ELECTRICAL FRIEDERICKSZ TRANSITION IN A NEMATIC CELL WITH PERIODIC POLAR ANCHORING ENERGY

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The threshold value for the electrical Friedericksz transition in a nematic liquid crystal cell with the periodic energy of director anchoring with the cell surface has been derived, and the above-threshold spatial distribution of the director in the applied electric field has been determined. The threshold value was shown to depend nonmonotonously on the number s of anchoring energy periods across the cell length. The above-threshold distribution of the director at integer s traces a periodic variation of the anchoring energy. The amplitude of the director's periodic deviation grows with the reduction of the ratio between the cell thickness and the anchoring energy period.

1. Introduction

A considerable interest in the uniform threshold reorientation of the director in nematic liquid crystals (NLCs) by external fields is associated with a widespread use of nematic cells as basic components in electron-optical devices [1–3]. In an external electric or magnetic field, there can also emerge a threshold spatially periodic structure of the director field in an NLC cell. In particular, in works [4, 5], the emergence of a spatial periodic structure of the director was studied at its threshold reorientation from the planar into the homeotropic state in a flexoelectric nematic cell with infinitely rigid boundary conditions. The influence of the finite energy of anchoring of the director with the surface in a flexoelectric NLC on the threshold and the period of a spatial distribution of the director that arises at the director reorientation in an electric field from the planar state into the homeotropic one [6] and *vice versa* [7], as well as at the planar-planar director reorientation [8], was considered. It was also found that, depending on the ratio between the Frank elastic moduli K_1 and K_2, the threshold spatially periodic structure of the director can arise in the absence of flexoelectric properties as well, at both the planar-homeotropic [6, 9–12] and planar-planar [13] director reorientations. The threshold periodic-in-space reorientation of the director in a homeotropic nematic cell in a light field with spatially modulated intensity was considered in work [14]. In work [15], a possibility to obtain a spatial periodic structure of the director at its planar-planar reorientation in a light field was demonstrated. The appearance of spontaneous spatial periodic distortions of the director in a planarly oriented NLC cell and the influence of the elastic constant K_{24} on them were considered in works [16–18].

Although the spatial periodic reorientation of the director is a bulk effect, its character depends, however, considerably on the strength and the type of NLC interaction with the cell surfaces. In the works mentioned above, the conditions imposed on the director were assumed uniform, with either finite or infinite anchoring energy. In work [19], the influence of a modulation of the easy-orientation axis of the director at the cell surface on the spatial periodic distribution of the director field in a flexoelectric nematic was studied. In work [20], spontaneous transitions between two orientational states in a semiinfinite nematic with periodically alternating planar and homeotropic boundary conditions at the surface were studied. The influence of a periodicity in the energy of director anchoring with the cell surface on the phase transition nematic–isotropic liquid was considered in work [21]. The interaction between substrates in an NLC cell with periodically alternating planar and homeotropic boundary conditions was studied in work [22], and the influence of the anchoring energy periodicity and thermal fluctuations of the director on this interaction in work [23]. In work [24], the emergence of domain structures in thin nematic films with periodic homeotropic planar boundary conditions was examined.

In this work, the emergence of a spatial periodic structure of the director in an NLC cell with a periodic distribution of the anchoring energy, which is induced by a constant electric field, has been considered.

2. Director Equations

Consider a plane-parallel cell with a nematic liquid crystal confined by the planes $z = -L/2$ and $z = +L/2$. The initial planar orientation of the director is along the axis Ox. The cell is embedded into an external uniform static electric field; the field strength vector $\mathbf{E}_0$ is directed along the axis Oz. We assume that the anchoring energy of the director with the cell surfaces at its reorientation in the plane xOy (the azimuthal energy) is infinitely high, and the anchoring energy connected with director deviations in the plane xOz (the polar energy) is a periodic function of the coordinate y. For definiteness, it can be presented in the form

$$W(y) = W_0 + V\cos\frac{2\pi y}{d}, \tag{1}$$

where W_0 and V are constants ($0 < V < W_0$). Then, in the one-constant approximation, the free energy of an NLC cell can be written down in the form

$$F = F_{\rm el} + F_E + F_S,$$

$$F_{\rm el} = \frac{1}{2}\int\limits_V \Big\{K\left((\operatorname{div}\mathbf{n})^2 + (\operatorname{rot}\mathbf{n})^2\right) -$$

$$-K_{24}\operatorname{div}\big(\mathbf{n}\operatorname{div}\mathbf{n} + [\mathbf{n}\times\operatorname{rot}\mathbf{n}]\big)\Big\}\,dV,$$

$$F_E = -\frac{\varepsilon_a}{8\pi}\int\limits_V \left(\mathbf{nE}\right)^2 dV,$$

$$F_S = -\frac{1}{2}\int\limits_S W(y)(\mathbf{ne})^2\,dS, \tag{2}$$

where $F_{\rm el}$ is the Frank elastic energy, F_E is the contribution to the free energy from the electric field, F_S is the surface free energy of a nematic selected in the form of the Rapini potential [25], K and K_{24} are the elastic constants of the nematic, $\mathbf{n}$ is the director, $\varepsilon_a = \varepsilon_\parallel - \varepsilon_\perp > 0$ is the anisotropy of the static dielectric permittivity, and $\mathbf{e}$ is a unit vector of the easy-orientation axis of the director at the cell surface ($\mathbf{e}\parallel Ox$).

As was shown in work [6] in the one-constant approximation, the Friedericksz transition occurs in the plane "external electric field–initial director orientation"provided that the flexopolarization is absent and the azimuthal anchoring energy is infinitely large. Therefore, we assume below that the reorientation of the director occurs in the plane xOz. Since the system is translation-invariant along the Ox-axis, the director in the NLC bulk can be searched in the form

$$\mathbf{n} = \mathbf{i}\cos\theta(y,z) + \mathbf{k}\sin\theta(y,z), \tag{3}$$

where $\mathbf{i}$ and $\mathbf{k}$ are the unit vectors of the Cartesian coordinate system. Minimizing the free energy (2) with respect to the angle θ, we obtain the equation

$$\frac{\partial^2\theta}{\partial y^2} + \frac{\partial^2\theta}{\partial z^2} + \epsilon E_z^2\sin\theta\cos\theta = 0 \tag{4}$$

and the corresponding boundary conditions

$$\left[\pm K\frac{\partial\theta}{\partial z} + \left(W_0 + V\cos\frac{2\pi y}{d}\right)\sin\theta\cos\theta\right]_{z=\pm L/2} = 0, \tag{5}$$

where $\epsilon = \dfrac{\varepsilon_a}{4\pi K}$. As is seen from Eq. (4) and boundary conditions (5), the elastic coefficient K_{24} gives no contribution to the spatial reorientation of the director in the adopted geometry of the problem.

Equations (4) and (5) for the director must be solved together with the electrostatic equations rot $\mathbf{E} = 0$ and div $\mathbf{D} = 0$ for the electric field $\mathbf{E} = (0, E_y, E_z)$ in the cell bulk.

3. Electric Field Threshold

Let the linear dimension D of a cell along the axis Oy be much larger than the cell thickness L, so that the boundary effects at the lateral cell surfaces can be neglected. We impose the cyclic boundary conditions with respect to the coordinate y on the function θ: $\theta(y + D, z) = \theta(y, z)$. Since the function $W(y)$ is even (see Eq. (1)), the function $\theta(y, z)$ can be taken in the form

$$\theta(y,z) = \sum_{n=0}^{\infty}\theta_n(z)\cos(q_n y)\,, \tag{6}$$

where $q_n = 2\pi n/D$.

To determine the threshold value for director reorientation, it is enough to confine the consideration to the linearized problem. Substituting expansion (6) into Eqs. (4) and (5) linearized in the angle θ with regard for the mutual linear independence of the functions $\cos(q_n y)$, we obtain the equation for the unknown coefficients $\theta_n(z)$,

$$\frac{d^2\theta_n}{dz^2} + (\epsilon E_0^2 - q_n^2)\theta_n = 0 \tag{7}$$

and the corresponding boundary conditions

$$\left[\pm K\frac{d\theta_n}{dz} + W_0\theta_n + V\sum_{m=0}^{\infty}\alpha_{nm}\theta_m\right]_{z=\pm L/2} = 0,$$

$$n = 0, 1, 2, \dots , \tag{8}$$

where

$$\alpha_{nm}(s) = \frac{s}{2\pi}(2 - \delta_{n0})\times$$

$$\times\sin(\pi s)\left[\frac{(-1)^{n+m}}{s^2-(n+m)^2} + \frac{(-1)^{n-m}}{s^2-(n-m)^2}\right], \tag{9}$$

and $s = D/d$ is the number of anchoring energy periods contained along the cell length.

Substituting the general solution of Eq. (7), $\theta_n(z) = a_n\cos(\sqrt{\epsilon E^2 - q_n^2}\, z)$, into the boundary conditions (8), we obtain the following system of homogeneous algebraic equations for the unknown coefficients a_n:

$$\left(\cos\xi_n - \frac{2}{\varepsilon}\,\xi_n\sin\xi_n\right)a_n + v\sum_{m=0}^{\infty}\alpha_{nm}a_m\cos\xi_m = 0,$$

$$n = 0, 1, 2, \dots . \tag{10}$$

Here, the notations $\varepsilon = \frac{W_0 L}{K}$, $v = \frac{V}{W_0}$, $\xi_n = \sqrt{p^2 - \left(\frac{\pi n L}{D}\right)^2}$, and $p = \frac{\pi}{2}\frac{E_0}{E_\infty}$ were used, and $E_\infty = \frac{\pi}{L\sqrt{\epsilon}}$ is the threshold value for the Fredericksz transition at the infinitely rigid uniform anchoring of the director with the cell surface [1, 3].

A condition for the system of equations (10) to have a nontrivial solution brings about a determinant equation for the electric field threshold,

$$\Delta = \begin{vmatrix} L_0 & v\alpha_{01} & v\alpha_{02} & \dots \\ v\alpha_{10} & L_1 & v\alpha_{12} & \dots \\ v\alpha_{20} & v\alpha_{21} & L_2 & \dots \\ \vdots & \vdots & \vdots & \ddots \end{vmatrix} = 0\,, \tag{11}$$

where $L_n = 1 + v\alpha_{nn} - \frac{2}{\varepsilon}\,\xi_n\tan\xi_n$.

We consider small nonuniformities in the anchoring energy, which satisfy the condition $v \ll 10\varepsilon L^2/D^2$. Then $v\alpha_{ml} \ll g_1 < g_2 < g_3 < \dots$ for all $m, l = 0, 1, 2, 3, \dots$. We seek the solution of Eq. (11) in the form $p = p_0 + \delta$, where p_0 is the solution at $v = 0$, and $|\delta| \ll p_0$. Supposing the quantity δ to be of the same order of magnitude as $v\alpha_{ml}$, we present the diagonal elements of Eq. (11) in the form

$$L_0 = v\alpha_{00} - f_0\delta - h_0\delta^2 + o(\delta^2) \sim v\alpha_{00},$$

$$L_n = g_n + v\alpha_{nn} + O(\delta), \quad \text{if} \quad n \geqslant 1, \tag{12}$$

where

$$f_0 = \frac{\varepsilon}{2p_0}\left(1 + \frac{2}{\varepsilon} + \frac{4p_0^2}{\varepsilon^2}\right), \quad h_0 = \frac{2}{\varepsilon}(1+\varepsilon)\left(1 + \frac{\varepsilon^2}{4p_0^2}\right), \tag{13}$$

$$g_n = 1 - \frac{2}{\varepsilon}\sqrt{p_0^2 - \left(\frac{\pi n L}{D}\right)^2}\tan\sqrt{p_0^2 - \left(\frac{\pi n L}{D}\right)^2}. \tag{14}$$

Consider the determinant Δ of the N-th order in Eq. (11). Expanding it along the bottom row, we obtain

$$\Delta_N = L_{N-1}\Delta_{N-1} + v\sum_{i=0}^{N-2}(-1)^{N+i+1}\alpha_{N-1,i}M_{N-1,i}\,, \tag{15}$$

where M_{ij} is the complementary minor of the ij-th element in the determinant. It is easy to see that the minor

$$M_{N-1,0} = (-1)^N v\alpha_{0,N-1}\,L_{N-2}L_{N-3}\dots L_1 + o(v\alpha_{ml})$$

is a quantity of the first order of smallness in $v\alpha_{0,N-1}$, and all minors $M_{N-1,i}$ $(1 \leqslant i \leqslant N-2)$ are small quantities of the order of $v^2\alpha_{ml}^2$. Confining expansion (15) to the terms of the order of $v^2\alpha_{ml}^2$, we obtain the recurrent formula

$$\Delta_N = L_{N-1}\Delta_{N-1} -$$

$$-v^2\alpha_{N-1,0}\alpha_{0,N-1}\,L_1L_2\dots L_{N-2} + o(v^2\alpha_{ml}^2) \tag{16}$$

which connects the determinants of the N-th and $(N-1)$-th orders. After applying the obtained recurrent relation N times to expression (15), the latter looks like

$$\Delta_N = L_1L_2\dots L_{N-1}\left(L_0 - v^2\sum_{i=1}^{N-1}\frac{\alpha_{i0}\alpha_{0i}}{g_i}\right). \tag{17}$$

Finding the solution of Eq. (11) and taking Eq. (17) into account, we obtain the value for δ and the threshold of orientational instability for the director,

$$E_{0th} = E_{W_0} + \frac{2v}{\pi} E_\infty \left[\frac{\alpha_{00}}{f_0} - v \left(\frac{h_0 \alpha_{00}^2}{f_0^3} + \frac{2}{f_0} \sum_{i=1}^{\infty} \frac{\alpha_{0i}^2}{g_i} \right) \right], \tag{18}$$

where $E_{W_0} = 2p_0 E_\infty/\pi$ is the threshold for the Friedericksz transition in the case of uniform finite anchoring energy W_0, and p_0 is the minimal positive root of the equation $p_0 \tan p_0 = \varepsilon/2$. As our calculations demonstrate, the sum over i in expression (18) converges; therefore, we may confine the consideration to a finite number of terms with an arbitrary prescribed accuracy.

In Fig. 1, the dependences of the dimensionless threshold field E_{0th}/E_∞ on the number s of anchoring energy periods contained along the cell length calculated numerically by formula (18) are depicted for various ratios L/D at $\varepsilon = 100$ and $v = 0.1$. One can seen that the electric field threshold oscillates as the parameter s grows, with the oscillation amplitude diminishing to the threshold value E_{W_0} for the Friedericksz transition obtained in the case of uniform ($s \to \infty$) finite anchoring energy W_0. At $s \gtrsim 5$, the threshold values become practically independent of the L/D-ratio at the fixed parameters ε and v. The threshold value is maximal at $s = 0$ (in the case of uniform finite anchoring energy $W_0 + V$) and, in accordance with expressions (18) and (9), can be presented in the form

$$E_{W_0+V} = \frac{2E_\infty}{\pi} \left(p_0 + \frac{v}{f_0} - \frac{h_0 v^2}{f_0^3} \right), \tag{19}$$

where the parameters f_0 and h_0 are defined by expressions (13).

If the cell length includes an integer number of anchoring energy periods, i.e. if $s = k \in N$, then $\alpha_{00}(k) = 0$, $\alpha_{0i}^2(k) = \delta_{ik}/4$, and, in accordance with expressions (18), the threshold is

$$E_{0th} = E_{W_0} - \frac{v^2 E_\infty}{\pi f_0 g_k}, \tag{20}$$

where the quantities f_0 and g_k are determined by formulas (13) and (14), respectively. Unlike the case of arbitrary s-values, only the terms that are quadratic in v give a contribution to the threshold value. As is seen from Fig. 1 and formula (20), the threshold values $E_{0th}(k)$ turn out lower by magnitude than the value E_{W_0} for the case of uniform finite anchoring energy W_0. If the integer number k of anchoring energy periods across the cell length increases, the threshold value $E_{0th}(k)$ grows monotonously, and, at $k \gg 1$, it is approximately determined by the formula

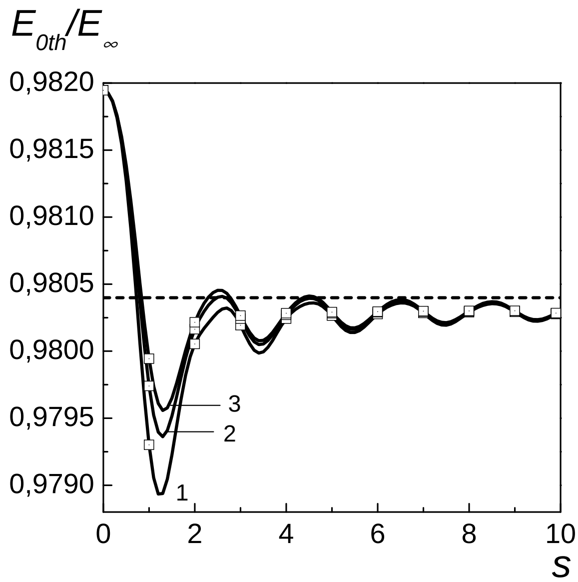

Fig. 1. Dependences of the electric field threshold E_{0th}/E_∞ on the parameter s at $\varepsilon = 100$, $v = 0.1$ (solid curve) and various $L/D =$ 0.03 (*1*), 0.04 (*2*), and 0.05 (*3*). The squares mark the threshold values at integer s. Dashed line corresponds to the threshold in the case of the uniform finite anchoring energy W_0

$$E_{0th} = E_{W_0} - \frac{v^2 E_\infty}{\pi f_0 \left(1 + \frac{2\pi}{\varepsilon}\frac{L}{d}\right)}.$$

Again, in the case of integer values $s = k$, the angle of a director deviation from its nonperturbed orientation is determined by only two terms in series (6), namely,

$$\theta(y,z) = \theta_0(z) + \theta_1(z)\cos(2\pi y/d). \tag{21}$$

Hence, there arises a spatial periodic structure of the director along the axis Oy, and the structure period is equal to that of the anchoring energy.

4. Spatial Distribution of a Director

Now, let us find a spatial distribution of the director in the cell, provided that the Friedericksz transition threshold is exceeded a little. Let the electric field $\mathbf{E}$ in the nematic bulk be expressed in terms of the potential $\varphi(y,z) = -(z + \psi(y,z))E_0$, where $\psi(y,z)$ is an unknown function. From the equation div $\mathbf{D} = 0$ and the boundary conditions for the electric field at the cell surface, we obtain an equation for the function $\psi(y,z)$ and the corresponding boundary conditions,

$$\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}=-2\frac{\varepsilon_a}{\varepsilon_\perp}\sin\theta\cos\theta\frac{\partial\theta}{\partial z}\,,$$

$$\left.\frac{\partial\psi}{\partial z}\right|_{z=\pm L/2}=0\,. \tag{22}$$

Consider the case where the cell length includes an integer number of anchoring energy periods, i.e. $D/d=k\in N$. If the anchoring energy nonuniformity is low, so that the condition $v\ll 10\varepsilon L^2/d^2$ is satisfied, we take the function $\theta(y,z)$ in the form (21), where $|\theta_1|\ll|\theta_0|$. Imposing the periodic condition $\psi(y+d,z)=\psi(y,z)$ on the function $\psi(y,z)$, we seek it in a form similar to expression (21),

$$\psi(y,z)=\psi_0(z)+\psi_1(z)\cos(2\pi y/d),\quad \text{where}\quad |\psi_1|\ll|\psi_0|. \tag{23}$$

According to the problem symmetry, the following relations must be satisfied: $\theta(y,-z)=\theta(y,z)$ and $\psi(y,-z)=\psi(y,z)$.

Substituting Eqs. (21) and (23) into expressions (4), (5), and (22) and confining the consideration to the terms of the order of θ^3, we obtain the following equations and boundary conditions for the unknown functions:

for $\theta_0(z)$,

$$\frac{d^2\theta_0}{dz^2}+\epsilon E_0^2\Big(\theta_0-\frac{2}{3}\theta_0^3+2\theta_0\frac{d\psi_0}{dz}\Big)=0, \tag{24}$$

$$\left[\pm K\frac{d\theta_0}{dz}+W_0\Big(\theta_0-\frac{2}{3}\theta_0^3\Big)+\frac{1}{2}V\theta_1(1-2\theta_0^2)\right]_{z=\pm L/2}=0, \tag{25}$$

for $\psi_0(z)$,

$$\frac{d^2\psi_0}{dz^2}=-2\frac{\varepsilon_a}{\varepsilon_\perp}\theta_0\frac{d\theta_0}{dz}\,, \tag{26}$$

$$\left.\frac{d\psi_0}{dz}\right|_{z=\pm L/2}=0, \tag{27}$$

for $\theta_1(z)$,

$$\frac{d^2\theta_1}{dz^2}+\Big(\epsilon E_0^2-\frac{4\pi^2}{d^2}\Big)\theta_1=0, \tag{28}$$

$$\left[\pm K\frac{d\theta_1}{dz}+W_0\theta_1+V\theta_0\right]_{z=\pm L/2}=0, \tag{29}$$

and for $\psi_1(z)$,

$$\frac{d^2\psi_1}{dz^2}-\frac{4\pi^2}{d^2}\psi_1=-2\frac{\varepsilon_a}{\varepsilon_\perp}\left(\theta_0\frac{d\theta_1}{dz}+\theta_1\frac{d\theta_0}{dz}\right), \tag{30}$$

$$\left.\frac{d\psi_1}{dz}\right|_{z=\pm L/2}=0. \tag{31}$$

Integrating Eq. (24) once over z, and taking systems (26) and (27) into account, we obtain

$$\frac{L^2}{4p^2}\left(\frac{d\theta_0}{dz}\right)^2=(1+\alpha\theta_{0s}^2)(\theta_{0m}^2-\theta_0^2)-\gamma(\theta_{0m}^4-\theta_0^4), \tag{32}$$

where $\theta_{0s}=\theta_0(z=\pm L/2)$, $\theta_{0m}=\theta_0(z=0)$ is the maximal value of the function $\theta_0(z)$ reached at the cell center ($z=0$), $\alpha=2\frac{\varepsilon_a}{\varepsilon_\perp}$, and $\gamma=\frac{1}{3}+\frac{\varepsilon_a}{\varepsilon_\perp}$.

Integrating Eq. ((32), we find

$$\frac{2p|z|}{L}\sqrt{1+\alpha\theta_{0s}^2}=$$

$$=\left(1+\frac{3\gamma\theta_{0m}^2}{4(1+\alpha\theta_{0s}^2)}\right)\arccos\frac{\theta_0}{\theta_{0m}}+\frac{\gamma\theta_0\sqrt{\theta_{0m}^2-\theta_0^2}}{4(1+\alpha\theta_{0s}^2)}\,. \tag{33}$$

Provided that $v\ll 10\varepsilon L^2/d^2$, the amplitude $\theta_1(z)$ of the first harmonic, which is determined by system (28) and (29), looks like

$$\theta_1(z)=-\frac{v\theta_{0s}}{\cos\xi_k}\cos\frac{2\xi_k z}{L}. \tag{34}$$

Consider the case of strong anchoring between the director and the cell surface ($\varepsilon\gg1$). From the boundary conditions (25) with regard for Eq. (32) and expressions (34), we obtain, to an accuracy of quantities linear in $1/\varepsilon$,

$$\theta_{0s}=\frac{2p\theta_{0m}}{\varepsilon}\left(1+\frac{v^2}{2}\right)\left(1-\frac{\gamma}{2}\theta_{0m}^2\right). \tag{35}$$

Substituting this value into Eq. (33) written down for the cell surfaces $z=\pm L/2$, we obtain, in the linear in $1/\varepsilon$ approximation,

$$\theta_{0m}^2=\frac{4}{3\gamma}\left(\frac{E_0}{E_{0th}}-1\right),\quad E_0\geqslant E_{0th}, \tag{36}$$

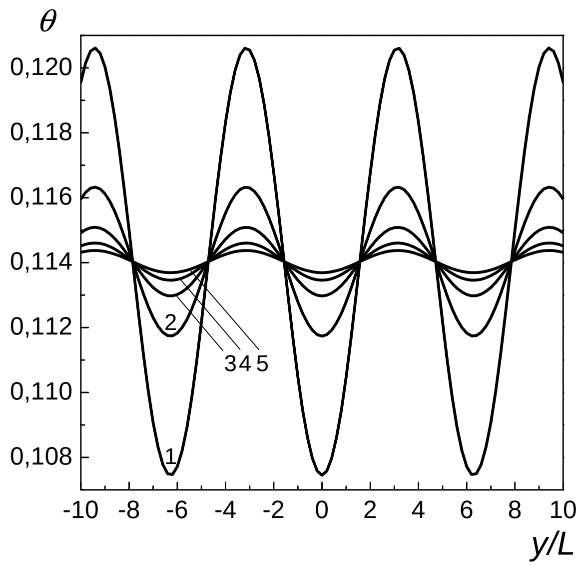

Fig. 2. Director deviation angle at the cell center, $z = 0$, as a function of the coordinate y for various $L/d = 0.1$ (*1*), 0.2 (*2*), 0.3 (*3*), 0.4 (*4*), and 0.5 (*5*)

where $E_{0th} = E_{W_0}\left(1 - \dfrac{v^2}{\varepsilon}\right)$ is the threshold value (see Eq. (20)) for the electric field, written down to an accuracy of linear in $1/\varepsilon$ quantities. In the limiting case of absolutely rigid anchoring between the director and the substrates ($W = \infty$), the value of θ_{0m}, as follows from Eq. (36), coincides with that found in work [26].

Substituting expression (35) into Eq. (33) and keeping small terms up to the order of θ_{0m}^3, we obtain the expression for the function $\theta_0(z)$

$$\theta_0(z) = \theta_{0m}\cos\Big(\frac{\pi z E_0}{LE_\infty}\Big) +$$

$$+\frac{\gamma\theta_{0m}^3}{8}\sin\Big(\frac{\pi z E_0}{LE_\infty}\Big)\left[6\frac{\pi z E_0}{LE_\infty} + \sin\Big(2\frac{\pi z E_0}{LE_\infty}\Big)\right], \qquad (37)$$

where θ_{0m} is given by formula (36).

From systems (26), (27) and (30), (31), taking expressions (34) and (37), respectively, into account, we find explicit forms for the functions $\psi_0(z)$ and $\psi_1(z)$:

$$\psi_0(z) = \frac{\varepsilon_a L\theta_m^2}{4\varepsilon_\perp}\left(\cos\Big(\frac{\pi E_0}{E_\infty}\Big)\frac{2z}{L} - \frac{E_\infty}{\pi E_0}\sin\Big(\frac{2\pi z E_0}{LE_\infty}\Big)\right), \qquad (38)$$

$$\psi_1(z) = -\frac{\varepsilon_a v p L\theta_m^2}{\varepsilon_\perp \varepsilon\cos\xi_k}\left\{\left[\frac{2\cos p\cos\xi_k}{(\pi L/d)^2} - \right.\right.$$

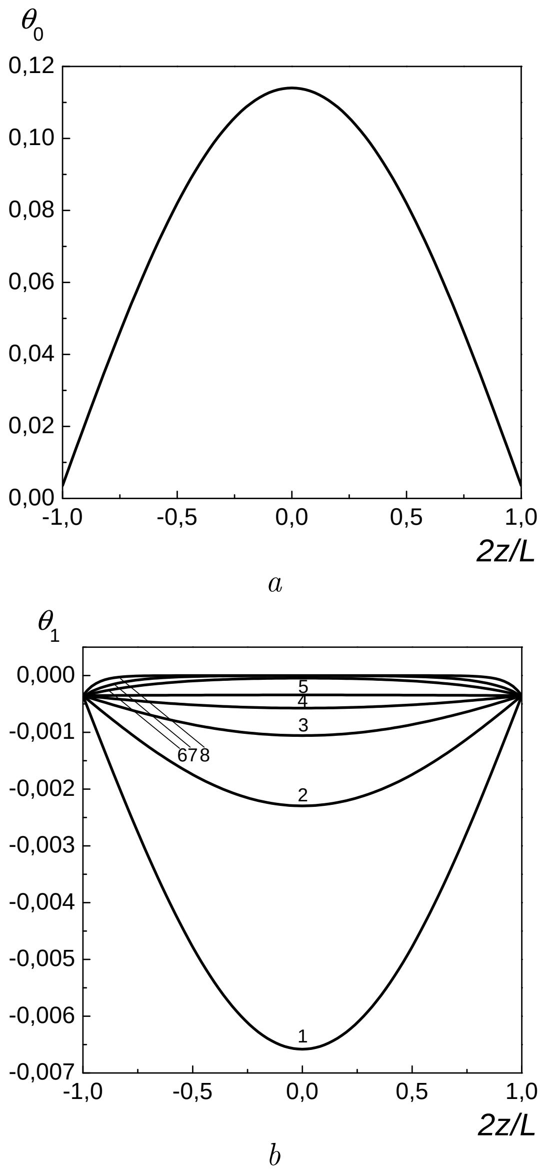

Fig. 3. Dependences of the quantities θ_0 (*a*) and θ_1 (*b*) on the reduced cell thickness $2z/L$ at $\varepsilon = 100$, $v = 0.1$, $E_0 = 1.01E_{0th}$, and various $L/d = 0.1$ (*1*), 0.2 (*2*), 0.3 (*3*), 0.4 (*4*), 0.5 (*5*), 1 (*6*), 2 (*7*), and 5 (*8*)

$$\left. - \frac{\cos(p-\xi_k)}{(p-\xi_k)^2 + (\pi L/d)^2} - \frac{\cos(p+\xi_k)}{(p+\xi_k)^2 + (\pi L/d)^2}\right]\times$$

$$\times\frac{\pi L\,\mathrm{sh}(2\pi z/d)}{d\,\mathrm{ch}(\pi L/d)} - \frac{(p-\xi_k)\sin\big(2(p-\xi_k)z/L\big)}{(p-\xi_k)^2 + (\pi L/d)^2} -$$

$$\left. - \frac{(p+\xi_k)\sin\big(2(p+\xi_k)z/L\big)}{(p+\xi_k)^2 + (\pi L/d)^2}\right\}. \qquad (39)$$

Formulas (38), (39), and (23) describe the distribution of the electric field potential in the cell bulk.

Formula (21) together with expressions (34) and (37) describe the distribution of the director field above the threshold of orientational instability. In Fig. 2, the director distributions at the cell center are depicted for a number of ratios L/d. In calculations, we put $\varepsilon = 100$, $v = 0.1$, and $E_0 = 1.01E_{0th}$.

The function $\theta_0(z)$ in formula (21) for the solution $\theta(y, z)$ depends implicitly (through θ_{0m} (36)) on the nonuniformity parameter v. Therefore, this function cannot be considered as corresponding to the uniform finite anchoring energy W_0. The function $\theta_0(z)$ describes the director field at $W(y) = W_0$ only in the limiting case $v \to 0$. Note also that the function $\theta_0(z)$ does not depend on the ratio L/d at any parameter v and has a profile depicted in Fig. 3,*a*.

The amplitude $\theta_1(z)$ of the spatial periodic director distribution (see formula (34)), which arises along the axis Oy, is linear in the parameter of anchoring energy nonuniformity v and essentially depends (through the parameter ξ_k) on the parameter L/d (the ratio between the cell thickness and the period of anchoring energy). In Fig. 3,*b*, the dependences $\theta_1(z)$ calculated for various values of this parameter are shown. As is seen from the figure, the absolute value of θ_1 monotonously grows, as the ratio L/d decreases. The most substantial dependence of θ_1 on L/d is observed at small values of this parameter.

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ЕЛЕКТРИЧНИЙ ПЕРЕХІД ФРЕДЕРІКСА В НЕМАТИЧНІЙ КОМІРЦІ З ПЕРІОДИЧНОЮ ПОЛЯРНОЮ ЕНЕРГІЄЮ ЗЧЕПЛЕННЯ

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Р е з ю м е

Отримано значення порога і запороговий просторовий розподіл директора в електричному полі в нематичній комірці з періодичною енергією зчеплення директора з її поверхнею. Показано, що значення порога немонотонно залежить від числа s періодів енергії зчеплення, що вкладаються на довжині комірки. Запороговий розподіл директора при цілих значеннях s відслідковує періодичну зміну енергії зчеплення. Амплітуда періодичного відхилення директора росте зі зменшенням відношення товщини комірки до періоду енергії зчеплення.