ON ASYMPTOTIC REGGE TRAJECTORIES OF HEAVY MESON RESONANCES

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We performed the analysis of the asymptotic behavior of Regge trajectories of nonstrange and strange mesons and found that the width of heavy hadrons for these trajectories cannot linearly depend on their mass. Such a finding clearly demonstrates that a widely spread belief on the linear mass dependence of the resonance width contradicts the linearity of Regge trajectories on the Mandelstam variable s. Using the data on masses and widths for ρ_{J--}, ω_{J--}, a_{J++}, and f_{J++} mesons with the spin values $J \leq 6$ and for K^*_J mesons with $J \leq 5$, we extracted the parameters of the asymptotically linear Regge trajectories predicted by the finite-width model of quark gluon bags. It is shown that the parameters obtained for the data sets B and D are consistent with the cross-over temperature determined by the lattice QCD simulations at the vanishing baryonic density and with the kinetic freeze-out temperature of early hadronizing particles found in relativistic heavy ion collisions at and above the highest SPS energy. Comparing the resonance width of sets B and D evaluated at the masses of Z and W bosons, respectively, we discovered that the calculated width values match that of the gauge bosons. We argue that such matches provide us with indirect, but the first experimental evidence for the compositeness of Z and W bosons. Based on these findings, we assume that Z, W, and Higgs bosons have the Regge trajectories which are similar to the asymptotic trajectories of the studied mesons. The predictions for the masses and widths of the Regge partners of Z and W bosons and for the mass dependence of the widths of Higgs boson Regge partners along with the values for the mass and width of the scalar Higgs mesons are made as well.

1. Introduction

Since its first applications in particle physics and till present days, the method of Regge poles remains a very reliable tool to describe a variety of nonperturbative processes in quantum chromodynamics (QCD). In particle physics, it was introduced at the beginning of the 1960s [1], and it is widely used up to now to describe the high-energy interactions of hadrons and nuclei. This method establishes an important connection between the high-energy scattering and the spectrum of particles and resonances. It was also a starting point to develop the dual and string models of hadrons. Although an apogee of the Regge method in particle physics ended after the beginning of the QCD era, some of its principal findings await for the QCD-based explanations.

It is well known that a Regge trajectory $J = \alpha(s)$ which is usually expressed in terms of the center-of-mass energy squared s of colliding particles can be also used in the t- or u-channels in terms of the Mandelstam variables t or u, respectively, due to a crossing symmetry of strong interaction. $\alpha(s)$ represents a set of leading Regge poles on the complex plane of angular momentum. An astonishing (approximate) linearity of Regge trajectories for the known hadronic states of mass M_h and spin J_h, i.e. $J_h = \alpha(M_h^2) = \alpha_0 + \alpha_0' M_h^2$, remains one of the unresolved problems of QCD despite very promising results obtained within the phenomenological planar models [2, 3]. Although there is no consensus [4] up to now on a linearity of the established Regge trajectories of hadrons due to the lack of the experimental data on heavy resonances of the spin above 6, it is, however, necessary to mention that, in the 1960s and 1970s, a lot of efforts was invested in the investigations of the asymptotic behavior of Regge trajectories for $|s| \to \infty$ based on their analytic properties in the complex s-plane [5, 6]. Under rather general assumptions, it was discovered [7] that asymptotic Regge trajectories cannot increase faster than a linear function of s, and, at the same time, they cannot increase slower than the square root of s for $|s| \to \infty$. Such a result restricts the nonlinear behavior from above, but it does not allow to rule out the nonlinear s-behavior that is weaker than 1 and stronger than $\frac{1}{2}$.

It is interesting that a strong argument in favor of the linear asymptotic behavior of Regge trajectories of free hadrons was recently obtained [8] within the exactly solvable statistical model for quark gluon (QG) bags [9]. This is the finite-width model (FWM), since it allows one to account for a finite width of QG bags. Quite generally, the FWM demonstrates that free QG bags of mass $M_r \geq M_0 \approx 2.5$ GeV that belong to a continuous mass-volume spectrum of the Hagedorn type [10] should have the mean width $\Gamma_r \sim \sqrt{M_r}$. Such a behavior can be provided by the asymptotically linear Regge trajectory [7, 8] only. Moreover, the FWM shows that such a behavior of the mean width of QG bags, but with the temperature-dependent coefficient, remains valid at high temperatures.

The main purpose of this work is to extract the parameters of the asymptotic Regge trajectory predicted by the FWM using the experimental data for nonstrange and strange mesons. In our analysis, we use the data on ρ_{J--}, ω_{J--}, a_{J++}, and f_{J++} mesons for spin values $J \leq 6$ and the data on K^*_J for $J \leq 5$ which are known with the highest accuracy. It is an extension of the approach suggested recently in [11] onto the strange mesons and the gauge bosons (Z and W). In contrast to the usual type of analysis which is reduced to searches for a connection between the spin and the mass of trajectory members, we use the full Regge trajectory in the complex s-plane. This allows us to simultaneously describe the masses and the widths of involved mesons. Such a task is very important nowadays, since a great significance of the width of heavy resonances or bags for the realistic equation of state of strongly interacting matter [8, 9, 12] and for a description of the fast equilibration process of heavy baryons/antibaryons [13–15] and kaons/antikaons [15] in relativistic heavy ion collisions was realized only recently. This kind of analysis is necessary for the further development of the string model of hadrons and for the improvement of such transport codes as the hadron string dynamics [16] and the UrQMD model [17] by including the finite width of heavy resonances. Here, we demonstrate also that a widely spread belief [14, 15, 18] that the width of heavy hadronic resonances linearly depends on their mass is simply inconsistent with the existence of linear Regge trajectories. In addition, we would like to connect the asymptotic Regge trajectories of hadrons with that for W, Z, and Higgs bosons.

The work is organized as follows. Section 2 contains a brief analysis of the asymptotic Regge trajectory properties which demonstrates that the resonance width is proportional to its mass for the asymptotically nonlinear trajectories only. In Section 3, we discuss two hypotheses to be verified for the nonstrange mesons and define the corresponding data sets to be fitted. The details of the fitting procedure are specified in Section 4, whereas the discussions of the obtained results for the nonstrange and strange mesons are, respectively, given in Sections 5 and 6. Section 7 is devoted to the analysis of the Regge trajectories of W, Z, and Higgs bosons, while Section 8 contains our conclusions.

2. Asymptotic Behavior of Bosonic Regge Trajectories

In the pre-QCD era, a lot of efforts was put forward [5, 7] into the determination of the Regge trajectory asymptotics for hadronic resonances. In our analysis, we follow [7] and adopt the most general assumptions on the trajectory: (I) $\alpha(s)$ is an analytic function, having only the physical cut from $s = s_0$ to $s = \infty$; (II) $\alpha(s)$ is polynomially restricted at the whole physical sheet; (III) there exists a finite limit of the phase trajectory at $s \to \infty$. Using these assumptions, it was possible to prove [7] that, as $s \to \infty$, the asymptotic behavior of a Regge trajectory at the whole physical sheet is

$$\alpha_u(s) = -G^2\,[-s]^{\nu} \;, \quad \text{with} \quad \frac{1}{2} \leq \nu \leq 1 \,. \qquad (1)$$

Here, the function $G^2 > 0$ should increase slower than any power in this limit, and its phase must vanish as $|s| \to \infty$. Clearly, $\nu = 1$ is an upper bound for the asymptotic behavior, while $\nu = \frac{1}{2}$ is its lower bound.

Since our main interest here is related to the asymptotically linear trajectories, we restrict ourselves to trajectories of the form

$$\alpha(s) = g^2\,[-(-s)^{\nu} + q(s)] \;, \quad \text{with} \quad \frac{1}{2} \leq \nu \leq 1 \,, \qquad (2)$$

where g is a real constant, and the correction $q(s)$ increases slower than $|s|^{\nu}$ in the limit $|s| \to \infty$, i.e. $|q(s)|/|s|^{\nu} \to 0$ in this limit. Since, at the resonance position, $s = s_r = |s_r|\, e^{i\,\phi_r}$ in the complex s-plane, the trajectory defines its spin J_r, one obtains $\mathrm{Im}\,[\alpha(s_r)] = 0$.

This condition allows one to determine the phase of the physical trajectory from the equation

$$\sin\left(\nu\,\phi_r + \pi(1-\nu)\right) = -\mathrm{Im}[q(s_r)]|s_r|^{-\nu} \to 0\,, \tag{3}$$

where we used $(-1)^\nu = e^{-i\pi\nu}$ to get the physical branch that corresponds to a resonance (more details can be found in [7]). We consider a formal solution $\phi_r = \pi\left(1-\frac{1}{\nu}\right) - \frac{1}{\nu}\,\mathrm{Im}[q(s_r)]|s_r|^{-\nu}$ of Eq. (3) which corresponds in the complex energy plane $E = \sqrt{s_r} \equiv M_r - i\frac{\Gamma_r}{2}$ to a resonance having the mass $M_r > 0$ and the width $\Gamma_r > 0$. The mass and the width of a resonance belonging to trajectory (2) are defined as

$$M_r = |s_r|^{1/2}\cos\frac{\phi_r}{2} \quad \text{and} \quad \Gamma_r = -2\,|s_r|^{1/2}\sin\frac{\phi_r}{2}\,. \tag{4}$$

It is clear that the positive values of resonance mass and width correspond to the inequalities $-\frac{\pi}{2} < \frac{\phi_r}{2} < 0$ which lead in the limit $|s_r| \to \infty$ to the conditions $\frac{1}{2} < \nu < 1$.

For the limiting cases $\nu = 1$ and $\nu = \frac{1}{2}$, the positive values of width and mass, respectively, are determined by the small correction $\frac{1}{\nu}\,\mathrm{Im}[q(s_r)]|s_r|^{-\nu}$ in (3). Equations (3) and (4) clearly demonstrate us that, only for asymptotically nonlinear trajectories (2), the resonance width is proportional to its mass, i.e. $\Gamma_r = -2\,M_r\tan\left(\frac{\phi_r}{2}\right)$ and $\phi_r \to \pi\left(1-\frac{1}{\nu}\right)$ in the limit $|s_r| \to \infty$. Contrarily, for the linear trajectory $\nu = 1$, the resonance width behaves as

$$\Gamma_r = 2\,\mathrm{Im}[q(s_r)]|s_r|^{-1/2} \sim \frac{2\,\mathrm{Im}[q(s_r)]}{|s_r|}\,M_r\,, \tag{5}$$

where we used expression (4) for the resonance mass in the last step. Since the function $q(s)$ is a small correction to the linear s-dependence, one concludes from the right-hand side of Eq. (5) that, for asymptotically linear trajectories, the width of heavy resonances cannot be proportional to their mass, since the ratio $\frac{\mathrm{Im}[q(s_r)]}{|s_r|} \to 0$ for large $|s_r|$. Thus, we obtain a very important conclusion that only the asymptotically nonlinear Regge trajectories (2) with ν lying between $\frac{1}{2}$ and 1 lead to the linear mass dependence of the resonance width, i.e. $\Gamma_r \sim M_r$, whereas the asymptotically linear Regge trajectories (2) with $\nu = 1$ generate a weaker mass dependence of the width. However, this general analysis cannot determine either the form of the function $q(s)$ or the range of s, at which such an asymptotic behavior is valid.

Fortunately, both of these questions can be answered within the FWM [8]. Thus, the FWM tells us that, for excited resonances that belong to the continuous part of the mass-volume spectrum of QG bags, the width dependence (5) starts to develop already for $M_r \geq M_0 \approx 2.5$ GeV. It predicts also that $q(s) \sim s^{3/4}$ which leads to $\Gamma_r \sim \sqrt{M_r}$. Such a conclusion gives a natural explanation of the observed huge deficit [19] in the number of hadronic resonances compared to the statistical bootstrap model [10]. Using these results, we conclude that the linear mass dependence of the resonance width assumed in [18] cannot be used to model the decay of heavy QG bags. It can be also that the application of the obtained results to the decay of Hagedorn states [14, 15] can give a quantitatively different outcome.

3. Constructing the Data Sets for Nonstrange Mesons

The FWM predicts the existence of a universal Regge trajectory for heavy QG bags. However, the determination of parameters of such a trajectory is immediately faced with two principal difficulties. The first of these difficulties is that the universal trajectory corresponds to heavy (excited) resonances with mass above $M_0 \approx 2.5$ GeV [8], while the experimental data in this region are absent. The second one is related to the fact that the usual fitting procedure is not suited to this task.

For the analysis, we choose the best studied trajectories [4, 20] with natural parity $P = (-1)^J$: ρ_{J--}, a_{J++} mesons of isospin 1 and spin $J \leq 6$, ω_{J--}, f_{J++} mesons of isospin 0 and spin $J \leq 6$, and the strange mesons $K^*_{J^P}$ of isospin $\frac{1}{2}$ and spin $J \leq 5$. In this section and in the next two ones, we analyze the nonstrange mesons in order to outline the way of constructing the data sets and the procedure to extract the Regge trajectory parameters, respectively, while the inspection of strange mesons is reserved for Section 6.

Nowadays, there are three mesons on each of the trajectories of nonstrange mesons [21]. These mesons are well suited to our purpose since, firstly, the parameters of their trajectories are close to each other [4, 20], and, secondly, the masses of a_{6++} and f_{6++} mesons are 2.45 ± 0.13 GeV and 2.465 ± 0.05 GeV, respectively, i.e. their masses are close to the value of M_0. Since here we would like to simultaneously fit the masses and the widths of resonances, we restrict ourselves to the analysis of ρ_{J--}, a_{J++}, ω_{J--}, and f_{J++} mesons because all of them belong to the parent (i.e., main) trajectories and because only the light hadronic resonances among other hadrons consisting of u and d quarks are well studied as compared to these mesons.

Although the trajectories of ρ_{J--}, a_{J++}, ω_{J--}, and f_{J++} mesons are similar, they are not identical. Since there is no *a priori* knowledge on which trajectory is closer to the asymptotic one, we cannot reject any of the data points by claiming that one of the data set is wrong. Also, we cannot simply fit all trajectories by the same set of parameters, since the masses and widths of the mesons with same value of resonance spin J are rather different in many cases, and their error bars even do not overlap. Therefore, our first task is to determine the correct set of data to be fitted with the corresponding errors.

The nonzero difference of meson masses of the same spin is a reflection of the chiral symmetry breaking in the confined phase. It is expected that, for excited mesons, the effect of chiral symmetry breaking gets weaker [22, 23]. This expectation is in line with the FWM prediction of the universal trajectory existence. However, from the practical point of view, it is necessary to account for the effect of chiral symmetry breaking in the fitting procedure. Clearly, the natural measure of the chiral symmetry breaking, which has to be included into the fitting, is the mass difference of $\delta M_J^o = |M_{\omega_J} - M_{\rho_J}|$ for odd values of J and $\delta M_J^e = |M_{f_J} - M_{a_J}|$ for even J values. Since the mass differences δM_J^o and δM_J^e are much smaller than the masses of the corresponding mesons, i.e. the chiral symmetry breaking effect is small, then it seems reasonably to expect that the universal trajectory should be located very close to or inside of the mass interval of mesons having the same spin. The same, of course, should be true for the resonance width. In addition, it is necessary to account for the experimental errors of resonance masses and widths.

Therefore, **hypothesis A** to be verified by the fit of experimental data is as follows: for the spin J, the mass and the width defined by the universal trajectory are located within the interval

$$M_r^{\exp} \in \left[\min\{M_J^{\min} - \delta M_J^{\min}; M_J^{\max} - \delta M_J^{\max}\};\right.$$
$$\left.\max\{M_J^{\min} + \delta M_J^{\min}; M_J^{\max} + \delta M_J^{\max}\}\right], \quad (6)$$
$$\Gamma_r^{\exp} \in \left[\min\{\Gamma_J^{\min} - \delta \Gamma_J^{\min}; \Gamma_J^{\max} - \delta \Gamma_J^{\max}\};\right.$$
$$\left.\max\{\Gamma_J^{\min} + \delta \Gamma_J^{\min}; \Gamma_J^{\max} + \delta \Gamma_J^{\max}\}\right], \quad (7)$$

respectively. Here, $M_J^{\min}$ and $\delta M_J^{\min}$ ($M_J^{\max}$ and $\delta M_J^{\max}$) denote the minimal (maximal) value of meson mass of spin J and its experimental error, respectively. In other words, for each J, the mass (width) of the universal Regge trajectory is assumed to be located inside the widest interval that can be constructed from masses (widths) of two mesons and their experimental errors. Thus, our fitting of hypothesis **A** relies on the maximal uncertainty in the experimental mass and width values, which, on the one hand, allows us to account for the experimental splitting in the masses and the widths of resonances and, on the other hand, to reduce an individual influence of each of four trajectories analyzed. Such an assumption allows us to determine the mean values of mass and width to be fitted for each J as

$$M_r^{\exp} \equiv \frac{1}{2}\left[\min\{M_J^{\min} - \delta M_J^{\min}; M_J^{\max} - \delta M_J^{\max}\} + \right.$$
$$\left. + \max\{M_J^{\min} + \delta M_J^{\min}; M_J^{\max} + \delta M_J^{\max}\}\right], \quad (8)$$
$$\Gamma_r^{\exp} \equiv \frac{1}{2}\left[\min\{\Gamma_J^{\min} - \delta \Gamma_J^{\min}; \Gamma_J^{\max} - \delta \Gamma_J^{\max}\} + \right.$$
$$\left. + \max\{\Gamma_J^{\min} + \delta \Gamma_J^{\min}; \Gamma_J^{\max} + \delta \Gamma_J^{\max}\}\right], \quad (9)$$

and their errors as

$$\delta M_r \equiv \frac{1}{2}\left[\max\{M_J^{\min} + \delta M_J^{\min}; M_J^{\max} + \delta M_J^{\max}\} - \right.$$
$$\left. - \min\{M_J^{\min} - \delta M_J^{\min}; M_J^{\max} - \delta M_J^{\max}\}\right], \quad (10)$$
$$\delta \Gamma_r \equiv \frac{1}{2}\left[\max\{\Gamma_J^{\min} + \delta \Gamma_J^{\min}; \Gamma_J^{\max} + \delta \Gamma_J^{\max}\} - \right.$$
$$\left. - \min\{\Gamma_J^{\min} - \delta \Gamma_J^{\min}; \Gamma_J^{\max} - \delta \Gamma_J^{\max}\}\right]. \quad (11)$$

Set **A** defined by Eqs. (9)–(11) from the experimental data is shown in the second column of Table 1. As one can see from this column, the errors of the $J = 1$ mass, $J = 3$ mass, and $J = 3$ width are essentially smaller than other errors, and, as we will see in the next section, they essentially affect the results of fitting. Therefore, in order to weaken such a dependence, we would like to verify **hypothesis B** that, for spin J, the universal Regge trajectory passes through the corridor $\pm\Delta$ taken from the arithmetical average of masses and widths of the corresponding mesons (set **B**).

We chose $\Delta = 0.035$ GeV. It is, however, clear that the scaling of Δ does not change the location of a χ^2 minimum in the space of parameters, although it changes the value of mean deviation squared per number of degrees of freedom χ^2 and the error bars of the fitting parameters.

Comparing sets **A** and **B** (see columns $A_{\exp}$ and $B_{\exp}$ in Table 1), one can see that the corresponding mass and width values, except for Γ_6, are very close to each set of data. The difference in the uncertainties of these sets allows us to study the stability of the fitting parameters.

4. Fitting Procedure

For the analysis, we choose the simplest parametrization of the Regge trajectory which satisfies requirements (I)–(III) and obeys the FWM predictions [8] that $q(s) \sim s^{3/4}$:

$$\alpha(s) = \alpha_0 + g_{\rm R}^2 \left[s + A_{\rm R}(-s)^{3/4} - iB_{\rm R}\right]. \quad (12)$$

Note that the term $(-s)^{3/4}$ in (12) has a correct behavior in the complex s-plane [7]. Two additional parameters to

T a b l e 1. Masses and widths (both given in GeV) of data sets A, B, C, and D along with the results obtained by their fitting (see more details in the text). The column $A_{\rm exp}$ contains the data points and their errors for hypothesis A. In the columns $B_{\rm exp}$ and $D_{\rm exp}$, there are data points corresponding to hypotheses B and D, respectively, while the data points have the same error Δ. The column $C_{\rm exp}$ consists of the experimental data on masses and widths of ρ_{1--}, ρ_{3--}, and ρ_{5--} mesons which are given to demonstrate the typical results of the four-parameter fit. The mass and the width of the resonance with $J = 6$ for the column $D_{\rm fit}$ are predicted using the fitting parameters. The last row contains the corresponding χ^2 per number of degrees of freedom to show the quality of the fit

	$A_{\rm exp}$	$A_{\rm fit}$	$B_{\rm exp}$	$B_{\rm fit}$	$C_{\rm exp}$	$C_{\rm fit}$	$D_{\rm exp}$	$D_{\rm fit}$
$M_1 \pm \delta M_1$	0.7754 ± 0.00734	0.7749	0.7758	0.7619	0.7690 ± 0.0009	0.76897	0.8142	0.8111
$\Gamma_1 \pm \delta\Gamma_1$	0.0792 ± 0.0708	0.0342	0.0788	0.0510	0.1490 ± 0.0010	0.14895	0.0697	0.0489
$M_2 \pm \delta M_2$	1.2971 ± 0.0233	1.3087	1.2970	1.2936	–	–	1.3400	1.3377
$\Gamma_2 \pm \delta\Gamma_2$	0.1451 ± 0.0423	0.1286	0.1445	0.1704	–	–	0.1292	0.1529
$M_3 \pm \delta M_3$	1.6770 ± 0.0140	1.6893	1.6779	1.6748	1.6888 ± 0.0021	1.6891	1.7100	1.7180
$\Gamma_3 \pm \delta\Gamma_3$	0.1645 ± 0.0135	0.1743	0.1645	0.2297	0.1610 ± 0.0100	0.1638	0.1627	0.2045
$M_4 \pm \delta M_4$	2.0101 ± 0.0191	2.0028	2.0095	1.9895	–	–	2.0213	2.0324
$\Gamma_4 \pm \delta\Gamma_4$	0.2820 ± 0.0620	0.2049	0.2750	0.2700	–	–	0.2493	0.2401
$M_5 \pm \delta M_5$	2.2725 ± 0.0925	2.2753	2.2900	2.2632	2.3300 ± 0.0350	2.270	2.3207	2.3076
$\Gamma_5 \pm \delta\Gamma_5$	0.3625 ± 0.1375	0.2282	0.3600	0.3010	0.4000 ± 0.1000	0.177	0.2993	0.2671
$M_6 \pm \delta M_6$	2.4500 ± 0.1300	2.5171	2.4575	2.5056	–	–	–	2.5576
$\Gamma_6 \pm \delta\Gamma_6$	0.4000 ± 0.2500	0.2489	0.3275	0.3285	–	–	–	0.2809
χ^2	–	0.7739	–	1.3109	–	4.	–	0.5774

the asymptotic linear trajectory (2), α_0 and $B_{\rm R}$, define the real and imaginary parts of $\alpha(0)$ at $s = 0$, respectively, i.e. $\mathrm{Re}(\alpha(0)) \equiv \alpha_0$ and $\mathrm{Im}(\alpha(0)) \equiv -g_{\rm R}^2 B_{\rm R}$. The constant $A_{\rm R}$ defines the phase ϕ_r of a resonance in the complex energy plane as

$$\sin(\phi_r) = A_{\rm R} \sin\left(\tfrac{3}{4}(\pi - \phi_r)\right) \sqrt{\frac{\cos\left(\frac{\phi_r}{2}\right)}{M_r}} + \\ + \frac{B_{\rm R}}{M_r^2} \cos^2\left(\frac{\phi_r}{2}\right) . \quad (13)$$

Clearly, this equation is an explicit form of Eq. (3) for trajectory (12). As we discussed in Section 2, $-\pi < \phi_r < 0$, which leads to the inequality $B_{\rm R} < -A_{\rm R} \sin\left(\frac{3}{4}(\pi - \phi_r)\right) \left[M_r^2 + \frac{\Gamma_r^2}{4}\right]^{3/4}$ that should hold for masses, widths, and phases of all resonances belonging to trajectory (12).

In fact, we used more complicated parametrizations of the trajectory $\alpha(s)$ than that of Eq. (12), but they did not give any improvement of the fit. In particular, in order to avoid the singularity of the intercept $\frac{d\,\alpha(s)}{d\,s}$ at $s = 0$, we also considered $(-(s + C_0))^{3/4}$ with a complex constant C_0 instead of the term $(-s)^{3/4}$ in (12). However, this modification increases the overall value of χ^2 per number of degree of freedom, since the reduction in the number of degree of freedom for one or two units has a dominant effect.

The spin of the resonance at its position in the complex s-plane $s_r = |s_r|\, e^{i\,\phi_r}$ is given by the expression

$$J_r = \mathrm{Re}\,(\alpha_{\rm R}(s_r)) = \alpha_0 + g_{\rm R}^2 M_r^2 \times \\ \times \frac{\left[\sin\left(\frac{1}{4}(3\pi + \phi_r)\right) - \frac{B_{\rm R}}{M_r^2}\cos^2\left(\frac{\phi_r}{2}\right)\cos\left(\frac{3}{4}(\pi - \phi_r)\right)\right]}{\cos^2\left(\frac{\phi_r}{2}\right)\sin\left(\frac{3}{4}(\pi - \phi_r)\right)} , \quad (14)$$

whereas its mass M_r and width Γ_r are defined by Eqs. (4). As one can see from Eqs. (4) and (14), the parameter $B_{\rm R}$ enters into these equations only in the combination $\frac{B_{\rm R}}{M_r^2}$. This fact clearly demonstrates the importance of the $B_{\rm R}$ parameter for small values of resonance mass, while, for large values of M_r, it generates a small correction to the asymptotic behavior of the trajectory.

Equation (14) can be rewritten in the form

$$M_r = \frac{1}{g_{\rm R}}\left[\frac{(J_r - \alpha_0)\cos^2\left(\frac{\phi_r}{2}\right)\sin\left(\frac{3}{4}(\pi - \phi_r)\right)}{\sin\left(\frac{1}{4}(3\pi + \phi_r)\right) - g_{\rm R}^2 B_{\rm R}\cos^2\left(\frac{\phi_r}{2}\right)\cos\left(\frac{3}{4}(\pi - \phi_r)\right)}\right]^{1/2}, \quad (15)$$

which is more convenient for finding out the resonance mass for known spin J_r and phase ϕ_r. The advantage of Eq. (15) is that its right-hand side does not depend on M_r. This allows us to simplify the searches for the minimum of the χ^2-function

$$\chi^2(A_{\rm R}, B_{\rm R}, g_{\rm R}, \alpha_0) = \\ = \frac{1}{N_{\rm dof}} \sum_r \left[\frac{[M_r - M_r^{\rm exp}]^2}{\delta M_r^2} + \frac{[\Gamma_r^{\rm exp} + 2M_r \tan\left(\frac{\phi_r}{2}\right)]^2}{\delta\Gamma_r^2}\right], \quad (16)$$

by solving numerically the system of equations (4), (15) for two unknown variables ϕ_r and M_r of a resonance

T a b l e 2. The parameters of the asymptotic Regge trajectory (12) obtained by the fitting of sets A, B, C, C^B, and D given in Table 1. The errors correspond to a standard (one σ) deviation from the data point values of the corresponding set

Parameter	$A_{\rm fit}$	$B_{\rm fit}$	$C_{\rm fit}$	$C^B_{\rm fit}$	$D_{\rm fit}$
α_0 (GeV)^0	0.4260 ± 0.0120	0.4250 ± 0.0180	0.4490 ± 0.007	0.300 ± 0.015	0.3770 ± 0.049
$g_{\rm R}$ (GeV)$^{-1}$	0.8815 ± 0.0049	0.8667 ± 0.0155	0.906 ± 0.006	0.867 ± 0.016	0.8646 ± 0.0161
$A_{\rm R}$ (GeV)$^{1/2}$	−0.287± 0.0110	−0.377± 0.0218	−0.157± 0.008	−0.385 ± 0.019	−0.327± 0.0340
$B_{\rm R}$ (GeV)^2	0.1033 ± 0.0504	0.1327 ± 0.0119	−0.050± 0.007	0.0734 ± 0.013	0.1221 ± 0.0091

having spin J_r for a given set of $A_{\rm R}, B_{\rm R}, g_{\rm R}$, and α_0 values. Here, $N_{\rm dof}$ denotes the number of independent degrees of freedom in the fitting, $M_r^{\rm exp}$ and $\Gamma_r^{\rm exp}$ are, respectively, the mass and the width of the resonance of spin J_r taken from the data sets defined in a preceding section, whereas δM_r and $\delta \Gamma_r$ are the corresponding uncertainties. The minimum of χ^2-function (4) was found by the independent variation of the fitting parameters $A_{\rm R}, B_{\rm R}, g_{\rm R}$, and α_0.

It is necessary to stress here that such a procedure provides an exact treatment of the resonance width in contrast to a popular approximate relation [6]

$$\Gamma_r \approx {\rm Im}(\alpha(M_r^2)) \left[M_r\, {\rm Re} \left(\frac{d\alpha(s)}{d\ s} \bigg|_{s=M_r^2} \right) \right]^{-1}, \qquad (17)$$

which can be used for very heavy resonances only, while, for ρ_{1--} and ω_{1--}, it deviates from the exact result by 20–30 %.

5. Results for Nonstrange Mesons

Using the above equations, we performed the four-parameter fit of the data set **A** defined by Eqs. (9)–(11). The results are given in Tables 1 and 2 and shown in Figs. 1–3. Although the $\chi^2_A \approx 0.774$ value is smaller than 1, the close inspection shows that, in contrast to the excellent fit of resonance masses, the fit of their widths seems not very satisfactory. From Figs. 2 and 3, one can clearly see that the data set **A** for widths is perfectly described for $J_r \leq 3$ only, whereas the obtained width of resonances Γ_r with $J_r > 3$ formally provides the minimum of χ^2-function because $\Gamma_r \in [\Gamma_r^{\rm exp} - \delta\Gamma_r; \Gamma_r^{\rm exp}]$, but the behavior of Γ_r does not reproduce the trend of the data for $J_r > 3$. The reason for such a behavior is that the mass and width uncertainties of set **A** are essentially smaller for the resonances of spin $J_r \leq 3$ than for that of higher spin values. We obtained exactly the same behavior, while examining the individual trajectories of ρ_{J--}, ω_{J--}, a_{J++}, and f_{J++} mesons. The last column of Table 1 shows the results of the ρ_{J--} trajectory fit. One can see that the masses and the widths of ρ_{1--} and ρ_{3--} mesons are reproduced perfectly, whereas the mass of ρ_{5--} meson is almost two standard deviations off from its experimental mean value, and its width is about two and half standard deviations off the mean experimental value of a width. An evident origin for such an outcome of the fit is rooted in very small experimental errors of ρ_{1--} and ρ_{3--} mesons as compared with the errors of ρ_{5--} meson. Clearly, the large value of $\chi^2_C \approx 4$ for the ρ_{J--} trajectory is generated by the ρ_{5--} meson data points.

After realizing this fact, we examined the stability of the obtained results. For this purpose, we formulated hypothesis **B** and analyzed it. Comparing the data sets **A** and **B**, it is clearly seen from Table 1 that the main difference between them is due to the value of errors: in contrast to set **A**, all errors for set **B** are chosen to be equal to Δ (democratic choice). The results of the fitting with set **B** are given in Tables 1 and 2 and shown in Figs. 1–3.

From Table 1, one can see that the obtained fit corresponds much better to the data trend of set **B**. In fact, there are only two data points, Γ_3 and Γ_5, which are about 60 MeV off the corresponding experimental values. Such a result seems to be a remarkable success for trajectory (12) which is expected to be valid in the limit $|s| \rightarrow \infty$.

From Table 2, it is seen that the most sensitive parameter to a change of data sets is $A_{\rm R}$, whereas α_0 and $g_{\rm R}$ are almost insensitive parameters, and $B_{\rm R}$ demonstrates a moderate sensitivity. It is clear from this table that the values of the fitting parameters α_0 and $g_{\rm R}$ are in good agreement with the corresponding parameters obtained by other groups [4, 20, 22].

It is worth to note that if we apply hypothesis **B** to the description of the ρ meson trajectory (column $C_{\rm exp}$ in Table 1) and, thus, define set $\mathbf{C}^B$, then, with the accuracy $\Delta \approx 54$ MeV, the χ^2 value of such a set becomes equal to χ^2_C of the fit obtained for this trajectory with the experimental errors (see column $C_{\rm fit}$ in Table 1). Other parameters for case $\mathbf{C}^B$ are given in Table 2, from which one can see that the parameters

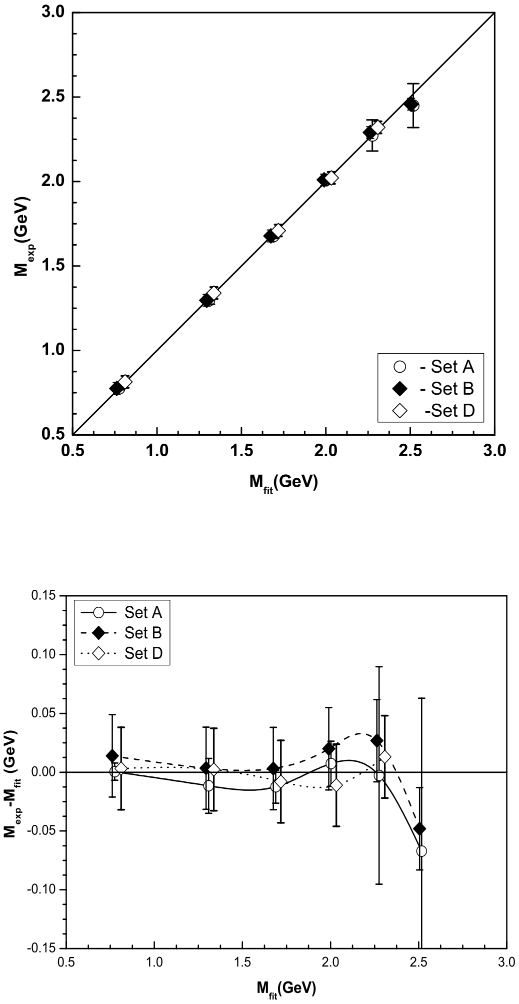

Fig. 1. Comparison between the resonance masses of sets **A**, **B**, and **D** and their fit by the asymptotic Regge trajectory (12). The upper panel shows the data points as a function of the corresponding mass value of the fit. For most of the data points, the error bars are smaller than the symbols. The lower panel shows the difference of data and the fit as a function of the fit value of the resonance mass. The curves in the lower panel are the cubic splines and are shown to guide the eyes

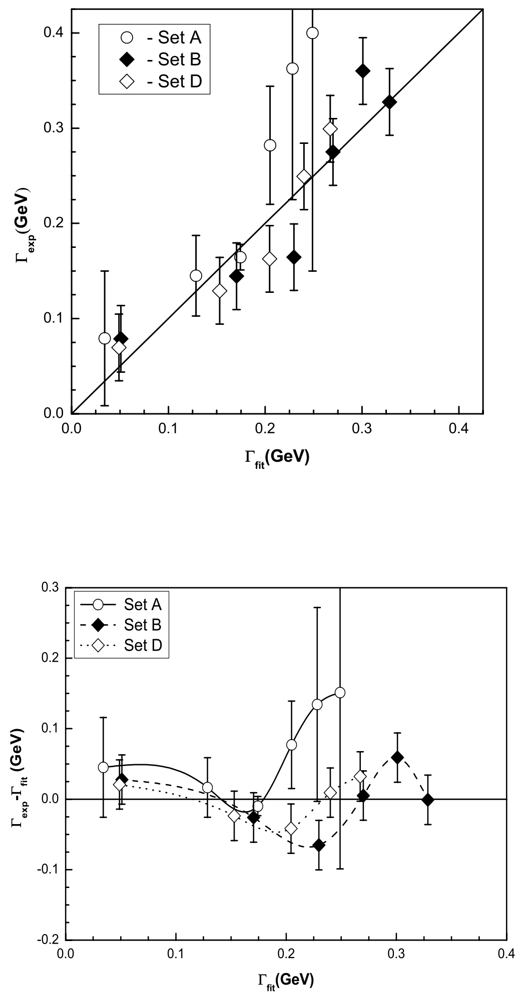

Fig. 2. Comparison between the resonance widths of sets **A**, **B**, and **D** and their fit by the asymptotic Regge trajectory (12). The upper panel shows the data points as a function of the corresponding width value of the fit. The lower panel shows the difference of data and the fit as a function of the fit value of the resonance width. The curves in the lower panel are the cubic splines and are shown to guide the eyes

which are important in the asymptotics $|s| \to \infty$, $g_{\rm R}$ and $A_{\rm R}$, are very close to the values obtained for set **B** for all nonstrange resonances, whereas the found $B_{\rm R}$ value is close to that of the fit with set **C**. From such a comparison, we conclude that the definition of the data set based on the democratic choice of errors grasps not only the average mass and width values for four analyzed nonstrange meson trajectories, but also it reflects the asymptotic behavior of each of these mesonic Regge trajectories.

Now it is necessary to choose the best fit among the results found for sets **A** and **B**. Although for the common error $\Delta = 35$ MeV, the found value of $\chi^2_B(35\text{MeV}) \approx 1.3109$ for set **B** is slightly larger than that for set **A**, one cannot simply favor fit **A** on the basis of a smaller χ^2 value. The problem is that one can increase Δ above the critical value $\Delta_c = 35\,\text{MeV}\left[\frac{\chi^2_B(35\text{MeV})}{\chi^2_A}\right]^{1/2} \approx 45.55$ MeV, and, in this way, it is possible to reduce $\chi^2_B(\Delta > \Delta_c)$ below χ^2_A. Clearly, the re-scale of the common error Δ would not change the values of fitting parameters and the minimum location for set **B**. Therefore, it seems that hypothesis **B** with the common error $\Delta > \Delta_c$ is more probable than hypothesis **A**, but rather small value of χ^2_A does not allow us to simply reject it. Thus, we need some additional criterion to favor one of these hypotheses.

Such a criterion is provided by the FWM which predicts the asymptotic behavior of the width of large/heavy QG bags on the basis of the lattice QCD data. Indeed, the FDM [8] allows one to extract the asymptotic mass dependence of the QG bag width $\Gamma_{\rm R}$ of the mean volume $V_r = \frac{V_0}{M_0} M_r$ and mass M_r,

$$\Gamma_{\rm R}(M_r) = 2\, C_\gamma \sqrt{\frac{2\,\ln(2)\, T_{\rm co}^5\, V_0}{M_0} M_r}\,, \qquad (18)$$

from a variety of lattice QCD data [24–26] for the vanishing baryonic density. Here, $V_0 = 1$ fm^3 is the minimal volume of large QG bags, $T_{\rm co}$ is the cross-over temperature at zero baryonic density, and the constant C_γ weakly depends on the number of elementary degrees of freedom in the analyzed lattice QCD model [8]: $C_\gamma \approx 1.28$ corresponds to the pure gluodynamics for the $SU(2)_C$ color group [24], $C_\gamma \approx 1.22$ describes the $SU(3)_C$ color group lattice QCD data with two quark flavors [25], and the value $C_\gamma \approx 1.3$ corresponds to the recent lattice QCD data for the $SU(3)_C$ color group with three quark flavors [26].

Equating the asymptotic form of the width from Eq. (18) with those obtained from the fitting of sets **A** and **B**, we can determine the corresponding value of the cross-over temperature and compare it with the established value. For the large resonance mass limit $M_r \to \infty$ in Eq. (4), one finds the asymptotic behavior of the resonance phase and width

$$\phi_r\Big|_{M_r\to\infty} \;\to\; A_{\rm R}\,\frac{\sin\left(\frac{3}{4}\pi\right)}{\sqrt{M_r}} \;\to\; -0\,, \qquad (19)$$

$$\Gamma_r\Big|_{M_r\to\infty} \;\to\; -M_r\,\phi_r \;\to\; -A_{\rm R}\sqrt{\frac{M_r}{2}}\,. \qquad (20)$$

From Eqs. (18) and (20), we find out that

$$T_{\rm co} = \left[\frac{A_{\rm R}^2\, M_0}{\ln(2)(4\,C_\gamma)^2\, V_0}\right]^{1/5}. \qquad (21)$$

Since we can neglect the small contribution of $s\bar{s}$-state (s is the strange quark) in the ω mesons, the mesons involved in sets **A** and **B** are built from the light u and d quarks. The corresponding value of C_γ is 1.22. Taking this value of C_γ and using the $A_{\rm R}$ values from Table 2 with the corresponding fitting errors, we determine the possible values of the cross-over temperature for each set: $T^A_{\rm co} = 157.1 \pm 2.2$ MeV and $T^B_{\rm co} = 175.15 \pm 4.05$ MeV. This result rules out hypothesis **A**, since its cross-over temperature is lower even than the chemical freeze-out temperature of most abundant hadrons $T_{\rm chem} = 165 \pm 5$ MeV extracted from the nucleus-nucleus collisions at RHIC energy $\sqrt{s_{NN}} = 130$ GeV [27, 28]. On the other hand, the cross-over temperature $T^B_{\rm co}$ for set **B** is in good agreement with the freeze-out temperature $T_{\rm early} = 170 \pm 5$ MeV of early hadronizing particles for which the kinetic and chemical freeze-outs occur simultaneously at the very moment of their hadronization. As we can see from the values of $T^B_{\rm co}$ and $T_{\rm early}$, this is the case for set **B**. Thus, the data on the early freeze-out temperature favor hypothesis **B**.

The early freeze-out temperature was established for the first time for J/ψ, ψ' mesons and $\Omega^\pm$ hyperons at the SPS laboratory energy $E^{\rm lab}_{NN} = 158$ GeV [29] and for ϕ mesons and $\Omega^\pm$ hyperons at the RHIC energy $\sqrt{s_{NN}} = 130$ GeV [30]. The same conclusion on the early hadronization phenomenon is supported by the recent analysis of the transverse momentum spectra of J/ψ, ϕ mesons and $\Omega^\pm$ at the top RHIC energy $\sqrt{s_{NN}} = 200$ GeV [31]. The complete systematic study on the early kinetic freeze-out of these hadrons and their hadronization process can be found in the recent review article [32].

We also would like to draw attention to the discrepancies between the experimental data on the resonance width and their fits. Their mass dependence is shown in Fig. 3. As one can deduce from Fig. 3, the difference between each data set and its fit has some periodic structure, which is more clearly seen for sets **B** and **D** (for more details, see [11]). The developed approach would allow us to accurately extract such a fine structure, if the experimental uncertainties were smaller. Therefore, to firmly establish this fine structure of the resonance width as compared with the asymptotic Regge trajectory, we need much more accurate data for all analyzed mesons. However, if this fine structure, indeed, exists, then the mean width of the mesons of spin $J = 7$ (and mass $M_7 \approx 2.723 \pm \Delta_c$ GeV) should be $\Gamma_7 \approx 0.3 \pm \Delta_c$ GeV instead of the with the set **B** fit value $\Gamma^B_7 \approx 0.36 \pm \Delta_c$ GeV, while the mean width of the meson of spin $J = 8$ (and mass $M_8 \approx 2.923 \pm \Delta_c$ GeV) should match the fit **B** value $\Gamma^B_8 \approx 0.384 \pm \Delta_c$ GeV, where, for an *a* priori uncertainty, we used the critical value of the common error Δ_c for set **B**. Hopefully, these predictions can be used to justify the existence of the fine structure in the analyzed Regge trajectories.

6. Results for Strange Mesons

Using the main conclusion of the preceding section and the experimental data [21], we constructed the data set **D** by taking the arithmetic average of masses and widths of the strange mesons K^*_{JP} of parity $P = (-1)^J$ and ρ_{J--}, a_{J++}, ω_{J--}, f_{J++} for the same spin value J and again choosing the common error $\Delta = 0.035$ GeV (democratic choice). Hence, **hypothesis D** is similar to **hypothesis B**, but it utilizes the larger set of experimental data. We also performed the four-parameter fit of set **D** and give its results in Tables 1 and 2 and Figs. 1–3. As one can see from the columns $B_{\rm exp}$ and $D_{\rm exp}$ of Table 1, the maximal difference between the data set is about 60 MeV, i.e it is less than 2Δ for the width Γ_5, whereas it is smaller than Δ in all other cases. Despite the fact that the number of independent degrees of freedom for set $D_{\rm exp}$ is 6, i.e. it is smaller than for sets $A_{\rm exp}$ and $B_{\rm exp}$, the higher stability of the data points of this set leads to a smaller value of the resulting χ^2 and to a wider range of error bars for the fitting parameters, as it is seen from Table 2.

Here, we show how one can use the fitting results for set **D**. One can determined the data of the K^*_J meson of spin J from the evident symbolic expression $3D_{\rm fit} - 2B_{\rm fit}$ by taking the data from the corresponding rows of Table 1. Thus, for K^*_5, one obtains: $M_5^{fit} \approx 2.396 \pm \Delta$ GeV and $\Gamma_5^{fit} \approx 0.199 \pm \Delta$ GeV, whereas their experimental values [21] are $M_5 \approx 2.382 \pm 0.033$ GeV and $\Gamma_5 \approx 0.178 \pm 0.069$ GeV, respectively. This example demonstrates that our prediction for the mass and the width of K^*_6 meson $M_6^{fit} \approx 2.662 \pm \Delta$ GeV and $\Gamma_6^{fit} \approx 0,186 \pm \Delta$ GeV is rather realistic, and it is not sensitive to the fine structure of the width, although the latter is clearly seen in Fig. 3 for set **D**.

As an independent check up of the obtained results, we determine the cross-over temperature at the vanishing baryonic density for QCD with the $SU(3)_C$ color group and with three quark flavors using Eq. (21) and $C_\gamma \approx 1.3$ [8]: $T^D_{\rm co} = 161.2 \pm 6.7$ MeV, which is well consistent with the early freeze-out temperature $T_{\rm early} = 170 \pm 5$ MeV of J/ψ, ϕ, ψ' mesons, and $\Omega^\pm$ hyperons found out for the laboratory energies at [29] and above [30,31] $E^{\rm lab}_{NN} = 158$ GeV and with the chemical freeze-out temperature of the most abundant hadrons $T_{\rm chem} = 165 \pm 5$ found at the RHIC energy $\sqrt{s_{NN}} = 130$ GeV [27,28]. Note also that the cross-over temperature $T^D_{\rm co}$ is in good agreement with the cross-over temperature obtained for the renormalized Polyakov loop $T^{LQCD}_{\rm co} = 170 \pm 4 \pm 3$ MeV by the recent lattice QCD simulations (see Table 3 in [33]).

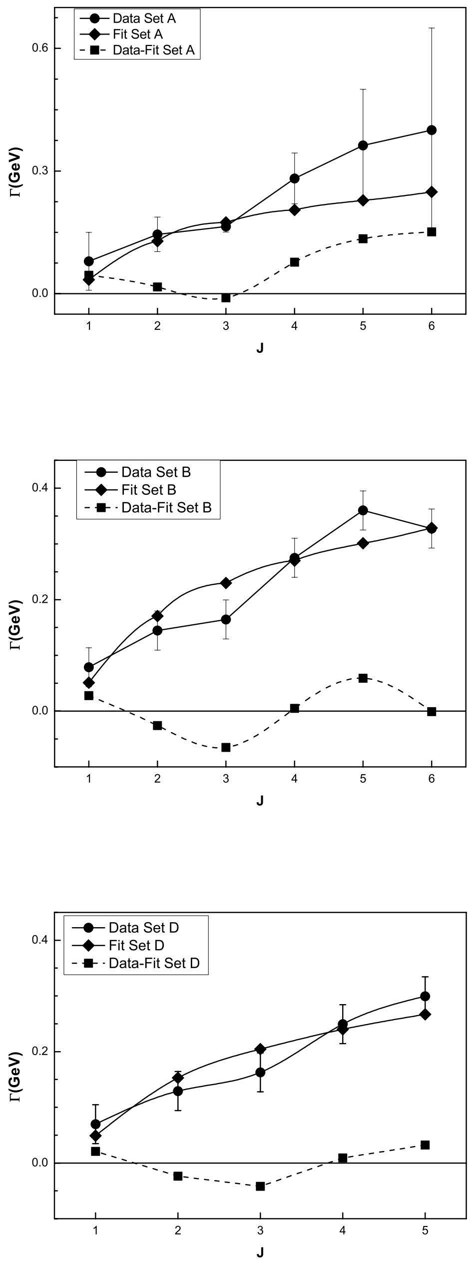

Fig. 3. The upper part of each plot shows the resonance width of mesons as a function of their spin J. The lower part of each plot shows the difference of the resonance width of mesons and the obtained value of the fit as functions of the resonance spin. The upper, middle, and lower panels correspond to sets **A**, **B**, and **D**, respectively. The curves in all panels are shown to guide the eyes

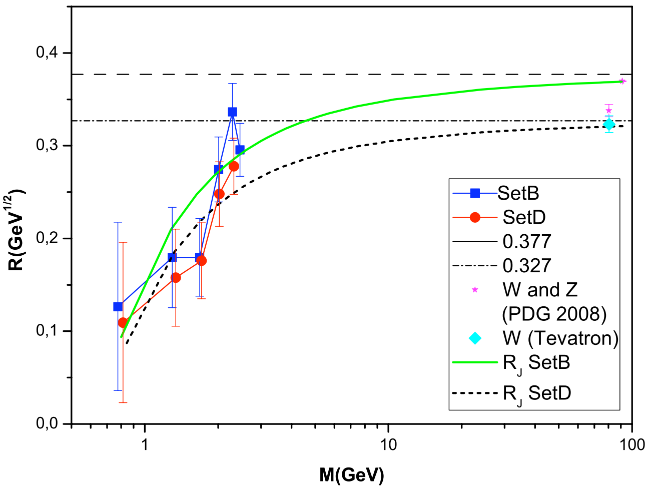

Fig. 4. Behavior of the ratio $R = \Gamma\sqrt{\frac{2}{M}}$ (in GeV$^{1/2}$) as a function of the resonance mass M is shown for sets **B** (squares) and **D** (circles), for W boson (the full estimate taken from [21] $R^W = 0.338$ GeV$^{1/2}$ and the Tevatron data $R^W = 0.323$ GeV$^{1/2}$ [21]) and for Z boson ($R^Z = 0.37$ GeV$^{1/2}$). The solid (short dashed) curve corresponds to the ratio R of the asymptotic Regge trajectory for the fitting parameters $B_{\rm fit}$ ($D_{\rm fit}$) of Table 2

7. Comparison with Heavy Gauge Bosons

After the completion of our analysis for the nonstrange and strange mesons, we decided to check the fact whether the widths of W and Z bosons are somehow related to the found parameters of the asymptotic Regge trajectories of light mesons, since the gauge bosons are the heaviest bosons available for examination. The results of such a comparison are shown in Fig. 4 for the ratio $R = \Gamma\sqrt{\frac{2}{M}}$ as a function of the resonance mass M. From this figure, one can see that, despite the very small error bars of R^Z for Z boson, it is just located at the curve obtained for set **B**, whereas the corresponding point for W boson is located within 2.75 standard deviations from the ratio found for set **D**. We have to stress that the average values for the W boson mass and width taken from [21] account for the results of experiments with the nonoverlapping error bars. If, however, we take only the data of two Tevatron experiments which, indeed, overlap, then the obtained ratio R just matches the set **D** value! Hence, it is very hard to regard such good double match as a simple double coincidence, but there are two additional facts. First, the mesons forming the data set **B** are dominantly composed of u and d quarks and antiquarks and have a small contribution of $s\bar{s}$ states. Therefore, if the resulting asymptotic Regge trajectory of set **B** corresponds to some particles, they should dominantly decay into s quark and its analog in the third quark family, b quark, via $s\bar{s}$ and $b\bar{b}$ states. Is it a coincidence that the hadronic decays of Z boson with s and b quarks go mainly via $s\bar{s}$ and $b\bar{b}$ states, whereas the Z boson decays via s quark (through B_s^0 meson) and via b quark (through B^+ meson), have, respectively, rather small partial branching ratios 1.56 ± 0.13 % and 6.1 ± 0.14 % of the total width [21]? Second, in constructing set **D**, the masses and the widths of mesons with isospin values 0 (ω, f mesons), 1 (ρ, a mesons), and $\frac{1}{2}$ (K^* mesons) were taken for each spin value with the weight $\frac{1}{3}$ according to our democratic choice. Therefore, if the resulting asymptotic Regge trajectory of set **D** corresponds to some particles, they should decay into the mesons containing s quark with a probability of about $\frac{1}{3}$. Is it again a coincidence that the partial branching ratio of W boson via the s quark ($s\bar{c}$ for W^- boson), whose ratio R^W for the Tevatron data [21] belongs to the corresponding value of set **D**, is about 32^{+13}_{-11} % [21], i.e. within the existing error bars it is close to one third? Note that if, in the set **D** construction, we took other weight for the K^* mesons than $\frac{1}{3}$, say $\frac{1}{2}$, then we could not describe R^W so well. Therefore, we believe there are too many coincidences, which, if it is the case, require four independent explanations.

In our opinion, all these "coincidences" can be explained simultaneously by the following hypothesis: the widths of Z and W bosons are described by the asymptotic Regge trajectories of mesons because these bosons belong to the Regge trajectory of the form (12). The idea of Reggeization of W and Z bosons was suggested long ago [34], but the concrete predictions out of it were made much later [35]. There several possible physical explanations for the origin of the Regge trajectories of Z and W bosons, but the most promising ones are provided by the technicolor model [36], the t quark condensate model [37], and the model of haplons suggested recently by H. Fritzsch [38]. Instead of discussing which of these models gives a better explanation of the found facts, we prefer to consider the possible physical consequences of the suggested hypothesis which can be verified at LHC.

Thus, from Eqs. (4) and the review of the particle properties [21], one can determine the trajectory phase for W and Z bosons $\phi_W \approx -0.02608$ and $\phi_Z \approx -0.02736$. Since these phases are rather small, it is possible to use the asymptotic expressions (19) and (20). For instance, from (20), one can get an estimate for the parameter $A_{\rm R}$ for W and Z bosons as $A_{\rm R}^W \approx -0.3377$ GeV$^{1/2}$ and $A_{\rm R}^Z \approx -0.3695$ GeV$^{1/2}$, which, in fact, deviate from the corresponding values for sets **D** and **B** by about 3 % and 2 %, respectively. Similarly, one can approximate (15) for $J = 1$ as

$$M_r(J) \approx \frac{1}{g_{\rm R}} \sqrt{J - \alpha_0}\,. \qquad (22)$$

Since the ratio R^Z for Z boson belongs to the set **B** curve, and R^W for W boson is very close to the set **D** curve (see Fig. 4), we can use their parameters α_0 in order to estimate the asymptotic value of the slope parameter $g_{\rm R}^2$ for the corresponding Regge trajectories and get $g_{\rm R}^Z \approx 8.028 \cdot 10^{-3}$ GeV^{-1} and $g_{\rm R}^W \approx 9.688 \cdot 10^{-3}$ GeV^{-1}. These numbers allow us to find the effective string tension from the traditional expression

$$\sigma_{\rm str} \approx \frac{1}{2\,\pi\, g_{\rm R}^2} \qquad (23)$$

and to get $\sigma_{\rm str}^Z \approx (49.7)^2$ GeV^2 and $\sigma_{\rm str}^W \approx (41.2)^2$ GeV^2. These rough estimates show us that compared to the string tension in QCD which is about $(0.42)^2$ GeV^2 [39], the values of the effective string tension for W and Z bosons are about 10^4 times larger, which, probably, evidences that the confinement mechanism of composite W and Z bosons may differ from the confinement mechanism existing in hadrons.

It is clear that Eq. (22) is also valid for all natural values of spin J. Therefore, one can use (22) and (20) to predict the mass and the width of the excited states of W and Z bosons using the expressions ($r \in \{W, Z\}$)

$$M_r(J) \;\approx\; M_r(1) \sqrt{\frac{J - \alpha_0}{1 - \alpha_0}}\,, \qquad (24)$$

$$\Gamma_r(J) \;\approx\; \Gamma_r(1) \sqrt[4]{\frac{J - \alpha_0}{1 - \alpha_0}}\,, \qquad (25)$$

where, for Z (W) boson, one should use the α_0 value for set **B** (**D**) from Table 2. Then we obtain $M_Z(3) \approx 199$ GeV, $\Gamma_Z(3) \approx 3.76$ GeV, $M_Z(5) \approx 265.7$ GeV, $\Gamma_Z(5) \approx 4.35$ GeV for Z boson and $M_W(3) \approx 165$ GeV, $\Gamma_W(3) \approx 3$ GeV, $M_W(5) \approx 219$ GeV, $\Gamma_W(5) \approx 3.46$ GeV for W boson. Clearly, the values of R ratio found for these estimates of the W and Z boson Regge partners belong, respectively, to the set **D** and set **B** curves shown in Fig. 4. For a comparison, we recall that the original predictions based on the Reggeization mechanism of W and Z bosons suggested in [35] gave much larger mass and width values for the excited gauge bosons $M_W(3) \approx M_Z(3) \approx 1$ TeV, $\Gamma_W(3) \approx \Gamma_Z(3) \approx 200$ GeV. Therefore, the observation of one of these states would not only provide us with a good test for the validity of the hypothesis that the gauge bosons, indeed, form the Regge trajectories, but would also distinguish our hypothesis from that of Ref. [35].

A few words should be said about the reasons why none of these states were ever observed in $e^+ + e^-$ and $p + \bar{p}$ collisions. First of all, these states have a sizable width, and, hence, their observation is technically difficult because it is hard to distinguish them from the background. Second, the states with spin $J \geq 3$ require a thorough analysis of higher partial waves which, for large values of collision energy $\sqrt{s}$, are kinematically suppressed as compared with that of the S-wave [6]. Even the recent searches for W' boson performed by the LHC experiments are aimed at the mass scale at and above 1 TeV, see, for instance, [40, 41]. We, however, would like to stress, first, that the predicted range of the W and Z Regge partners is far below the experimentally studied area and, second, the difficulties of their experimental detection may tremendously increase, if the main decays of the W and Z Regge partners go via the low-lying Regge states of the same trajectory.

Similar to [35], we extend our hypothesis on the Regge trajectories of W and Z bosons to the Higgs mesons. However, in contrast to [35], we assume that the Higgs trajectory, if it exists, has the form (12), and its parameters are of the same order of magnitude as those we found for W and Z bosons. In fact, the line of arguments of [34] can be used to justify such an assumption for the Regge partners of the Higgs trajectory with the spin values $J = 0, 2, 4, ...$ In addition, we note that, if the studied Regge trajectories are, indeed, universal, then it is our educated guess that the parameters of the Higgs trajectory, if it exists, should be similar to those of the analyzed trajectories.

Earlier, we saw that the spin of W and Z bosons is very sensitive to the values of mass, $g_{\rm R}$ and α_0. Hence, one has to apply them to the Higgs trajectory with great care. Moreover, we cannot blindly use them for the scalar Higgs bosons, since, in this case, one would simply get an imaginary mass value from (22) for α_0 values assumed for W and Z bosons. On the other hand, the dependence of the width on the mass and the coefficient $A_{\rm R}$ is not too sensitive. Therefore, we assume that the coefficient $A_{\rm R}^{\rm H}$ of the Higgs trajectory is close to the arithmetic average of the corresponding coefficients for W and Z bosons, i.e. $A_{\rm R}^{\rm H} \approx \frac{1}{2}(A_{\rm R}^W + A_{\rm R}^Z) \approx -0.354 \pm 0.027$ GeV$^{1/2}$. Consequently, we can predict that the mass dependence of the scalar Higgs trajectory width is $\Gamma_{\rm H} = -A_{\rm R}^{\rm H} \sqrt{\frac{M_{\rm H}}{2}}$. Using the predictions for the Higgs width as a function of its mass [42], we found that the values $M_{\rm H} \approx 251.5 \pm 2.6$ GeV and $\Gamma_{\rm H} \approx 3.97 \pm 0.22$ GeV of the scalar Higgs mass and width are consistent with our hypothesis for the standard model results, whereas the values $M_{\rm H} \approx 1335 \pm 247$ GeV and $\Gamma_{\rm H} \approx 9.136 \pm 0.895$ GeV are consistent with the minimal supersymmetric extension of the standard model with $\tan\beta = 30$ in the scenario with maximal mixing [42]. Since the parameter α_0 for the Higgs trajectory is unknown, we can only predict the spin dependence of the mass and width of the Higgs Regge partners

$$M_{\rm H}(J) \;\approx\; M_{\rm H}\sqrt{\frac{J-\alpha_0}{-\alpha_0}}\,, \qquad (26)$$

$$\Gamma_{\rm H}(J) \;\approx\; \Gamma_{\rm H}\sqrt[4]{\frac{J-\alpha_0}{-\alpha_0}}\,, \qquad (27)$$

for $J = 2, 4, ...$ provided that $\alpha_0 < 0$. Note that although the last inequality looks bit unusual, it does not contradict any consequences of the Froissart theorem [6]. Since our estimate for the standard model Higgs mass is essentially larger than its upper bound based on the standard model predictions [21], this fact, probably, evidences for the need to extend the standard model.

If, however, we take the results of the global recent electroweak fit of the Standard Model, that $M_{\rm H} = 82.8^{+30.2}_{-23.3}$ GeV ($M_{\rm H} = 119.4^{+13.4}_{-4.0}$ GeV) obtained without (with) using information from direct Higgs searches [43], then the width values for these predictions are $\Gamma = 2.735^{+0.15}_{-0.046}$ GeV ($\Gamma = 2.278^{+0.383}_{-0.347}$ GeV). These are the numbers that can be compared with the experimental findings. Hopefully, the experimental searches for the Higgs bosons at LHC will allow us either to make more definite predictions or to disprove our hypotheses.

8. Conclusions

In this work, we analyzed the asymptotic behavior of Regge trajectories of nonstrange and strange mesons. With the help of the approach developed in [7], we showed that a widely circulating belief that the width of heavy hadrons linearly depends on their mass simply contradicts the existence of asymptotically linear Regge trajectories which is expected to be the case by the open string model [44, 45], the closed string model [44], the anti-de-Sitter conformal field theory [46], and the FWM [9].

We analyzed the common data sets for masses and widths of ρ_{J--}, ω_{J--}, a_{J++} and f_{J++} mesons for the spin values $J \leq 6$ and of K^*_J mesons for $J \leq 5$ in order to elucidate the parameters of the asymptotically linear Regge trajectories that are consistent with the FWM predictions. Thus, we verified hypotheses **A** and **B** for nonstrange mesons and hypothesis **D** for the strange ones which differently define the data uncertainties. Since the worked out fitting procedure employs the exact expressions for the resonance mass and width derived from the Regge trajectory in the complex energy plane, we obtained a high-quality fit. Thus, it is shown that the data sets for nonstrange mesons, **A** and **B**, can be fitted with a rather small value of χ^2 per degree of freedom of about 0.774, and the χ^2 for the data set **D** is 0.577. We used the results of the fit to estimate the cross-over temperature value based on the predictions of the FWM and obtained $T^A_{\rm co} = 157.1 \pm 2.2$ MeV for set **A**, $T^B_{\rm co} = 175.15 \pm 4.05$ MeV for set **B** and $T^D_{\rm co} = 161.2 \pm 6.7$ MeV for set **D**. As it is argued, the cross-over temperature obtained from set **A** is incompatible with the early freeze-out temperature of J/ψ, ϕ, ψ' mesons, and $\Omega^{\pm}$ hyperons found out for the laboratory energies at and above $E^{\rm lab}_{NN} = 158$ GeV, whereas the cross-over temperature of sets **B** and **D** are consistent with the early freeze-out temperature of these particles.

A detailed examination of the mass dependence of the resonance width for the analyzed sets of data led us also to a conclusion of a possible fine (periodic) structure compared with the asymptotic behavior of the Regge trajectory. This result might be of a great interest for the string models of hadrons. However, to firmly establish the existence of such a structure, we need, firstly, a higher accuracy of experimental data and, secondly, a thorough verification of the predictions made for the widths and the masses of the nonstrange resonances of spin 7 and 8. Using the fit for set **D**, we predicted the mass and the width of K^*_6 meson.

Comparing the resonance widths of sets **B** and **D** evaluated at the mass values of Z and W bosons, respectively, we found very close matches between these width values and those of the gauge bosons. We argue that such matches give us the indirect, but, nevertheless, first experimental evidence for the compositeness of Z and W bosons. To explain these facts, we suggested a hypothesis that Z and W bosons have the asymptotic Regge trajectories whose functional dependence on the invariant mass squared is similar to that of the nonstrange and strange mesons analyzed here. Using this hypothesis, we showed that the effective string tension for Z and W bosons is about 10^4 times larger than that of QCD which may evidence that the confinement mechanism of composite gauge bosons differs from the confinement mechanism existing in hadrons. Such a hypothesis allowed us to estimate the masses and the widths of Regge partners of these bosons.

Following the logic of [34, 35] and the results obtained for Z and W bosons, we suggested a hypothesis that the scalar Higgs boson can have Regge partners as well. With the help of the scalar Higgs boson width predictions, we estimated its mass and width $M_{\rm H} \approx 251.5 \pm 2.6$ GeV and $\Gamma_{\rm H} \approx 3.97 \pm 0.22$ GeV which are compatible with the standard model conjecture for the Higgs boson width, whereas the values $M_{\rm H} \approx 1335 \pm 247$ GeV and $\Gamma_{\rm H} \approx 9.136 \pm 0.895$ GeV are consistent with the minimal supersymmetric extension of the standard model with $\tan\beta = 30$ in the scenario with maximal mixing. Because our estimate for the standard model Higgs mass is essentially larger than its upper bound based on the other predictions of this model [21, 43], we conclude that this fact evidences for a necessity to extend the standard model.

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1. G.F. Chew and S.C. Frautschi, Phys. Rev. Lett. **7**, 394 (1961); V.N. Gribov, ZhETF **41**, 667 (1961).
2. A. Dubin, A.B. Kaidalov, and Yu. Simonov, Phys. Lett. **B** 323, 41 (1994).
3. A.B. Kaidalov and Yu.A. Simonov, Phys. Lett. **B** 477, 163 (2000).
4. see a detailed discussion in A. Tang and J.W. Norbury, Phys. Rev. **D** 62, 016006 (2000).
5. more references can be found in M. Ida, Prog. Theor. Phys. **40**, 901 (1968); R.W. Childers, Phys. Rev. Lett. **21**, 868 (1968).
6. P.D.B. Collins, An *Introduction to Regge Theory and High-Energy Physics* (Cambridge Univ. Press, Cambridge, 1977).
7. A.A. Trushevsky, Ukr. J. Phys. **22**, 353 (1977).
8. K.A. Bugaev, V.K. Petrov, and G.M. Zinovjev, Phys. Rev. C **79**, 054913 (2009); K.A. Bugaev and G.M. Zinovjev, Ukr. J. Phys. **55**, 586 (2010); and K.A. Bugaev, arXiv:0906.4273 [nucl-th].
9. K.A. Bugaev, V.K. Petrov, and G.M. Zinovjev, Europhys. Lett. **85**, 22002 (2009); arXiv:0801.4869 [hep-ph]; K.A. Bugaev, arXiv:0809.1023 [nucl-th].
10. R. Hagedorn, Suppl. Nuovo Cimento **3**, 147 (1965).
11. K.A. Bugaev, E.G. Nikonov, A.S. Sorin, and G.M. Zinovjev, JHEP **02**, 059 (2011); arXiv:1012.0150 [hep-ph].
12. D.B. Blaschke and K.A. Bugaev, Fizika B **13**, 491 (2004); Phys. Part. Nucl. Lett. **2**, 305 (2005).
13. C. Greiner and S. Leupold, J. Phys. G **27**, L95 (2001).
14. C. Greiner *et al.*, J. Phys. G **31**, S725 (2005).
15. J. Noronha-Hostler, C. Greiner, and I.A. Shovkovy, Phys. Rev. Lett. **100**, 252301 (2008); J. Phys. G **37**, 094017 (2010); J. Noronha-Hostler, M. Beitel, C. Greiner, and I.A. Shovkovy, Phys. Rev. C **81**, 054909 (2010); J. Noronha-Hostler and C. Greiner, arXiv:1008.5075 [nucl-th] and references therein.
16. W. Ehehalt and W. Cassing, Nucl. Phys. A **602**, 449 (1996); J. Geiss, W. Cassing, and C. Greiner, Nucl. Phys. A **644**, 107 (1998); W. Cassing and E.L. Bratkovskaya, Phys. Rep. **308**, 65 (1999).
17. S.A. Bass *et al.*, Prog. Part. Nucl. Phys. **41**, 225 (1998).
18. F. Lizzi and I. Senda, Phys. Lett. B **244** 27 (1990); I. Senda, Phys. Lett. B **263** 270 (1991); F. Lizzi and I. Senda, Nucl. Phys. B **359** 441 (1991).
19. W. Broniowski, W. Florkowski, and L.Y. Glozman, Phys. Rev. **D 70**, 117503 (2004).
20. see, for instance, P. Desgrolard, M. Giffon, E. Martynov, and E. Predazzi, Eur. Phys. J. C **18**, 555 (2001) and references therein.
21. C. Amsler *et al.*, Phys. Lett. B **667**, 1 (2008) [http://pdg.lbl.gov].
22. A.B. Kaidalov, in *At the Frontier of Particle Physics: Handbook of QCD*, edited by M. Shifman (World Scientific, Singapore, 2002) (arXiv:0103011 [hep-ph]).
23. L.Ya. Glozman, Acta Phys. Pol. B **35**, 2985 (2004).
24. J. Engels, F. Karsch, J. Montway, and H. Satz, Nucl. Phys. B **205**, 545 (1982).
25. T. Celik, J. Engels, and H. Satz, Nucl. Phys. B **256**, 670 (1985).
26. M. Cheng et al., Phys. Rev. D **77**, 014511 (2008).
27. P. Braun-Munzinger, D. Magestro, K. Redlich, and J. Stachel, Phys. Lett. B **518**, 41 (2001).
28. J. Cleymans, B. Kämpfer, M. Kaneta, S. Wheaton, and N. Xu, Phys. Rev. C **71**, 0409071 (2005).
29. K.A. Bugaev, M. Gazdzicki, and M.I. Gorenstein, Phys. Lett. B **523**, 255 (2001); K.A. Bugaev, J. Phys. G **28**, 1981(2002); M.I. Gorenstein, K.A. Bugaev, and M. Gazdzicki, Phys. Rev. Lett. **88**, 132301 (2002).
30. K.A. Bugaev, M. Gazdzicki, and M.I. Gorenstein, Phys. Rev. C **68**, 017901 (2003); Phys. Lett. B **544**, 127 (2002).
31. S.V. Akkelin, P. Braun-Munzinger, and Yu.M. Sinyukov, Phys. Rev. C **81**, 034912 (2010).
32. P. Braun-Munzinger and J. Wambach, Rev. Mod. Phys. **81**, 1031 (2009).
33. Y. Aoki *et al.*, JHEP **06**, 088 (2009).
34. A.A. Anselm and V.N. Gribov, Nucl. Phys. B **61**, 253 (1973).
35. A.A. Anselm and M.G. Ryskin, Phys. Lett. B **329**, 300 (1994).
36. E. Fahri and L. Susskind, Phys. Rep. **84**, 277 (1981); K. Yamawaki, Prog. Theor. Phys. Suppl. **167**, 127 (2007).

37. V.A. Miransky and K. Yamawaki, Mod. Phys. Lett. A **4**, 1043 (1989).
38. H. Fritzsch, arXiv:1010.1428 [hep-ph] 8 p.
39. A. Mocsy and P. Petreczky, PoS LAT2007 **216** (2007).
40. The CMS Collaboration, Phys. Lett. B **698**, 21 (2011); arXiv:1012.5945 [hep-ex].
41. The CMS Collaboration, arXiv:1103.0030 [hep-ex].
42. see a review M. Carena and H.E. Haber, Prog. Part. Nucl. Phys. **50**, 63 (2003) and references therein.
43. J. Haller, arXiv:1006.003 [hep-ph].
44. E.V. Shuryak, Prog. Part. Nucl. Phys. **62**, 48 (2009).
45. I. Senda, Z. Phys. C **55**, 331 (1992).
46. see, for instance, A. Karch *et al.*, Phys. Rev. D **74**, 015005 (2006) and references therein.

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ПРО АСИМПТОТИКУ ТРАЄКТОРІЙ РЕДЖЕ ВАЖКИХ МЕЗОННИХ РЕЗОНАНСІВ

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Р е з ю м е

Проведено аналіз асимптотичної поведінки траєкторій Редже недивних та дивних мезонів і знайдено, що для цих траєкторій ширина важких адронів не може лінійно залежати від їхньої маси. Такий результат явно демонструє, що широко поширена впевненість у лінійній залежності ширини резонансів від їхньої маси суперечить лінійності траєкторії Редже за мандельстамівською змінною s. Використовуючи дані по масам та ширинам для ρ_{J--}, ω_{J--}, a_{J++} і f_{J++} мезонів зі спінами $J \leq 6$ і для K_J^* мезонів зі спінами $J \leq 5$, ми визначили параметри асимптотично лінійних траєкторій Редже, які передбачено моделлю кінцевої ширини для кварк-глюонних мішків. Показано, що параметри, отримані для набору даних B і D, сумісні з температурою кросовера, визначеною в симуляціях ґраткової КХД за нульових значень баріонної щільності та з температурою кінетичного фрізаута частинок, що рано адронізуються, знайденої в релятивістських зіткненнях важких іонів за найвищої енергії SPS та за енергій вищих за неї. Порівнюючи ширину резонансів для наборів даних B і D, які, відповідно, були обчислені для мас Z і W бозонів ми знайшли, що обчислені значення ширин збігаються з ширинами калібрувальних бозонів. Наведено аргументи на користь того, що такий збіг дає нам непряме, але перше експериментальне свідчення про складений характер Z та W бозонів. На основі цих результатів ми припустили, що Z, W та Хіггс бозони мають траєкторії Редже, подібні до асимптотичних траєкторій проаналізованих мезонів. Зроблено передбачення для мас і ширин Редже партнерів Z і W бозонів та для масової залежності ширини Редже партнерів бозонів Хіггса, а також дано значення для маси і ширини скалярних мезонів Хіггса.