INELASTIC ELECTRON SCATTERING FORM FACTOR OF ISOSCALAR ($T = 0$) AND ISOVECTOR ($T = 1$) PARTICLE-HOLE STATES IN ^{12}C AND ^{16}O

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Inelastic longitudinal and transverse electron scattering form factors of low-lying $T = 0$, $T = 1$ particle-hole states of ^{12}C and ^{16}O are studied in the framework of the Tamm–Dancoff approximation (TDA). The Hamiltonian with the Michigan-three-Yukawa (M3Y) potential is diagonalized. To obtain a good agreement with the experimental data, the ground state is corrected by including the admixture from higher orbits with regard for the core polarization effects.

1. Introduction

Electoexcitation of nuclei is very important to study their structure, where the measured form factors are compared with those calculated in the framework of some nuclear models. For closed-shell nuclei, the structure of low-lying excited states is studied mainly through particle-hole models using Tamm–Dancoff Approximation (TDA) and Random-Phase Approximation (RPA). According to TDA, the ground state is treated within the independent particle model (closed shell), while the excited states of closed-shell nuclei are described as a linear combination of particle-hole excitations which are created by the excitation of a nucleon from the closed (filled) shell to a higher unoccupied shell leaving a hole within the closed shell. Asymmetry between the ground and excited states is removed in the RPA, where both states allowed having particle-hole pairs.

The inelastic electric and magnetic electron scattering with the use of nuclear structure models including the particle-hole excitation was first studied in [1], where the diagonalization of the residual two-body force was restricted to a subspace of configurations with one particle-one hole ($1p-1h$) with energy $1\hbar\omega$. The authors [2] reformulated the schematic model developed in [3] which is based on the particle-hole interaction to study the inelastic transverse form factor for the electromagnetic excitation of the giant resonance in nucleus ^{12}C. An extensive comparison of the TDA and RPA particle-hole models was performed in [4], where the available experimental data on the inelastic scattering of high energy electrons by the closed-shell nuclei ^{12}C, ^{16}O, and ^{40}Ca within particle-hole models of nuclear excitation were analyzed. The $T = 1$ (isovector) single particle-hole states of ^{12}C on the basis of the harmonic oscillator shell model were studied in [2] and later on in [5], where the configuration mixing was included via a Serber–Yukawa residual interaction.

The first 3^- level in ^{40}Ca and ^{208}Pb nuclei in the framework of TDA and RPA models have studied in [6], where the measured and calculated reduced transition probabilities B($E3\uparrow$) were compared. The transverse M4, M2, and M1 form factors for the inelastic electron scattering by ^{48}Ca nucleus using the independent Second RPA (SRPA) which includes the ($2p-2h$) excitation only were studied in [7] .

In the present work, the form factor of the electron scattering transition from the ground state to excited states in ^{12}C and ^{16}O are studied in the framework of the Tamm–Dancoff Approximation. The Hamiltonian with the Michigan-three-Yukawa (M3Y) potential is diagonalized. The admixture of higher configurations is also considered by including the single particle orbits up to $2s$-$2p$ for ^{12}C and ^{16}O, and the core polarization effects are taken into account to reproduce the experimental data.

2. Formalism

According to TDA, the ground state of a closed shell nucleus is taken to be a closed (filled) shell, while the excited state wave function can be constructed as a linear combination of pure basis (unperturbed) or uncorrelated wave functions as in [2]:

$$|\Psi_\alpha\rangle = \sum_{k=1}^{g} \chi_{k\alpha}^{JT} \left|\psi_k^{(0)}\right\rangle_{JMTT_Z}, \tag{1}$$

where $k \equiv (ab^{-1})$ with labels (a) for particles with quantum numbers $(n_a\ell_a j_a)$ and (b) for holes with quantum numbers $(n_b\ell_b j_b)$, g is the number of a state, and α is the order of a nuclear state. Inserting the linear combination into the Schrödinger equation and using the orthonormality of the $\left|\psi_k^{(0)}\right\rangle$, s give the secular equation

$$\sum_{k}^{g} \left\langle \psi_{k'}^{(0)}\right| H \left|\psi_k^{(0)}\right\rangle \chi_{k\alpha} = E_\alpha \chi_{k'\alpha}. \tag{2}$$

It can be diagonalized to yield the energy eigenvalues E and eigenvectors $\chi_{k\alpha}$.

We have

$$\sum_{k}^{g} \left\langle \psi_{k'}^{(0)}\right| H \left|\psi_k^{(0)}\right\rangle = \sum_{k=1}^{g} \left\langle \psi_{k'}^{(0)}\right| H^{(0)} + H^{(1)} \left|\psi_k^{(0)}\right\rangle =$$

$$= E_k^{(0)} \delta_{k'k} + \left\langle \psi_{k'}^{(0)}\right| H^{(1)} \left|\psi_k^{(0)}\right\rangle, \tag{3}$$

where $E_k^{(0)} = (e_a - e_b)$ is the unperturbed energy, and $H^{(1)} = V(1,2)$ is the residual interaction.

The residual interaction used in this work is the Michigan sum of three-range Yukawa (M3Y) potentials taken from [8]. This potential is derived from the fitting of parametrized three Yukawa radius-dependent form of central, spin-orbit, and tensor forces with the harmonic oscillator matrix element of the Reid soft-core potential [9]. It should be noted that the authors used usually different versions and parametrizations of a proximity potential to reproduce the experimental data (see, e.g., [10]).

The M3Y potential $V(1,2)$ includes the terms [8]

$$V(1,2) = V_{\mathrm{C}} + V_{\mathrm{LS}} + V_{\mathrm{T}}, \tag{4}$$

where

$$V_{\mathrm{c}} = \sum_{i=1}^{3} V_i Y(r/R_i), \quad V_{\mathrm{so}} = \sum_{i=1}^{3} V_i Y(r/R_i)\mathbf{S}\cdot\boldsymbol{\ell},$$

$$V_{\mathrm{t}} = \sum_{i=1}^{3} V_i r^2 Y(r/R_i) S_{12}$$

V_{c}, V_{so}, and V_{t} are central, spin-orbit, and tensor parts of the M3Y potential, respectively.

According to the M3Y potential the two-body interaction parameters, V_i and R_i, take special values given in Table (1) [11].

The nuclear longitudinal and transverse form factors are expressed in terms of reduced matrix elements of the electron scattering operator as [12]

$$|F_J^\eta(q)|^2 = \frac{4\pi}{Z^2}\frac{1}{2J_i+1}\left|\sum_T (-1)^{T_f - T_{f_z}} \begin{pmatrix} T_f & T & T_i \\ -T_{f_z} & M_T & T_{i_z} \end{pmatrix} \times \right.$$

$$\left. \times \left\langle J_f T_f \right| \| \hat{T}_{JT}^\eta(q) \| \left| J_i T_i \right\rangle \right|^2 |F_{\mathrm{c.m}}(q)|^2 |F_{\mathrm{f.s}}(q)|^2, \tag{5}$$

where the matrix elements are reduced both in the angular momentum J and isospin T, and η selects the longitudinal (L), transverse electric (el), and transverse magnetic (m) operators, respectively.

The last two terms in Eq. (5) are the correction factors of the center-of-mass (c.m) and a finite nucleon size (f.s) given by [13] and [14], respectively:

$$F_{\mathrm{c.m}}(q) = e^{q^2 b^2/4A}, \tag{6a}$$

$$F_{\mathrm{f.s}}(q) = e^{-a_p q^2/4}. \tag{6b}$$

Here, $a_p = 0.4 fm^2$, b is the oscillator length parameter (or size parameter), and A is the nuclear mass number.

The many-particle states matrix element reduced in the spin-isospin space can be written in terms of a single-particle matrix element as [15], [16]

$$\left\langle J_f T_f \right| \| \hat{T}_{JT}^\eta \| \left| J_i T_i \right\rangle = \sum_{\substack{j_a j_b \\ t_a t_b}} \chi_{ab^{-1}}^{JT} \langle a| \left\| \hat{T}_{JT}^\eta \right\| |b\rangle, \tag{7}$$

where $\chi_{ab^{-1}}^{JT}$ is the eigenvector obtained from the diagonalization of the (TDA) Hamiltonian matrix element in the presence of the M3Y potential. The states $|b\rangle$ and

$|a\rangle$ are the single-particle wave functions of the initial and final states, respectively.The single-particle transition operator $\hat{T}^{\eta}$ depends on whether the single nucleon is a proton or a neutron, so the single-particle matrix element can be written in terms of a single-particle matrix element reduced in spin only as [12]

$$\left\langle a \right| \parallel \hat{T}^{\eta}_{JT} \parallel \left| b \right\rangle =$$

$$= \sqrt{\frac{2T+1}{2}} \sum_{t_z} P_T(t_z) \left\langle n_a \ell_a j_a \parallel \hat{T}^{\eta}_{J,\, t_z} \parallel n_b \ell_b j_b \right\rangle . \quad (8)$$

Here,

$$P_T(t_z) = \begin{cases} 1, & \text{for} \quad T = 0, \\ (-1)^{1/2 - t_z}, & \text{for} \quad T = 1. \end{cases} \quad (9)$$

The reduced single-particle matrix element of the longitudinal operator can be written as [17]

$$\langle n_a \ell_a j_a \| \hat{T}^{L}_{J,t_z} \| n_b \ell_b j_b \rangle = e(t_z) P_J(e\ell, \ell_a, \ell_b) C_J(j_a, j_b) \times$$

$$\times \langle n_a \ell_a |\, j_J(qr)\, | n_b \ell_b \rangle , \quad (10)$$

where

$$P_J(e\ell, \ell_a, \ell_b) = \frac{1}{2} \left[1 + (-1)^{\ell_a + J + \ell_b} \right] \quad (11)$$

and

$$C_J(j_a, j_b) = (-1)^{j_a + 1/2} \sqrt{\frac{(2j_a+1)(2j_b+1)(2J+1)}{4\pi}} \times$$

$$\times \begin{pmatrix} j_a & J & j_b \\ 1/2 & 0 & -1/2 \end{pmatrix} . \quad (12)$$

The reduced single-particle matrix element of the transverse magnetic operator is given by [12]

$$\langle n_a \ell_a j_a \parallel \hat{T}^{m}_{J,t_z} \parallel n_b \ell_b j_b \rangle =$$

$$= i \mu_N \left[g_\ell(t_z) O^m_J(a,b) + g_s(t_z) S^m_J(a,b) \right] , \quad (13)$$

where the contribution of the orbital, O^m_J, and spin, S^m_J, parts are given explicitly as follows [12]:

$$O^m_J(a,b) = P_J(m, \ell_a, \ell_b) C_J(j_a, j_b) A_J(j_a, j_b) \sqrt{J(J+1)} \times$$

$$\times \langle n_a \ell_a | \, \frac{1}{r} j_J(qr) \, | n_b \ell_b \rangle , \quad (14)$$

$$S^m_J(a,b) = \frac{1}{2} \left[J(J+1) \right]^{-1/2} C_J(j_a, j_b) P_J(m, \ell_a, \ell_b) \times$$

$$\times \{ J \left[B(j_a, j_b) - J - 1 \right] \, \langle n_a \ell_a | \, \frac{1}{r} j_J(qr) \, | n_b \ell_b \rangle -$$

$$- q\, B(j_a, j_b) \, \langle n_a \ell_a | \, j_{J-1}(qr) \, | n_b \ell_b \rangle \} \quad (15)$$

with

$$P_J(m, \ell_a, \ell_b) = \frac{1}{2} \left[1 + (-1)^{\ell_a + \ell_b + J + 1} \right] , \quad (16)$$

$$A_J(a,b) = \left[1 + \frac{B(a,b)}{J} \right] \left[1 - \frac{B(a,b)}{J+1} \right] , \quad (17)$$

$$B(a,b) = 2 + D(j_a, \ell_a) + D(j_b, \ell_b), \quad (18)$$

$$D(j, \ell) = j(j+1) - \ell(\ell+1) - 3/4. \quad (19)$$

In our calculations of the form factor, the ground state wave function is modified to include an admixture of higher configurations (extension of the ground-state wave function) enhancing the collectivity, which seems to be very important [18]:

$$|b\rangle = \gamma \, |n_b \, l_b \, j_b\rangle + \delta \, \left| (n_b + 1) \, l_b j_b \right\rangle . \quad (20)$$

Here, γ and δ are mixing parameters with $\gamma^2 + \delta^2 = 1$.

By extension, we mean the configuration mixing between the closed shell $|(n, l\, j)^m, 00\rangle$ and $|(n+1, l\, j)^m, 00\rangle$ configurations, such that the ground-state wave function becomes: $|00\rangle = \gamma \, |(n, l\, j)^m, 00\rangle + \delta \, |(n+1, l\, j)^m, 00\rangle$, where $m = 2(2j+1)$ represents the number of nucleons that fill the state. However, the configuration mixing is actually due to the strong interaction among nucleons. Thus, the physical meaning of the extension is a correlation (i.e. the interaction among nucleons) in the ground state.

Example: – for ^{12}C,

$$|00\rangle = \gamma \left| (1s_{1/2})^4 (1p_{3/2})^8, 00 \right\rangle + \delta \left| (2s_{1/2})^4 (2p_{3/2})^8, 00 \right\rangle ,$$

and all possible $1p - 1h$ configurations are obtained in this case by promoting particles from $1s_{1/2}$-$1p_{3/2}$ or $2s_{1/2}$-$2p_{3/2}$ shells to $1p_{1/2}$, $2s$-$1d$ and $2p$-$1f$ shells. The quantity $|\gamma|^2$ represents the probability that the ground state is described by the configuration $\left| (1s_{1/2})^4 (1p_{3/2})^8, 00 \right\rangle$, while $|\delta|^2$ represents the probability that the ground state is described by the configuration $\left| (2s_{1/2})^4 (2p_{3/2})^8, 00 \right\rangle$. It should be noted that the extension of the ground state wave function modifies only the radial part of the reduced single-particle matrix element, while the angular part remains unchanged.

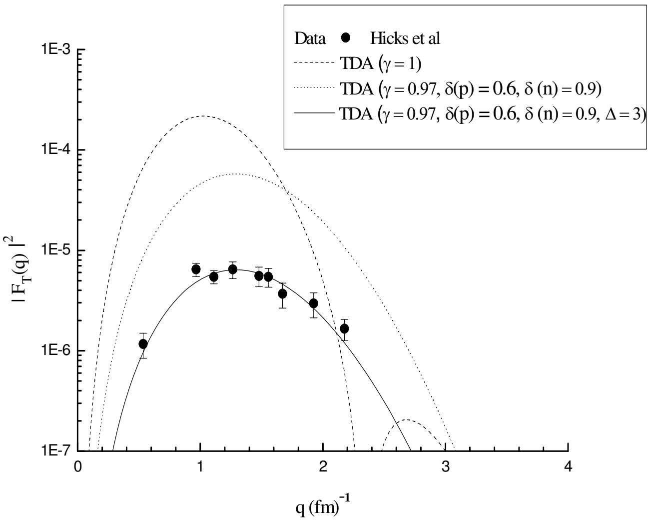

Fig. 1. Transverse M2 form factor for the 2_1^- ($T = 0$) state at $E_x = 11.83$ MeV in ^{12}C nucleus. Data are taken from [19]

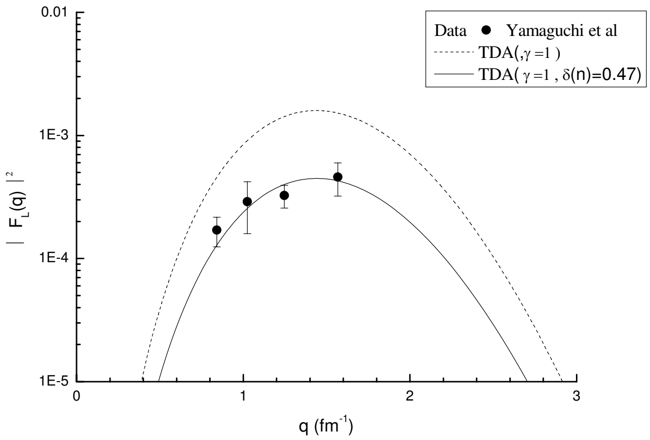

Fig. 2. Longitudinal C3 form factor for the 3_1^- ($T = 1$) state at $E_\chi = 18.60$ (MeV) in ^{12}C nucleus. Data are taken from [20]

3. Results and Discussion

The test of nuclear models is carried out by the comparison of the results of theoretical calculations with the corresponding experimental values.

In this work, the form factors of electron scattering transition from the ground state to an excited state in ^{12}C and ^{16}O are compared with those calculated according to the TDA model.

The diagonalization of the TDA model Hamiltonian is carried out, by using Eq. (2), in the space of one particle-one hole ($1p$-$1h$) states which include the harmonic oscillator orbits ($1s_{1/2}$, $1p_{3/2}$, $1p_{1/2}$, $1d_{5/2}$, $2s_{1/2}$, $1d_{3/2}$, $1f_{7/2}$, $2p_{3/2}$, $2p_{1/2}$, $1f_{5/2}$) to obtain eigenvalues and eigenvectors (amplitude $\chi_{(ab)}$), in the presence of the (M3Y) potential taken from [8] with the Reid parameter given in Table (1) taken from [11].

The resulting values of the amplitudes are used to calculate the reduced many-particle matrix elements of the electron scattering form factors which are functions of the momentum transferred $\mathbf{q}$ according to Eq. (7). The reduced single-particle matrix element in this equation is calculated according to Eq. (10) for the Coulomb scattering and Eq. (13) for the magnetic scattering. The squared form factors for the longitudinal or transverse magnetic scattering are calculated according to Eq. (5) which involves the corrections for the center-of-mass motion and the finite size of a nucleon.

The admixture of higher configurations is also considered by including the single-particle orbits up to $2s$-$2p$ for ^{12}C and ^{16}O, by using the reduced single-particle matrix element given in Eq. (20). Core polarization effects are also taken into account, by using the effective values of charge and g-factors which are different from those of free nucleons.

In our calculation of the single-particle matrix elements, the harmonic oscillator wave function with the size parameters $b = 1.64$ and 1.83 is adopted for ^{12}C and ^{16}O, respectively.

Our (TDA) calculations for the negative parity ($T = 0$) at J_n^π, E_χ (MeV) $= 2_1^-, 11.83$ and ($T = 1$) at $3_1^-, 18.6$, in ^{12}C predict the energy eigenvalues at 17.621 and 19.068 MeV, respectively.

Figure 1 shows the M2 form factor for the isoscalar 2_1^- state at $E_\chi = 11.83$ (MeV) in ^{12}C nucleus. The calculation of the form factor in the framework of TDA is shown by the dashed curve which overestimates the experimental data taken from [19]. The dotted curve shows the effect of the addition of a more collectivity to this state by a modification of the ground state wave function to include $2s$ and $2p$ shells with $\gamma = 0.97$, as well as taking the core polarization effects into account by introducing the effective g_s-factors for a proton $g_s^{\rm eff}(p) = \delta(p) g_s^{\rm free}(p)$ and for a neutron $g_s^{\rm eff}(n) = \delta(n) g_s^{\rm free}(n)$, where $\delta(p) = 0.6$ and $\delta(n) = 0.9$.

Both the energy eigenvalue and the magnetic quadrupole form factor of this state are deviated greatly from the experimental data, which denotes the need of the contribution of $2p$-$2h$ and higher particle-hole excitations. The solid curve is obtained by reducing the amplitudes χ by a factor of 3, i.e., using a modified amplitudes χ', where $\chi' = \chi/\Delta$ with $\Delta = 3$. This reduction in the χ, s may be ascribed to the two particle–two hole ($2p$-$2h$) and higher excitations that are neglected in our calculations.

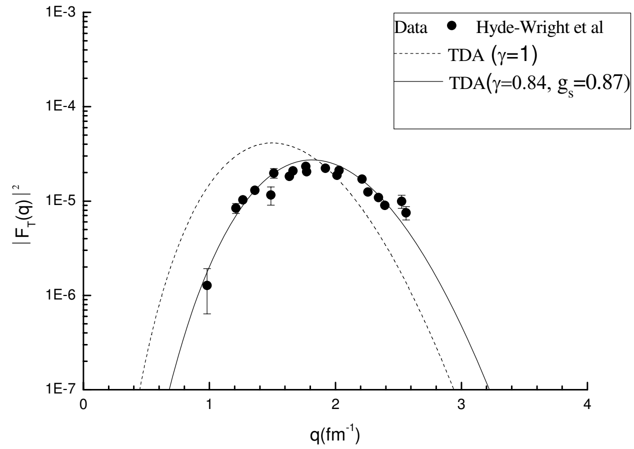

Fig. 3. Transverse M4 form factor for the 4_1^- $(T = 0)$ state at $E_\chi = 17.79$ (MeV) in ^{16}O nucleus. Data are taken from [21]

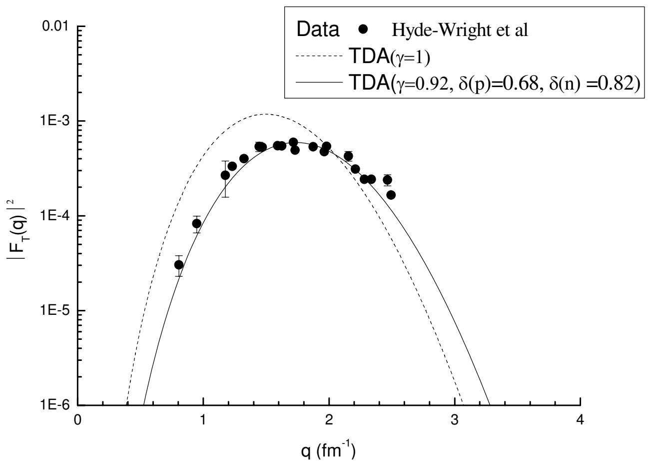

Fig. 4. Transverse M4 form factor for the 4_1^- $(T = 1)$ state at $E_\chi = 18.98$ (MeV) in ^{16}O nucleus. Data are taken from [21]

Figure 2 shows the longitudinal C3 form factor for the isovector $(T = 1)$ 3_1^- state at $E_\chi = 18.6$ MeV. The dashed curve represents the model space calculation. In our work, the experimental data [20] are well described with regard for the core polarization effects by introducing the effective charge for a neutron only ($e^{\text{eff}}(n) = \delta(n)e^{\text{free}}(n)$ with $\delta(n) = 0.47$). The calculations of the energies of the negative parity states $(T = 0)$ at $J^\pi T(E_\chi, \text{MeV}) = 4^-0(17.79)$ and $(T = 1)$ at $4^-1(18.98)$ in ^{16}O are carried out in the framework of TDA, and they are found to be 20.6 and 19.4 MeV, respectively.

Figure 3 shows the transverse M4 form factor for the excitation to the $J^\pi T(E_\chi, \text{MeV}) = 4^-0(17.79)$ state. The dashed curve is our result obtained within the particle-hole model TDA which is shifted to the left of the data. The solid curve is the form factor calculated with core-polarization effects by giving an effective value to the g_s-factors of a proton and a neutron which is less than that of a free nucleon by a factor $\delta = 0.87$, as well as by using the extended model space configuration to give a more collective description to this state with $\gamma = 0.84$. A very good agreement with the experimental data taken from [21] is obtained.

Figure 4 shows the transverse M4 form factor for the excitation to the isovector 4_1^- state at 18.98 MeV. The dashed curve represents our calculation in the framework of the particle-hole configuration (TDA). To shift the theoretical curve toward the experimental data, a correlation in the ground state must be taken into account by including an admixture of higher configurations. The solid curve shows the results of calculations in the extended space with $\gamma = 0.92$, as well as those obtained with the introduction of core-polarization effects by using the effective g-factors, ($g_s^{\text{eff}}(p) = \delta(p)g_s^{\text{free}}(p)$, and $g_s^{\text{eff}}(n) = \delta(n)g_s^{\text{free}}(n)$, where $\delta(p) = 0.69$ and $\delta(n) = 0.82$). This leads to a very good description of the experimental data taken from [21].

4. Conclusion

The calculation of the energies and electron scattering form factors for closed-shell nuclei ^{12}C and ^{16}O in the framework of the Tamm–Dancoff approximation gives a reasonable agreement with the experimental data. The correlation in the ground state is introduced by including an admixture from higher harmonic oscillator orbits using a mixing parameter, and the core polarization effects are taken into account by giving effective values to the charges and g-factors of nucleons which are different from their free values.

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ФОРМФАКТОРИ НЕПРУЖНОГО РОЗСІЯННЯ ЕЛЕКТРОНІВ ДЛЯ ІЗОСКАЛЯРНОГО ($T = 0$) ТА ІЗОВЕКТОРНОГО ($T = 1$) СТАНІВ ЧАСТИНКА–ДІРКА В ^{12}C І ^{16}O

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Р е з ю м е

Досліджено формфактори непружного розсіяння електронів у поздовжньому і поперечному напрямках для низьколежачих $T = 0$ і $T = 1$ станів частинка–дірка в ^{12}C і ^{16}O в наближенні Тамма–Данкова. Діагоналізовано гамільтоніан з Мічіган-3-Юкава потенціалом. Для одержання гарного узгодження з експериментом основний стан змінено домішуванням вищележачих станів з урахуванням ефекту поляризації остова.