EXCITON CONDENSATION IN QUANTUM WELLS. SELF-ORGANIZATION AGAINST BOSE-CONDENSATION

V.I. SUGAKOV

Institute for Nuclear Research, Nat. Acad. of Sci. of Ukraine
(47, Prosp. Nauky, Kyiv 03680, Ukraine)

PACS 71.35.-y, 89.75.Fb

The analysis of interpretations of the experimental data on the emission spectra of excitons in double quantum wells is carried out. Features of both the spatial distribution of the emission and its behavior depending on the temperature and the pumping intensity are explained by the appearance of the condensed phase of excitons caused by their interaction. The explanation does not involve the Bose–Einstein condensation of excitons. The spatial distribution of the exciton density in the condensed phase depends on the exciton lifetime and is a consequence of self-organization processes in the non-equilibrium system. The distribution of excitons over trapped and free states and its influence on the emission spectra are investigated. The hydrodynamic equations for interacting excitons are obtained. The existence of soliton-like states (autosolitons) outside of the spinodal decomposition region is shown.

1. Introduction

While seeking the Bose–Einstein condensation of excitons in double quantum wells, the several non-trivial effects were observed. The very large exciton lifetime is a special feature of excitons in double quantum wells in the presence of an electric field directed in parallel to the normal to the quantum well plane [1]. The effect occurs due to the separation of electrons and holes into different wells, which causes a very weak overlapping of their wave functions and the damping of the mutual recombination. The large lifetime allows one to create a high concentration of excitons at small pumpings and to study a manifestation of the effects of exciton-exciton interaction. Excitons with electrons and holes localized in different wells are called "indirect excitons." The nonzero dipole moment of indirect excitons should causes their mutual repulsion, which complicates the creation of the condensed phase of excitons. These properties and the fact that excitons have the integer spin and a small mass suggested the creation of the Bose–Einstein condensed phase of excitons in double quantum wells and stimulated the search for this phase. The investigations gave a number of new results. In the emission spectra of indirect excitons from a double quantum well based on AlGaAs, the narrow band with unusual properties was observed and studied [2–4]. The band appears at the some threshold pumping. The temperature dependence of the emission intensity at a fixed pumping is described by a straight line, which crosses the abscissa axis at the temperature, at which the emission intensity is equal to zero, i.e., at the temperature which is the threshold temperature at the given pumping. At a fixed temperature and a low pumping, the dependence of the emission on the pumping is ultralinear. The authors [3, 4] built a phase diagram "threshold pumping–temperature". The phase diagram has the peculiarity: the dependence of the threshold pumping on the temperature does not tend to zero as $T \to 0$.

The nontrivial results were found for a spatial distribution of the exciton emission from a quantum well. In works [5, 6], the appearance of a ring was observed in the emission from a double quantum well outside the laser spot, in which the excitons were excited, was observed. The ring radius exceeded significantly the exciton diffusion length. The explanation of the appearance of a ring was given in [7, 8] under the assumption that holes are captured by the well more effectively than electrons, and, in addition, there are donors in the crystal, which create some concentration of free electrons. As a result, a region rich by holes arises in the quantum well in a vicinity of the laser spot. Outside of this region, the quantum well is enriched by free electrons. On the boundary of the region, the processes of recombination take place, which causes the creation of excitons on the ring and the appearance of a spatial distribution of the emission in the form of a ring.

Intriguing facts appear under the investigation of the spatial distribution of the exciton density. Different spatial inhomogeneous structures were observed in the emission of indirect excitons at the pumping greater than some critical value. For example, the separation of the emission ring into fragments periodically localized along the ring was observed in [5]. In work [9], the excitation of the quantum well was carried out through a window in a metallic electrode. The authors found out a periodic structure in the luminescence spectrum in the form

of islands situated along the ring under the ring perimeter.The number of islands grew with the window radius.

The indirect excitons can be created not only in double quantum wells, but also in a wide quantum well in the presence of a strong electric field. The electric field divides electrons and holes between different sides of the quantum well. As a result, the excitons with charges strongly separated in the space and with a large lifetime are created [10]. On such dipole excitons in a wide quantum well, the effects similar to those on excitons in double quantum wells were observed [11]. Recently, Timofeev and coauthors [12] presented examples of the structures in the emission spectra for different forms of windows in the electrode: a rectangle, two circles, and others. In work [13], the authors created an additional periodic potential for excitons, by choosing the form of the electrode. It was found that, besides the periodic structure imposed by external conditions, the partition of the emission into fragments was observed in the direction, where the potential is almost uniform. This direction is perpendicular to that, along which the system has a periodic potential.

The phenomena of the symmetry loss and the creation of structures in the emission spectra of indirect excitons stimulated a series of theoretical investigations [14–19]. The authors of work [14] considered the instability, which arises under the variation of level occupations by particles with the Bose–Einstein statistics. Namely, the growth of the occupation of a level with zero moment should stimulate the transitions of excitons to this level. But the density of excitons was found greater, and the temperature was found lower than the values observed on the experiments. Some authors explain the appearance of the periodicity by the Bose condensation of excitons [15, 16]. A suggestion to describe the system by a nonlinear Schrödinger equation was advanced in [17], and a possibility of the Mott transition in systems was discussed in [18]. In these works, the main efforts were applied to the ascertainment of the basic possibility for the formation of a periodicity without a specific application of the results to the explanation of other properties of the systems (the dependence of properties on a pumping, temperature, and other parameters).

The another approach was developed in works [20–24, 28] for an explanation of the experiments concerning the excitons in double quantum wells. The approach is based on the following assumptions.

1. There exists the condensed phase of indirect excitons caused by the attractive interaction between excitons. As was already mentioned, there is the dipole-dipole repulsion interaction between excitons. But the simple calculations show that, at not very far distances between quantum wells, when the dipole moment of an exciton is not too large, the exchange and van der Waals interactions exceed the dipole-dipole repulsion at certain distances between excitons. The existence of the attractive interaction between excitons is confirmed by the calculations of biexcitons [25–27] and by studies of many-exciton systems [20].

2. The finite value of the exciton lifetime plays an important role in the formation of a spatial distribution of exciton condensed phases. As usual, the exciton lifetime exceeds significantly the duration of the establishment of a local equilibrium. For this reason, the lifetime of excitons is suggested to be equal to infinity in the solutions of many problems. But, taking the finiteness of exciton lifetime into account is necessary in the study of the spatial distribution of phases in two-phase systems, because the exciton lifetime is less than the time of the establishment of the equilibrium between phases. The last time is determined by slow diffusion processes and is great. Just the finite exciton lifetime restricts the maximal size of the condensed phase of excitons and causes the correlation in positions of separate regions of the condensed phases. Thus, the created spatial structures are non-equilibrium, and they are a consequence of the processes of self-organization in non-equilibrium systems. The appearance of a instability and the creation of a periodic distribution of the exciton density at high densities of excitons with attractive interaction were shown in work [30] from the viewpoint of self-organization processes.

The theory developed in works [20–24, 28] has explained almost all features of the manifestation of indirect excitons. So, the behavior of a narrow band observed in [4] as a function of the pumping and the temperature was considered in [21]. Works [20, 22–24] present the explanation of the spatial distribution of the exciton density founded in [5, 9] and its dynamics at variations in the pumping and temperature. The experimental "threshold–temperature" phase diagram [3, 5] was explained in [21]. While developing the theory, two approaches of the theory of phase transitions were used: the model of nucleation (Lifshitz–Slyozov) and the model of spinodal decomposition (Cahn–Hilliard). These models were generalized for particles with a finite lifetime, which is of importance for the interpretation of experimental results. The involvement of Bose–Einstein statistics for excitons is not required for the explanation of experiments. Moreover, the condensation under consideration occurs in the real space, and it is not the Bose–Einstein condensation. The theory explains all observed

results except two ones presented below, which were not considered jet in the framework of the presented theory.

1) It was shown experimentally in work [31] that the maximum of the frequency dependence of the emission from the region between the islands is lower, than the maximum of the emission frequency from the islands, so from the region, where the exciton density is large. The difference of the frequencies is small, it is less than the width of the emission band. But, on the base of these data, the authors drew conclusion that there is the repulsion interaction between excitons. This result contradicts the main assumptions of of the model of works [20–24, 30] about the presence of attractive interaction between excitons, which causes the creation of the condensed phase.

2) In works [12, 32, 33], a coherence was observed in the emission spectra from different regions of an island of the condensed phase [33] or even from different islands [12, 32]. The coherence was revealed in the interference of the emission from different spatial points.

Effect 2 is not considered in the presented paper. For its explanation, a microscopic model of the condensed phase is needed. But the following qualitative explanation can be given. In experiments, the interference of electromagnetic waves is observed, rather than that of wave functions. Since the electromagnetic wave and the scattered field are coherent, the interference of the emission from two islands can occur as a result of the superposition of the electromagnetic field emitted by some island and the same field scattered by another island. It was shown in [34–36] that the strong correlation between densities at different points takes place in the case where the condensed phase exists. It is a reason for the interrelation of the wave emitted from some point and the wave scattered by other region. We mention a sharp maximum of the Fourier transformation of the correlation function. But the quantitative calculations require the data on a microscopic model of the condensed phase of excitons, particularly, the numerical value of the polarizability is needed.

The explanation of effect 1 is given in the next section.

2. Distribution of Excitons over Localized and Delocalized States

As we mentioned already, the periodically situated islands were observed in the emission spectrum from the ring outside the laser spot [5]. According to [31], the frequency of the emission from the region with large exciton concentration (from islands) is higher than that from the region with lower concentration (from the region between islands). Based on this result, the authors of work [31] made conclusion that the interaction between excitons is repulsive, and, therefore, the formation of the condensed phase by the attractive interaction is impossible. This contradicts the main assumption of works [20–24], though these works explain many experiments. We now remove this contradiction, taking the presence of localized excitons into account. Residual donors, acceptors, and defects create the random fluctuating potential, which can be a reason for the appearance of localized levels. Till now, the explanation of the creation of localized states is not determined definitely, but their existence is confirmed by the presence of an emission in the region of the frequencies less than the frequency of the exciton band emission. At a low temperature and a small pumping, the main part of the band consists of the emission from defect centers. According to works [4, 5], the band width of the emission in a vicinity of the bottom of the indirect exciton band in double quantum wells on the base of AlGaAs has an order of (1.5–2) meV. By the Timofeev group [4], the narrow band with width of the order of $(0.2 \div 0.3)$ meV was discovered on the short-wave side of the spectrum. The band appears at increasing the pumping, and it is related to the exciton condensed phase. In the works by Butov *et al.* [5, 31], this narrow band was not observed, and the emission spectra of localized and delocalized states are not separated. So, in [5,31], the shape of the joint emission of the condensed phase and localized states is investigated. Let us consider the relation between the contribution to the emission band from free excitons and excitons localized on defects.

The exciton states (free and localized) are distributed over levels after the creation of electrons and holes by an external irradiation and their subsequent recombination and relaxation. As usual, the relaxation time is much less than the exciton lifetime. Thus, it may be suggested that the excitons are in the state of quasithermodynamical equilibrium. But such an equilibrium cannot be established for deep localized states at low temperatures, because the transitions from the low levels to upper ones have a low probability. Therefore, we assume that the free excitons are in the state of thermodynamical equilibrium, and we will find the distribution of localized excitons from the kinetic equations. Let us consider the distribution of free and localized excitons under conditions of a steady-state irradiation. In the calculations, we assume that 1) the defect center can capture only a single exciton, 2) the temperature is so low that the region of the exciton band population (the value of order of κT) is much less than the bandwidth, which is

formed by free excitons and the excitons trapped by defects; then the system of excitons can be described by the density of free excitons n, that is the number of excitons in unit area of a quantum well. The system of localized states will be described by the density of localized excitons in unit interval of the energy – $n_l(E)$. The energy distribution of defect levels will be described by the function $\rho(E)$, which determines the number of levels in unit area and in unit interval of the energy. The defect levels are occupied due to the transitions of excitations from the free exciton states to the defect ones. The inverse processes from the defect states to the exciton band take place as well. At small concentrations of defects, we can neglect the transitions of excitations between defect states. The system of kinetic equations for the level population has the form

$$\frac{dn}{dt} = G - \frac{n}{\tau_{\rm ex}} - \int W_{{\rm ex},l}(E) n (\rho(E) - n_l(E)) dE$$

$$+ \int W_{l,{\rm ex}}(E) n_l(E) n_{cE} dE, \quad (1)$$

$$\frac{dn_l(E)}{dt} = -\frac{n_l(E)}{\tau_l(E)} + W_{{\rm ex},l}(E) n (\rho(E) - n_l(E)) -$$

$$- W_{l,{\rm ex}}(E) n_l(E) n_{cE}, \quad (2)$$

where $\tau_{\rm ex}$ and $\tau_l(E)$ are the lifetimes of the free and trapped excitons, respectively, G is the pumping (the number of the excitons created for unit time and in unit area of a quantum well), $W_{{\rm ex},l}(E)$ and $W_{l,{\rm ex}}(E)$ are the probabilities of the direct and inverse transitions in unit time from the exciton band to the defect levels with the energy E at a single exciton in unit area. The factor $(\rho(E) - n_l(E))$ in formula (1) corresponds to the fact that the transitions are possible only on the free states of defect levels. The last term in formula (1) describes the transitions from defect states to the exciton band. The probability of this transition is proportional to the transition probability for a single exciton, the population of defect levels, and the density of excitons in a defect n_{cE}. At a single exciton in the defect, the value of n_{cE} is inversely proportional to the surface area occupied by the defect.

Let us consider the energy distribution of excitations under a steady-state irradiation. For that, let us to substitute the value $n_l(E)$ obtained from Eq. (2) into Eq. (1). As a result, we obtain the equation for the exciton density,

$$n_L + n = G\tau_{\rm ex}, \quad (3)$$

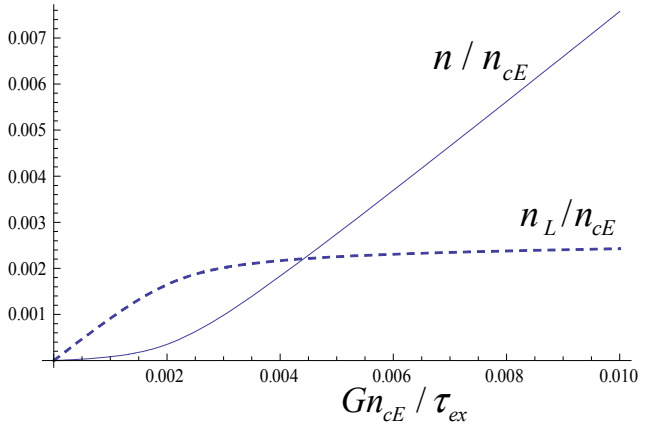

Fig. 1. Dependence of the density of free (solid line) and trapped (dashed line) excitons on the pumping. The parameters of the system: $T = 2$ K, $N_l = 0.0032\ n_{cE}$, $W_{{\rm ex},l}\tau_{\rm ex} n_{cE} = 10000$; $\alpha = 300\ ({\rm eV})^{-1}$

where n_L is the total number of excitons localized on defects,

$$n_L = n\tau_{\rm ex} \int\limits_{-\infty}^{0} \frac{W_{{\rm ex},l}\, \rho(E) dE}{W_{{\rm ex},l}(n + n_{cE} \exp(E/\kappa T))\tau_l(E) + 1}. \quad (4)$$

In this formula, we took the connection between the direct and inverse processes into account: $W_{l,{\rm ex}} = W_{{\rm ex},l} \exp(E/\kappa T)$, where the energy of a defect level E is reckoned from the bottom of the exciton band ($E < 0$). Further calculations need the information about the numerical values of the coefficients in Eq. (3). To obtain a qualitative picture of the distribution of excitations, we consider the following model. The dependence of the energy spectrum density of defect states on the energy has exponential form, namely $\rho(E) = \alpha N_l \exp(\alpha E)$, where N_l is the density of defect centers, the matrix element of the transition from an exciton state to a defect $W_{{\rm ex},l}$ does not depend on E, and the lifetimes of excitations for different defects coincide, $\tau_{\rm ex} = \tau_l$. The results of calculations of the exciton density and the density of trapped excitons are presented in Fig. 1. The values of parameters are shown in the capture to the figure.

At the size of a trap of 10^{-6} cm, the parameter n_{cE} has the order of 10^{12} cm^{-2}. The concentration of defects for the parameters chosen in calculations (see Fig. 1) equals 3.2×10^9 cm^{-2}. The width of the band created by traps has the order of 0.003 eV. At a small pumping, the emission band is determined by the emission from traps. At a high pumping, the occupation of the trap levels become saturated. According to Fig. 1, the

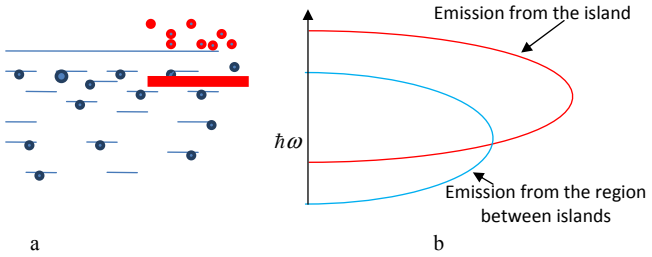

Fig. 2. Distribution of excitations in the traps and in the states of the exciton band. The thick line *a* corresponds to the energy per single exciton in the condensed phase. On the right *b*), the upper line describes the emission from islands (emission of both the condensed phase and trapped excitons), the low line describes the emission of trapped excitons

concentration of excitations is of the order of 2×10^9 cm^{-2} under the saturation. At high concentrations and under the attractive interaction, the excitons create the condensed phase. The energy per single exciton in the condensed phase decreases as compared with the energy of free excitons (the thick line in Fig. 2).

Therefore, it is favorable for excitons to create the condensed phase, but the gain of energy (the value of the order of 0.0002 eV) is less than the bandwidth. However, because the exciton density in islands is larger than the density of excited states of traps, and the emission energy of excitons is higher than the emission energy of traps, the energy of the emission from the islands is shifted to a short-wave side. However, the excitons cannot leave the condensed phase (islands) and move to the traps (to the states with lower energy), since the levels of traps are occupied. Thus, the emission frequency of the condensed phase is larger than the emission frequency of the gas phase, *even at the attractive interaction between excitons.*

It should be noted that, in work [5], the emission band from the condensed phase is wider than the band that was observed in work [4], where another method of creation of excitons was applied. Maybe, this is related to the fact that, in work [5], the excitons were created in the region of the $p - n$ transition: from one side, the electrons approach this the region, and the holes move to the region from other side. The excitons are situated in a space between the regions rich by electrons from one side, and by holes from another side. To combine into excitons, electrons and holes should surely go through the region of the condensed phase. Therefore, this region contains the charges, which create an electric field and cause the broadening of the emission band.

3. Hydrodynamics of Excitons in the Condensed Phase

The hydrodynamic equations of excitons were obtained and analyzed in work [37]. We will obtain the hydrodynamic equations of excitons in the condensed phase generalizing the Navier–Stokes equations. The system is characterized by the exciton density $n \equiv n(\mathbf{r}, t)$ and the velocity of the exciton liquid $\mathbf{u} \equiv \mathbf{u}(\mathbf{r}, t)$. The equation of continuity is rewritten in the form

$$\frac{\partial n}{\partial t} + \mathrm{div}(n\mathbf{u}) = G - \frac{n}{\tau_{\mathrm{ex}}}. \tag{5}$$

In the comparison with the typical equation of a liquid, the presented equation for excitons contains the terms that describe the pumping and the finite lifetime of excitons.

The equation of motion of a unit volume of the exciton liquid reads

$$\frac{\partial mnu_i}{\partial t} = -\frac{\partial \Pi_{ik}}{\partial x_k} - \frac{mnu_i}{\tau_{sc}}, \tag{6}$$

where m is the exciton mass, Π_{ik} is the tensor of the exciton flux density,

$$\Pi_{ik} = P_{ik} + mnu_iu_k - \sigma'_{ik}, \tag{7}$$

where P_{ik} is the pressure tensor, σ'_{ik} is the viscosity stress tensor.

In comparison with the typical Navier–Stokes equation, the braking of the exciton liquid by phonons and defects is introduced in Eq. (6) using the scattering time τ_{sc}. In Eq. (6), we neglected a change of the momentum caused by the creation and the annihilation of excitons. Indeed, the change of the momentum per unit time and in unit volume owing to the disappearance of excitons has the order of mnu/τ_{ex}. Since $\tau_{\mathrm{ex}} \gg \tau_{sc}$, this value is much less than the last term in formula (6). The change of the momentum due to the addition of new excitons by a pumping is small too.

Introducing the coefficients of viscosity and using Eq. (5), Eq. (6) can be rewritten in the form

$$\rho\left(\frac{\partial u_i}{\partial t} + \left(u_k\frac{\partial}{\partial x_k}\right)u_i\right) = -\frac{\partial P_{ik}}{\partial x_k} + \eta\Delta u_i +$$

$$+(\varsigma + \eta/3)\left(\frac{\partial}{\partial x_i}\right)\mathrm{div}\mathbf{u} - \frac{\rho u_i}{\tau_{sc}}. \tag{8}$$

Let us consider the tensor of pressure. To find the connection of the tensor with other parameters, it is necessary to use the equation of state. We suggest that the

state of local equilibrium in the system can be described by the free energy depending on the spatial coordinate. Let us present the functional of the free energy in the form

$$F = \int d\mathbf{r} \left(\frac{K}{2} (\nabla n)^2 + f(n) \right). \tag{9}$$

The first term in the integrand describes the energy of innhomogeneities.

At the given presentation of the free energy, the pressure tensor is determined by the formula [38]

$$P_{\alpha\beta} = \left(p_0 - \frac{K}{2} (\nabla n)^2 - Kn\Delta n \right) \delta_{\alpha\beta} + K \frac{\partial n}{\partial x_\alpha} \frac{\partial n}{\partial x_\beta}, \tag{10}$$

where $p_0 = nf'(n) - f(n)$ is the equation of condensed state.

In view of (10), we rewrite Eq. (8) in the form

$$\frac{\partial u_i}{\partial t} + u_k \frac{\partial u_i}{\partial x_k} + \frac{1}{m} \frac{\partial}{\partial x_i} \left(-K\Delta n + \frac{\partial f}{\partial n} \right) + \nu \Delta u_i +$$

$$+ (\varsigma/m + \nu/3) \left(\frac{\partial}{\partial x_i} \right) \mathrm{div}\mathbf{u} + \frac{u_i}{\tau_{sc}} = 0. \tag{11}$$

Equations (5) and (11) are the hydrodynamic equations for the exciton system. Equation (11) differs from the hydrodynamic equation investigated in [37] by the presence of the third term, which describes the condensed phase. We consider the case where the condensed phase exists. It follows from the estimations made in work [37] that the terms with the viscosity coefficients can be omitted. Then, for processes slowly varying in time in the linear approximation in the velocity, we obtain the velocity $\mathbf{u}$ from Eq. (11). Substituting it in Eq. ((5), we obtain

$$\frac{\partial n}{\partial t} + \mathrm{div}\mathbf{j} = G - \frac{n}{\tau_{\mathrm{ex}}}, \tag{12}$$

where

$$\mathbf{j} = n\mathbf{u} = -\frac{\tau_{sc} n}{m} \boldsymbol{\nabla} \left(-K\Delta n + \frac{\partial f}{\partial n} \right), \tag{13}$$

or $\mathbf{j} = -M\nabla\mu$, where $\mu = \delta F/\delta n$ is the chemical potential of the system, $M = nD/\kappa T$ is the mobility, and $D = \kappa T \tau_{sc}/m$ is the diffusion coefficient of excitons.

Therefore, the equation for the exciton density reads

$$\frac{\partial n}{\partial t} = D\Delta n + \frac{D}{\kappa T} (-Kn\Delta^2 n - K\boldsymbol{\nabla} n \cdot \boldsymbol{\nabla} n \Delta n) +$$

$$+ \frac{D}{\kappa T} \boldsymbol{\nabla} \cdot \left(n \frac{\partial^2 f}{\partial n^2} \boldsymbol{\nabla} n \right) + G - \frac{n}{\tau_{\mathrm{ex}}}. \tag{14}$$

Just in the form of (14) in the model of spinodal decomposition, we investigated the spatial distribution of the exciton density under the condensation of excitons at various dependences f on n [22, 23, 34, 39, 40]. Under some restrictions imposed on the functional F, the uniform solution is unstable, and the spatial structure arises in the system. For the system under study, the instability appears if the function $f(n)$ has a minimum corresponding to the condensed phase. In the abovementioned works, the examples of such functions were given. Here, we analyze another dependence $f(n)$, which is often used in the theory of phase transitions. We choose the density of free energy in the form

$$f = \kappa T n (\ln(n/n_a) - 1) + a\frac{n^2}{2} + b\frac{n^4}{4} + c\frac{n^6}{6}, \tag{15}$$

where a, b, and c are constants. The existence of the condensed phase requires that the value of b be negative. Three last terms describe the exciton-exciton interaction, and the first term was introduced in order to describe the system at small exciton concentrations.

Let us introduce the dimensionless parameters $\tilde{n} = n/n_o$, where $n_o = (a/c)^{1/4}$, $\tilde{b} = b/(ac)^{1/2}$, and $\tilde{\mathbf{r}} = \mathbf{r}/\xi$, where $\xi = (K/a)^{1/2}$ is the coherence length, $\tilde{t} = t/t_0$, where $t_0 = \frac{\kappa T K}{D n_o a^2}$, $D_1 = \frac{\kappa T}{a n_o}$, $\tilde{G} = G t_0 n_0$, $\tilde{\tau}_{\mathrm{ex}} = \tau/t_0$. As a result, Eq. (12) is reduced to the form (we drop the symbol $\sim$)

$$\frac{\partial n}{\partial t} = D_1\Delta n - n\Delta^2 n - \boldsymbol{\nabla}(n\boldsymbol{\nabla}\Delta n) + n\Delta n(1 + 3bn^2 + 5n^4) +$$

$$+ (\boldsymbol{\nabla} n)^2 (1 + 9bn^2 + 25n^4) + G - \frac{n}{\tau_{\mathrm{ex}}}. \tag{16}$$

At a steady-state uniform pumping, Eq. (16) has the uniform stationary solution $n = G\tau_{\mathrm{ex}}$. By the conventional method with the linearization of Eq. (16) with respect to small deviations from the uniform solution, we obtain that the uniform solution is stable, if the exciton density is less than some critical value n_c, which is determined by solving the equation

$$(D_1/n_c + 1 + 3bn_c^2 + 5n_c^4)/n_c + 2/\sqrt{n_c \tau_{\mathrm{ex}}} = 0. \tag{17}$$

The critical value of the pumping equals $G_c = n_c \tau_{\mathrm{ex}}$. At a higher pumping, a periodic structure arises. The wave number k_c of the structure and the lattice period λ_c are determined from the relation

$$k_c = 2\pi/\lambda_c = 1/(n_c \tau_{\mathrm{ex}})^{1/4}, \tag{18}$$

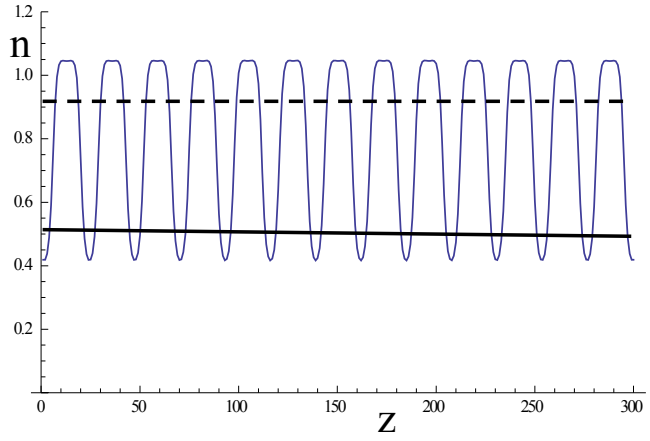

Fig. 3. Spatial dependence of the exciton density at various values of the pumping: $G = 0.0056$ for the continuous line, $G = 0.007$ for the periodic line, and $G = 0.00924$ for the dashed line. $D_1 = 0.03$, $b = -1.9$

and n_c depends slightly on $\tau_{\rm ex}$. Therefore, the lattice period is proportional to $(\tau_{\rm ex})^{1/4}$). This result was obtained in work [30]. At the pumping larger than the threshold value ($G > G_{\rm c1}$), the periodic distribution of the exciton density appears. In Fig. 3, the stationary value of exciton density for a one-dimensional system $n(\mathbf{r}, t) \equiv n(z, t)$) for three values of pumping is presented. The periodic solution exists in the some interval of the pumping $G_{\rm c1} < G < G_{\rm c2}$.

The solution presented in Fig. 3 is obtained from Eq. (16) under the initial conditions $n(z, 0) = 0$ and the boundary conditions $n'(0, t) = n'(L, t) = n''(0, t) = n''(L, t) = 0$, where L is the size of the system. The problem was solved also at different boundary conditions. Generally speaking, at a great size of the system, the structure of the solution (the period and the amplitude of the superlattice) does not depend on the boundary conditions at some distance from the boundary. At the given parameters, the periodic solution exists at $0.0056 < G < 0.0092$. Outside of this region, the solution describes a uniform system: the gas phase at the pumping less than the lower boundary value and the condensed phase at the pumping greater than the upper boundary value.

For a proof of the main equation (14) the last term in Eq. (6), which describes the loss of the momentum of excitons due to their scattering on phonons and defects, is of importance. Just this term describes the processes, which cause a decay of the exciton flux. From the viewpoint of the possibility of the appearance of superfluidity, the situation for excitons is more complicated than that for liquid helium and for the atoms of alkali metals at ultralow temperatures. In the last systems, the phonons are the intrinsic component of the spectrum of the system. The interaction between phonons are the interaction between of atoms of the system and causes no change of the total momentum of the system and its motion as the whole. Phonons and defects for excitons, are external subsystems, which brake the motion of excitons. Therefore, to create the exciton superfluidity state, it is necessary that the value of τ_{sc} grow significantly. It is possible for exciton polaritons, which weakly interact with phonons; we note that there is a certain experimental evidence of the superfluidity [41]. For the indirect excitons, there is no microscopic theory of the condensed phase and elementary excitations in it. Therefore, it is not shown the possibility to remove the effects of external factors (phonons, defects). So, the question about the existence of the superfluidity for indirect excitons remains open. As for the explanation of the experimental spike [42] in the emission of indirect excitons after the switch-off of a pumping, it is ambiguous. According to work [42], the effect occurs due to a stimulated population of levels caused by the Bose–Einstein statistics of excitons. But the spike can appear as a result of the increase in the exciton lifetime due to the removal of Auger processes (see calculations in [21]).

Thus, the peculiarities observed at large densities of indirect excitons can be explained by the phase transitions in the system of particles with attractive interaction and the finite value of the lifetime without involvement of the Bose-Einstein condensation.

4. Exciton Autosolitons

As was shown, at $n < n_{\rm c1} (G < G_{\rm c1})$, the uniform solution of Eq. (16) is stable. But, in some limits of a pumping at $G < G_{\rm c1}$, there exists a stationary solution localized in space for the exciton density distribution. For example, with the parameters used in calculations of the exciton distribution in Fig. 3, the threshold value of the pumping equals $G_{\rm c1}$ =0.0056789; but, at a steady-state pumping, there is the spatial nonuniform solution of Eq. (16) at $G < G_{\rm c1}$ in the form of an isolated spike. It can be obtained, by solving Eq. (16) at the pumping, which consists of a constant value G_0 and an additional pulse dG with maxima at the some point of the space and at some time moment:

$$dG = s \exp[-w(z - z_0)^2] \exp[-p(t - t_0)^2]. \qquad (19)$$

Here, s, w, and p are parameters. Formula (19) describes a pulse of the pumping, which acts during some time interval with the maximum at the point z_0.

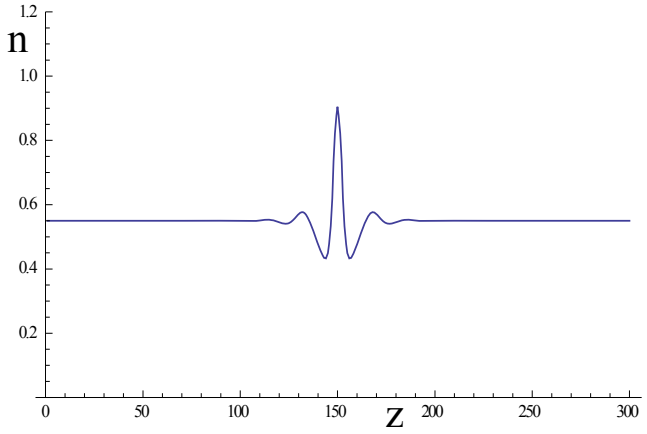

Fig. 4. Spatial dependence of the exciton density at the pumping $G = 0.0055 < G_{c1}$; $D_1 = 0.03$, $b = -1.9$

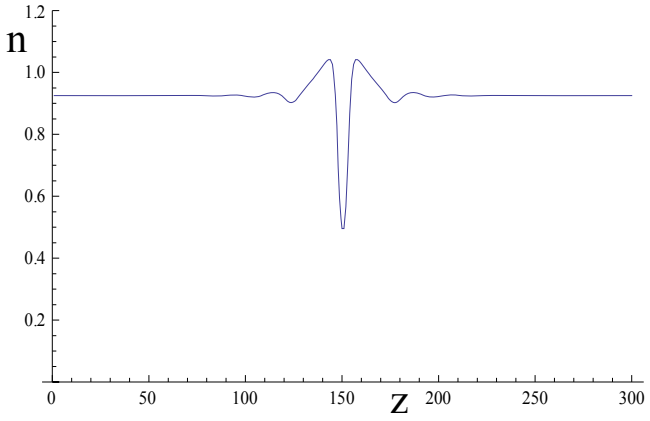

Fig. 5. Spatial dependence of the exciton density at $G = 0.0095 > G_{c2}$; $D_1 = 0.03$, $b = -1.9$

The solution of Eq. (16) obtained under the application of the addition pulse (19) in the region $z_0 = L/2$, has the form presented in Fig. 4 at $t \to \infty$.

The solution exists as $t \to \infty$, i.e., at the times when the action of the addition pulse is already absent. The shape of the spike $n(z)$ does not depend on the parameters s, w, and p, except cases where at least one of these parameters tends to zero and becomes less than some value. In addition, the solution in the form presented in Fig. 4 arises also, if the additional pulse is absent, but there is some distribution of the exciton density at the initial time $t = 0$:

$$n(z, 0) = s_0 \exp(-w(z - z_0)^2). \tag{20}$$

We can verify by direct calculations that the solution presented in Fig. 4 in the form of a localized distribution of the density is stable. We call the state that describes this solution by an excitonic autosoliton. The coordinate dependence of the exciton density will be designated by $n_{as}(z)$. The autosolitons exist in the some region of the pumping $G_{cas} < G < G_{c1}$. The solutions in the form of an autosoliton exist side-by-side with the uniform solutions.

The excitonic autosolitons correspond to solitary solutions of the nonlinear equations for excitons (16). The name "**auto**soliton" is introduced according to [43]; it underlines that the solitary waves arise in the dissipative system in a contrast to "solitons" which appear in conservative systems. The solutions in the form of autosolitons are degenerate: if there is a solitary solution $n_{as}(z)$, then $n_{as}(z - z_0)$ will be also a solution at arbitrary z_0 (in an infinite medium). But, if there is an external field in the system, which creates a spatially variable additional potential for excitons, the solitary excitation moves. Thus, at the linear spatial dependence of the additional potential energy of excitons, the term $\delta V = -dz$ should be added in the formula of the free energy (15).In this case, Eq. (16) has a solution in the form of autowaves $n_{as}(z-vt)$, where v is the autowave velocity. For periodic solutions at $G > G_{c1}$, such autowaves were investigated in work [40].

Localized solutions exist also in the some region, where the pumping is greater than the value, at which the periodic structures arise. The solutions exist under an additional pumping pulse in the form (19), but at $s < 0$. The example of such a solution is presented in Fig. 5. These structures are appear in the form of a dip and can be called antiautosolitons.

To explain the appearance of the autosoliton-type solution, we recall that the phase transition are investigated. As is known, there exists the region between a spinodal and a binodal, in which the creation of a nucleus of the new phase is needed for the phase transition. The size of the nucleus should exceed some critical value. The above-obtained criteria for the appearance of a uniform solution at $n < n_{c1}$ from the equation (17) under small fluctuations determine, in fact, the boundary of the spinodal. The existence of nonuniform solutions in the form of autosolitons at the pulses larger than some critical value corresponds to the appearance of the nucleus outside of the spinodal boundary. Since we are studying the non-equilibrium processes for unstable particles, the spinodal and binodal regions depend on the lifetime of particles. In addition, for stable particles, a radius of the new phase increases with time. For example, according to the Lifshitz–Slyozov theory [44], the radius grows as $t^{1/3}$ in a three-dimensional system. In a contrast, for the

particles with the finite lifetime, the shape of the density distribution of particles does not depend on the time.

It should be noted that, in the approach of nucleation-growth to the condensation of excitons with a generalization onto unstable particles [20–24], the existence of the condensed-phase islands outside of the spinodal region is taken into account automatically, because this approach consider fluctuations, which are not presented in Eq. (16).

5. Conclusions

In this work, several problems of the theory of indirect exciton condensation in quantum wells on the base of AlGaAs are considered, and the analysis of the experimental data on the spectra of indirect excitons in quantum wells is fulfilled.

In particular, it is shown that the results of the experiment [31] where the emission spectrum from the condensed-phase region is shifted to the short-wave side as compared with the emission spectra from the gas-phase region, do not contradict the model with an attractive interaction between excitons. It is taken into account that the emission spectrum is formed both by excitons of the condensed phase and by excitons trapped on defects, whose population is saturated with increase in the pumping.

In the model of spinodal decomposition, the solutions in the form of solitary states (exciton autosolitons and antiautosolitons) are found. In the model of nucleation-growth, the solutions correspond to nuclei of the condensed phase in the gas phase and to nuclei of the gas phase in the condensed phase.

The hydrodynamic equations for excitons are constructed with regard for the interaction between excitons. Difficulties of the formation of the superfluidity state of excitons caused by the presence of phonons and defects are analyzed. It is follow from the analysis fulfilled in the work that there are no experimental data obtained under investigations of indirect excitons in the quantum wells on the base of AlGaAs, which cannot be explained without an involvement of the Bose–Einstein condensation of excitons.

The models considered in the work are phenomenological. As we underlined earlier [22, 23], the used approach is valid also, if the condensed phase is an electron-hole liquid. In the spinodal approach, the quantity n in the equation for the exciton density (14) should be considered as the density of electron-hole pairs. The model of nucleation-growth, which we used, by studying the exciton condensation in [20–24], is similar to the methods used in [45–47] in the investigation of electron-hole droplets in germanium and silicon. But, in contrast to works [45–47], we took into account the correlation in the spatial positions of droplets (islands in the two-dimensional case), which allows us to describe the various structures, mutual positions of droplets, their concentration, and other properties observed in quantum wells on the base of AlGaAs.

1. T. Fukuzava, E.E. Mendez, and J.M. Hong, Phys. Rev. Lett. **64**, 30066 (1990).
2. A.V. Larionov, V.B. Timofeev, J. Hvam, and K. Soerensen, JETP Lett. **71**, 117 (2000).
3. A.A. Dremin, A.V. Larionov, and V.B. Timofeev, Fiz. Tverd. Tela **46**, 168 (2004).
4. V.B. Timofeev, Usp. Fiz. Nauk **175**, 315 (2005).
5. L.V. Butov, A.C. Gossard, and D.S. Chemla, Nature **418**, 751 (2002).
6. B.D. Snoke, S. Denev, Y.Liu *et al.*, Nature **418**, 754 (2002).
7. L.V. Butov, Solid State Commun. **127**, 89, (2003).
8. D. Snoke, Y. Liu, L. Pfeifer, and K. West, Solid State Commun. **127**, 187 (2003).
9. A.V. Gorbunov and V.B. Timofeev, JETP Lett. **83**, 146 (2006); V.B. Timofeev, Usp. Fiz. Nauk **176**, 651 (2006).
10. V.V. Solov'ev, I.V. Kukushkin, V. Smet *et al.*, JETP Lett. **83**, 533(2006).
11. A.V. Gorbunov and V.B. Timofeev, JETP Lett. **84**, 329 (2006).
12. V.B. Timofeev, A.V. Gorbunov, and D.A. Demin, Fiz. Nizk. Temper. **37**, 229 (2011).
13. M. Remeika, J.C. Graves, A.T. Hammark *et al.*, Phys. Rev. Lett. **102**, 186803 (2009).
14. L.S. Levitov, B.D. Simons, and L.V. Butov, Phys. Rev. Lett. **94**, 176404 (2005).
15. A.V. Paraskevov and T.V. Khabarova, Phys. Lett. A **368** 151 (2007).
16. R.B. Saptsov, JETP Lett. **86**, 687 (2008).
17. C.S. Liu, H.G. Luo, and W.C. Wu, J. Phys.: Condens. Matter **18**, 9659 (2006).
18. V.S. Babichenko, arXiv/cond-mat: 0706.0994 (2007).
19. V.K. Mukhomorov, Fiz. Tverd. Tela **52**, 225 (2010).
20. V.I. Sugakov, Ukr. Fiz. Zh. **49**, 1117 (2004); Solid State Commun. **134**, 63 (2005).
21. V.I. Sugakov, Fiz. Tverd. Tela **48**, 1868 (2006); Phys. of the Solid State **48**, 1984 (2006); Fiz. Nizk. Temper. **32**, 1449 (2006).
22. V.I. Sugakov, Phys. Rev. B **76**, 115303 (2007).

23. A.A. Chernyuk and V.I. Sugakov, Phys. Rev. B **74**, 085303, (2006).
24. V.I. Sugakov and A.A. Chernyuk, JETP Lett. **85**, 570 (2007).
25. M.Y.I. Tan, N.D. Drummord, and I. Needs, Phys. Rev. B **71**, 033303 (2005).
26. Ch. Schindler and R. Zimmermann, Phys. Rev. B **78**, 045313 (2008).
27. A.D. Meyertholen and M.M. Folger, Phys. Rev. B **78**, 235307 (2008).
28. A.A. Chernyuk and V.I. Sugakov, Solid State Comm. **149**, 2185 (2009).
29. Yu.E. Lozovik and O.L. Berman, JETP Lett. **64**, 573 (1996).
30. V.I. Sugakov, Fiz. Tverd. Tela **28**, 2441 (1986); V.I. Sugakov, *Lectures in Synergetics* (World Scientific, Singapore, 1998).
31. Sen Yang, A.V. Mintsev, A.T. Hammack, and L.V. Butov, Phys. Rev. B **75**, 033311 (2007)
32. A.V. Gorbunov and V.B. Timofeev, JETP Lett. **84**, 390 (2006).
33. Sen Yang, A.T. Hammack, M.M. Fogler, and L.V. Butov, Phys. Rev. Lett. **97**, 187402 (2006).
34. V.I. Sugakov, Solid State Commun. **106**, 705 (1998).
35. A. Ishikawa, T. Ogawa, and V. Sugakov, Phys. Rev. B **64**, 144301 (2001).
36. A. Ishikawa and T. Ogawa, Phys. Rev. E **65**, 026131 (2002).
37. B. Link and G. Bayn, Phys. Rev. Lett. **69**, 2959 (1992).
38. M.R. Swift, E. Orlandini, W.R. Osborn, and J.M. Yeomans, Phys. Rev. E **54**, 5041 (1996).
39. V.I. Sugakov, Phase Trans., **75**, 953 2002.
40. V.I. Sugakov, J. Phys.: Condens. Matter **21**, 275803 (2009).
41. J. Kasprzak, M. Richard, S. Kundermann, A. Baas, P. Jeambrun, J.M.J. Keeling, F.M. Marchetti, M.H. Szymanska, R. Andre, J.L. Staehli, V. Savona, P.B. Littlwood, B. Deveaud, and Le Si Dang, Nature **443**, 409 (2006).
42. L.V. Butov, A. Imagoglu, A.V. Mintsev, K.L. Campman, and A.C. Gossard, Phys. Rev. B **59**, 1625 (1999); L.V. Butov, A.L. Ivanov, A. Imagoglu, R.B. Littlewood, A.A. Shashkin, V.T. Dolgopolov, K.L. Campman, and A.C. Gossard, Phys. Rev. Lett. **86**, 5608 (2005).
43. V.S. Kerner, V.V. Osipov, Soviet Physics Uspekhi **33**, 679 (1990).
44. I.M. Lifshitz and V.V. Slyozov, J. Phys. Chem. Solids **19**, 35 (1961).
45. R.N. Silver, Phys. Rev. B **11**, 1569 (1975).
46. R.M. Westervelt, Phys. Status Solidi B **74**, 727 (1976).
47. V.S. Bagaev, N.V. Zamkovets, L.V. Keldysh, N.N. Sybel'din, and V.A. Tsvetkov, Sov. Phys. JETP **43**, 783 (1976).

Received 29.07.11

КОНДЕНСАЦІЯ ЕКСИТОНІВ У КВАНТОВИХ ЯМАХ. САМООРГАНІЗАЦІЯ ПРОТИ БОЗЕ-КОНДЕНСАЦІЇ

В.Й. Сугаков

Р е з ю м е

Проведено аналіз інтерпретації експериментальних даних по спектрах випромінювання екситонів з подвійних квантових ям. Особливості у просторовому розподілі випромінювання та в його поведінці залежно від температури та інтенсивності накачки пояснюються з точки зору існування конденсовної фази екситонів, зумовленої взаємодією між екситонами, а не їх бозе–ейнштейнівською конденсацією. Структура просторового розподілу густини екситонів у конденсованій фазі залежить від часу життя екситонів і є наслідком процесів самоорганізації в нерівноважних системах. Залежно від накачки досліджено розподіл екситонів по делокалізованих та локалізованих станах і його вплив на спектри випромінювання. Знайдено рівняння гідродинаміки екситонів, яке враховує взаємодію між екситонами. Показано існування екситонних солітоноподібних станів (автосолітонів) за межами області спінодального розпаду.