref. [46] and were calculated more accurately in Ref. [50], including the impact of the Schwinger conductivity σ_{f}. Considering the electromagnetic modes with the circular polarization $\lambda = \pm$, one could obtain

$$[\dot{\mathcal{E}}^{(n)}]_b = \frac{d\ln k_{\mathrm{h}}(t)}{dt}\frac{\Delta(t)}{4\pi^2}\left(\frac{k_{\mathrm{h}}(t)}{a(t)}\right)^{n+4} \times \sum_{\lambda=\pm1}\lambda^n E_\lambda(\xi(t), s(t)), \tag{25}$$

$$[\dot{\mathcal{G}}^{(n)}]_b = \frac{d\ln k_{\mathrm{h}}(t)}{dt}\frac{\Delta(t)}{4\pi^2}\left(\frac{k_{\mathrm{h}}(t)}{a(t)}\right)^{n+4} \times \sum_{\lambda=\pm1}\lambda^{n+1} G_\lambda(\xi(t), s(t)), \tag{26}$$

$$[\dot{\mathcal{B}}^{(n)}]_b = \frac{d\ln k_{\mathrm{h}}(t)}{dt}\frac{\Delta(t)}{4\pi^2}\left(\frac{k_{\mathrm{h}}(t)}{a(t)}\right)^{n+4} \times \sum_{\lambda=\pm1}\lambda^n B_\lambda(\xi(t), s(t)), \tag{27}$$

where $k_{\mathrm{h}}(t)$ is the momentum of the mode that crosses the horizon at time t

$$k_{\mathrm{h}}(t) = \max_{t'\le t}\Big\{a(t')H(t')\big[|\xi(t')| + \sqrt{\xi^2(t') + s^2(t') + s(t')}\big]\Big\}, \tag{28}$$

$$\xi(t) = \frac{dI}{d\phi}\frac{\dot{\phi}}{2H}, \quad s(t) = \frac{\sigma_{\mathrm{f}}(t)}{2H}, \tag{29}$$

the parameter Δ is the exponential of the integrated conductivity,

$$\Delta(t) \equiv \exp\left(-\int_{-\infty}^{t}\sigma_{\mathrm{f}}(t')dt'\right), \tag{30}$$

and

$$E_\lambda(\xi, s) = \frac{e^{\pi\lambda\xi}}{r^2(\xi, s)}\Big|(ir(\xi,s) - i\lambda\xi - s) \times W_{-i\lambda\xi,\frac{1}{2}+s}(-2ir(\xi,s)) + W_{1-i\lambda\xi,\frac{1}{2}+s}(-2ir(\xi,s))\Big|^2, \tag{31}$$

$$G_\lambda(\xi, s) = \frac{e^{\pi\lambda\xi}}{r(\xi, s)}\Big\{\Re e\Big[W_{i\lambda\xi,\frac{1}{2}+s}(2ir(\xi,s)) \times W_{1-i\lambda\xi,\frac{1}{2}+s}(-2ir(\xi,s))\Big] - s\left|W_{-i\lambda\xi,\frac{1}{2}+s}(-2ir(\xi,s))\right|^2\Big\}, \tag{32}$$

$$B_\lambda(\xi, s) = e^{\pi\lambda\xi}\left|W_{-i\lambda\xi,\frac{1}{2}+s}(-2ir(\xi,s))\right|^2 \tag{33}$$

with $r(\xi, s) = |\xi| + \sqrt{\xi^2 + s + s^2}$ and the Whittaker function $W_{\kappa,\mu}(y)$. For large ξ, these expressions can be expanded in series in inverse powers of $r(\xi, s)$, which is convenient for numerical computations.

Equations (22)–(24) form an infinite chain, because the equation of motion for the nth order function always contains at least one function with the $(n+1)$th power of the curl. However, this chain can be truncated at some large order $n_{\max}$ due to a simple observation. For a large power $n \gg 1$ of the spatial curl, the main contribution to $\mathcal{E}^{(n)}$, $\mathcal{B}^{(n)}$, $\mathcal{G}^{(n)}$ is made by modes with the largest possible momentum which is the momentum of the mode crossing the horizon at the moment of time under consideration $k_h(t)$. Then one can write

$$\mathcal{E}^{(n_{\max}+1)} \approx \left(\frac{k_h(t)}{a(t)}\right)^2 \mathcal{E}^{(n_{\max}-1)} \tag{34}$$

and similar relations for $\mathcal{B}^{(n_{\max}+1)}$, $\mathcal{G}^{(n_{\max}+1)}$ which allows us to truncate the chain at some order $n_{\max}$. We have chosen to relate the quantities with orders differing by 2, because they have the same symmetry properties under the spatial inversion. After the truncation, we get a closed system of ordinary differential equations that describe the self-consistent evolution of classical observables in the form of quadratic functions of the electric and magnetic fields with an arbitrary power of the curl.

3. Chiral Magnetic Effect

In the previous section, we obtained a coupled system describing the generation of electromagnetic fields and charged fermions during the axion inflation with a special attention to the production of the chiral asymmetry in the fermionic sector. However, the latter may also have an impact on electromagnetic fields due to the *chiral magnetic effect* phenomenon. It leads to a nonzero electric current proportional to the magnetic field with the proportionality coefficient related to the chiral asymmetry (for a review, see Ref. [90] and references therein):

$$\boldsymbol{j}_{\mathrm{CME}} = \frac{e^2}{2\pi^2}\,\mu_5 \boldsymbol{B}. \tag{35}$$

Here, μ_5 is the chiral chemical potential which is a conjugated quantity to $N_5 = \int n_5\, d^3\boldsymbol{x}$ in equilibrium thermodynamics. In other words, it characterizes the average energy that one needs to spend in order to increase the chiral charge of the system N_5 by unity.

In order to determine μ_5, we assume that fermions produced by the Schwinger effect rapidly thermalize and can be characterized by the equilibrium Fermi–Dirac distribution function with some temperature T. Although such an approximation is not well justified far from the end of the inflation, when the number of produced fermions is very low, it seems to be reasonable during the final part of the axion inflation, where the generated electromagnetic field is strong, and the Schwinger pair creation effectively produces many fermions. In any case, such an equilibrium approximation will allow us to estimate definitely only qualitatively and not much precisely the chirality production and the impact of the chiral magnetic effect on the electromagnetic field generation during the axion inflation.

Since, there were no fermions at the beginning of the inflation and the Schwinger effect can produce them only in particle-antiparticle pairs, the plasma is quasineutral, and the electric chemical potential $\mu = 0$. Then the momentum distribution functions for the right- and left-handed particles have the form:

$$f_{R/L}(p) = \frac{1}{\exp\left(\frac{p \mp \mu_5}{T}\right) + 1}, \tag{36}$$

i.e., the chemical potential for right-handed particles equals μ_5, while, for left-handed ones, it is equal to $-\mu_5$. The distribution functions for the corresponding antiparticles $\bar{f}_{R/L}$ can be obtained from Eq. (36) by the replacement $\mu_5 \to -\mu_5$. Then the chiral charge and the total energy density of fermions can be expressed in terms of T and μ_5 as follows:

$$n_5 = \int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\left[f_R(p) - f_L(p) - \bar{f}_R(p) + \bar{f}_L(p)\right] = \\ = \frac{1}{3}\mu_5 T^2 + \frac{1}{3\pi^2}\mu_5^3, \tag{37}$$

$$\rho_{\mathrm{f}} = \int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\, p\, \left[f_R(p) + f_L(p) + \bar{f}_R(p) + \bar{f}_L(p)\right] = \\ = \frac{7\pi^2}{60}T^4 + \frac{1}{2}\mu_5^2 T^2 + \frac{1}{4\pi^2}\mu_5^4. \tag{38}$$

Here, we used the fact that the antiparticle of a right-handed particle has left-handed chirality and vice versa. We would like to note that Eqs. (37) and (38) give the exact results of integration of the Fermi–Dirac functions without any assumptions about the relative magnitude of μ_5 and T. These nonlinear relations allow us to use interchangeably two sets of variables, either n_5 and $\rho_{\rm f}$ or μ_5 and T. In order to consider the chiral magnetic effect, the latter set of variables is more convenient, since current (35) depends on μ_5.

Using Eqs. (16) and (18) together with relations (37) and (38), we derive the following equations of motion for T and μ_5:

$$\dot{T}+HT=\frac{1}{\pi^2 T\, D(T,\,\mu_5)}\left[\left(\pi^2T^2+3\mu_5^2\right)\sigma_{\rm f}\mathcal{E}^{(0)}+\right.$$
$$\left.+\,3\mu_5\left(\pi^2T^2+\mu_5^2\right)\frac{e^2}{2\pi^2}\mathcal{G}^{(0)}\right], \tag{39}$$
$$\dot{\mu}_5+H\mu_5=-\frac{1}{D(T,\,\mu_5)}\left[2\mu_5\sigma_{\rm f}\mathcal{E}^{(0)}+\right.$$
$$\left.+\left(\frac{7\pi^2}{5}T^2+3\mu_5^2\right)\frac{e^2}{2\pi^2}\mathcal{G}^{(0)}\right], \tag{40}$$

where

$$D(T,\,\mu_5)=\frac{7\pi^2}{15}T^4+\frac{2}{5}\mu_5^2T^2+\frac{1}{\pi^2}\mu_5^4=$$
$$=\frac{7\pi^2}{15}\left(T^2+\frac{3}{7\pi^2}\mu_5^2\right)^2+\frac{32}{35\pi^2}\mu_5^4>0. \tag{41}$$

In order to determine the impact of the chiral magnetic effect on the gauge-field production, we add current (35) to the left-hand side of the Maxwell equation (12). This current is similar to the fourth term in Eq. (12) since it is also proportional to $\boldsymbol{B}$. Therefore, we have to effectively replace

$$I'(\phi)\dot{\phi}\;\to\;I'(\phi)\dot{\phi}+\frac{e^2}{2\pi^2}\mu_5 \tag{42}$$

everywhere in the equations of motion for electromagnetic fields, in particular, in Eqs. (22)–(24) which then take the form

$$\dot{\mathcal{E}}^{(n)}+[(n+4)H+2\sigma_{\rm f}]\,\mathcal{E}^{(n)}-2\left(I'(\phi)\dot{\phi}+\frac{e^2}{2\pi^2}\mu_5\right)\mathcal{G}^{(n)}+$$
$$+\,2\mathcal{G}^{(n+1)}=[\dot{\mathcal{E}}^{(n)}]_b, \tag{43}$$
$$\dot{\mathcal{G}}^{(n)}+[(n+4)H+\sigma_{\rm f}]\,\mathcal{G}^{(n)}-\mathcal{E}^{(n+1)}+\mathcal{B}^{(n+1)}-$$
$$-\left(I'(\phi)\dot{\phi}+\frac{e^2}{2\pi^2}\mu_5\right)\mathcal{B}^{(n)}=[\dot{\mathcal{G}}^{(n)}]_b, \tag{44}$$
$$\dot{\mathcal{B}}^{(n)}+(n+4)H\,\mathcal{B}^{(n)}-2\mathcal{G}^{(n+1)}=[\dot{\mathcal{B}}^{(n)}]_b. \tag{45}$$

Note that the parameter ξ in Eqs. (25)–(33) is replaced by

$$\xi_{\rm eff}=\xi+\frac{e^2}{4\pi^2}\frac{\mu_5}{H} \tag{46}$$

and, thus, μ_5 modifies also the boundary terms.

4. Numerical Results

Let us now specify the inflationary model and consider the chirality generation in it numerically. We use the simplest quadratic inflaton potential

$$V(\phi)=\frac{m^2\phi^2}{2}, \tag{47}$$

which well describes the behavior of a wide class of inflaton potentials close to their minima. This is important, since the most intense generation of the electromagnetic fields and charged fermions occurs close to the end of the inflation, when the inflaton indeed approaches the minimum of its potential. For definiteness, in the numerical analysis, we take $m=6\times10^{-6}\,M_{\rm P}$.

The axial coupling constant $I(\phi)$ is taken in a linear form

$$I(\phi)=\beta\frac{\phi}{M_{\rm p}} \tag{48}$$

with a dimensionless parameter β. This is the typical expression for the coupling function for the axion-like inflaton field. For the numerical analysis, we use values $\beta=10$–25. For smaller values, the generated fields are very weak, while larger values would lead to the generation of big non-Gaussianities in the primordial scalar power spectrum [94].

The initial conditions for the inflaton field and its time derivative are chosen from the requirement that the inflation stage lasts at least 60 e-foldings, i.e., $\phi(0)\approx15.5\,M_{\rm P}$ and $\dot{\phi}(0)=-\sqrt{2/3}MM_{\rm P}$. The latter expression was derived in the slow-roll approximation for potential (47). Note that only the last 10–15 e-foldings are important for magnetogenesis and the production of fermions; however, the initial conditions should be specified well before this moment. Zero initial values for all electromagnetic quantities $\mathcal{E}^{(n)}$, $\mathcal{B}^{(n)}$, and $\mathcal{G}^{(n)}$, as well as for the fermion energy density $\rho_{\rm f}$ and chiral charge n_5, were assumed.

One important comment on the introduction of the chiral chemical potential should be made. The nonlinear relations (37) and (38), generally speaking, do not allow one to determine μ_5 and T for arbitrary values of $\rho_{\rm f}$ and n_5. In fact, assuming that Eqs. (37) and (38) are satisfied, the following relation is valid:

$$\frac{3\sqrt{\pi}\,n_5}{2\sqrt{2}\,\rho_{\rm f}^{3/4}}=\frac{1+\frac{\pi^2T^2}{\mu_5^2}}{\left(1+\frac{2\pi^2T^2}{\mu_5^2}+\frac{7\pi^4T^4}{15\mu_5^4}\right)^{3/4}}. \tag{49}$$

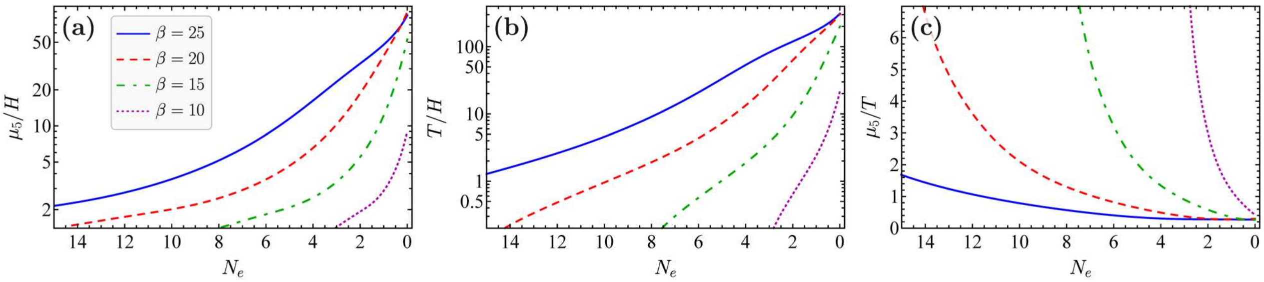

Fig. 1. The chiral chemical potential in Hubble units μ_5/H [panel (a)], the effective temperature in Hubble units T/H [panel (b)], and their ratio μ_5/T [panel (c)] in the dependence on the number of e-foldings counted from the end of the inflation N_e for four different values of the axial coupling parameter: $\beta = 10$ (purple dotted lines), $\beta = 15$ (green dashed-dotted lines), $\beta = 20$ (red dashed lines), and $\beta = 25$ (blue solid lines)

One can easily check that the right-hand side of this equation lies in the interval [0, 1] for all possible values of μ_5 and T. Consequently, if, for given values of ρ_f and n_5, the combination $\frac{3\sqrt{\pi}\,n_5}{2\sqrt{2}\,\rho_\mathrm{f}^{3/4}} > 1$, Eqs. (37), (38) cannot be inverted, and μ_5 and T cannot be identified. In this situation, we conclude that fermions cannot be characterized by the thermal Fermi–Dirac distribution function. This indeed happens in our numerical analysis at very early times, when the particle production is very slow. It is quite natural, since, for low densities of particles, their thermalization is very unlikely in the exponentially expanding Universe.

In order to overcome this difficulty, at 60 e-foldings to the end of the inflation, we impose the initial conditions for ρ_f and n_5 rather than for μ_5 and T. (Of course, in this case, we are not able to account for the chiral magnetic effect in the Maxwell equations; however, for low fermion densities, this effect can be neglected.) We numerically solve our system of equations for the electromagnetic functions (22)–(24) together with Eqs. (16) and (18) for ρ_f and n_5 till some moment of time, when the condition $\frac{3\sqrt{\pi}\,n_5}{2\sqrt{2}\,\rho_\mathrm{f}^{3/4}} < 1$ is satisfied. At this moment of time, we are able to invert Eqs. (37), (38) and determine the corresponding values of μ_5 and T. They are taken as the initial conditions for the second stage, where we solve Eqs. (39) and (40) for μ_5 and T together with Eqs. (43)–(45) for the electromagnetic bilinear function which now involve the chiral magnetic effect.

We present the results for μ_5 and T generated in our model as functions of the number of e-foldings to the end of the inflation in Fig. 1. Panel (a) shows the chiral chemical potential in the Hubble units, panel (b) shows the temperature in the Hubble units, and panel (c) represents their ratio. The lines of different colors and styles correspond to four different values of the axial coupling constant: $\beta = 10$ (purple dotted lines), $\beta = 15$ (green dashed-dotted lines), $\beta = 20$ (red dashed lines), and $\beta = 25$ (blue solid lines).

First of all, we would like to note that both the temperature and chiral chemical potential grow in time, which corresponds to the fact that more fermions are produced by the Schwinger effect, and more fermions change their chirality due to the chiral anomaly. However, the ratio μ_5/T decreases in time meaning that the relative fraction of chiral imbalance $n_5 = n_R - n_L$ compared to the total fermion number $n_\mathrm{tot} = n_R + n_L$ decreases in time. This can be explained by the fact that, near the end of the inflation, the electromagnetic field becomes very strong, and the Schwinger effect produces a lot of fermions. Being initially equally distributed between the chiralities, they flip their chirality due to the chiral anomaly which results in the chiral imbalance. However, the latter process is less intense, since it is proportional to the second power of the electromagnetic field, while the Schwinger pair production term in Eq. (16) is proportional to the third power of the electromagnetic field. Thus, more particles are produced in total, than particles that change their chirality due to the chiral anomaly. Thus, we can conclude that the chiral imbalance n_5/n_tot becomes less, as time passes.

Figure 1, (c) implies that the ratio of the final chiral chemical potential to the temperature at the end of the inflation weakly depends on the axial coupling parameter β and equals approximately $\mu_5/T \sim 0.27$–0.3. This can be translated into the chiral asymmetry by the following considerations. For $\mu_5/T \ll 1$,

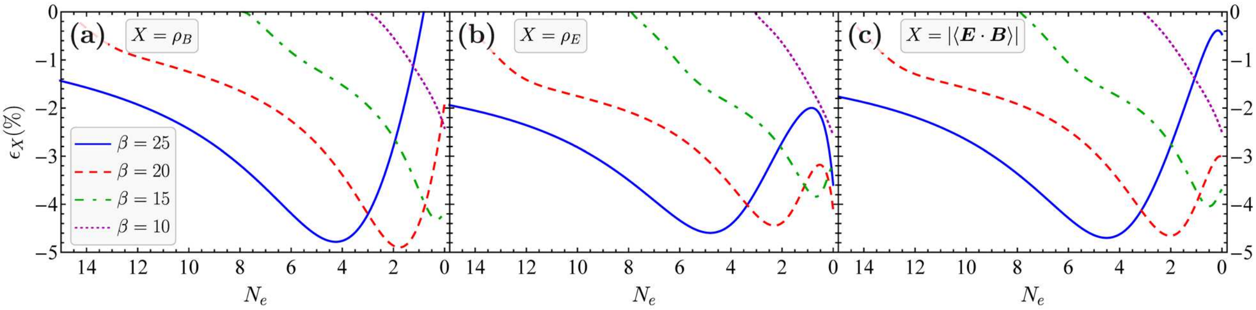

Fig. 2. The relative change of the magnetic energy density ρ_B [panel (a)], electric energy density ρ_E [panel (b)], and Chern-Pontryagin density $|\langle \boldsymbol{E} \times \boldsymbol{B}|$, caused by the presence of the chiral magnetic effect, in the dependence on the number of e-foldings counted from the end of the inflation N_e for four different values of the axial coupling parameter: $\beta = 10$ (purple dotted lines), $\beta = 15$ (green dashed-dotted lines), $\beta = 20$ (red dashed lines), and $\beta = 25$ (blue solid lines)

the total number density of right- and left-handed fermions equals

$$n_{\text{tot}} = n_R + n_L = \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \Big[f_R(p) + f_L(p) + \\ + \bar{f}_R(p) + \bar{f}_L(p) \Big] = \frac{3\zeta(3)}{\pi^2} T^3 + \mathrm{O}(\mu_5^2 T), \tag{50}$$

where $\zeta(3)$ is the Riemann zeta function. Then, taking into account that, in the same approximation, $n_5 = n_R - n_L \approx \mu_5 T^2/3$, we obtain

$$\eta = \frac{n_5}{n_{\text{tot}}} = \frac{n_R - n_L}{n_R + n_L} \approx \frac{\pi^2}{9\zeta(3)} \frac{\mu_5}{T} \approx 0.91 \frac{\mu_5}{T}. \tag{51}$$

Thus, we conclude that an excess of one chirality with respect to the other can reach 25%. Thus, the axion inflation is, indeed, an efficient chirality generator.

Now, let us discuss the impact of the chiral magnetic effect on the evolution of the electromagnetic field. For this purpose, we solved Eqs. (43)–(45) in the presence of the chiral magnetic effect and compared the results with the solutions of Eqs. (22)–(24), where this effect is switched off. The relative deviation

$$\epsilon_X = \frac{X - X_{\text{ref}}}{X_{\text{ref}}} \times 100\%, \tag{52}$$

where $X = \{\rho_E, \rho_B, |\langle \boldsymbol{E} \cdot \boldsymbol{B} \rangle|\}$ and X_{ref} is the solution without the chiral magnetic effect. We show these relative deviations in Fig. 2 again for four values of β considered above.

Analyzing this plot, we conclude that the impact of the chiral magnetic effect is rather weak, since the deviation is of order 1–5%. Qualitatively, it leads to a decrease of the electromagnetic field (note the negative values of ϵ_X). This is natural, since the electromagnetic field is the source of chiral asymmetry (due to the chiral anomaly), while the chiral magnetic effect represents the backreaction of this asymmetry on the electromagnetic field. If this backreaction were to decrease the electromagnetic field, one would get an instability in the system. As we discussed above, the axial coupling with the inflaton field and the chiral magnetic effect have similar structure. However, during the inflation, the latter is much less important than the former, and a general tendency to increase the electromagnetic field is observed.

5. Conclusion

In this work, we studied the generation of electromagnetic fields and charged fermions during the axion inflation accounting for the backreaction of generated fields on the inflaton evolution, the Schwinger effect, and the chirality production due to the chiral anomaly. Moreover, we have considered the chiral magnetic effect and analyzed its impact on magnetogenesis. For this purpose, we employed the gradient-expansion formalism previously proposed by three of us in Refs. [46, 50]. Since it operates with bilinear electromagnetic functions in the position space, it involves all physically relevant modes at once and thus allows us to treat the highly nonlinear phenomena listed above.

Qualitatively, the picture of the chirality production is the following. Far from the end of the inflation, the electromagnetic field is weak, and the Schwinger pair production is very slow (see, e.g., Ref. [50]). As a result, the number density of produced particles is small, and we cannot assume that they are thermalized. Indeed, the estimates in Ref. [48] for the de

Sitter spacetime and a constant electromagnetic field confirm that the thermalization does not occur. However, close to the end of the inflation, the inflaton rolls faster, and, therefore, the electromagnetic field becomes very strong (for sufficiently large β, the electromagnetic energy density can be comparable with that of the inflaton, see Ref. [46, 50]). Then it is natural to assume that the charged particles which are very intensively produced due to the Schwinger effect quickly achieve the thermal distribution. In this assumption, we introduced the effective temperature T and chiral chemical potential μ_5 which characterizes the chiral asymmetry of fermions. Although both quantities grow in time due to the particle and chirality productions, the ratio μ_5/T is a decreasing function of time. Indeed, at earlier times, there is a small number density of fermions, and the chiral anomaly transfers most of them into one chiral state resulting in $\mu_5 \gg T$. At later times, however, more and more fermions are created by the Schwinger effect (which produces the same amount of particles with both chiralities), and the chiral anomaly cannot transfer all of them into one chirality. As a result, the ratio μ_5/T decreases.

We have performed our numerical analysis in a simple inflationary model with quadratic potential $V(\phi) = m^2\phi^2/2$ and for a linear axial coupling function $I(\phi) = \beta\phi/M_{\mathrm{P}}$ with a dimensionless parameter β. We showed that, despite the fact that μ_5/T rapidly decreases in time during the last few e-foldings of the inflation, the final residual value of this quantity at the end of the inflation weakly depends on the coupling parameter β and is of order 0.27–0.3. Numerically, this corresponds to 25–27% of the chiral imbalance in the fermion number density. Thus, we conclude that the axion inflation is an efficient chirality generator.

The nonzero value of the chemical potential μ_5 leads to a new contribution to the electric current – the chiral magnetic effect $\boldsymbol{j} \propto \mu_5 \boldsymbol{B}$ which backreacts on the electromagnetic field evolution. We analyzed numerically this impact and concluded that such a backreaction leads to only a few percent change in the electric and magnetic energy densities. Thus, the impact of the chiral magnetic effect on the gauge field evolution is less important than the gauge field interaction with the inflaton. We would expect, however, that the produced chiral asymmetry may be very important in the postiflationary evolution of the magnetic field, since, at that time, the interaction with the inflaton will not play any role. Since our gradient-expansion formalism is applicable only during the inflation, some other methods are needed in order to investigate the postinflationary evolution in this system. Hopefully, this problem will be addressed elsewhere.

Let us note once more that the results obtained in this work are based on the assumption that, starting from a certain moment, a few e-foldings prior to the end of the inflation, the fermions become thermalized and can be characterized by the temperature and chemical potential. Technically, such an assumption was done in order to estimate the value of μ_5 and then analyze the impact of the produced chiral asymmetry on the electromagnetic field. However, since this impact is very weak, one can neglect the chiral magnetic effect during the inflation and study the chirality production without any assumptions about the fermion distribution. The most efficient tool to perform such an analysis is the kinetic approach. We plan to address this issue in the further studies.

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ГЕНЕРАЦІЯ ХІРАЛЬНОЇ
АСИМЕТРІЇ ПІД ЧАС АКСІОННОЇ ІНФЛЯЦІЇ

Ми дослідили генерацію хірального заряду під час аксіонної інфляції, де псевдоскалярне поле інфлатона ϕ взаємодіє аксіально з електромагнітним полем через доданок $(\beta/M_p)\phi\,\boldsymbol{E}\cdot\boldsymbol{B}$ з безрозмірною константою зв'язку β. Щоб описати еволюцію електромагнітного поля та визначити величину $\langle\boldsymbol{E}\cdot\boldsymbol{B}\rangle$, яка завдяки хіральній аномалії є джерелом хіральної асиметрії під час інфляції, ми використовуємо формалізм градієнтного розкладу. Він працює з набором вакуумних середніх від білінійних електромагнітних функцій і дозволяє враховувати зворотну реакцію згенерованих полів на еволюцію інфлатона, а також швінгерівське народження заряджених ферміонів. Крім того, ми припускаємо, що згенеровані ферміони термалізуються та внаслідок хірального магнітного ефекту дають внесок до електричного струму, $\boldsymbol{j}_{\rm CME}=e^2/(2\pi^2)\mu_5\boldsymbol{B}$, де μ_5 – хіральний хімічний потенціал, який кількісно визначає створену хіральну асиметрію. Розв'язуючи систему рівнянь для поля інфлатона, масштабного фактора, білінійних функцій електромагнітного поля та хіральної густини заряду (хірального хімічного потенціалу), ми знаходимо, що генерація хіральної асиметрії є досить ефективною, що відображається у великих значеннях хірального хімічного потенціалу в кінці аксіонної інфляції.

Ключові слова: аксіонна інфляція, формалізм градієнтного розкладу, ефект Швінгера, хіральна аномалія, хіральна асиметрія.