PERMITTIVITY OF PLASMA IN RANDOM FIELDS OF MODERATE INTENSITY

V.I. ZASENKO,[1] A.G. ZAGORODNY,[1] J. WEILAND[2]

[1]Bogolyubov Institute for Theoretical Physics, Nat. Acad. of Sci. of Ukraine
(14b, Metrologichna Str., Kyiv 03680, Ukraine)

[2]Department of Electromagnetics,
Chalmers University of Technology and Euroatom-VR Association
(41296 Göteborg, Sweden)

PACS 52.65.-y, 52.40.Db

The permittivity of plasma in the electric field of random waves of moderate intensity is given in terms of the particle transition probability between two points of the phase space. The transition probability was found as an approximate solution of the Fokker–Planck equation. Validity of this analytical approximation was verified by the direct simulation of the particle diffusion in a field of random waves.

1. Introduction

One of the mechanisms of wave decay in plasma is a resonance interaction between particles and a wave. To a certain degree, the decay of a wave due to the resonance interaction is a more delicate effect than the existence of a wave by itself. Plasma oscillations can be described by equations of hydrodynamics when the movement of plasma components is considered as fluid flows. Along with this, the wave decay is caused by the interaction of a wave and only those particles that move with a velocity close to the phase velocity of the wave. This is manifested in that the oscillation frequency is given through an integral of the particle distribution function and, therefore, is not too much sensitive to its specific form, and the wave decrement is given by a derivative of distribution functions at the resonance velocity. Consequently, a decrement is much more sensitive to a particular form of the distribution function in the resonance region than an eigenfrequency.

When linear waves are considered, it is supposed that a perturbation of the distribution function by a field is small. However, along with this, it is necessary to demand a small perturbation of their derivative, and this is a stronger condition for the distribution function perturbation. In fact, the condition of a small field is not equivalent to the smallness of a distribution function derivative perturbation, and the validation of the latter should be fulfilled in every particular case. It was shown in the early work of Dawson [1] that the smallness of a derivative perturbation is correct only on time intervals less than the bounce time of a particle trapped into the potential well of a wave. This corresponds to the condition that the perturbation of derivatives remains small as compared with the derivative of an unperturbed function. It is obvious that, for any small but finite field, there is a time interval beyond which a perturbation of derivatives is not further negligible. In other words, the criterion of small perturbation is not uniform and is not met for any time.

Coming over from one regular wave to a superposition of several waves, we may find that the situation become even more complex. The main property of linear solutions is that their superposition is a solution as well. So the decrement of a wave, *whether it is considered as a linear one,* should not depend on the presence of other waves. Considering a motion of resonant particles in a field that is formed by a superposition of waves, we note that it becomes stochastic, i.e., it gains a new feature inappropriate for one wave. It seems this is the first non-linear effect that should be accounted for weak fields even if the waves may be still taken as linear in all other aspects. In a case of several waves, the restriction on an amplitude of a wave, in order that it may be treated as a linear one is more rigorous than that for one wave; and, in a case of many waves, such a restric-

 ISSN 2071-0194. Ukr. J. Phys. 2011. Vol. 56, No. 7

tion becomes much more rigorous. Any excitation in real systems could hardly obey this restriction. Thus, it is necessary to have a more rigorous description of resonant particles behavior in the case where a lot of waves are excited, as far this is typical of plasma in the turbulent state.

Electrodynamic properties of plasma are described by its permittivity. It is modified in the presence of an external field or eigenwaves, and such a modification increases with growth of the field strength. However, even for a weak field of many 'linear' waves, we may expect a modification of the permittivity, as far as a particle motion can not be further considered as free, or even as regular. Instead of this, the behavior of resonant particles in a superposition of waves becomes stochastic. For this reason, we may not consider waves as completely linear, and our aim is to calculate a propagator of particles that reflects a distortion of the free particle motion by its stochastic behavior in the resonance region.

The effects of orbits diffusion in a turbulent field was described in early work of Dupree [2], where an approach to the calculation of the renormalized plasma permittivity was proposed. His approach was criticized, however, for an assumption that a particle behavior is described by the same diffusion law on various time scales that are more and less than the field correlation time. Despite this indication of inconsistency, his estimation for a wave decay due to the orbit diffusion is still used till nowadays.

Our aim is to find the particle propagator (transition probability of a particle between two points of the phase space) on different time scales in the presence of a random wave field. In our previous works, we studied the transition probability of a particle in the velocity space, and an approximate analytical solution was found and checked in direct simulations. Indeed, for the small Kubo number $K \ll 1$, a distortion of the free particle propagator $\delta(x-x'-v(t-t'))\delta(v-v')$ by a weak field can be neglected. However, for stronger fields, when the Kubo number becomes of the order of 1, it is necessary to take a modification of the propagator into account. In this work, we will generalize it from the velocity space to the full coordinate-velocity phase space and use it to construct the plasma permittivity in the presence of random waves of moderate intensity (i.e., for the Kubo number of the order of 1).

2. Basic Equation

To calculate the permittivity of plasma in an external wave field, we take the equation for a distribution function of particles in the velocity and coordinate space in the form

$$\left(\frac{\partial}{\partial t}+v\frac{\partial}{\partial x}\right)F(x,v,t)+\frac{e}{m}E(x,t)\frac{\partial}{\partial v}F(x,v,t)=0, \quad (1)$$

where the total field is the sum

$$E(x,t)=E_1(x,t)+E_s(x,t) \quad (2)$$

of a regular self-consistent electric field $E_1(x,t)$ and an external random electric field $E_s(x,t)$, which is supposed to be a Gaussian process homogeneous in space and time. We take the total distribution function in correspondence with the field representation, as the sum of three terms

$$F(x,t)=F_0(x,t)+F_1(x,t)+F_s(x,t), \quad (3)$$

where $F_0(x,t)$ is the averaged distribution function in the absence of $E_1(x,t)$, $F_0(x,t)+F_1(x,t)$ is the averaged total distribution function , and $F_s(x,t)$ is an excitation of the distribution function by the external field.

3. Permittivity

In accordance with the representation of the field, we write the equation for the distribution function (1) in the following form:

$$\left(\frac{\partial}{\partial t}+v\frac{\partial}{\partial x}\right)F(x,v,t)+\frac{e}{m}E_s(x,t)\frac{\partial}{\partial v}F(x,v,t)=$$

$$=-\frac{e}{m}E_1(x,t)\frac{\partial}{\partial v}(F_0(x,v,t)+F_s(x,v,t)). \quad (4)$$

The equation for the self-consistent field reads

$$\mathrm{div}E_1=4\pi e\int dvF_1(x,v,t). \quad (5)$$

A solution for the distribution function can be given through the transition probability of a particle between two points of the phase space. The equation for the transition probability W has the form

$$\left(\frac{\partial}{\partial t}+v\frac{\partial}{\partial x}\right)W(x,v,t;x',v',t')+$$

$$+\frac{e}{m}E_s(x,t)\frac{\partial}{\partial v}W(x,v,t;x',v',t')=0. \quad (6)$$

The initial condition for the transition probability is as follows:

$$W(x,v,t';x',v',t')=\delta(x-x)\delta(v-v'). \quad (7)$$

Whether the solution of Eq. (4) is found and then averaged, the excitation of the distribution function F_1 takes the form

$$F_1(x,v,t) = -\frac{e}{m}\int\limits_0^t dt' \int dx'dv'\bar{W}(x,v,t;x',v',t')\times$$

$$\times E_1(x',t')\frac{\partial}{\partial v'}F_0(x',v',t'), \quad (8)$$

Now the plasma permittivity can be given in terms of the averaged transition probability $\bar{W}$:

$$\varepsilon_k(t) = \delta(t) - i\frac{4\pi e^2}{mk^2}\int dvdv'\bar{W}_k(v,v',t)k\frac{\partial}{\partial v'}F_0(v',0). \quad (9)$$

Thus, to obtain the plasma permittivity, we should find firstly the averaged transition probability.

4. Transition Probability

Taking Eqs. (6) and (7) into account, the equation for the averaged transition probability $\bar{W}$ can be given in a form [5]

$$\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial x}\right)\bar{W}(x,v,t;x',v',t') =$$

$$= \left(\frac{e}{m}\right)^2 \frac{\partial}{\partial v}\int\limits_{t'}^t d\tau \int dydu\bar{W}(x,v,t;y,u,\tau)\times$$

$$\times\langle E(x,t)E(y,\tau)\rangle\frac{\partial}{\partial u}\bar{W}(y,u,\tau;x',v',t') \quad (10)$$

with the same initial conditions

$$\bar{W}(x,v,t';x',v',t') = \delta(x-x)\delta(v-v').$$

Equation (10) is a non-linear integro-differential equation, and it has to be simplified in order to obtain an explicit solution. When random fields are absent, the solution of this equation is given by the free particle propagator. As was shown in our previous paper [3] for moderate fields, which correspond to the Kubo number of the order of 1, this equation is reduced to the equation of diffusion with a convective term:

$$\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial x}\right)\bar{W}(x-x',v,v',t-t') =$$

$$= \left(\frac{e}{m}\right)^2 \frac{\partial}{\partial v}\int\limits_{t'}^t d\tau\langle E^2\rangle_{v\tau,\tau}\frac{\partial}{\partial v}\bar{W}(x-x',v,v',t-t').$$

After the Fourier transformation with respect to the coordinate, this equation takes a form

$$\frac{\partial\bar{W}_k(v,v_0,t)}{\partial t} + ik\bar{W}_k -$$

$$-\frac{\partial}{\partial v}D(v,t)\frac{\partial}{\partial v}\bar{W}_k(v,v_0,t) = 0, \quad (11)$$

where the time-dependent diffusion coefficient is given through the field correlation function

$$D(v,t) = \left(\frac{e}{m}\right)^2\int\limits_0^t\langle E_s^2\rangle_{v\tau,\tau}d\tau. \quad (12)$$

The approximate solution for the transition probability can be given as

$$\bar{W}_k(v,v_0,t) = C(t)\times$$

$$\times\exp\left[-ikv + ik\int\limits_{v_0}^v du\int\limits_0^t d\tau a_k(v,v_0,\tau)\right]\times$$

$$\times\exp\left[-\left(\int\limits_{v_0}^v du\frac{1+a_k(v,v_0,\tau)}{2\sqrt{\int\limits_0^t D(u,\tau)d\tau}}\right)^2\right], \quad (13)$$

where the following notation is used:

$$a_k(v,v_0,\tau) = \frac{2ik}{v-v_0}\int\limits_0^t(t-\tau)D(u,\tau)d\tau.$$

The normalization coefficient is given by

$$C^{-1} = \int dv\exp\left[-\left(\int\limits_{v_0}^v\frac{du}{2\sqrt{\int\limits_0^t D(u,\tau)d\tau}}\right)^2\right].$$

The approximate solution (13) is a generalization of the transition probability in the velocity space, which was calculated earlier in [3]. Thus, the permittivity of plasma in external random fields is given explicitly by Eqs. (9) and (13).

As an illustration, let us consider a reduction of the obtained results to the well-known equations. If $D = \mathrm{const}$, this solution is reduced to

$$\bar{W}_k(v,v_0,t) = \frac{1}{2\sqrt{\pi Dt)}}\exp(-ikv - (1/3)Dk^2t^3)\times$$

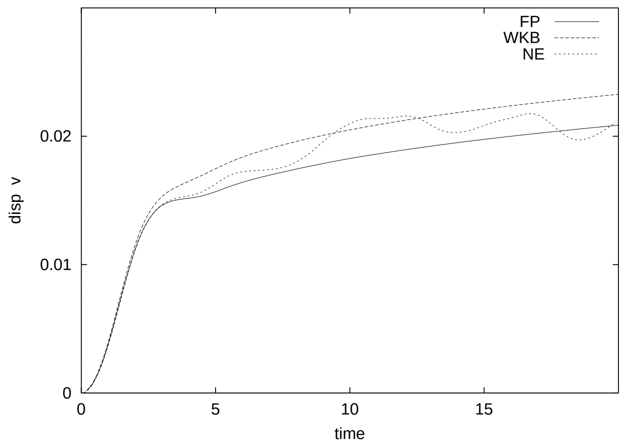

Fig. 1. Evolution of the velocity dispersion for resonant particles and a moderate field. Initial stage. Time is given in units of a wave period

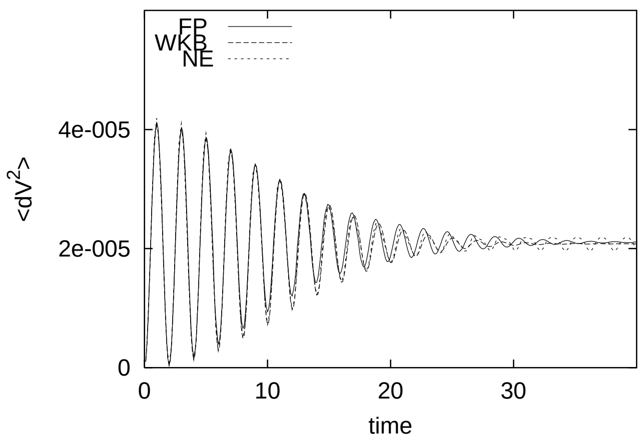

Fig. 2. Velocity dispersion for non-resonant particles and a weak field. Initial stage

$$\times \exp\left[-\frac{(v - ikDt^2 - v_0)^2}{4Dt}\right].$$

It has the same form as the exact solution of Eq. (11) with a constant diffusion coefficient. If we neglect the dependence on the velocity, we obtain the results of Dupree [2].

In the limit of zero external fields, this solution is reduced to the free paricle propagator. For $D = 0$,

$$\bar{W}_k(v, v_0, t) = \exp(-ikv)\delta(v - v_0),$$

which gives the well-known linear permittivity of plasma.

5. Comparison with the Results of Simulation

As an amendment, we give here some illustrative material discussed in more details in our previous works. It shows that a temporal evolution of the velocity dispersion calculated through the approximate transition probability is in a fairly good agreement with the results of direct simulation. We believe that such agreement validates the approximate solution for the transition probability up to moderate field strengths (the Kubo number $K = 2.5$). An approximate form for the transition probability between two points in the velocity space was proposed in our paper [3], where the comparison of the results of analytical calculation and simulation was made. Later, it was shown that the transition probability in such form takes the particle trapping into account and describes the non-resonant interaction (which is much weaker than the resonant one) as well [4]. Here, the comparison of the results of direct simulation (NE), approximate analytical solution of the Fokker–Planck eqation (WKB), and numerical solution of the Fokker–Planck eqation (FP) for various fields is given.

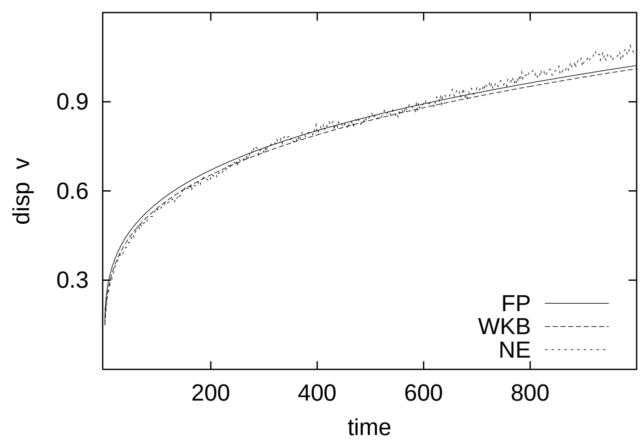

Fig. 3. Evolution of the velocity dispersion for resonant particles and a moderate field. Longer time interval

6. Conclusions

The permittivity of plasma in the electric field of random waves of a moderate intensity is given via the particle transition probability between two points of the phase space. The transition probability was found as an approximate solution of the Fokker–Planck equation. The validity of this analytical approximation was supported by a direct simulation of the particle diffusion in a field of random waves.

Two of the authors (VIZ and AGZ) are grateful to Chalmers University of Technology for their hospitality. The work is partly supported by DFFD-RFFI project F40.2/108.

1. J. Dawson, Phys. Fluids **4**, 869 (1961).

2. T.H. Dupree, Phys. Fluids **9**, 1773 (1966).

3. V. Zasenko, A. Zagorodny, and J. Weiland, Phys. Plasmas **12**, 062311 (2005).

4. V. Zasenko, A. Zagorodny, and J. Weiland, Ukr. J. Phys. **53**, 517 (2008).

5. S.A. Orszag and R.H. Kraichnan, Phys. Fluids **10**, 1720 (1967).

Received 10.03.11

ДІЕЛЕКТРИЧНА ПРОНИКНІСТЬ ПЛАЗМИ
У ВИПАДКОВИХ ПОЛЯХ ПОМІРНОЇ
ІНТЕНСИВНОСТІ

В.І. Засенко, А.Г. Загородній, Я. Вейланд

Р е з ю м е

Діелектричну проникність плазми у випадкових полях помірної інтенсивності подано через ймовірність переходу частинки між двома точками фазового простору. Ймовірність переходу знайдено як наближений розв'язок рівняння Фоккера–Планка. Коректність цього аналітичного наближення підтверджується прямим моделюванням дифузії частинок у полі випадкових хвиль.