THE PARAMETRIC SPACE OF THE TWO-HIGGS-DOUBLET MODEL AND SAKHAROV'S BARYOGENESIS CONDITIONS

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The electroweak phase transition in the Two-Higgs-Doublet Model is investigated. The Gibbs potential at a finite temperature is computed with regard for the one-loop plus ring diagram contributions. The strong first-order phase transition satisfying Sakharov's baryogenesis conditions is determined for the values of scalar field masses allowed by experimental data. The relation between the model parameters supplying the phase transition to be of the first order is derived. It is shown that a sequence of phase transitions is also possible. A comparison with results of other authors is done.

1. Introduction

Nowadays, it is well known that the Minimal Standard Model (MSM) of elementary particles cannot explain the baryon asymmetry observed in the Universe. The mechanism of generation of this asymmetry from the initially symmetric state was proposed by A.D. Sakharov ([1], see also review [2]) and today is formulated as three baryogenesis conditions:

1. Baryon number non-conservation.
2. C- and CP-symmetry violation.
3. Deviation from thermal equilibrium.

In electroweak theory, a deviation from the thermal equilibrium can be provided by the electroweak phase transition (EPT). The investigations [3–5] showed that the EPT in the MSM is strong enough for the Higgs boson mass values that are incompatible with the modern experimental bound $m_h \geq 114.4$ GeV. Monte Carlo simulations were used to study the EPT in the effective three-dimensional gauge theory [4]. It was shown that a critical point exists in this model. For the experimentally allowed m_h values, the EPT becomes of the second order. In [5], the EPT in the MSM in presence of external magnetic and hypermagnetic fields was investigated. It was concluded that the third baryogenesis condition is not fulfilled, and the EPT becomes of the second order in strong fields. Thus, a deviation from the thermal equilibrium in the MSM is not strong enough, and Sakharov's baryogenesis scenario is not realized.

In connection with this, the investigation of the EPT in extensions of the Standard Model is of substantial interest. One of the extensions is the Two-Higgs-Doublet Model (THDM) [6, 7]. The THDM predicts four additional scalar particles: a neutral particle H, a pair of charged fields $H^{\pm}$, and a pseudoscalar particle A_0. As compared with the MSM, the THDM contains more free parameters in the scalar sector.

The EPT in the THDM was investigated in [8]. The obtained results were revised in [9], by using an improved approximation (avoidance of the high-temperature expansion, inclusion of the ring diagrams, and taking the experimentally measured mass of the top quark into account). It was concluded that the third baryogenesis condition is fulfilled in the THDM for some parameter values.

In the recent paper [10], possible scenarios of the model behavior during the cooling were investigated. It was shown that the realization of a certain scenario strongly depends on the parameter values of the scalar potential. The authors have computed the Gibbs potential in the THDM in a simple approximation considering only the first non-trivial finite-temperature corrections to the tree-level potential. Within this potential, they observed a few interesting phenomena:

1. A sequence of phase transitions (a second-order EPT breaks the electroweak symmetry; then a first-order EPT occurs).

2. Exotic CP-breaking or charge-breaking minima can be realized during the cooling.

The main goal of the present paper is to determine the domain of the THDM parameters, for which the strong first-order phase transition happens, and the third baryogenesis condition holds. We use the approximation of thermal equilibrium. Following [3, 5, 9], we compute the Gibbs potential in the one-loop order and include contributions of the ring diagrams. This consistent approximation, ensuring the minima of the effective potential to be real, allows us to check the results of [10].

We answer the question which is the domain in the space of model parameters that ensures the strong first-order phase transition. To estimate that, we introduce a certain relation between the model parameters and show that if this relation is satisfied, the system undergoes a strong first-order phase transition.

In [8, 9], the special restrictions on the THDM parameters were imposed, and the authors assumed that these restrictions ensure the symmetry breaking along the $\tan\beta = v_2/v_1 = 1$ direction. Here, v_i are the vacuum expectation values of the doublets. We do not restrict our investigation to this specific case. Below, we show that the noted assumption is not true in general, and a sequence of phase transitions may occur. Moreover, the phase transitions happening may be either of the first- or second-order. In any case, the jump of the order parameter may differ essentially from that observed in [9].

The paper is organized as follows. Section 2 contains the necessary information on the Lagrangian and the parametrization we use. Section 3 is devoted to the computation of the Gibbs potential. In Section 4, we present the obtained results on the phase transition. We also discuss the results obtained in [9, 10] and present the relation between parameters ensuring the first-order phase transition. Concluding remarks are given in Section 5. The appendices contain a special information used in the main text.

2. Lagrangian

The THDM Lagrangian differs from the MSM one in the scalar and Yukawa sectors. It can be written as

$$\mathcal{L} = \mathcal{L}_{\rm H} + \mathcal{L}_f + \mathcal{L}_g + \mathcal{L}_{\rm Yuk} + \mathcal{L}_{\rm gauge\,fixing+ghost}.$$

The scalar sector is

$$\mathcal{L}_{\rm H} = \frac{1}{2}\sum_{i=1}^{2}\left|\left(\partial_\mu - \frac{ig}{2}\sigma_a A^a_\mu - \frac{ig\prime}{2}Y_{\varphi_i}B_\mu\right)\varphi_i\right|^2 - V, \quad (1)$$

where V is a scalar potential. To simplify the analysis, we restrict our consideration to CP-conserving vacua only. We consider the potential which possesses the Z_2 symmetry [7, 11],

$$V = \sum_{i=1}^{2}\left[-\frac{1}{2}\mu_i^2\varphi_i^\dagger\varphi_i + \lambda_i(\varphi_i^\dagger\varphi_i)^2\right] + \lambda_3\left(\mathrm{Re}[\varphi_1^\dagger\varphi_2]\right)^2 +$$
$$+\lambda_4\left(\mathrm{Im}[\varphi_1^\dagger\varphi_2]\right)^2 + \lambda_5\left(\varphi_1^\dagger\varphi_1\right)\left(\varphi_2^\dagger\varphi_2\right), \quad (2)$$
$$\varphi_i = \begin{pmatrix}\sqrt{2}a_i^+ \\ c_i + id_i\end{pmatrix}.$$

This means the invariance with respect to the transformation

$$\varphi_1 \to -\varphi_1, \quad \varphi_2 \to \varphi_2.$$

The neutral scalar fields c_i acquire the non-zero vacuum expectation values (VEV) v_i and break the $SU(2)\times U_Y(1)$ symmetry giving masses to gauge bosons and fermions. The mass mixing of the a_i, c_i, and d_i fields takes place. The spectrum of physical particles is obtained by the substitution

$$\chi^+ = a_1^+\cos\gamma + a_2^+\sin\gamma,\ H^+ = -a_1^+\sin\gamma + a_2^+\cos\gamma,$$
$$h = c_1\cos\alpha + c_2\sin\alpha,\ H = -c_1\sin\alpha + c_2\cos\alpha,$$
$$\chi_3 = d_1\cos\delta + d_2\sin\delta,\ A_0 = -d_1\sin\delta + d_2\cos\delta, \quad (3)$$

where $\chi^\pm$ and χ_3 are the Goldstone modes. The expressions for α, γ, and δ, and their dependence on $v_{1,2}$ are adduced in Appendix A2.

In what follows, $v_{1,2}$ denote *arbitrary shifts* of $c_{1,2}$ fields. These shifts define a "jump" of the order parameter during the phase transition. The extremum points of the potential are denoted as $v_{0\,1,2}$. They are defined as

$$\left.\frac{\partial V}{\partial c_i}\right|_{c_i=v_{0i}} = 0. \quad (4)$$

We consider the case where the VEVs are non-zero for both doublets, i.e. $v_{0\,1,2} \neq 0$ (see Appendix A1).

The gauge-boson masses are

$$m_W^2 = \frac{g^2}{4}v^2, \quad m_Z^2 = \frac{g^2+g'^2}{4}v^2, \quad v^2 = v_1^2 + v_2^2.$$

In [8–10], a similar scalar sector is considered. The main difference is that there are the additional terms $\mu_3^2\varphi_1^\dagger\varphi_2+$ h.c. softly violating the Z_2 symmetry. As was noted in [8], the influence of these terms on the EPT strength is small (though their presence allows one to introduce an additional CP violation). Another distinction is the parametrization of the scalar field couplings (see Appendix A3).

As we mentioned above, the analysis in [8, 9] was simplified by restricting the possible tree-level parameter values to

$$\mu_1^2 = \mu_2^2, \quad \lambda_1 = \lambda_2. \tag{5}$$

In what follows, we consider this case separately.

The general parametrization for the Yukawa interaction is

$$\mathcal{L}_{\rm Yuk} = -\sum_{f_{\rm L}}\sum_{i=1}^{2}\Big\{G_{d,i}\left[\bar{f}_{\rm L}\varphi_i(f_d)_{\rm R} + (\bar{f}_d)_{\rm R}\varphi_i^\dagger f_{\rm L}\right] + \\ + G_{u,i}\left[\bar{f}_{\rm L}\varphi_i^c(f_u)_{\rm R} + (\bar{f}_u)_{\rm R}\varphi_i^{c\dagger} f_{\rm L}\right]\Big\}, \tag{6}$$

$$f_{\rm L} = \frac{1-\gamma_5}{2}\begin{pmatrix} f_u \\ f_d \end{pmatrix}, \quad f_{\rm R} = \frac{1+\gamma_5}{2}f.$$

Here, $\varphi_i^c = i\sigma_2\varphi_i^\dagger$. The heavy quarks – t and b – only are of importance for the Gibbs potential, so we neglect the CKM mixing.

This Yukawa Lagrangian leads to the existence of flavor-changing neutral currents (FCNC) [7]. According to the Glashow–Weinberg theorem [12], the dangerous processes with the FCNCs can be excluded at the tree-level if all fermions of a given electric charge couple to no more than one Higgs doublet. The most popular parametrizations which respect these restrictions are the THDM type I and the THDM type II [11, 13]:

- in the THDM type I, all fermions are decoupled from the second doublet, i.e. $G_{d,2} = G_{u,2} = 0$;
- in the THDM type II, the u, c, and t quarks couple to the first doublet, while d, s, and b couple to the second doublet, i.e. $G_{d,1} = G_{u,2} = 0$.

We consider the THDM type II parametrization. The main reason is that it represents a low-energy limit of the Minimal Supersymmetric Standard Model.

There is an interesting feature in the THDM type II. It follows from the expressions for the quark masses at the tree-level minimum of the potential:

$$m_{t,0} = G_{t,1}v_{01}, \quad m_{b,0} = G_{b,2}v_{02}. \tag{7}$$

Let v_{02} be one or two orders of magnitude smaller than v_{01}. Then $G_{t,1}$ and $G_{b,2}$ couplings have to be of the same order of magnitude to preserve the quark mass ratio. This is an essential difference from the MSM and the THDM type I. By noting this, we find that, in the THDM type II case, it is necessary to include the b quark contributions to the Gibbs potential and Debye masses.

To compare our results with that of [9], we also consider the case of $G_{t,1} = G_{t,2}$. In this parametrization, the contribution of the b quark to the Gibbs potential is also negligibly small. We will refer to this together with restrictions (5) as the *doublet-universal* parametrization.

In the present paper, all calculations are carried out in the Feynman–'t Hooft gauge. The gauge-fixing functions are

$$G^a = \frac{1}{\sqrt{\xi}}\left(\partial^\mu A^a_\mu + \xi\frac{ig}{4}\sum_{i=1}^{2}\left(\varphi_i^\dagger\sigma^a\varphi_{0i} - \varphi_{0i}^\dagger\sigma^a\varphi_i\right)\right),$$

$$G = \frac{1}{\sqrt{\xi}}\left(\partial^\mu B_\mu + \xi\frac{ig'}{4}\sum_{i=1}^{2}\left(\varphi_i^\dagger\varphi_{0i} - \varphi_{0i}^\dagger\varphi_i\right)\right),$$

$$\varphi_{0i} = \begin{pmatrix} 0 \\ v_i \end{pmatrix}. \tag{8}$$

Then, the gauge-fixing part of the Lagrangian reads

$$\mathcal{L}_{\rm gauge\,fixing} = -\frac{1}{2}\left(\sum_{a=1}^{3}G^{a2} + G^2\right). \tag{9}$$

The quadratic terms in the Faddeev–Popov sector are

$$\mathcal{L}_{\rm ghost} = -\bar{u}^+(\partial^2 + \xi m_W^2)u^- - \bar{u}^-(\partial^2 + \xi m_W^2)u^+ - \\ -\bar{u}_Z(\partial^2 + \xi m_Z^2)u_Z - \bar{u}_A\partial^2 u_A, \tag{10}$$

where ξ is a gauge-fixing parameter. For arbitrary ξ, the gauge-boson propagator is

$$iD^{\mu\nu}(p) = -\frac{i}{p^2 - m^2 + i\epsilon}\left(g^{\mu\nu} + (\xi - 1)\frac{p^\mu p^\nu}{p^2 - \xi m^2}\right).$$

The fermion and gauge field sectors of the Lagrangian are taken to be

$$\mathcal{L}_f = i\sum_{f_{\rm L}}\bar{f}_{\rm L}\gamma^\mu\left(\partial_\mu - \frac{ig}{2}\sigma_a A^a_\mu - \frac{ig'}{2}Y_{f{\rm L}}B_\mu\right)f_{\rm L} + \\ +i\sum_{f_{\rm R}}\bar{f}_{\rm R}\gamma^\mu\left(\partial_\mu - ig'Q_f B_\mu\right)f_{\rm R},$$

$$\mathcal{L}_{\rm g} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}F_a^{\mu\nu}F^a_{\mu\nu}. \tag{11}$$

We proceed with the calculation of the Gibbs potential.

3. Gibbs Potential

The calculation of the Gibbs potential was discussed in numerous papers [14, 15] (see also review [16]). We compute the effective potential in the following standard way:

$$V_G(v_i) = V_{\rm tree} + V^1_{\rm vac}(v_i) + V^1_T(v_i), \tag{12}$$

where $V_{\rm tree}$ is the tree-level potential, $V^1_{\rm vac}$ is the one-loop correction at zero temperature, V^1_T is the one-loop finite-temperature correction. The tree-level part is obtained by substituting φ_{0i} into (2) and reads

$$V_{\rm tree}(v_i) = -\frac{1}{2}(\mu_1^2 v_1^2 + \mu_2^2 v_2^2) + \lambda_1 v_1^4 + \lambda_2 v_2^4 + \\ +(\lambda_3+\lambda_5) v_1^2 v_2^2. \qquad (13)$$

Now, we consider $V^1_{\rm vac}$ and V^1_T.

3.1. One-loop contributions at zero temperature

The regularized contribution of a field of mass m to V_G is [14]

$$V^1_{\rm v}(m) = \frac{1}{64\pi^2}\left(\frac{m^2}{s_0} + \\ + m^4\left(\ln(s_0 m^2) - 3/2 + \gamma - \frac{i\pi}{2}\right)\right). \qquad (14)$$

We use Schwinger's proper time regularization with the regularization parameter s_0. It has to be set to zero at the end of calculations.

The general expression for the scalar field mass is

$$m^2_\pm = B_1 v_1^2 + B_2 v_2^2 + B_3 \pm \\ \pm\sqrt{(C_1 v_1^2 + C_2 v_2^2 + C_3)^2 + (D_1 v_1 v_2)^2}. \qquad (15)$$

Here, B_i, C_i, and D_i are some combinations of the tree-level VEVs and couplings. There are four pairs of the scalar fields: h and H, $\chi^\pm$ and $H^\pm$, χ_3 and A_0. The sign in Eq. (15), "$-$" or "$+$", corresponds to the mass of one field of a pair. For example, we have the "$-$" sign in case of the χ_3 mass and the "$+$" sign for the A_0 mass (see Appendix A2). In the sum of both field contributions, the term

$$\frac{1}{2}(m_+^4 - m_-^4)\ln\left(\frac{m_+^2}{m_-^2}\right) \qquad (16)$$

appears. It is cancelled out by the term coming from a high-temperature expansion, when the finite-temperature corrections are taken into account. However, we do not use this expansion in our calculations, and, therefore, the explicit cancellation does not occur. This term results in cumbersome quantum corrections to V_G.

The contribution coming from fermions, gauge bosons, and ghosts is given by (14) with regard for the factor A,

$$A = \begin{cases} -1\times 4\times 3, & \text{quark},\\ -2, & \text{ghost},\\ 3+\xi^2, & \text{gauge boson}.\end{cases} \qquad (17)$$

This factor accounts for the number of degrees of freedom and the color states of fields.

We choose the renormalization conditions preserving the tree-level vacuum energy value, VEVs, and mass terms. They are taken to be

$$V_{\rm v}(v_{0i}) = V_{\rm tree}(v_{0i}), \qquad \left.\frac{\partial V_{\rm v}}{\partial v_i}\right|_{\rm vac} = 0,$$
$$\left.\frac{\partial^2 V_{\rm v}}{\partial v_1^2}\right|_{\rm vac} = -\mu_1^2 + 12\lambda_1 v_{01}^2 + 2(\lambda_3+\lambda_5)v_{02}^2,$$
$$\left.\frac{\partial^2 V_{\rm v}}{\partial v_2^2}\right|_{\rm vac} = -\mu_2^2 + 12\lambda_2 v_{02}^2 + 2(\lambda_3+\lambda_5)v_{01}^2,$$
$$\left.\frac{\partial^2 V_{\rm v}}{\partial v_1 \partial v_2}\right|_{\rm vac} = 0. \qquad (18)$$

Since the renormalized contributions from the scalar sector are cumbersome, we do not adduce them here. They could be obtained easily by using a symbolic calculation software. The renormalized contributions of a fermion, a gauge boson, or a ghost field read

$$V^{1,\rm r}_{\rm v}(m) = \frac{A}{64\pi^2}\left(m^4\left(\ln\left(\frac{m^2}{m^2_{\rm vac}}\right) - \frac{1}{2}\right) + \frac{m^4_{\rm vac}}{2} - \\ -(m^2 - m^2_{\rm vac})^2\right), \qquad (19)$$

where $m_{\rm vac}$ is the field mass value at $v_i = v_{0i}$.

The complete temperature-independent part of the Gibbs potential is

$$V_{\rm v}(v_i) = V_{\rm tree} + V^{1,\rm r}_{{\rm v},\,h,H} + V^{1,\rm r}_{{\rm v},\,\chi^\pm,H^\pm} + V^{1,\rm r}_{{\rm v},\,\chi_3,A_0} + \\ +2(3+\xi^2)V^{1,\rm r}_{\rm v}(m_W) + (3+\xi^2)V^{1,\rm r}_{\rm v}(m_Z) - \\ -4V^{1,\rm r}_{\rm v}(\sqrt{\xi}m_W) - 2V^{1,\rm r}_{\rm v}(\sqrt{\xi}m_Z) - \\ -12V^{1,\rm r}_{\rm v}(m_t) - 12V^{1,\rm r}_{\rm v}(m_b), \qquad (20)$$

where $V^{1,\rm r}_{{\rm v},\,h,H}$, $V^{1,\rm r}_{{\rm v},\,\chi^\pm,H^\pm}$, and $V^{1,\rm r}_{{\rm v},\,\chi_3,A_0}$ are the scalar field contributions.

3.2. One-loop contributions at finite temperatures

Finite-temperature corrections are calculated, by using the Matsubara formalism. For the contribution of one bosonic degree of freedom, we have [15]

$$V^{\rm b}_T(m) = \frac{T^4}{2\pi^2}\int\limits_0^\infty dx\, x^2 \ln\left(1 - \exp(-\sqrt{x^2+\beta^2 m^2})\right), \qquad (21)$$

where β is the inverse temperature. We recall that Matsubara's frequencies for ghost fields are even [17]. For one fermionic degree of freedom, we have

$$V_T^{\rm f}(m) = \frac{T^4}{2\pi^2}\int\limits_0^\infty dx x^2 \ln\left(1+\exp(-\sqrt{x^2+\beta^2 m^2})\right). \tag{22}$$

The degree-of-freedom factors for fermions and ghosts stand in (17). The contribution of the massive gauge field is

$$V_T^{\rm gauge}(m) = 3V_T^{\rm b}(m) + V_T^{\rm b}(\sqrt{\xi}m). \tag{23}$$

The last term in (23) cancels a part of the ghost field contribution.

3.3. Ring diagram contributions

As is well known, an imaginary part of the one-loop Gibbs potential arises at small v_i. It comes from the scalar sector contributions at finite temperature and indicates the instability of the system. Gauge bosons are massless in the symmetric phase that leads to infrared divergences. These shortcomings of the one-loop effective potential can be avoided by adding the ring-diagram (Fig. 1) contributions [15, 18–20]. These diagrams introduce additional finite-temperature corrections to the masses of bosons that result in the terms of the order $\sim g^3$ or $\sim \lambda_i^{3/2}$ in V_G.

The ring-improved finite-temperature correction to the Gibbs potential in the scalar field case is

$$V_T^{\rm b}(m(T)) = \frac{T^4}{2\pi^2}\int\limits_0^\infty dx\, x^2 \ln\left(1-\exp(-\sqrt{x^2+\beta^2 m^2(T)})\right),$$

$$m^2(T) = m^2 + \delta m^2(T), \tag{24}$$

where $\delta m^2(T)$ denotes the Debye mass of a field. This correction ensures that the imaginary part for the finite-temperature contribution is absent. The Debye mass is defined through the polarization tensor Π of a field taken in the infrared limit [20]

$$\delta m^2(T) = \Pi(k_0 = 0, \bar{k} \to 0), \tag{25}$$

where k is the four-momentum of a field.

For gauge fields, the Debye mass is defined as $-\Pi_{00}$ in the infrared limit. The ring-diagram contribution to V_G from each massive gauge boson is [3]

$$V_{\rm g}^{\rm ring}(m) = -\frac{T}{12\pi}\left((m^2+\delta m^2(T))^{3/2} - m^3\right). \tag{26}$$

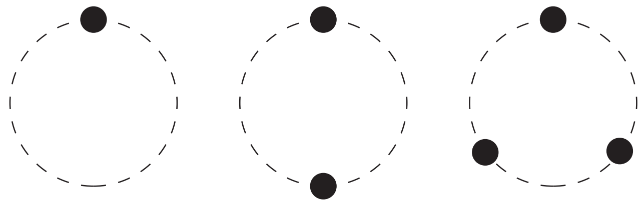

Fig. 1. Ring-diagram contributions for scalar field. Black blobs denote Debye masses

The one-loop Debye masses of the Higgs fields in the THDM type II are

$$\delta m_h^2(T) = T^2\left((2\lambda_1 + \frac{1}{2}G_t^2)\cos^2\alpha + (2\lambda_2 + \frac{1}{2}G_b^2)\sin^2\alpha + \frac{\lambda_3+\lambda_4+4\lambda_5}{6} + \frac{3g^2+g'^2}{16}\right),$$

$$\delta m_H^2(T) = T^2\left((2\lambda_1 + \frac{1}{2}G_t^2)\sin^2\alpha + (2\lambda_2 + \frac{1}{2}G_b^2)\cos^2\alpha + \frac{\lambda_3+\lambda_4+4\lambda_5}{6} + \frac{3g^2+g'^2}{16}\right). \tag{27}$$

For other scalar fields, the Debye masses are given by similar expressions. The difference is that the angle α is replaced by γ, δ from Eq. (3).

The complete Higgs-sector contribution to the finite-temperature part of V_G is

$$V_T^{\rm s}(v_i) = V_T^{\rm b}(m_h(T)) + V_T^{\rm b}(m_H(T)) + 2V_T^{\rm b}(m_{\chi^\pm}(T)) + 2V_T^{\rm b}(m_{H^\pm}(T)) + V_T^{\rm b}(m_{\chi_3}(T)) + V_T^{\rm b}(m_{A_0}(T)). \tag{28}$$

The gauge-boson Debye masses are

$$\delta m_W^2(T) = 2g^2T^2, \quad \delta m_Z^2(T) = \frac{11g'^4+5g^4}{4(g^2+g'^2)}T^2. \tag{29}$$

For Faddeev–Popov ghosts, the Debye mass is zero in the leading order in T.

In Appendix B, we give the Debye masses of all fields. We note that these corrections are ξ-independent.

The finite-temperature gauge-field and ghost contribution to V_G is

$$V_T^{\rm g}(v_i) = 6V_T^{\rm b}(m_W) + 3V_T^{\rm b}(m_Z) - 2V_T^{\rm b}(\sqrt{\xi}m_W) - V_T^{\rm b}(\sqrt{\xi}m_Z) + 2V_{\rm g}^{\rm ring}(m_W) + V_{\rm g}^{\rm ring}(m_Z). \tag{30}$$

For the final expression of V_G in the THDM, we have

$$V_G(v_i) = V_{\rm v}(v_i) + V_T^{\rm s}(v_i) + V_T^{\rm g}(v_i) - 12V_T^{\rm f}(m_t) - 12V_T^{\rm f}(m_b). \tag{31}$$

The minimum value of V_G is gauge-independent. The ξ-dependence is cancelled out between the gauge boson,

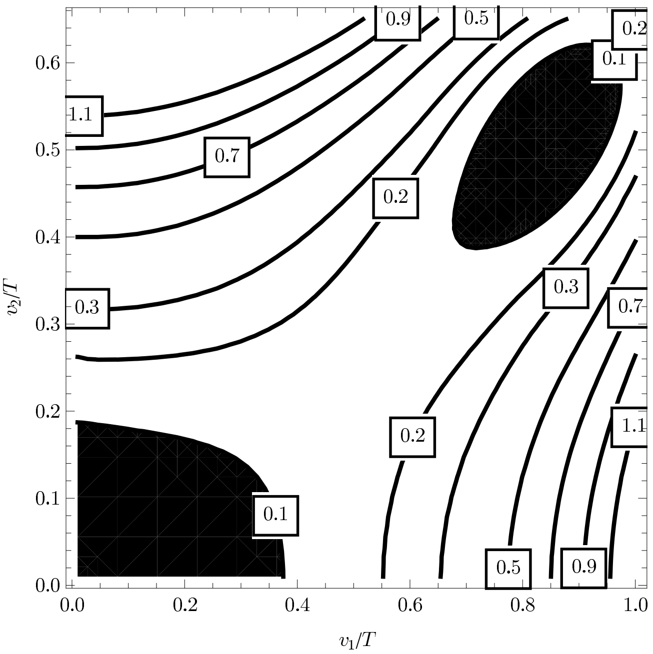

Fig. 2. Two distinct minima signaling the EPT of the first order are realized. The critical temperature is $T_c = 125.65$ GeV

Goldstone field, and ghost contributions. Physical quantities are also gauge-invariant [21].

However, the Gibbs potential is gauge-dependent [14] at arbitrary values of v_i. For numerical calculations, one has to choose the value for ξ. We set $\xi = 1$. In this case, the Goldstone field masses equal to the masses of corresponding gauge bosons.

4. Phase Transition

The third Sakharov condition is fulfilled if the first-order EPT is realized, and the order parameter jump is greater than 1 [2]:

$$\frac{\delta v}{T_c} > 1, \tag{32}$$

where T_c is the critical temperature.

In the THDM, there are two order parameters – v_1/T and v_2/T. In general, several jumps of the order parameters may occur, i.e. single phase transitions, as well as sequences of phase transitions, may happen. During a series of EPTs, the system goes to an intermediate vacuum state, where the symmetry is broken for one doublet only. We discuss possible scenarios of phase transitions in the THDM and find the domain in the parameter space, for which $\delta v/T_c$ is large.

We consider $\lambda_{1,2,3,4,5}$ and one of the VEVs $v_{01,2}$ as the free parameters of the model (see Appendix A1).

4.1. Possible scenarios

The plots of $100(V_G(v_1, v_2) - V_G(0, 0))/T^4$ versus v_1/T, v_2/T are shown in the figures. The black areas represent lower values of the Gibbs potential. The parameter values for the figures can be found in Appendix C.

First, we consider the THDM type II.

1. Fig. 2. The form of V_G indicates a strong first-order EPT. The order parameter jump is

$$\delta v = \frac{\sqrt{\delta v_1^2 + \delta v_2^2}}{T_c} = 1.01.$$

For this set of parameter values, the tree-level scalar field masses are

$$m_h = 119 \text{ GeV}, \quad m_H = 131 \text{ GeV}, \quad m_{H^\pm} = 181 \text{ GeV}, \quad m_{A_0} = 338 \text{ GeV}.$$

This scenario is the most favorable for successful baryogenesis.

2. Fig. 3. The sequence of phase transitions is generated. The weak first-order EPT breaking the symmetry along the directions $\tan\beta = 0$ or $\tan\beta = +\infty$ happens (the former case is shown in Fig. 3,*a*). Then next weak first-order EPT follows (Fig. 3,*b*). The Gibbs potential minimum is now located along the $0 < \tan\beta < +\infty$ direction in the (v_1, v_2) plane. The scalar field masses in the tree-level approximation are

$$m_h = 114 \text{ GeV}, \quad m_H = 132 \text{ GeV}, \quad m_{H^\pm} = 181 \text{ GeV}, \quad m_{A_0} = 266 \text{ GeV}.$$

The successful baryogenesis cannot be realized. The thermal equilibrium approximation used could be unreliable. This is because of non-equilibrium processes happening after the first phase transition. In fact, this series of phase transitions, as concerns its consequences, could substitute one strong enough first-order phase transition. The calculation of characteristics for a phase transition of such type requires other methods and additional investigations.

3. Fig. 4. The sequence of phase transitions happens. A strong first-order EPT breaking of the symmetry along the $\tan\beta = 0$ or $\tan\beta = +\infty$ direction is generated (the former case is shown in the figure). Then the system undergoes a second-order phase transition, the $\tan\beta$ value being finite and non-zero. The scalar field masses are

$$m_h = 120 \text{ GeV}, \quad m_H = 201 \text{ GeV}, \quad m_{H^\pm} = 322 \text{ GeV}, \quad m_{A_0} = 429 \text{ GeV}.$$

This scenario is acceptable for the successful baryogenesis.

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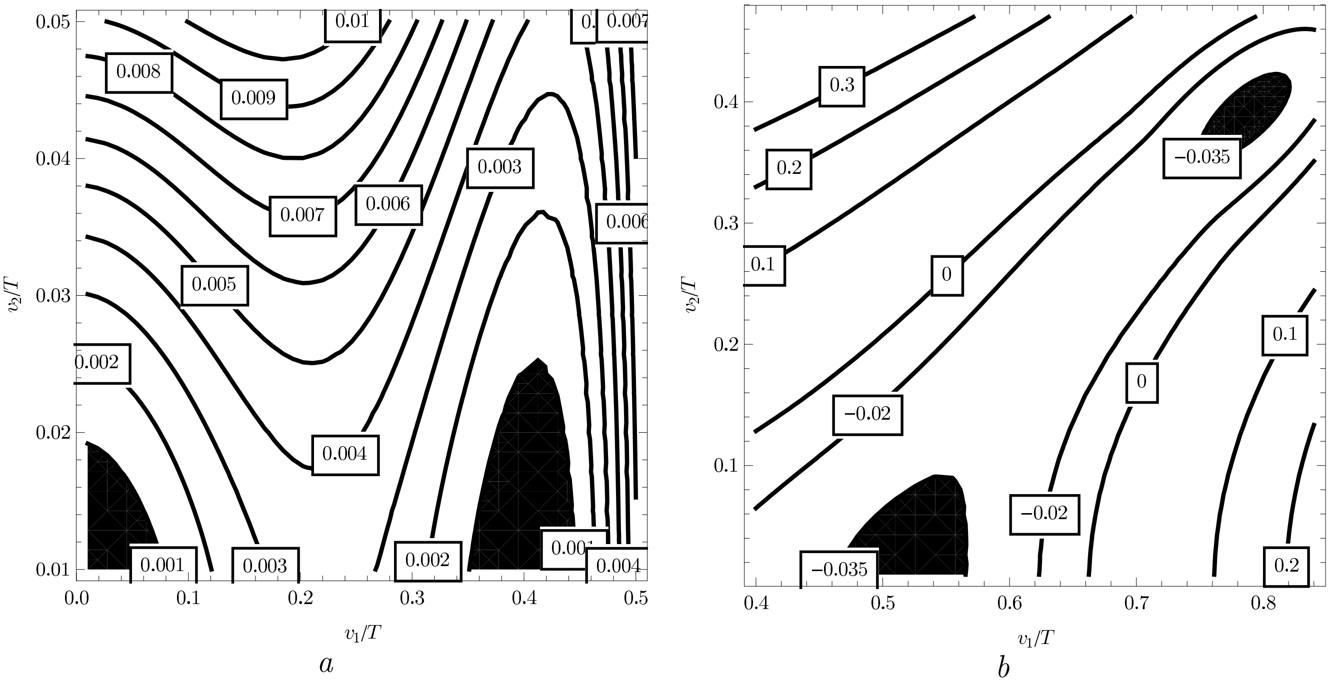

Fig. 3. Sequence of the first-order phase transitions. The critical temperatures are (a) $T_c = 128.18$ GeV, (b) $T_c = 127.66$ GeV

4. A sequence of second-order phase transitions is possible. Such scenario is realized, for example, if the scalar field masses at the tree level are

$m_h = 114$ GeV, $\quad m_H = 162$ GeV, $\quad m_{H^\pm} = 181$ GeV, $m_{A_0} = 276$ GeV.

In this case, the baryogenesis is not realized.

These results also hold for the THDM type I qualitatively and, in most cases, quantitatively. The main difference from the THDM type II is that the b quark influence on the Gibbs potential is much weaker. Therefore, a quantitatively different picture should be expected for small $\tan\beta_0 = v_{02}/v_{01}$ values, namely $\tan\beta_0 \leq 0.1$. For this domain of the parameter space, the b quark contribution to V_G in the THDM type II is non-negligible.

Note that the parameter values considered correspond to Sector I of the parameter space defined in [10]. In this sector, the authors observed a second-order EPT. They proposed that a more detailed investigation based on a consistent corrected effective potential could predict the EPT of the first order. This is just what we have observed.

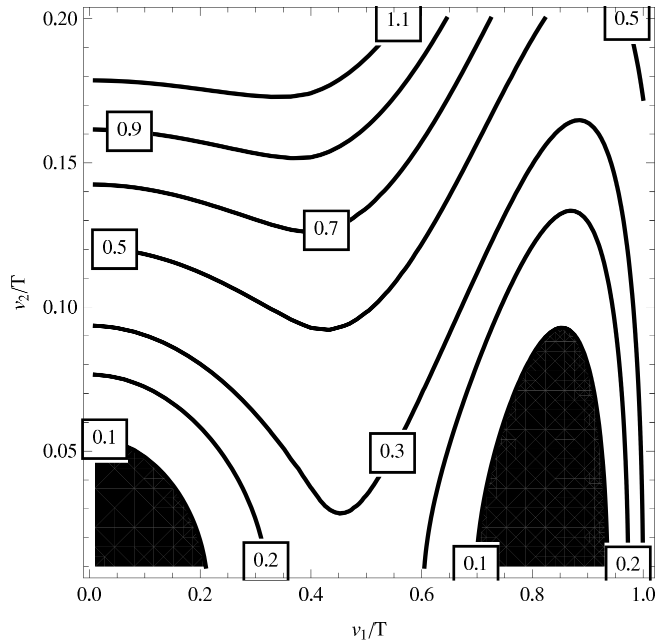

Fig. 4. The first-order EPT in the sequence of phase transitions. The critical temperature is $T_c = 153.3$ GeV

4.2. The doublet-universal parametrization

Since there is a possibility of a sequence of phase transitions, it is interesting to discuss the assumption about the symmetry breaking made in [8, 9]. In [9], a strong first-order EPT was observed. It was assumed that the symmetry breaking happens along the $\tan\beta = 1$ direction, if restrictions (5) are imposed. This allows one to

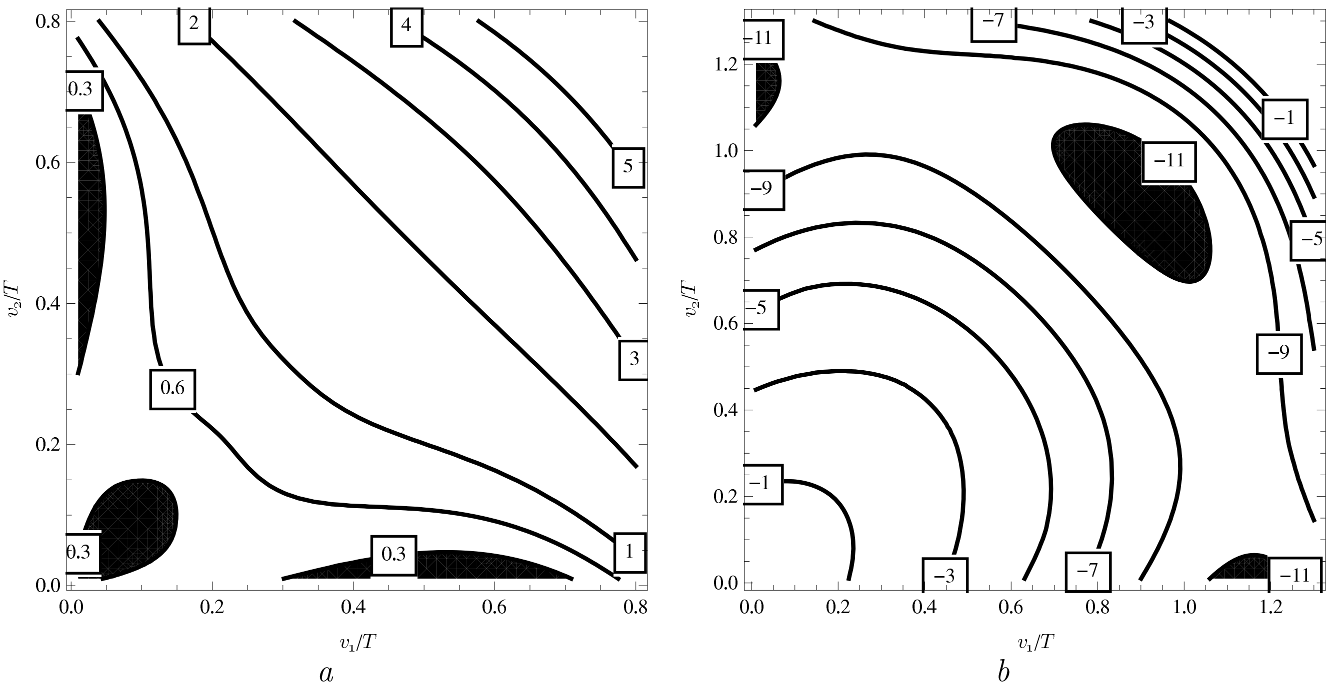

Fig. 5. Sequence of the first-order EPTs is realized. The critical temperatures are (a) $T_c = 192$ GeV; (b) $T_c = 154$ GeV

express all free parameters of the potential in terms of the tree-level masses of fields. The domain of the model parameter space considered in [9] corresponds to the condition for scalar field masses

$$m_H = m_{H^\pm} = m_{A_0}. \tag{33}$$

In [9], the tree-level masses of scalar fields were used as free parameters, instead of the couplings. However, the mass values do not define uniquely the values of couplings. Really, let us consider the h field mass case. The tree-level expression of m_h at the minimum point of the potential is

$$m_h^2 = 4\lambda_1 v_{01}^2 + 4\lambda_2 v_{02}^2 -$$
$$-4\sqrt{(\lambda_2 v_{02}^2 - \lambda_1 v_{01}^2)^2 + (v_{01} v_{02}(\lambda_3 + \lambda_5))^2}.$$

By applying (5), we obtain

$$m_h^2 = 4v_{01}^2\left(2\lambda_1 - |\lambda_3 + \lambda_5|\right). \tag{34}$$

Since $\lambda_3 < 0$ (see Appendix A2), there exist two possible values of λ_5 corresponding to the same value of m_h, namely

$$\lambda_5^\pm = -\lambda_3 \pm (2\lambda_1 - \frac{m_h^2}{4v_{01}^2}). \tag{35}$$

It appears that, at $\lambda_5 = \mathrm{Min}(\lambda_5^+, \lambda_5^-)$ for the parameter values resulting in (33), the symmetry is broken along the $\tan\beta = 1$ direction, as it was assumed. However, if λ_5 is taken to be $\mathrm{Max}(\lambda_5^+, \lambda_5^-)$, then the evolution of the system is completely different. The system undergoes the sequence of first-order phase transitions. The minima with broken symmetry appear along the $\tan\beta = 0$ and $\tan\beta = +\infty$ directions. After that, the Gibbs potential develops another minimum along the $\tan\beta = 1$ direction. This sequence is shown in Fig. 5. We can conclude that the assumption mentioned is not always true. These model parameters correspond to the scalar field mass values

$$m_h = 120 \text{ GeV}, \quad m_H = m_{H^\pm} = m_{A_0} = 250 \text{ GeV}.$$

Note also that, in this parametrization, three degenerate vacuum states may coexist.

4.3. Relation between model parameters

It can be seen from Appendix C that a small change of any parameter value may result in a significant change in the system's evolution (compare the scenarios shown in Figs. 2 and 3). So, it is interesting to determine any relation between model parameters which provides a large $\delta v/T_c$ value, and, hence, a strong first-order phase transition. Let us turn to this problem.

From geometric reasons, it is natural to assume that a jump of the order parameter is large enough if the non-trivial vacuum appears in the $0 < \tan\beta < +\infty$ direction. This happens if the symmetry is broken for both Higgs doublets simultaneously. The analytical form of this condition reads

$$\begin{cases} \mu_1^2(T) = 0, \\ \mu_2^2(T) = 0. \end{cases} \quad (36)$$

The values of couplings ensuring this condition can be found by using the high-temperature expansion of (21) and (22).

Since the quantum corrections are cumbersome in the THDM, we discuss the application of (36) considering a toy model as an example. Then we apply this relation to the THDM type II.

The toy model has to possess two main properties of the THDM – the spontaneous symmetry breaking and the mass mixing of scalar fields. The Lagrangian we use is

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi_1\partial^\mu\varphi_1 + \frac{1}{2}\partial_\mu\varphi_2\partial^\mu\varphi_2 - V,$$

$$V = -\frac{1}{2}(\mu_1^2\varphi_1^2 + \mu_2^2\varphi_2^2) + \lambda_1\varphi_1^4 + \lambda_2\varphi_2^4 + \lambda_3\varphi_1^2\varphi_2^2, \quad (37)$$

where φ_i are real scalar fields. The Lagrangian is invariant with respect to the transformation $\varphi_i \longrightarrow -\varphi_i$.

Let us shift φ_i by arbitrary values v_i and obtain the mass eigenstates as those in Eq. (3). We denote a new pair of fields as $h_{1,2}$. The tree-level VEVs v_{0i} are obtained from the relations

$$\begin{cases} -\mu_1^2 + 4\lambda_1 v_1^2 + 2\lambda_3 v_2^2 = 0, \\ -\mu_2^2 + 4\lambda_2 v_2^2 + 2\lambda_3 v_1^2 = 0. \end{cases}$$

The tree-level potential is

$$V_{\rm tree} = -\frac{1}{2}(\mu_1^2 v_1^2 + \mu_2^2 v_2^2) + \lambda_1 v_1^4 + \lambda_2 v_2^4 + \lambda_3 v_1^2 v_2^2. \quad (38)$$

The one-loop corrections to the Gibbs potential are given by Eqs. (14) and (21). We use the $\overline{MS}$ renormalization scheme. Then the one-loop vacuum corrections can be written as

$$V^1_{\rm vac}(m) = \frac{1}{64\pi^2} m^4 \ln\left(\frac{m^2}{\mu^2}\right).$$

The high-temperature expansion of (21) looks as follows:

$$V_T^{\rm b}(m) = -\frac{\pi^2 T^4}{90} + \frac{T^2 m^2}{24} - \frac{T m^3}{12\pi} -$$
$$-\frac{1}{64\pi^2} m^4 (\ln\frac{m^2}{T^2} - 5.41).$$

For the one-loop Gibbs potential, we have

$$V_G^{\rm toy} = V^1_{\rm vac}(m_{h_1}) + V^1_{\rm vac}(m_{h_2}) +$$

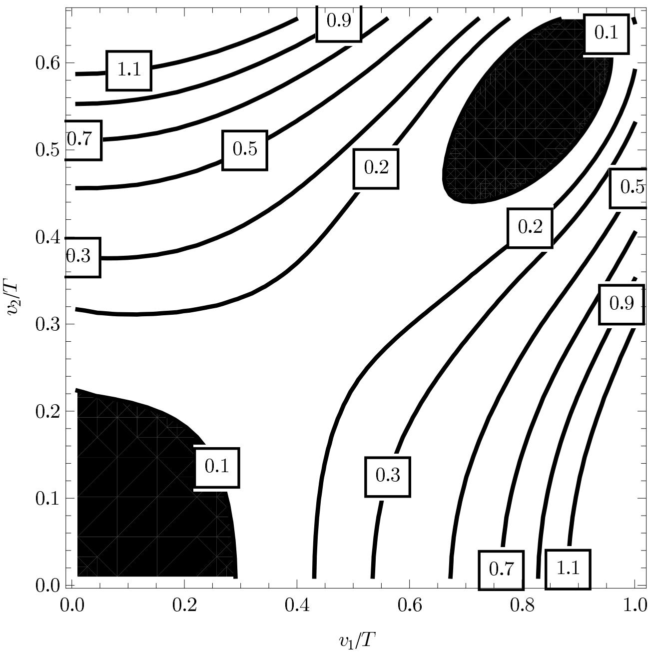

Fig. 6. The phase transition in the THDM type II with the optimal parameter values. The critical temperature is $T_c = 125.39$ GeV

$$+V_T^{\rm b}(m_{h_1}) + V_T^{\rm b}(m_{h_2}). \quad (39)$$

The functions $\mu_{1,2}(T)$ are the factors at $v_{1,2}^2$ in (39):

$$\mu_1^2(T) = \mu_1^2 + \frac{1}{64\pi^2}(12\lambda_2\mu_1^2 + 2\lambda_3\mu_2^2)\times$$
$$\times\left(\ln\frac{T^4}{\mu^4} + 10.82\right) - \frac{T^2}{12}(12\lambda_1 + 2\lambda_3),$$
$$\mu_2^2(T) = \mu_2^2 + \frac{1}{64\pi^2}(12\lambda_1\mu_2^2 + 2\lambda_3\mu_1^2)\times$$
$$\times\left(\ln\frac{T^4}{\mu^4} + 10.82\right) - \frac{T^2}{12}(12\lambda_2 + 2\lambda_3). \quad (40)$$

There are seven parameters in the toy model – $\lambda_{1,2,3}$, $\mu_{1,2}$, $v_{01,2}$. Four of them are free. Note that we took the condition $v_{01}^2 + v_{02}^2 = \text{const}$ into account. The way of using (36) is the following. We take arbitrary numerical values for any three parameters (for example, $\lambda_{1,2,3}$), and the value of the remaining parameter (v_{01}) is determined by solving the system (36).

The fulfillment of (36) is sufficient for the symmetry to be broken along the $0 < \tan\beta < +\infty$ direction.

4.4. THDM case

Let us apply the procedure described above to the THDM type II. In (2), there are nine parameters – $\lambda_{1,2,3,4,5}$, $\mu_{1,2}$, $v_{01,2}$. Six of them are free (we choose

$\lambda_{1,2,3,4,5}$ and v_{01}). We set the values of the couplings (listed in Appendix C), then determine $\mu_{1,2}(T)$, and solve (36) to obtain the v_{01} value for each set of the parameters. We will refer to the parameters obtained this way as the *optimal* parameters. The Gibbs potential with the optimal parameters taken is shown in Fig. 6. The $\lambda_{1,2,3,4,5}$ values are the same as those in the case shown in Fig. 2, and the jump of the order parameter is somewhat larger: $\delta v/T_c = 1.02$.

If the parameters are optimal, and the fields A_0, $H^{\pm}$ are heavy enough ($m_{H^{\pm}} > 80$ GeV, [22]), the strong first-order EPT is realized in the THDM. For small deviations from the optimal parameters, the third Sakharov condition is still fulfilled. As a result, we see that the baryogenesis condition is satisfied in a wide domain of the parameter space. Five of the six free parameters of the model can be set to the values which are constrained by modern experimental bounds and stability requirements for the tree-level potential. Then the remaining parameter can be calculated by using (36).

5. Conclusions

In the present paper, we have investigated the electroweak phase transition in the Two-Higgs Doublet Model on the base of the ring-improved one-loop Gibbs potential. We found that there is a wide domain in the parameter space of the model, for which the third Sakharov's baryogenesis condition is fulfilled. The values of the parameters entering the potential correspond to the scalar field masses that are compatible with modern experimental constraints. The parameter values for this domain can be found by using the introduced relation (36).

The EPT kind depends strongly on the tree-level parameter values. We have observed that the single phase transitions, as well as the sequences of transitions of the first and second orders, may happen.

We have concluded from our analysis that the restrictions $\mu_1^2 = \mu_2^2$, $\lambda_1 = \lambda_2$ do not ensure that the symmetry breaking is realized along the $\tan\beta = 1$ direction, as it was proposed in [8,9]. We have seen also that, in the model studied in [8,9], three degenerate vacua may coexist. Some of them can be realized as overcooled states.

Nowadays, there are few essential experimental constraints on the THDM parameter values: the lower bound on the mass of a charged scalar field in the MSSM and the bounds on $\tan\beta$ (see review in [22]). In this situation, the parameter values, for which the successful baryogenesis is possible, could be used as certain reference points in the study of a model extending the MSM.

APPENDIX A
Information on the scalar sector

In this appendix, we present the constraints on parameters of the scalar sector, the expressions for scalar field masses and the relations between parameters in different parametrizations.

1. *Tree-level potential properties*

Six of the nine scalar sector parameters are free due to the minimum conditions for the tree potential

$$-\mu_1^2 + 4\lambda_1 v_{01}^2 + 2(\lambda_3+\lambda_5)v_{02}^2 = 0,$$
$$-\mu_2^2 + 4\lambda_2 v_{02}^2 + 2(\lambda_3+\lambda_5)v_{01}^2 = 0, \quad \text{(A1)}$$

and the VEV v_0 is known:

$$v_{01}^2 + v_{02}^2 = v_0^2 = (246\,\text{GeV})^2.$$

For the stability of the tree-level vacuum, the potential value at large v_i values has to be positive. This translates into Sylvester's criterion for the quadratic form

$$\lambda_1 v_1^4 + \lambda_2 v_2^4 + (\lambda_3+\lambda_5)v_1^2 v_2^2.$$

Then the scalar field couplings are restricted to the conditions

$$\lambda_1 > 0, \quad \lambda_2 > 0, \quad 4\lambda_1\lambda_2 > (\lambda_3+\lambda_5)^2. \quad \text{(A2)}$$

In the present paper, we consider the case where the potential minimum is realized at $v_{01} \neq 0$, $v_{02} \neq 0$. This is because other cases $v_{01} \neq 0$, $v_{02} = 0$ or vice versa can be reduced to the MSM case with several additional fields, and no mass mixing is present.

2. *Scalar field masses*

From (3), we derive the expressions for the angles α, γ, and δ.

The α angle and the h, H masses are

$$\tan 2\alpha = \frac{A_3}{A_2 - A_1},$$
$$m_{h,\,H}^2 = A_1 + A_2 \mp \sqrt{(A_2-A_1)^2 + A_3^2},$$
$$A_1 = -\frac{1}{2}\mu_1^2 + 6\lambda_1 v_1^2 + (\lambda_3+\lambda_5)v_2^2,$$
$$A_2 = -\frac{1}{2}\mu_2^2 + 6\lambda_2 v_2^2 + (\lambda_3+\lambda_5)v_1^2,$$
$$A_3 = 4v_1v_2(\lambda_3+\lambda_5). \quad \text{(A3)}$$

The δ angle and the χ_3, A_0 masses read

$$\tan 2\delta = \frac{B_3}{B_2 - B_1},$$
$$m_{\chi_3,\,A_0}^2 = B_1 + B_2 \mp \sqrt{(B_2-B_1)^2 + B_3^2},$$
$$B_1 = -\frac{1}{2}\mu_1^2 + (2\lambda_1 + \xi(g^2+g'^2)/8)v_1^2 + (\lambda_4+\lambda_5)v_2^2,$$
$$B_2 = -\frac{1}{2}\mu_2^2 + (2\lambda_2 + \xi(g^2+g'^2)/8)v_2^2 + (\lambda_4+\lambda_5)v_1^2,$$
$$B_3 = 2v_1v_2(\lambda_3 - \lambda_4 + \xi(g^2+g'^2)/8). \quad \text{(A4)}$$

The γ angle and the $\chi^{\pm}$, $H^{\pm}$ masses are calculated to be

$$\tan 2\gamma = \frac{2C_3}{C_2 - C_1},$$

$$m^2_{\chi\pm,\,H^\pm} = C_1 + C_2 \mp \sqrt{(C_2 - C_1)^2 + 4C_3^2},$$
$$C_1 = -\frac{1}{2}\mu_1^2 + (2\lambda_1 + \xi g^2/8)v_1^2 + (\lambda_4 + \lambda_5)v_2^2,$$
$$C_2 = -\frac{1}{2}\mu_2^2 + (2\lambda_2 + \xi g^2/8)v_2^2 + (\lambda_4 + \lambda_5)v_1^2,$$
$$C_3 = v_1 v_2(\lambda_3 + \xi g^2/8). \quad \text{(A5)}$$

At $v_i = v_{0i}$, the A_0 and $H^\pm$ masses are gauge-invariant, and the Goldstone fields masses are $\sqrt{\xi}m_Z$, $\sqrt{\xi}m_W$. The masses of the h, H, A_0, and $H^\pm$ fields are

$$m_h^2 = 4\lambda_1 v_{01}^2 + 4\lambda_2 v_{02}^2 -$$
$$-4\sqrt{(\lambda_2 v_{02}^2 - \lambda_1 v_{01}^2)^2 + (v_{01}v_{02}(\lambda_3 + \lambda_5))^2},$$
$$m_H^2 = 4\lambda_1 v_{01}^2 + 4\lambda_2 v_{02}^2 +$$
$$+4\sqrt{(\lambda_2 v_{02}^2 - \lambda_1 v_{01}^2)^2 + (v_{01}v_{02}(\lambda_3 + \lambda_5))^2},$$
$$m_{A_0}^2 = 2(\lambda_4 - \lambda_3)(v_{01}^2 + v_{02}^2),$$
$$m_{H\pm}^2 = -2\lambda_3(v_{01}^2 + v_{02}^2). \quad \text{(A6)}$$

These expressions yield the following restriction on the scalar field couplings:

$$\lambda_3 < 0, \quad \lambda_4 - \lambda_3 > 0. \quad \text{(A7)}$$

These constraints ensure that the scalar fields A_0 and $H^\pm$ are physical ones.

3. Different parametrizations

The expression for the THDM potential used in [10] is

$$V = -\frac{1}{2}\left[m_{11}^2 x_1 + m_{22}^2 x_2 + m_{12}^2(x_3 + x_3^\dagger)\right] +$$
$$+\frac{\tilde\lambda_1 x_1^2 + \tilde\lambda_2 x_2^2}{2} + \tilde\lambda_3 x_1 x_2 + \tilde\lambda_4 x_3 x_3^\dagger + \frac{\tilde\lambda_5(x_3^2 + x_3^{\dagger 2})}{2}, \quad \text{(A8)}$$

where

$$x_1 = \varphi_1^\dagger\varphi_1, \quad x_2 = \varphi_2^\dagger\varphi_2, \quad x_3 = \varphi_1^\dagger\varphi_2, \quad \varphi_i = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_i \end{pmatrix},$$

and we consider the CP-conserving case only. By comparing this expression with (2) and (13), we obtain the relations between the parameter values in different parametrizations:

$$m_{11}^2 = 2\mu_1^2, \quad m_{22}^2 = 2\mu_2^2, \quad \tilde\lambda_1 = 8\lambda_1, \quad \tilde\lambda_2 = 8\lambda_2,$$
$$\tilde\lambda_3 = 4\lambda_5, \quad \tilde\lambda_4 = 2(\lambda_3 - \lambda_4), \quad \tilde\lambda_5 = 2(\lambda_3 + \lambda_4). \quad \text{(A9)}$$

Note that the parameter m_{12} is absent in the potential investigated in the present paper.

APPENDIX B
Debye masses

Finite-temperature corrections to the h and H masses are given by (27). The Debye masses of the $\chi^\pm$, $H^\pm$, χ_3, and A_0 fields are calculated to be

$$\delta m^2_{\chi^\pm} = T^2\left((2\lambda_1 + \frac{1}{2}G_t^2)\cos^2\gamma + (2\lambda_2 + \frac{1}{2}G_b^2)\sin^2\gamma + \right.$$
$$\left. + \frac{\lambda_3 + \lambda_4 + 4\lambda_5}{6} + \frac{3g^2 + g'^2}{16}\right),$$

$$\delta m^2_{H^\pm} = T^2\left((2\lambda_1 + \frac{1}{2}G_t^2)\sin^2\gamma + (2\lambda_2 + \frac{1}{2}G_b^2)\cos^2\gamma + \right.$$
$$\left. + \frac{\lambda_3 + \lambda_4 + 4\lambda_5}{6} + \frac{3g^2 + g'^2}{16}\right),$$
$$\delta m^2_{\chi_3} = T^2\left((2\lambda_1 + \frac{1}{2}G_t^2)\cos^2\delta + (2\lambda_2 + \frac{1}{2}G_b^2)\sin^2\delta + \right.$$
$$\left. + \frac{\lambda_3 + \lambda_4 + 4\lambda_5}{6} + \frac{3g^2 + g'^2}{16}\right),$$
$$\delta m^2_{A_0} = T^2\left((2\lambda_1 + \frac{1}{2}G_t^2)\sin^2\delta + (2\lambda_2 + \frac{1}{2}G_b^2)\cos^2\delta + \right.$$
$$\left. + \frac{\lambda_3 + \lambda_4 + 4\lambda_5}{6} + \frac{3g^2 + g'^2}{16}\right). \quad \text{(B1)}$$

For the THDM type I and for the doublet-universal parametrization, we have the following differences:

1. THDM type I. The G_b coupling is always small as compared to the G_t coupling and can be neglected.

2. The doublet-universal parametrization. In this case, G_b can be neglected. Also, one has to make substitutions into (B1) using the prescription

$$G_t^2 \sin^2\alpha, \gamma, \delta \to G_t^2(\sin\alpha, \gamma, \delta + \cos\alpha, \gamma, \delta)^2,$$
$$G_t^2 \cos^2\alpha, \gamma, \delta \to G_t^2(\sin\alpha, \gamma, \delta - \cos\alpha, \gamma, \delta)^2.$$

We present the contributions to the gauge boson Debye mass from different sectors of the model. The Z boson Debye mass is

$$\delta m_Z^2 = \delta m_Z^{s\,2} + \delta m_Z^{f\,2} + \delta m_Z^{g\,2},$$
$$\delta m_Z^{s\,2} = \frac{T^2}{3}\frac{g^4 + g'^4}{g^2 + g'^2}, \quad \delta m_Z^{f\,2} = \frac{T^2}{12}\frac{3g^4 + 29g'^4}{g^2 + g'^2},$$
$$\delta m_Z^{g\,2} = \frac{2}{3}T^2\frac{g^4}{g^2 + g'^2}.$$

For the W boson mass, we have

$$\delta m_W^2 = \delta m_W^{s\,2} + \delta m_W^{f\,2} + \delta m_W^{g\,2},$$
$$\delta m_W^{s\,2} = \frac{1}{3}g^2T^2, \quad \delta m_W^{f\,2} = g^2T^2, \quad \delta m_W^{g\,2} = \frac{2}{3}g^2T^2.$$

The s, f, and g superscripts denote the contributions coming from the scalar, fermion, and gauge boson sectors, respectively.

APPENDIX C
Parameter values for figures

In this appendix, we adduce the parameter values used in the calculations. The t and b mass values were taken 175 GeV and 4.2 GeV, respectively.

T a b l e 1. The parameter values for (2) used for plotting the Gibbs potential

fig.	λ_1	λ_2	λ_3	λ_4	λ_5	v_{01}/v_{02}
2	0.045	0.135	–0.27	0.675	0.27	1.91
3	0.045	0.135	–0.27	0.315	0.27	2
4	0.095	0.2375	–0.855	0.655	0.855	2.64
5	0.16	0.16	–0.52	0	0.72	1
6	0.045	0.135	–0.27	0.675	0.27	1.86

T a b l e 2. The parameter values resulting in a sequence of second-order phase transitions

λ_1	λ_2	λ_3	λ_4	λ_5	v_{01}/v_{02}
0.045	0.135	–0.27	0.675	0.27	1.22

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ПАРАМЕТРИЧНИЙ ПРОСТІР МОДЕЛІ З ДВОМА ХІГГС-ДУБЛЕТАМИ ТА УМОВИ САХАРОВА БАРІОГЕНЕЗУ

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Р е з ю м е

У роботі досліджено електрослабкий фазовий перехід у дводублетному розширенні Стандартної моделі елементарних частинок. Розраховано ефективний потенціал моделі при скінченній температурі в однопетельному наближенні з урахуванням внесків кільцевих діаграм. Показано, що жорсткий електрослабкий фазовий перехід першого роду, необхідний для виконання умови Сахарова баріогенезу, реалізується в дводублетній моделі при значеннях мас скалярних полів, які є сумісними із сучасними експериментальними обмеженнями. Виведено певне співвідношення між параметрами моделі, при виконанні якого фазовий перехід є переходом першого роду. Також показано, що в моделі можуть відбуватись каскадні фазові переходи. Проведено порівняння отриманих результатів з результатами інших авторів.