AVERAGE SIZE AND DENSITY OF CLUSTERS IN DUSTY PLASMA

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As is well known, metal and metal oxide dust particles either in combustion plasmas or in low-pressure gas-discharge plasmas are able to get a positive or negative charge. When the particles are identically charged, the Debye interparticle repulsion force arises. However, due to the additional interaction of grains with plasma electrons and ions, the resultant intergrain pair potential energy becomes non-Debye. As a result, it can cause local minima by certain plasma parameters. This case develops a quasimolecular structuralization, where two equally charged grains are spaced at a finite distance in equilibrium. The same principle of grain structuralization is valid for the multibody case where grains are associated in a crystal structure or clusters. The present work is devoted to the description of the dependence of the average size and the density of clusters on the interparticle potential. The general formula for the average cluster energy via the pair potential energy is derived.

1. Introduction

It is well known that metal and metal oxide dust particles either in combustion plasmas or in low-pressure gas-discharge plasmas are able to get a positive or negative charge [1–3]. When the particles are charged identically, the Debye interparticle repulsion force arises. However, due to the additional interaction of grains with plasma electrons and ions, the resultant intergrain pair potential energy becomes non-Debye. As a result, it can cause local minima by certain plasmas parameters. This case develops a quasimolecular structuralization, where two equally charged grains are spaced at a finite distance in equilibrium. The same principle of grain structuralization is valid for the multibody case where grains are associated in a crystal structure or clusters. It is obvious that, in the case of rapidly decreasing pair potentials, the dependence of the cluster radius on grain numbers inside it is expressed by $R \sim d_0\sqrt[3]{N}$, where d_0 – minimum of $\Phi(x)$, N – grain number. The present work is devoted to the description of the dependence of the average size and density of clusters on the interparticle potential.

2. Statement of the Problem

Let us consider that the intergrain interaction is governed only by the pair potential energy which is a function of the intergrain distance: $U = \Phi(|\mathbf{r}_2 - \mathbf{r}_1|)$. Then it is possible to calculate the N-grain cluster energy by the use of a summation procedure, by spacing grains in the spherical cluster volume uniformly at the average intergrain distance d. One can find the equilibrium cluster potential energy by the variation of d. Making use of this procedure for various numbers of grains, it is possible to obtain the consequent dependence for the cluster radius and the average cluster density: $R(N,d) = \frac{3}{4\pi}d\sqrt[3]{N}$, $n(N) = d(N)^{-3}$.

First, we find the interaction energy between the selected grain that is located at the distance r, $r \leq R$, away from the cluster center and the remaining grains inside it as the sum

$$\mathcal{E}(\mathbf{r}_i) = \sum_{\substack{j\neq i \\ |\mathbf{r}_j|\leq R}} \Phi\big(|\mathbf{r}_j - \mathbf{r}_i|\big), \qquad i = \overline{1\ldots N}. \tag{1}$$

For obtaining the whole cluster energy E, it is necessary to sum up the single-grain energy $\mathcal{E}$ over all grains and to divide it in half to exclude the repeating summands ($\Phi_{1,2} = \Phi_{2,1}$):

$$E = \frac{1}{2}\sum_{i=1}^{N}\mathcal{E}(\mathbf{r}_i). \tag{2}$$

In the calculation of $\mathcal{E}(\mathbf{r}_i)$, it is convenient to change the summation by the integration:

$$\mathcal{E}(\mathbf{r}_i, R, d) = \frac{1}{d^3}\int\limits_V \tilde{\Phi}(|\mathbf{r}_i - \mathbf{r}\,'|)\,dV\,',$$

$$\tilde{\Phi}(r) = \Phi(r)h(r),$$

$$h(r\,) = d\sum_{k=1}^{\infty}\delta(r - kd), \tag{3}$$

$\delta(r)$ is the Dirac function, and the integration is carried out over the whole cluster volume. Making use of spherical coordinates, we derive the following expression after several transformations:

$$\mathcal{E}(r_i,R,d)=\frac{2\pi}{r_i d^3}\int\limits_0^R\int\limits_{|r_i-r'|}^{r_i+r'}\tilde{\Phi}\big(x\big)x\,r'\,dr'\,dx. \tag{4}$$

As $r_i\in[0;R]$ and $r'\in[0;R]$, $\Rightarrow x\in[0;2R]$. Represent our potential $\tilde{\Phi}$ as the identity:

$$\tilde{\Phi}(x)=\int\limits_0^{2R}\tilde{\Phi}(x')\delta(x'-x)\,dx'.$$

Then (4) can be integrated without using $\Phi(x)$, and we express the single-grain energy $\mathcal{E}$ in the convolution form with an analytically defined function as

$$\mathcal{E}(r_i,R,d)=$$

$$=\frac{2\pi}{r_i d^3}\int\limits_0^R\int\limits_{|r_i-r'|}^{r_i+r'}\int\limits_0^{2R}\tilde{\Phi}\big(x'\big)x\,r'\delta(x'-x)\,dr'\,dx\,dx'=$$

$$=\frac{2\pi}{r_i d^3}\int\limits_0^{2R}\tilde{\Phi}\big(x'\big)\,\mathtt{J}(r_i,R,x')dx', \tag{5}$$

$$\mathtt{J}(r_i,R,x')=\int\limits_0^R\int\limits_{|r_i-r'|}^{r_i+r'}r'x\delta(x'-x)dr'\,dx. \tag{6}$$

The cluster energy will be presented from (2) by the simple change of the summation by the integration as opposed to the previous case:

$$E(R,d)=\frac{2\pi}{d^3}\int\limits_0^R\mathcal{E}(r,R,d)r^2\,dr=$$

$$=\frac{1}{d}\int\limits_0^{2R}\tilde{\Phi}\big(x'\big)\mathtt{K}(x',R,d)\,dx'=\sum_{k=1}^{\lfloor\frac{2R}{d}\rfloor}\Phi\big(kd\big)\mathtt{K}(kd,R,d)\,, \tag{7}$$

$$\mathtt{K}(x',R,d)=\frac{4\pi^2}{d^5}\int\limits_0^R\mathtt{J}(r,R,x')r\,dr. \tag{8}$$

Here, we do not present the complete derivation of $\mathtt{J}(r,R,x')$ and $\mathtt{K}(x',R,d)$ owing to the bulkiness of mathematical calculations. We just notice that, for obtaining $\mathtt{J}(r,R,x')$, it is necessary to integrate (6) over x firstly, to indicate the regions, where the integral is nonzero, and then to integrate the obtained expression over r' for nonzero regions. Making up these calculations for $r\in[0;R]$, we obtain

$$\mathtt{J}(r,R,x')=\frac{x'}{2}\begin{cases}4x'r, & x'\in[0;R-r);\\ R^2-(x'-r)^2, & x'\in(R-r;R+r].\end{cases} \tag{9}$$

The obtained expression can be rewritten as inequalities for r. Whereupon, it can be available to calculate integral (8). Let us represent the final result for $x'\in[0;2R]$:

$$\mathtt{K}(x',R,d)=\frac{4\pi^2}{d^5}\left(\frac{2x'^2R^3}{3}-\frac{x'^3R^2}{2}+\frac{x'^5}{24}\right). \tag{10}$$

Thus, using the method offered above, it can be easy to calculate the spherical cluster potential energy possessing the pair intergrain energy $\Phi(x)$, grains number, and average intergrain distance. One can calculate the equilibrium average intergrain distance d on the basis of the minimum cluster energy (7).

3. Results

For the visual interpretation of our results, we used the "dressed electrostatic potential" derived in work [4]:

$$\Phi(x)=\frac{Q_1Q_2}{D}\frac{e^{-x}}{x}\left(1-\delta\frac{x}{2}\right), \tag{11}$$

$x=r/D$, D – the Debye radius, and Q_1, Q_2 – charges of particles, which are supposed equal to 1. The parameter δ characterizes the grain polarization. It is equal to 1 at the complete polarization and to 0 in its absence.

In Fig. 1, we present the calculated dependences of the cluster radius and its average density (Fig. 2) versus the grain number in it at $D=1$. The dimension of the length is equal to the Debye radius dimension. It is seen for $\delta=1$ that the cluster becomes denser, as N increases. This could be explained by the presence of attracting forces between far-spaced grains. As δ decreases, the intergrain interaction is governed by the nearest neighbors. Thus, the average cluster density is defined by the minimum of $\Phi(x)$ completely.

In Fig. 3, we show the dependence $\mathcal{E}(r)$ at $N=10^4$, $\delta=1$, $(D=0.5,d=0.68)$, $(D=1,d=1.32)$, and

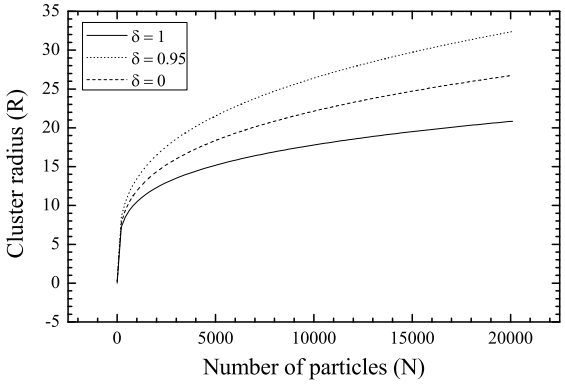

Fig. 1.

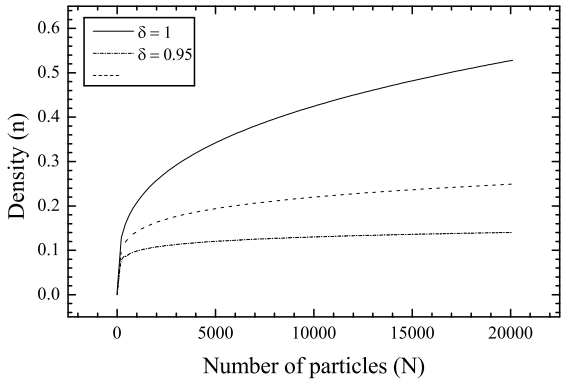

Fig. 2.

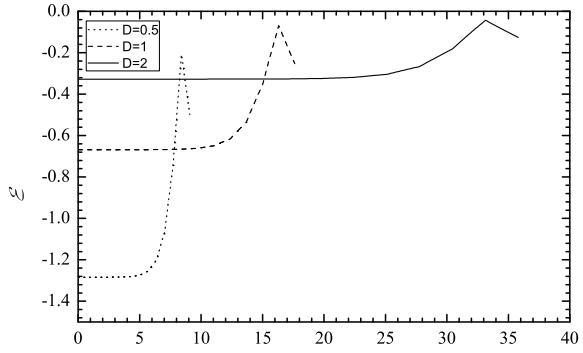

Fig. 3.

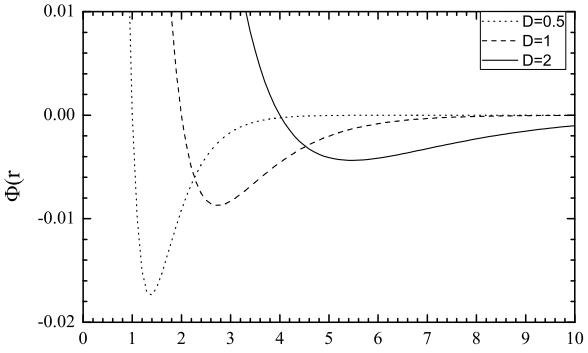

Fig. 4.

($D = 2, d = 2.68$). One can see, by approaching to the cluster edge, that $\mathcal{E}(r)$ increases and then decreases. We explain these facts by the example of $D = 1$. The average intergrain distance is equal to 1.32, that is a half of the minimum of the pair potential 2.68 (see Fig. 4). Therefore, the intergrain energy for the nearest neighbors is positive. But if we take the far-spaced grains into account, the energy becomes negative. By approaching the edge of the cluster, the number of far-spaced grains decreases, and, consequently, the energy is going up. By the further approaching to the edge, the number of nearest neighbors increases faster than that of far-spaced ones. As a result, $\mathcal{E}(r)$ is falling down.

The obtained results may be useful due to the ability to calculate the cluster intergrain force by the simple differentiation of $\mathcal{E}(r)$. The presence of this force will allow us to set and to solve up the consequent problem of internal cluster deformations and to reveal specific dynamic effects.

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СЕРЕДНІЙ РОЗМІР ТА ГУСТИНА КЛАСТЕРІВ У ПИЛОВІЙ ПЛАЗМІ

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Р е з ю м е

Широко відомо те, що частинки металів і їх оксидів у плазмі продуктів згоряння або газорозрядній плазмі здатні здобувати позитивний або негативний заряд. У випадку однаково заряджених частинок, між ними з'являються дебаєвські сили відштовхування. Однак завдяки додатковій взаємодії із плазмою, у виразі для результуючої міжчасткової енергії взаємодії можуть виникати мінімуми при деяких параметрах плазми. Це може привести до утворення стійкої конфігурації двох ча-

стинок, що перебувають на фіксованій відстані одна від одної. Той же принцип є вірним для випадку великої кількості частинок, які поєднуються в кристалічну структуру або кластери. Ця робота присвячена знаходженню залежності розміру й середньої густини пилового кластера від кількості частинок у ньому.