EFFECTIVE THREE-PHOTON VERTEX IN A DENSE FERMIONIC MEDIUM

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The tensor of an effective three-photon vertex has been calculated in the one-loop approximation for a medium with non-zero chemical potential. The tensor properties at various photon wavelengths and frequencies have been analyzed. The case of photon scattering by a magnetic field has been studied in detail. Possible applications of the results obtained have been discussed.

1. Introduction

Physical vacuum is an object of quantum-mechanical origin. A lot of effects resulting from just this origin have been theoretically calculated and experimentally measured more than fifty years ago. First of all, it is the Casimir effect [1], which, being a macroscopic manifestation of quantum-mechanical processes in vacuum, is not only an important correction to the theories of Universe's and elementary particle structures, but also has practical applications. Quantum-mechanical effects in vacuum are responsible for processes that violate the superposition principle; in particular, these are light-by-light scattering, Delbrück scattering, and others [2, 3].

Interaction of an electromagnetic field through vacuum is described by means of Feynman diagrams with an even number of photon lines, the so-called two-photon vertices. The existence of unpaired vertices in vacuum is forbidden by the Furry theorem [4]. An introduction of even small number of particles into vacuum makes only quantitative, but not qualitative corrections to the process of photon–photon interaction. Such a situation takes place, as far as the chemical potential in the system is less than the particle rest energy, i.e. the medium formed by the particles can be considered rarefied.

However, the description of the behavior of photons and an electromagnetic field in a dense fermionic medium requires that Feynman diagrams with an odd number of external photon lines should be included as well. It is so, because, provided that the medium is dense, the C-parity becomes broken: and, in this connection, the conditions for the Furry theorem are not satisfied. As a result, unpaired photon vertices may exist. In the absence of dense medium, the vacuum polarization phenomenon enables the existence of only paired photon vertices. Therefore, the physical phenomena, which the vertices of indicated type are responsible for, have been studied much more thoroughly than those that arise in dense media. However, for the description of fields in a dense medium to be correct, the application of, e.g., the tensor of a three-photon vertex is necessary in principle, because it dominates as $\mu \to \infty$ and allows new, quite specific physical processes to run. In particular, these are the decay of static fields into free photons, spontaneous generation of a magnetic field from an electric one, and direct interaction between free photons and electric and magnetic fields.

In works [5–7], the tensor of a three-photon vertex function was studied in the static-field approximation. The main results of those researches consist, first of all, in the substantiation of the essential non-conservation of vertex transversality, provided that the Feynman parametrization is used. The obtained characteristic features in the behavior of electromagnetic fields in the presence of a dense medium are also of importance. However, in this approximation, all effects stemming from the existence of a nonzero three-photon vertex can be observed only immediately from inside the medium itself. Similar processes play an appreciable role as a factor of evolution of large formations originating from dense fermionic media (white dwarfs and, especially, neutron stars). However, it is a very difficult task to observe the manifestations of the nonlinear behavior of an electromagnetic field as a result of quantum-mechanical effects in the medium in the static case in accessible experiments, because those manifestations are weak and indirect.

In this work, the three-photon vertex in a dense medium is studied. As a dense medium, a set of fermions is meant, for which the chemical potential μ is higher than the rest energy m. In particular, they may be electrons at moderate densities, baryons, and even quarks in the deconfinement phase. Not only static fields are studied, but also slowly changing ones, for which the inequality $\frac{k_4}{\sqrt{\mu^2-m^2}} < 1$, where k_4 is the photon frequency, is considered to be satisfied. This approximation is sufficient for many applications, because the high-frequency fields do not "feel" the medium, and the corresponding vertices are suppressed in the case of large k_4's. The attractiveness of this approximation as a research tool consists in that it describes the processes forbidden in the absence of a medium. The results of this work can be applied in the NICA and FAIR projects (where collisions between heavy nuclei are studied) to reliably and directly detect the very fact of the formation of a dense fermionic matter and to study the matter properties.

Here, we calculate and analyze the properties of the tensor corresponding to a three-photon vertex and reduce it to the cases of zero and low values of the photon frequency k_4. For separate static-field cases, we present exact expressions for the corresponding nonzero tensor components, as well as a simplified linear approximation for the case of low k_4. In addition, we analyze a special case, namely, the interaction between a classical magnetic field and free photons. Hence, a basis for a wide spectrum of possible researches has been developed.

The work consists of Introduction and four sections. In Section 2, the explicit form of the tensor of a three-photon vertex in a dense medium is presented for the static-field case. In Section 3, the tensor is calculated in the long-wave approximation. Section 4 is devoted to the study of a special case, the interaction between photons and a magnetic field. Section 5 contains the discussion of the results obtained and conclusions.

2. Three-photon Vertex in the Static-field Case

In this section, we aim at calculating the nonzero components of the three-photon vertex tensor in a medium; they exist in the one-loop approximation for the state of rest at zero temperature. In a (3 + 1)-dimensional space, this case was examined in more details in work [6]. The analytical expression for a three-photon vertex is expressed by the integral

$$\Pi_{\mu\nu\gamma}(k^{(1)},k^{(2)},k^{(3)}) = \delta(k^{(1)}+k^{(2)}+k^{(3)})\frac{e^3}{(2\pi)^3}\times$$

$$\times\mathrm{Tr}\int d^4p(\gamma_\mu G(p+k^{(1)})\gamma_\nu G(p-k^2)\gamma_\gamma G(p)+$$

$$+\gamma_\mu G(p)\gamma_\nu G(p+k^2)\gamma_\gamma G(p-k^{(1)})), \tag{1}$$

where μ, ν, and γ are indices running from one to four; γ_μ are the Dirac matrices; and $G(p)$ is the Green's function of a fermion that forms a vertex loop (usually, it is an electron),

$$G(p) = \frac{-i\hat{p}+m}{p^2+m^2}. \tag{2}$$

The p-momentum components for the Green's function are presented in the Euclidean coordinates, so that

$$p = \begin{cases} p_\rho, \rho = 1,2,3, \\ p_0 + i\mu, \rho = 0. \end{cases} \tag{3}$$

The single internal parameter is μ, the chemical potential of a medium; in the case of cold fermionic plasma, it depends only on the plasma density.

If $k_4 = 0$, by calculating integral (1), we obtain exact expressions for static fields. In this case, the form of the vertex function becomes considerably simpler, with only the following tensor components remaining nonzero [6]:

$$\Pi_{444} = \frac{ie^3}{2\pi^3}\left(4\sum_{i=1}^{3} J_2(k^{(i)}\right)+$$

$$+\sum_{n=1,n\neq l}^{3}\sum_{l=1}^{3} J_1(k^{(l)},k^{(n)})((k^{(l)})^2+(k^{(n)})^2+$$

$$+(k^{(l)},k^{(n)})-4m^2)-4J_3(k^{(l)},k^{(n)})), \tag{4}$$

$$\Pi_{ij4} = \frac{ie^3}{2\pi^3}\left(\sum_{n=1,n\neq l}^{3}\sum_{l=1}^{3} J_1(k^{(l)},k^{(n)})\times\right.$$

$$\times(k_j^{(1)}k_i^{(3)}-(k^{(1)}k^{(3)})\delta_i j)-4J_3(k^{(2)})\times$$

$$\times\left(\delta_{ij}+(k_j^{(1)}k_i^{(3)})\frac{(k^{(1)}k^{(3)})-(k^{(1)})^2-(k^{(3)})^2}{(k^{(1)}k^{(3)})^2}\right)\Bigg). \quad (5)$$

Here, J_1, J_2, and J_3 are the functions of the external momenta $k^{(1)}$, $k^{(2)}$, and $k^{(3)}$, the chemical potential μ, and the mass m; they are presented in Appendix. The analytical integration for the functions J_1, J_2, and J_3 to a final result can be fulfilled only in the case where all the momenta are collinear. Otherwise, asymptotic approximations can be derived for J_1 and J_3. The symmetric components Π_{i4j} and Π_{4ij} are obtained by making the corresponding substitutions $k^{(1)}\to k^{(2)}$, $k^{(2)}\to k^{(3)}$, and $k^{(3)}\to k^{(1)}$ for the former, and $k^{(1)}\to k^{(3)}$, $k^{(3)}\to k^{(2)}$, and $k^{(2)}\to k^{(1)}$ for the latter.

Passing in formulas (4) and (5) to small momenta ($\frac{k^{(1)}}{a}\ll 1$ and $\frac{k^{(2)}}{a}\ll 1$), where $a=\sqrt{\mu^2-m^2}$, we obtain the following asymptotic behavior for tensor components in the symmetric case $\frac{k^{(1)}}{k^{(2)}}\to 1$:

$$\Pi_{444}=\frac{e^3}{\pi^3}\theta(\mu^2-m^2)\sqrt{\mu^2-m^2}(6+$$

$$+\sum_{n>1}^{3}\sum_{l=1}^{2}\frac{\beta_{in}-\pi q_{in}}{\sin\beta_{in}}((2(1+\cos\beta_{12}))^{\frac{2-n}{2}}+$$

$$+(3-n)(1+\cos\beta_{12})-\cos\beta_{in}))+O\left(\frac{k^{(1)}}{a}\right), \quad (6)$$

$$\Pi_{ij4}=\frac{e^3}{\pi^3}\theta(\mu^2-m^2)\sqrt{\mu^2-m^2}(\delta_{ij}+$$

$$+\cos\alpha_{3j}\cos\alpha_{1i}\cos\beta_{13}-\cos\alpha_{1i}\cos\alpha_{1j}-$$

$$-\cos\alpha_{3i}\cos\alpha_{3j})+O\left(\frac{k^{(1)}}{a}\right), \quad (7)$$

where β_{in} is an angle between the vectors k^l and k^n, $\cos\alpha_{1i}$ is the direction cosine of the vector k^1, q_{in} are arbitrary integers that satisfy the equality $q_{12}+q_{13}+q_{23}=2$, and $\theta(\mu^2-m^2)$ is the Heaviside function.

For large momenta, $\left(\frac{k^{(1)}}{a},\frac{k^{(2)}}{a}\right)\gg 1$, and in the symmetric limit, formulas (4) and (5) give the following result for tensor components:

$$\Pi_{444}=O\left(\frac{k^{(1)}}{a}\right), \quad (8)$$

$$\Pi_{ij4}=O\left(\frac{k^{(1)}}{a}\right). \quad (9)$$

It testifies that, in the high-momentum and, accordingly, short-distance approximation, the medium becomes asymptotically transparent, and its influence on the field properties is insignificant. This takes place, because the vertex function is proportional to the chemical potential, the latter being a small parameter in the case of high momenta. However, for linear scales $r\geq(\mu^2-m^2)^{-\frac{1}{2}}$, the substitution of $\Pi_{\mu\nu\gamma}$ by its asymptotic value does not violate the adequacy of description for the properties of an electromagnetic field with an arbitrary strength distribution. Under such conditions, the quantity $\sqrt{\mu^2-m^2}/k$ is a large parameter, and there is no necessity to use the “adiabatic expansion”.

3. Three-photon Vertex in the Case of Low-frequency Photons

The expressions obtained for the components of the three-photon vertex tensor in the static case allow the effect of a medium on the electromagnetic field to be considered qualitatively and a few asymptotic approximations to be derived. In particular, the latter include the phenomenon of asymptotic transparency of the medium for short-wave photons and the inverse, “smearing” influence of large gradients at the description of the medium action on the static field. However, for the description of the medium effect on the interaction of free photons to be more detailed, the dynamic components of the tensor must be taken into consideration as well.

After calculating the trace, the vertex function looks like

$$\Pi_{\mu\nu\gamma}=\frac{ie^3}{(2\pi)^3\beta}\int d^3p\,dp_4\times$$

$$\times\left(\frac{f_{\mu\nu\gamma}}{[(p+k^{(1)})^2+m^2][(p-k^{(2)})^2+m^2][p^2+m^2]}+\right.$$

$$\left.+\frac{\widetilde{f}_{\mu\nu\gamma}}{[(p-k^{(1)})^2+m^2][(p+k^{(2)})^2+m^2][p^2+m^2]}\right), \quad (10)$$

where the function $f_{\mu\nu\gamma}$ depends on the variables $k^{(1)}$, $k^{(2)}$, p, and m. Let us write it down in the following form:

$$f_{\mu\nu\gamma}(k^{(1)},k^{(2)},p,m)=$$

$$=\Big\{[p_\nu+k_\nu^{(1)}][(p_\gamma-k_\gamma^{(2)})p_\mu-(p_\mu-k_\mu^{(2)})p_\gamma]+$$

$$+[p_\gamma + k_\gamma^{(1)}][(p_\nu - k_\nu^{(2)})p_\mu - (p_\mu - k_\mu^{(2)})p_\nu]+$$

$$+[p_\mu + k_\mu^{(1)}][(p_\gamma - k_\gamma^{(2)})p_\nu - (p_\nu - k_\nu^{(2)})p_\gamma]+$$

$$+[(\mathbf{p}+\mathbf{k}^{(1)})(\mathbf{p}-\mathbf{k}^{(2)}) + m^2]\times$$

$$\times[p_\gamma\delta_{\mu\nu} - p_\mu\delta_{\nu\gamma} + p_\nu\delta_{\mu\gamma}]-$$

$$-[(\mathbf{p}+\mathbf{k}^{(1)})\mathbf{p} + m^2][(p_\gamma - k_\gamma^{(2)})\delta_{\mu\nu}-$$

$$-(p_\mu - k_\mu^{(2)})\delta_{\nu\gamma} + (p_\nu - k_\nu^{(2)})\delta_{\mu\gamma}]-$$

$$-[\mathbf{p}(\mathbf{p}-\mathbf{k}^{(2)}) + m^2][(p_\gamma - k_\gamma^{(1)})\delta_{\mu\nu}-$$

$$-(p_\mu - k_\mu^{(1)})\delta_{\nu\gamma} + (p_\nu - k_\nu^{(1)})\delta_{\mu\gamma}]\Big\}, \qquad (11)$$

where $\mathbf{pk}^{(n)}$ is the scalar product of only spatial parts of these vectors. Another function, $\tilde{f}_{\mu\nu\gamma}$, is the complex conjugate of $f_{\mu\nu\gamma}$. For the first and second terms, the poles differ in only the sign of the chemical potential μ; i.e. the poles are $z_1^{\pm} = \pm i\varepsilon_0 - i\mu$, $z_2^{\pm} = \pm i\varepsilon_1 - i\mu - k_4^{(1)}$, and $z_3^{\pm} = \pm i\varepsilon_2 - i\mu + k_4^{(2)}$ for the former and $z_1^{\pm} = \pm i\varepsilon_0 + i\mu$, $z_2^{\pm} = \pm i\varepsilon_1 + i\mu - k_4^{(1)}$, and $z_3^{\pm} = \pm i\varepsilon_2 + i\mu + k_4^{(2)}$ for the latter. The quantities ε_i are the fermion energy with respect to each vertex photon, namely, $\varepsilon_0 = (p^2+m^2)^{1/2}$, $\varepsilon_1 = ((p+k^{(1)})^2+m^2)^{1/2}$, and $\varepsilon_2 = ((p-k^{(2)})^2+m^2)^{1/2}$.

Applying the residue method for the integration over dp_4 in formula (10), we obtain an expression consisting of three terms. Each term corresponds to a specific pole of the integrand in expression (10); it is composed of a scalar coefficient H_n, where n is the pole number, and a certain vector structure. In the case of low-frequency photons and, hence, low k_4-values, the determination of multipliers H_n demands that the linear approximation be used and the obtained expressions be expanded into power series of k_4. After carrying out the corresponding calculations, those multipliers can be expressed as follows:

$$H_1 \approx \frac{n_e(\varepsilon_0) - n_p(\varepsilon_0)}{2i\varepsilon_0[2\mathbf{pk}^{(1)} + (\mathbf{k}^{(1)})^2][-2\mathbf{pk}^{(2)} + (\mathbf{k}^{(2)})^2]};$$

$$H_2 \approx (n_e(\varepsilon_1) - n_p(\varepsilon_1))(2i\varepsilon_1[2\mathbf{pk}^{(1)} - (\mathbf{k}^{(1)})^2]\times$$

$$\times[-2\mathbf{p}(\mathbf{k}^{(2)} + \mathbf{k}^{(1)}) + (\mathbf{k}^{(2)})^2 - (\mathbf{k}^{(1)})^2])^{-1};$$

$$H_3 \approx (n_e(\varepsilon_2) - n_p(\varepsilon_2))(2i\varepsilon_2[2\mathbf{pk}^{(2)} - (\mathbf{k}^{(2)})^2]\times$$

$$\times(-2\mathbf{p}(\mathbf{k}^{(2)} + \mathbf{k}^{(1)}) + (\mathbf{k}^{(1)})^2 - (\mathbf{k}^{(2)})^2))^{-1}, \qquad (12)$$

where $n_e(\varepsilon_n)$ and $n_p(\varepsilon_n)$ are the electron and positron, respectively, density functions, which look like $n_e(\varepsilon_n) = [1 + \exp(\beta(\varepsilon_n - \mu)]^{-1}$, $n_p(\varepsilon_n) = [1 + \exp(\beta(\varepsilon_n + \mu)]^{-1}$.

The H_n-multipliers generate three groups of terms for each vertex function component, each of the latter including k_4 raised to a certain power ranging from 0 to 4. We adopt that the "temporal" momentum component is a small parameter; therefore, terms with k_4 raised to a power larger than two can be neglected. At the same time, the terms without k_4 comprise a static part of the components, which was calculated in work [6]. Hence, we can take into account the contribution made by the expression for static fields. Then, all we need is to calculate those terms, which depend linearly on k_4:

$$F_{444} \approx F_{444}^{\text{stat}} + \text{Re}\{H_1[\mathbf{pk}^{(2)}k_4^{(1)} + (\mathbf{pk}^{(1)} - \varepsilon_0^2)k_4^{(2)}]+$$

$$+H_2[(\varepsilon_0^2 + \mathbf{pk}^{(1)} + \varepsilon_1^2)k_4^{(2)}+$$

$$+(2\varepsilon_0^2 + \mathbf{pk}^{(2)} - \mathbf{k}^{(1)}\mathbf{k}^{(2)} + 2\varepsilon_1^2)k_4^{(1)}]-$$

$$-H_3[(\varepsilon_0^2 - \mathbf{pk}^{(2)} + \varepsilon_2^2)k_4^{(1)}+$$

$$+(2\varepsilon_0^2 - \mathbf{pk}^{(1)} - \mathbf{k}^{(1)}\mathbf{k}^{(2)} + 2\varepsilon_2^2)k_4^{(2)}]\}. \qquad (13)$$

The function F is coupled with Π by the relation

$$\Pi_{\mu\nu\gamma} = \frac{ie^3}{(2\pi)^3\beta}\int d^3p F_{\mu\nu\gamma}.$$

While calculating F_{ij4}, we should take into consideration that $i \neq 4$ and $j \neq 4$, so that $\delta_{i4} = \delta_{j4} = 0$. Moreover, we use the approximation, for which $k_4^{(a)}k_4^{(b)} = 0$. Therefore, we find

$$F_{ij4} \approx F_{ij4}^{\text{stat}} + \text{Re}\{\varepsilon_0\delta_{ij}(H_2\varepsilon_1 k_4^{(1)} + H_3\varepsilon_2 k_4^{(2)})+$$

$$+H_1[(\mathbf{pk}^{(2)}\delta_{ij} + 2p_ip_j - p_ik_j^{(2)} - p_jk_i^{(2)})k_4^{(1)}+$$

$$+(\mathbf{pk}^{(1)}\delta_{ij} + 2p_ip_j - p_ik_j^{(1)} - p_jk_i^{(1)})k_4^{(2)}]+$$

$$+H_2[((\mathbf{p}(2\mathbf{k}^{(1)} + \mathbf{k}^{(2)}) + (k^{(1)}))^2\delta_{ij}+$$

$$+2p_ip_j + p_ik_j^{(2)} + p_jk_i^{(2)})k_4^{(1)} - ((\mathbf{p}\ \mathbf{k}^{(1)}+$$

$$+(\mathbf{k}^{(1)})^2\delta_{ij} + 2p_ip_j + p_ik_j^{(1)} + p_jk_i^{(1)})k_4^{(2)}]-$$

$$-H_3[((\mathbf{p}(2\mathbf{k}^{(2)} + \mathbf{k}^{(1)}) - (\mathbf{k}^{(2)}))^2\delta_{ij}-$$

$$-2p_ip_j + p_ik_j^{(1)} + p_jk_i^{(1)})k_4^{(2)} + ((-\mathbf{p}\ \mathbf{k}^{(2)}+$$

$$+(\mathbf{k}^{(2)})^2\delta_{ij} + 2p_ip_j - p_ik_j^{(2)} - p_jk_i^{(2)})k_4^{(1)}]\}. \qquad (14)$$

In a similar way, we make transformations for other tensor elements. Taking into account that $F_{i44}^{\rm stat} = F_{ijl}^{\rm stat} = 0$ in the static case, we obtain

$$F_{ijl} \approx$$

$$\approx \mathrm{Im}\left[H_1\varepsilon_0\left\{\begin{matrix}\left\{\begin{matrix}(k_l^{(2)}\delta_{ij}+k_j^{(2)}\delta_{il}- \\ -k_i^{(2)}\delta_{jl}-2p_j\delta_{il})\end{matrix}\right\}k_4^{(1)}+ \\ +\left\{\begin{matrix}(k_l^{(1)}\delta_{ij}-k_j^{(1)}\delta_{il}+ \\ +k_i^{(1)}\delta_{jl}+2p_i\delta_{jl})\end{matrix}\right\}k_4^{(2)}\end{matrix}\right\}+\right.$$

$$+H_2\varepsilon_1\left\{\begin{matrix}\left\{\begin{matrix}(k_i^{(2)}\delta_{jl}-k_l^{(2)}\delta_{ij}-k_j^{(2)}\delta_{il}+(k_i^{(1)}+2p_i)\times \\ \times\delta_{jl}+(k_l^{(1)}+2p_l)\delta_{ij}-(k_j^{(1)}+2p_j)\delta_{il})\end{matrix}\right\}\times \\ \times k_4^{(1)} + \left\{\begin{matrix}(k_l^{(1)}\delta_{ij}-k_j^{(1)}\delta_{il}+ \\ +k_i^{(1)}\delta_{jl}+2p_i\delta_{jl})\end{matrix}\right\}k_4^{(2)}\end{matrix}\right\}+$$

$$\left.+H_3\varepsilon_2\left\{\begin{matrix}\left\{\begin{matrix}(k_j^{(1)}\delta_{il}-k_l^{(1)}\delta_{ij}-k_i^{(1)}\delta_{jl}+(k_j^{(1)}-2p_j)\times \\ \times\delta_{il}+(k_l^{(1)}-2p_l)\delta_{ij}-(k_i^{(1)}-2p_i)\delta_{jl})\end{matrix}\right\}\times \\ \times k_4^{(2)} + \left\{\begin{matrix}(k_l^{(2)}\delta_{ij}+k_j^{(2)}\delta_{il}- \\ k_i^{(2)}\delta_{jl}-2p_i\delta_{jl})\end{matrix}\right\}k_4^{(1)}\end{matrix}\right\}\right], \qquad (15)$$

$$F_{i44} \approx$$

$$\approx \mathrm{Im}[H_2\varepsilon_1[((k_i^{(1)} + 2p_i) - k_i^{(2)})k_4^{(1)}+$$

$$+(k_i^{(1)} + 2p_i)k_4^{(2)}]+$$

$$+H_3\varepsilon_2[((k_i^{(2)} - 2p_i) - k_i^{(1)})k_4^{(2)}+$$

$$+(k_i^{(2)} - 2p_i)k_4^{(1)}] - \varepsilon_0(H_2 + H_3)$$

$$[(k_i^{(2)} - 2p_i)k_4^{(1)} + (k_i^{(1)} + 2p_i)k_4^{(2)}]]. \qquad (16)$$

The expressions derived for all tensor elements in the long-wave approximation compose a system that describes such photon–photon processes as the photon decay into two photons, the decay of an electric field into free photons, the generation of a magnetic field in the presence of the electric one, and the interaction between a free photon and a magnetic field. The expressions found for the tensor components testify that the processes of creation of real photons take place only in alternating fields, because the corresponding tensor components include only dynamic parts. Unlike the static case, dynamic processes can be observed "from outside" the medium. Therefore, they are suitable for studying the microscopic volumes in a dense medium.

4. Interaction Between Photons and a Magnetic Field in a Medium

On the basis of the results obtained, let us consider a case of the interaction between photons and a classical magnetic field in the presence of fermionic plasma. In this case, we have a state of the external field without energy transfer and with a definite fixed spatial momentum, which can be designated as $(k_l^{(1)} + k_l^{(2)})$, where $l \in 1, 3$. Then, the element of the scattering cross-section looks like

$$S = F_{ijl}k_i^{(1)}k_j^{(2)}(k_l^{(1)} + k_l^{(2)}) + F_{44l}k_4^{(1)}k_4^{(2)}(k_l^{(1)} + k_l^{(2)})+$$

$$+F_{i4l}k_i^{(1)}k_4^{(2)}(k_l^{(1)} + k_l^{(2)})F_{4jl}k_4^{(1)}k_j^{(2)}(k_l^{(1)} + k_l^{(2)}). \qquad (17)$$

According to the assumption that k_4 is small, the term $F_{44l}k_4^{(1)}k_4^{(2)}(k_l^{(1)} + k_l^{(2)})$ approximately equals zero, whereas the components with k_4 in the terms $F_{i4l}k_i^{(1)}k_4^{(2)}(k_l^{(1)} + k_l^{(2)})$ and $F_{4jl}k_4^{(1)}k_j^{(2)}(k_l^{(1)} + k_l^{(2)})$ – all of them are dynamic contributions – are also reduced. Moreover, the field photons have no energy component. However, since they are written down as $(k_l^{(1)} + k_l^{(2)})$, the equality $k_4^{(1)} = -k_4^{(2)}$ must be satisfied. In other words, it is a process of free photon scattering by a magnetic field. Then, the nonzero tensor elements look like

$$F_{i4l} = F_{i4l}^{\rm stat}, \quad F_{4jl} = F_{4jl}^{\rm stat}, \qquad (18)$$

$$F_{ijl} \approx \mathrm{Im}k_4^{(1)}\times$$

$$\times\left\{H_1\varepsilon_0\left\{\begin{matrix}(k_l^{(2)} - k_l^{(1)})\delta_{ij} + (k_j^{(2)} + k_j^{(1)})\delta_{il}- \\ (k_i^{(2)} + k_i^{(1)})\delta_{jl} - 2p_i\delta_{jl} - 2p_j\delta_{il}\end{matrix}\right\}+\right.$$

$$+H_2\varepsilon_1[k_i^{(2)}\delta_{jl}-k_l^{(2)}\delta_{ij}-k_j^{(2)}\delta_{il}+p_l\delta_{ij}-p_j\delta_{il}]+$$

$$\left.+H_3\varepsilon_2[k_j^{(1)}\delta_{il}-k_l^{(1)}\delta_{ij}-k_i^{(1)}\delta_{jl}-p_l\delta_{ij}+p_i\delta_{jl}]\right\}. \quad (19)$$

One can see that all dynamics of the process is contained in the element F_{ijl}, whereas the other nonzero tensor components describe only the statics. In addition, since we consider the process of free photon scattering by a magnetic field, the dispersion relations must be satisfied. Therefore, $\mathbf{k}^{(1)}=\mathbf{k}^{(2)}=k$, because $|k_4^{(1)}|=|k_4^{(2)}|$.

We can take advantage of this property by multiplying the tensor by the corresponding vectors k. Then, we immediately obtain an expression for the scattering cross-section; it contains only scalar products $(\mathbf{k}^{(1)},\mathbf{k}^{(2)})$ that can be equalized to each other. The final expression for the effective scattering cross-section has the form

$$k_i^{(1)}k_j^{(2)}F_{ijl}(k_l^{(2)}+k_l^{(1)})\approx-\mathrm{Im}k_4^{(1)}(\mathbf{k})^2\times$$

$$\times\left\{2(1+\cos\gamma)(\cos\alpha+\cos\beta)\times\right.$$

$$\times\frac{n_e(\varepsilon_0)-n_p(\varepsilon_0)}{2i[2\mathbf{p}\cos\alpha+\mathbf{k}][-2\mathbf{p}\cos\beta+\mathbf{k}]}\mathbf{p}+$$

$$+(\mathbf{k}(1+\cos\gamma)+\mathbf{p}(\cos\alpha+\cos\beta)\times$$

$$\times\left(\frac{n_e(\varepsilon_1)-n_p(\varepsilon_1)}{2i[2\mathbf{p}\cos\alpha-\mathbf{k}][-2\mathbf{p}(\cos\beta+\cos\alpha)]}+\right.$$

$$\left.\left.+\frac{n_e(\varepsilon_2)-n_p(\varepsilon_2)}{2i[2\mathbf{p}\cos\beta-\mathbf{k}][-2\mathbf{p}(\cos\beta+\cos\alpha)]}\right)\right\}, \quad (20)$$

where γ is the angle between the vectors $k^{(1)}$ and $k^{(2)}$, and α and β are the angles between the vector p and the corresponding k. As follows from the general form of expression (20), the cross-section of photon scattering by a magnetic field is proportional to the squared absolute value of the photon wave vector. The behavior of this dependence can be used, while studying this scattering process experimentally. Notice that, for instance, in the case of an external magnetic field without a medium, the scattering cross-section is proportional to k^4 [9, 10].

5. Discussion of Results

Thus, we have obtained the explicit expressions for the components of the three-photon vertex tensor in two approximations, the static-field and low-frequency ones, and a special case of the interaction between free photons and a magnetic field is analyzed. The static-case approximation demonstrates the essence of why the nonlinear behavior of fields emerges in a dense medium. At the same time, it forms a necessary basis for the description of various processes considered above. The low-frequency approximation describes a large number of nonlinear photon–photon interactions and allows dynamic processes to be studied. We consider the photon scattering by a magnetic field as the most interesting case. Therefore, it was examined in more details. As a result, we derived an exact expression for the scattering cross-section. The obtained dependence for the cross-section is a quadratic function of the absolute value of photon momentum. It can be used to experimentally check the presence (formation) of a dense medium.

The phenomena arising in media with broken C-parity are of great interest, first of all, because they are an experimental confirmation of theoretical principles used in quantum electrodynamics. However, physical phenomena of such an origin can also find practical applications even today, mainly, as a detector of the presence of dense fermionic substances and a tool of their research. Using the asymptotic relations that describe processes in the medium, the internal characteristics of such media—first of all, the chemical potential—can be measured.

The processes in static fields, which were described in works [5, 6], can be a powerful factor in the evolution of formations consisting of superdense matter, such as white dwarfs and neutron stars. The static case is, first of all, a phenomenon of the decay of an electrostatic field into two quasistatic photons of a magnetic field. As a consequence, it gives rise to a spontaneous generation of magnetic fields in a fermionic plasma embedded into an electric field, as well as to the inverse phenomenon, namely, the generation of a solenoidal electric field from a magnetic one. Such an event is rather probable for pulsars with their superstrong magnetic field. However, the cited works did not aim at describing the processes that allow one to study the properties of fermionic media from outside.

The low-frequency approximation used in this work reveals new opportunities for taking the influence of a medium on the interaction between fields into consideration. This case can be reduced to a number of probable processes. In our opinion, the process of free photon scattering by a magnetic field is the most interesting. The scattering parameters depend only on the field characteristics and the chemical potential μ of a medium. Theoretically, this process allows one to measure μ by detecting scattered photons. The phenomenon of such a type can find application in the NICA and FAIR projects (where collisions between heavy nuclei are studied) both as a tool for the reliable detection of the very fact of the generation of a dense fermionic matter and to study the properties of this matter.

In this work, the process of photon scattering has been considered in the case of a static external magnetic field. However, the derived expressions can be applied with a high accuracy to real detector systems as well. The matter is that intense fields are generated following the mechanisms like a magnetic explosion generator or a high-current coil. Such devices create fields with the frequencies ranging from 10 to 200 kHz. These frequencies are negligibly low even in comparison with the frequencies of infra-red photons. Therefore, the external field can be adopted as approximately static in this case.

APPENDIX

Here, the explicit forms for the functions of external photon momenta in the case of static fields [6] are given:

$$J_1(k^{(1)},k^{(2)}) = \frac{i\pi^2\theta(\mu^2-m^2)}{k^{(1)}k^{(2)}}\left[\int\limits_0^{\varepsilon} T dp \ln\left|\frac{M(p,k^{(1)},k^{(2)})}{M(-p,k^{(1)},k^{(2)})}\right| - \right.$$
$$-\theta\left(\sqrt{\mu^2-m^2}-\frac{k^{(3)}}{2\sin\beta}\right)\int\limits_{\frac{k^{(3)}}{2\sin\beta}}^{\sqrt{\mu^2-m^2}} -iTdp\Big(\arcsin N(p,k^{(1)},k^{(2)}) -$$
$$\left.-\arcsin N(-p,k^{(1)},k^{(2)})\Big)\right], \tag{21}$$

$$J_2(k^{(1)},k^{(2)}) = \frac{i\pi^2\theta(\mu^2-m^2)}{k^{(1)}k^{(2)}}\sqrt{\mu^2-m^2}\times$$
$$\times\left[\ln\left|\frac{k^{(1)}-2\sqrt{\mu^2-m^2}}{k^{(1)}+2\sqrt{\mu^2-m^2}}\right|\times\right.$$
$$\left.\times\left(\frac{\sqrt{\mu^2-m^2}}{k^{(1)}}-\frac{k^{(1)}}{4\sqrt{\mu^2-m^2}}\right)-1\right], \tag{22}$$

$$J_3(k^{(1)},k^{(2)}) = \frac{i\pi^2\theta(\mu^2-m^2)}{k^{(1)}k^{(2)}}\left[\int\limits_0^{\varepsilon} Tp^2 dp \ln\left|\frac{M(p,k^{(1)},k^{(2)})}{M(-p,k^{(1)},k^{(2)})}\right| - \right.$$
$$-\theta\left(\sqrt{\mu^2-m^2}-\frac{k^{(3)}}{2\sin\beta}\right)\int\limits_{\frac{k^{(3)}}{2\sin\beta}}^{\sqrt{\mu^2-m^2}} -iTp^2dp\left(\arcsin N(p,k^{(1)},k^{(2)}) -\right.$$
$$\left.\left.-\arcsin N(-p,k^{(1)},k^{(2)})\right)\right], \tag{23}$$

The internal functions M, N, and T look like

$$M(p,k^{(1)},k^{(2)}) = (k^{(2)})^2 + k^{(1)}k^{(2)}\cos\beta -$$
$$-4p^2\sin^2\beta + 2pk^{(2)}\cos\beta + 2pk^{(1)} + |2p\cos\beta + k^{(2)}|T, \tag{24}$$

$$N(p,k^{(1)},k^{(2)}) = ((k^{(2)})^2 + k^{(1)}k^{(2)}\cos\beta -$$
$$-4p^2\sin^2\beta + 2k^{(2)}\cos\beta + 2pk^{(1)})\times$$
$$\times((k(1)-2p)\sin\beta\sqrt{4p^2-(k^{(2)})^2})^{-1}, \tag{25}$$

$$T = \frac{p}{\sqrt{(k^{(1)}+k^{(2)})^2-4p^2\sin^2\beta}}. \tag{26}$$

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ЕФЕКТИВНА ТРИФОТОННА ВЕРШИНА У ГУСТОМУ ФЕРМІОННОМУ СЕРЕДОВИЩІ

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Р е з ю м е

Обчислено тензор ефективної трифотонної вершини для випадку середовища з ненульовим хімічним потенціалом μ в однопетльовому наближенні. Проаналізовано властивості тензора залежно від довжини хвилі та частоти зовнішніх фотонів. Проведено детальне дослідження окремого випадку розсіяння вільних фотонів на магнітному полі. Обговорено можливі застосування отриманих результатів.