# ION-ACOUSTIC TURBULENCE IN PLASMAS

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The new properties of plasmas with ion-acoustic turbulence (IAT) related to the turbulent heating of particles are discussed. We expound the ideas of the coarsened theory of IAT, which allows one to describe the strong heating of particles. The description of the competition of the electron heating and the ion heating allows us to open the discussion about the finite existence time of IAT and to give some estimation of such time. It is also demonstrated that the usual model of IAT the yields time-dependent electric conductivity in a constant-strength electric field and a nonlinear time dispersion for the conductivity in a quasistationary field.

## 1. Introduction

Our paper is devoted to the theoretical discussion of some phenomena related to the time-dependent turbulent plasma heating. The state of turbulent plasma with ion-acoustic turbulence was discovered during the 1960s due to the pioneering researches of a group at I.V. Kurchatov Institute in Moscow and a group at the Kharkov Institute of Physics and Technology. This discovery had won the recognition in the USSR on the state's level: "Discovery No. 112: Phenomenon of turbulent heating and anomalous plasma resistance. Application OT-7595 from March 30, 1970, Priority: September 9, 1965. Published: October 23, 1972. The authors: M.V. Babykin, E.D. Volkov, P.P. Gavrin, B.A. Demidov, E.K. Zavoiskii, L.I. Rudakov, V.A. Skoryupin, V.A. Suprunenko, E.A. Sukhomlin, Ya.B. Fainberg and S.D. Fanchenko" [1]. Moreover, this discovery attracted the attention of many experimentalists and theoreticians among the plasma physicists all over the world. In particular, it is possible to speak about the creation of some especial kinetic approach in the 20th century to the description of the ion-acoustic turbulence (IAT) as a new plasma state. This description is based on the Cherenkov interaction of ion-acoustic waves (IAW) with charged plasma particles [2–4] (L.I. Rudakov, L.V. Korablev, 1966; L.M. Kovrizhnykh, 1966, 1967) and on the induced scattering of ion-acoustic waves by plasma ions [5, 6] (B.B. Kadomtsev, 1964, V.I. Petviashvili, 1963). The quasilinear approximation allowed one to predict the anisotropy of the angular distribution of the IAT pulsation that was really observed. The analysis of the induced scattering of IAW by ions yielded the prediction of the wide frequency spectrum of IAT pulsations: the so-called Kadomtsev–Petviashvili spectrum. The theoretically predicted quasilinear angular distribution of IAT pulsations obtained by Rudakov and Korablev was nonstationary. The further development of the quasistationary IAT theory was influenced by the hypothesis put forward by Sizonenko and Stepanov [7] about a regularization of the Rudakov–Korablev spectrum by nonlinear effects. Later, this hypothesis was proved as a theorem in the theory that involved simultaneously the Cherenkov interaction of IAW with charged particles and the induced scattering of IAW by ions [8].

We note that, during a long time, the theory of IAT was based on a model of plasmas with the same ratio of charge to mass for all ions. The important step in the development of the IAT theory of plasmas with unequal ion charge ($e_\alpha$) to mass ($m_\alpha$) ratios,

$$\frac{e_\alpha}{m_\alpha} \neq \frac{e_\beta}{m_\beta}, \tag{1}$$

was made later [9]. The present paper is devoted to the case of such plasmas.

The importance of the above-mentioned discovery No. 112 is connected with the heating of ions that is a very important problem in controlled thermonuclear fusion projects. But the theory of strong turbulent ion heating is consistent only when the integrals of ion-ion collisions are larger than the collision integrals of ions with the turbulent plasma pulsations, and it is possible to speak about the Maxwellian ion distribution. Such possibility is absent on the way to the thermonuclear temperature.

So we draw attention to the case of negligible two-particle ion-ion collisions. But, in this case, it is difficult to use the available theory of IAT, because the different versions of such theory are based on the hypothesis about the isotropic Maxwellian ion distribution in the main region of the velocity space.

On the other hand, in the case of plasma with a heavy-ion admixture, the paper [9] demonstrated the anisotropic bi-Maxwellian velocity distribution for this admixture. Moreover, the longitudinal temperature for the velocity component antiparallel to the constant-strength electric field $\mathbf{E}$, which heats plasma and realizes IAT, is sufficiently less than the transverse temperature for two other velocity components.

## 2. Ideas of the Coarsened IAT Theory

Further, we use, as our supposition [10], for the case of two-ion-component plasmas when the masses and densities of both species are comparable that the main body of the ion velocity distribution is bi-Maxwellian:

$$f_\alpha\left(V_x, V_y, V_z, t\right) =$$

$$= \frac{N_\alpha m_\alpha^{3/2}}{(2\pi)^{3/2}\,\kappa_{\mathrm{B}} T_{\perp\alpha}(t)\left(\kappa_{\mathrm{B}} T_{\|\alpha}(t)\right)^{1/2}} \times$$

$$\exp\left(-\frac{m_\alpha\left(V_x^2+V_y^2\right)}{2\kappa_{\mathrm{B}} T_{\perp\alpha}(t)} - \frac{m_\alpha V_z^2}{2\kappa_{\mathrm{B}} T_{\|\alpha}(t)}\right). \qquad (2)$$

Here, $\kappa_{\mathrm{B}}$ is the Boltzmann constant, $m_\alpha$, $N_\alpha$, $T_{\perp\alpha}(t)$, and $T_{\|\alpha}(t)$ are the mass, number density, transversal and longitudinal temperatures of ions of the $\alpha$ species $(\alpha = 1, 2)$, respectively. Our supposition about the bi-Maxwellian ion distribution is corroborated in our theory. Let us underline that the change of $T_{\perp\alpha}(t)$ and $T_{\|\alpha}(t)$ in time gives the change of the main body of the ion velocity distribution.

In the limits of such supposition, the used later coarsened description of the strong ion heating right up to conditions, when the ion-ion collisions are negligible, becomes consistent.

This allows us to examine the question about the competition of the processes of heating of electrons and ions. This new question in the theory of IAT leads to another question about the time of the existence of IAT as a phenomenon in the nonisothermal plasma.

It is important to mention that, in our approach to IAT, we use the so-called Coulomb model which takes only the Coulomb interaction of charged particles of a plasma into account. In accordance with this model, we can use the following usual expression for the plasma permittivity:

$$\varepsilon(\omega, \mathbf{k}) = 1 + \sum_s \frac{4\pi e_s^2}{m_s k^2} \int \frac{1}{\omega - \mathbf{kV}}\,\mathbf{k}\frac{\partial f_s}{\partial \mathbf{V}} d\mathbf{V} \equiv$$

$$\equiv 1 + \delta\varepsilon_e(\omega, \mathbf{k}) + \sum_{\alpha=1,2} \delta\varepsilon_\alpha(\omega, \mathbf{k}). \qquad (3)$$

In the first term, the summation is taken over each kind of charged plasma particles.

The variant expounded here of the IAT theory uses a number of results of the previous papers which retain, nevertheless, their importance.

Similar to our papers [9, 11], in which plasmas with two ion species were analyzed, we consider ion-acoustic waves with the spectrum

$$\omega = \omega(\mathbf{k}) = V_S k \left(1 + k^2 r_{\mathrm{De}}^2\right)^{-1/2}. \qquad (4)$$

Here, $r_{\mathrm{De}} = \left(\kappa_{\mathrm{B}} T_e / 4\pi e^2 N_e\right)^{1/2}$is is the electron Debye screening radius; $e$ is the electron charge; $N_e$ is the electron number density; $T_e$ is the electron temperature; $\omega_{\mathrm{L}} = \left(\omega_{\mathrm{L1}}^2 + \omega_{\mathrm{L2}}^2\right)^{1/2}$ with $\omega_{\mathrm{L}\alpha} = \left(4\pi e_\alpha^2 N_\alpha / m_\alpha\right)^{1/2}$ and $e_\alpha$ are the Langmuir frequency and charge of the $\alpha$-ion species; $\omega$ and $k$ are the frequency and the absolute value of the wave vector of ion-acoustic waves, respectively; and $V_S = \omega_{\mathrm{L}} r_{\mathrm{De}}$ is the velocity of an ion acoustic wave with long wavelength.

Now let us determine our quasistationary approach to the IAT theory. The basic equation of this approach is

$$\gamma(\mathbf{k}) \equiv \gamma_e(\mathbf{k}) + \sum_{\alpha=1,2} \gamma_\alpha(\mathbf{k}) + \gamma_{\mathrm{NL}}(\mathbf{k}) = 0. \qquad (5)$$

Here, $\gamma(\mathbf{k})$ is the increment of ion-acoustic waves, which is a sum of the electronic $\gamma_e$ and ionic $\gamma_\alpha$ contributions caused by the Cherenkov interaction of IAW according to electrons and ions, and the nonlinear ionic contribution $\gamma_{\mathrm{NL}}$ is connected with the induced scattering of IAW by ions. We must change the description of the last one in comparison with the previous one. In accordance to formula (13) from [9] for the nonlinear contribution $\gamma_{\mathrm{NL}}$ to the damping rate of ion-acoustic waves caused by their induced scattering by ions having unequal charge-to-mass ratios, we have

$$\gamma_{\mathrm{NL}}(\mathbf{k}) = \int \frac{d\mathbf{k}}{8\pi} N(\mathbf{k}) \frac{|\mathbf{k}-\mathbf{k}'|^2}{\omega_{\mathrm{L}}^4} \left(\frac{\mathbf{kk}'}{kk'}\right)^2 \omega(\mathbf{k})\,\omega(\mathbf{k}') \times$$

$$\times \frac{\partial \delta\left(\omega(\mathbf{k}) - \omega(\mathbf{k}')\right)}{\partial \omega(k)} \left(\frac{e_1}{m_1} - \frac{e_2}{m_2}\right)^2 \times$$

$$\times \frac{\omega_{\mathrm{L1}}^2 \delta\varepsilon_2^2(0, \mathbf{k}-\mathbf{k}') + \omega_{\mathrm{L2}}^2 \delta\varepsilon_1^2(0, \mathbf{k}-\mathbf{k}')}{\left[\delta\varepsilon_1(0, \mathbf{k}-\mathbf{k}') + \delta\varepsilon_2(0, \mathbf{k}-\mathbf{k}')\right]^2}. \qquad (6)$$

Here, $N(\mathbf{k}) = N(k)\,\Phi(\cos\theta_k)$ is the axisymmetric distribution of ion-acoustic waves over their wave vectors $\mathbf{k}$, $\theta_k$ is the angle between the wave vector and $e\mathbf{E}$,

$N(k)$ is the distribution of IAT pulsations over their wave numbers, $\Phi(\cos\theta_k)$ is their angular distribution, and $\delta\varepsilon_\alpha\left(0,\vec{k}-\mathbf{k}'\right)$ is the contribution of ions of the $\alpha$ species to the static longitudinal plasma permittivity. In accordance to the anisotropic bi-Maxwellian velocity distribution of $\alpha$-species of ions, we have

$$\delta\varepsilon_\alpha(0,\mathbf{k}-\mathbf{k}') =$$

$$= \frac{(4\pi e_\alpha^2 N_\alpha/\kappa_\mathrm{B})}{\left[(k_x-k_x')^2 T_{\perp\alpha} + \left(k_y-k_y'\right)^2 T_{\perp\alpha} + (k_z-k_z')^2 T_{\parallel\alpha}\right]}. \quad (7)$$

Our change of the $\gamma_\mathrm{NL}$ description is connected with the approximation of the expression for $\delta\varepsilon_\alpha(0,\mathbf{k}-\mathbf{k}')$. Our approximation is based on the result of paper [9] about the heating of a small-ion admixture, where it was demonstrated that the transverse temperature of the ion admixture increases faster than its longitudinal temperature. We use this result as a base of our assumption about the analogous result for every kind of ions. This assumption is corroborated in our theory.

So we ignore the longitudinal temperature in comparison with the transversal one, by describing the influence of the induced scattering of ion-acoustic waves by ions. Then we use the expression

$$\delta\varepsilon_\alpha(0,\mathbf{k}-\mathbf{k}') = \frac{4\pi e_\alpha^2 N_\alpha}{\kappa_\mathrm{B} T_{\perp\alpha}\left[(k_x-k_x')^2 + \left(k_y-k_y'\right)^2\right]} \equiv$$

$$\equiv \frac{1}{r_{\perp\alpha}^2\left[(k_x-k_x')^2 + \left(k_y-k_y'\right)^2\right]} \quad (8)$$

instead of the expression $\delta\varepsilon_\alpha(0,\mathbf{k}-\mathbf{k}') = r_{D\alpha}^{-2}(\mathbf{k}-\mathbf{k}')^{-2}$ in the theory with Maxwellian ion distributions. Here, $r_{D\alpha} = \left(\kappa_\mathrm{B} T_\alpha / 4\pi e_\alpha^2 N_\alpha\right)$ is the Debye radius of ions with temperature $T_\alpha$. Therefore, our proposed approximate theory of the nonlinear influence of scattering by ions differs from the IAT theory proposed about twenty years ago [9] in that the ion Debye radii are replaced with the transverse ion radii

$$r_{D\alpha}^{-1} = \sqrt{\frac{4\pi e_\alpha^2 N_\alpha}{\kappa_\mathrm{B} T_\alpha}} \to r_{\perp\alpha}^{-1} = \sqrt{\frac{4\pi e_\alpha^2 N_\alpha}{\kappa_\mathrm{B} T_{\perp\alpha}}}. \quad (9)$$

Since we have so coarsened the description of the induced scattering of IAW by ions, we can write

$$\gamma_\mathrm{NL}(\mathbf{k}) = \int \frac{d\mathbf{k}}{8\pi} N(\mathbf{k}) \frac{|\mathbf{k}-\mathbf{k}'|^2}{\omega_\mathrm{L}^4}\left(\frac{\mathbf{k}\mathbf{k}'}{kk'}\right)^2 \times$$

$$\times \frac{\omega_\mathrm{L1}^2 r_{\perp 1}^4 + \omega_\mathrm{L2}^2 r_{\perp 2}^4}{\left[r_{\perp 1}^2 + r_{\perp 2}^2\right]^2}\omega(\mathbf{k}) \frac{\partial\delta(\omega(\mathbf{k}) - \omega(\mathbf{k}'))}{\partial\omega(k)} \times$$

$$\times \left(\frac{e_1}{m_1} - \frac{e_2}{m_2}\right)^2 \omega(\mathbf{k}'). \quad (10)$$

The simple replacement of $r_{D\alpha}$ by $r_{\perp\alpha}$ allows us to use the result of the previous theory for the distribution of ion-acoustic pulsations on the absolute values of the wave vectors immediately:

$$N(k) = \sqrt{\frac{\pi}{2}} \frac{\omega_\mathrm{L}^6 r_\mathrm{De}^5}{\omega_\mathrm{Le}} \frac{1}{(\omega_{L1}^2 r_{\perp 1}^4 + \omega_\mathrm{L2}^2 r_{\perp 2}^4)} \times$$

$$\times \frac{\left(r_{\perp 1}^2 + r_{\perp 2}^2\right) Y(k r_\mathrm{De})}{\left[(e_1/m_1) - (e_2/m_2)\right]^2}, \quad (11)$$

$$Y(x) = \frac{x^{-4}}{(1+x^2)}\left[\ln\left(\frac{\sqrt{1+x^2}+1}{x}\right) - \frac{1}{(1+x^2)^{1/2}} - \right.$$

$$\left. - \frac{1}{3(1+x^2)^{3/2}}\right]. \quad (12)$$

Such simple replacement gives us the so-called turbulent Knudsen number

$$K_{N2} = 6\pi^2 \frac{|e| N_e \omega_\mathrm{Le}^2}{\omega_\mathrm{L}^8 r_\mathrm{De}} \left(\frac{e_1}{m_1} - \frac{e_2}{m_2}\right)^2 \times$$

$$\times \frac{\left(\omega_\mathrm{L1}^2 r_{\perp 1}^4 + \omega_\mathrm{L2}^2 r_{\perp 2}^4\right)}{\left(r_{\perp 1}^2 + r_{\perp 2}^2\right)^2} E \equiv \frac{E}{E_{N2}}. \quad (13)$$

By comparing it with 1, we can distinguish the limits of the weak and strong electric fields.

The formula for the electronic increment of IAW in the basic equation is the usual one (see, e.g., [2, 3]). As an illustration, we give such formula for the case of the Maxwellian electron distribution [9]:

$$\gamma(k,\theta_k) = \gamma_S(k)\left(\frac{\omega}{kV}\right)^3\left(-\left(\frac{\omega}{kV_S}\right) + \right.$$

$$+ \frac{2}{\pi}\cos\theta_k \int\limits_0^{\sin\theta_k} \frac{d\xi}{\left(\sin^2\theta_k - \xi^2\right)^{1/2}} \times$$

$$\times\left[\frac{\nu_E}{\nu_2\left(\sqrt{1-\xi^2}\right)}+\frac{\nu_1\left(\sqrt{1-\xi^2}\right)}{\sqrt{1-\xi^2}\nu_2\left(\sqrt{1-\xi^2}\right)}\right]\Bigg), \quad (14)$$

where

$$\nu_n(y)=\int_0^\infty\frac{k^3dk}{4\pi^2}\left(\frac{\omega}{kV_S}\right)^{4-n}\times$$

$$\times\int_{-1}^{+1}\frac{dx}{(y^2-x^2)^{1/2}}\left(\frac{x}{y}\right)^n\frac{\omega N(k,x)}{N_e\sqrt{m_e\kappa_\mathrm{B}T_e}},$$

$$\nu_E=(9\pi/8)^{1/2}|e|E/m_eV_S,$$

$$\gamma_S(k)=(\pi/8)^{1/2}kV_S(\omega_\mathrm{L}/\omega_\mathrm{Le}).$$

The last term in the basic equation $\gamma(\mathbf{k})=0$ is the ion decrement of IAW related to the Cherenkov interaction of waves with ions. The ion Cherenkov interaction takes only the Landau damping of IAW into account. Hence, for the ratio between the ion and electron damping rates of IAW, we can use the relation

$$\delta\left(\cos^2\theta_k\right)=\sum_{\alpha=1,2}\delta_\alpha\left(\cos^2\theta_k\right)=$$

$$=\sum_{\alpha=1,2}\frac{\omega_\mathrm{Le}}{\omega_{\mathrm{L}\alpha}}\left(\frac{r_\mathrm{De}^2}{r_{\perp\alpha}\sin^2\theta_k+r_{\|\alpha}\cos^2\theta_k}\right)^{1/2}\times$$

$$\times\exp\left(-\frac{\omega_\mathrm{L}^2r_\mathrm{De}^2}{2\omega_{\mathrm{L}\alpha}^2\left(r_{\perp\alpha}\sin^2\theta_k+r_{\|\alpha}\cos^2\theta_k\right)}\right). \quad (15)$$

Here, $\omega_\mathrm{Le}$ is the electron Langmuir frequency.

Now we can say that the basic equation allows us to obtain not only $N(k)$, but also the following nonlinear Abel-type integral equation for the angular distribution of IAT pulsations $\Phi(\cos\theta_k)$ (compare [9]):

$$\int_0^x dt\left[\frac{t}{x^2}\left(1+\varphi\left(x^2\right)+\Delta\left(x^2\right)\right)-1\right]\frac{t\Phi(t)}{(x^2-t^2)^{1/2}}=$$

$$=\frac{K_{N2}}{\lambda}x^2, \quad 0\le x\le 1, \quad (16)$$

where

$$\varphi\left(x^2\right)=\frac{1}{2}\Bigg(M_0-M_2+(6M_1-10M_3)x^4+$$

$$+(6M_1-10M_3)x^4-(M_o+6M_1-3M_2-8M_3)x^2+$$

$$+(M_0-3M_2)x^2\left(1-x^2\right)^{1/2}\ln\left(\frac{1+\sqrt{1+x^2}}{x}\right)\Bigg) \quad (17)$$

is the function describing the nonlinear influence of turbulence on its angular distribution because of $M_n=\int_0^1 x^n\Phi(x)\,dx$ and $\lambda\simeq 0.5$. The new angular dependence is connected with the function

$$\Delta\left(\cos^2\theta_k\right)=\sum_\alpha\Delta_\alpha\left(\cos^2\theta_k\right)=$$

$$=\sum_\alpha\cos\theta_k\frac{d}{d\theta_k}\int_0^{\theta_k}\frac{\sin\theta\delta_\alpha\left(\cos^2\theta\right)d\theta}{\left(\cos^2\theta-\cos^2\theta_k\right)} \quad (18)$$

resulting from the anisotropy of the ion velocity distribution.

In the limit $K_{N2}/\lambda\gg 1$, the solution of the nonlinear Abel-type integral equation was obtained previously [9] as

$$\Phi(x)=\frac{2K_{N2}}{\pi\lambda x^2}\frac{d}{dx}\int_0^x\frac{\varsigma^5d\varsigma}{\sqrt{x^2-\varsigma^2}\varphi\left(\varsigma^2\right)}. \quad (19)$$

This is a case of the strong electric field, where the function $\varphi\left(\varsigma^2\right)$ contains

$$M_n=\sqrt{2K_{N2}/\pi\lambda}b_n, \quad \text{where} \quad b_0=2.47;$$

$$b_1=1.84; \quad b_2=1.44; b_3=1.17. \quad (20)$$

Further in the discussion of the results connected with the turbulent plasma heating, we limit ourselves to the discussed case of the strong electric field.

## 3. Heating of Turbulent Plasma Particles

The results of the theory of IAT angular distribution allows us to look at the kinetic equations, which describe the distribution of the main group of ions in the ion velocity space. These equations were derived in [9, 12]. But since these kinetic equations describe the evolution to the bi-Maxwellian distribution, we propose the replacement $r_{D\alpha} \to r_{\perp\alpha}$. In the $(K_{N2}/\lambda) \gg 1$ limit in accordance with [12], we have

$$\frac{\partial f_\alpha}{\partial t} = d_\alpha \left[ A_\perp \left( \frac{\partial^2 f_\alpha}{\partial V_x^2} + \frac{\partial^2 f_\alpha}{\partial V_y^2} \right) + A_\parallel \frac{\partial^2 f_\alpha}{\partial V_z^2} \right], \quad (21)$$

where

$$d_\alpha = \frac{|e|\, E N_e \omega_{\rm L} r_{\rm De} \omega_{{\rm L}\alpha}^2 r_{\perp\alpha}^4}{m_\alpha N_\alpha \left(\omega_{\rm L1}^2 r_{\perp 1}^4 + \omega_{\rm L2}^2 r_{\perp 2}^4\right)} \quad (22)$$

and $A_\perp \cong 0.6$, $A_\parallel \cong 0.15$. The solution of these ion kinetic equations gives us the anisotropic bi-Maxwellian velocity distributions of both ion components. The transversal and longitudinal temperatures evolve according to the equations ($\alpha = 1, 2$)

$$\frac{d(\kappa_{\rm B} T_{\perp(\parallel)\alpha})}{dt} = 2 m_\alpha d_\alpha A_{\perp(\parallel)}. \quad (23)$$

Therefore, our disregard of $r_{\parallel\alpha}$ in comparison with $r_{\perp\alpha}$ in the case of strong electric fields gives us the accuracy of 25% for the description of the induced IAW scattering by ions. Such accuracy is better in the case of weak electric fields [10]. In view of Eqs. (23), we can write the following equation for the transversal temperatures:

$$\frac{dT_{\perp 1}}{dT_{\perp 2}} = \frac{m_1 d_1}{m_2 d_2}. \quad (24)$$

This equation in the limit of the strong electric field looks simply as

$$\frac{dT_{\perp 1}}{dT_{\perp 2}} = \frac{m_2 e_2^2 N_2^2}{m_2 e_2^2 N_2^2} \frac{T_{\perp 1}^2}{T_{\perp 2}^2}. \quad (25)$$

Then we have the simple solution

$$T_{\perp 1}^{-1}(t) - T_{\perp 1}^{-1}(t_0) = \frac{m_2 e_2^2 N_2^2}{m_1 e_1^2 N_1^2} \left( \frac{1}{T_{\perp 2}(t)} - \frac{1}{T_{\perp 2}(t_0)} \right). \quad (26)$$

If

$$\frac{T_{\perp 2}(t_0)}{T_{\perp 1}(t_0)} = \frac{m_2 e_2^2 N_2^2}{m_1 e_1^2 N_1^2} \quad (27)$$

at the initial time moment, then this ratio remains valid later at $t \geq t_0$.

Further we should like to discuss, as an illustration, the simple example of the equal temperatures of heated ions, when $T_{\perp 1}(t_0) = T_{\perp 2}(t_0) = T_\perp(t_0)$ and $m_1 e_1^2 N_1^2 = m_2 e_2^2 N_2^2$. In the opposite case $m_1 e_1^2 N_1^2 \neq m_2 e_2^2 N_2^2$ if we assume the monotonic growth of $T_{\perp 1}(t)$. Then we have $T_{\perp 1}(t \to \infty) \to \infty$ and

$$\frac{T_\perp(t_0)}{T_{\perp 2}(t \to \infty)} = \frac{m_2 e_2^2 N_2^2}{m_2 e_2^2 N_2^2 - m_1 e_1^2 N_1^2}. \quad (28)$$

So if $m_2 e_2^2 N_2^2 > m_1 e_1^2 N_1^2$, then $T_{\perp 2}(t)$ strives to its limiting saturation value.

To discuss the heating of two ion components with the equal temperatures in the regime without the saturation, we use the kinetic equations

$$\frac{\partial f_\alpha}{\partial t} = \bar{d}_\alpha \left[ A_\perp \left( \frac{\partial^2 f_\alpha}{\partial V_x^2} + \frac{\partial^2 f_\alpha}{\partial V_x^2} \right) + A_\parallel \frac{\partial^2 f_\alpha}{\partial V_z^2} \right], \quad (29)$$

where

$$\bar{d}_\alpha = \frac{|e|\, E N_e \omega_{\rm L} e_\alpha^2 N_\alpha^2}{\sum\limits_{\beta=1,2} m_\beta e_\beta^2 N_\beta^3} r_{\rm De}(t). \quad (30)$$

The solutions of these equations for the spatially uniform plasmas are the bi-Maxwellian distributions with the transversal and longitudinal temperatures which satisfy the equations

$$\frac{d\left(\kappa_{\rm B} T_{\perp(\parallel)\alpha}\right)}{dt} = 2 m_\alpha A_{\perp(\parallel)} \bar{d}_\alpha(t). \quad (31)$$

The right-hand sides of these two equations are time-dependent because of the time dependence of the electron temperature $T_e(t)$. In the case of the strong electric field, the growth of $T_e(t)$ is practically connected only with the turbulent Joule heating:

$$\frac{3}{2} N_e \frac{d(\kappa_{\rm B} T_e)}{dt} \cong \sigma E^2. \quad (32)$$

According to work [11] in the limit of the strong electric field, the turbulent electric conductivity can be written as

$$\sigma \cong 1.2 \frac{|e|\, N_e V_S}{E} \sqrt{K_{N2}/\lambda}. \quad (33)$$

Then we have

$$\frac{d(\kappa_{\rm B} T_e)^{3/4}}{dt} = 0.6 \frac{|e|\, E \omega_{\rm L} \sqrt{R}}{(4\pi e^2 N_e)^{1/4}}. \quad (34)$$

For the heating of ions with equal temperatures and without saturation, we have

$$R = \frac{K_{N2}}{\lambda} r_{\rm De} \to \tilde{R} = \frac{12\pi^2 |e|\, N_e E \omega_{\rm Le}^2 \bar{\omega}^2}{\omega_{\rm L}^8} \left( \frac{e_1}{m_1} - \frac{e_2}{m_2} \right)^2, \quad (35)$$

where

$$\bar{\omega}^2 = 4\pi \sum_{\beta=1,2} m_\beta e_\beta^2 N_\beta^3 \left( \sum_{\alpha=1,2} m_\alpha N_\alpha \right)^{-2}. \quad (36)$$

Therefore, we can write the following time dependence of the electron temperature:

$$\kappa_{\rm B}T_e\left(t\right)=\left(\left[\kappa_{\rm B}T_e\left(t_0\right)\right]^{3/4}+\left(\frac{E^2}{4\pi}\right)^{3/4}g_e^{3/4}\left[\omega_{\rm L}\left(t-t_0\right)\right]\right)^{4/3}. \quad (37)$$

Here,

$$g_e=\left[3.4\frac{\omega_{\rm Le}^2\bar{\omega}^2\omega_{\rm H}^4}{\omega_{\rm L}^8}\left(\frac{Z_1}{A_1}-\frac{Z_2}{A_2}\right)^2\right]^{2/3}, \quad (38)$$

and $Z_\alpha=e_\alpha/\left|e\right|$, $A_\alpha=m_\alpha/m_{\rm H}$, $m_{\rm H}$ is the hydrogen atom mass $\omega_{\rm H}^2=4\pi e^2N_e/m_{\rm H}$.

The regime of the strong electron heating $T_e\left(t\right)\gg T_e\left(t_0\right)$ is described by the formula

$$N_e\kappa_{\rm B}T_e\left(t\right)=\frac{E^2}{4\pi}g_e\left[\omega_{\rm L}\left(t-t_0\right)\right]^{4/3}. \quad (39)$$

Now we can obtain the explicit time dependence of the ion temperatures

$$N_\alpha\kappa_{\rm B}T_{\perp(\|)\alpha}\left(t\right)=N_\alpha\kappa_{\rm B}T_{\perp(\|)\alpha}\left(t_0\right)+$$

$$+\frac{1.2A_{\perp(\|)}m_\alpha e_\alpha^2N_\alpha^3}{\sum\limits_{\beta=1,2}m_\beta e_\beta^2N_\beta^3}\sqrt{g_e}\frac{E^2}{4\pi}\left[\omega_{\rm L}\left(t-t_0\right)\right]^{5/3}. \quad (40)$$

On the other hand, the electron temperature grows as $\left[\omega\left(t-t_0\right)\right]^{4/3}$. We see that the rate of growing of the temperature of ions can exceed that of the electron temperature growing up for an enough time. Moreover, this tendency allows one to see the possibility to violate the condition of the ion temperature smallness in comparison with the electron temperature. The last violation is the violation of the condition for the existence of ion acoustic waves and, therefore, the existence of the ion acoustic turbulence.

## 4. Estimation of the IAT Existence Time

Let us write the ratio

$$\frac{T_e\left(t\right)}{T_{\perp\alpha}\left(t\right)}=$$

$$=\frac{\left(\left[\kappa_{\rm B}T_e\left(t_0\right)\right]^{3/4}+\left(\frac{E^2}{4\pi}\right)^{3/4}g_e^{3/4}\left[\omega_{\rm L}\left(t-t_0\right)\right]\right)^{4/3}}{\left(\kappa_{\rm B}T_{\perp\alpha}\left(t_0\right)+\frac{E^2}{4\pi}\sqrt{g_e}1.2A_\perp\frac{m_\alpha e_\alpha^2N_\alpha^2N_e}{\sum\limits_{\beta=1,2}m_\beta e_\beta^2N_\beta^3}\left[\omega_{\rm L}\left(t-t_0\right)\right]^{5/3}\right)}. \quad (41)$$

This ratio of temperatures of electrons and ions allows us to obtain a numerical estimation of the existence time of the strong-field regime of the turbulent plasma heating. More transparent is the formula in the case of the strong electron and ion heating ($T_e\left(t\right)\gg T_e\left(t_0\right)$, $T_{\perp\alpha}\left(t\right)\gg T_{\perp\alpha}\left(t_0\right)$)):

$$\frac{T_e\left(t\right)}{T_{\perp\alpha}\left(t\right)}=\frac{\sqrt{g_e}\sum_\beta m_\beta e^2N_\beta^3}{1.2A_\perp N_e m_\beta e_\alpha^2N_\alpha^2\left[\omega_{\rm L}\left(t-t_0\right)\right]^{1/3}}. \quad (42)$$

We can try to obtain the necessary estimation after we set this ratio to five. Then we have

$$\left[\omega_{\rm L}\left(t_f-t_0\right)\right]=g_e^{3/2}\left[\frac{\sum\limits_{\beta=1,2}m_\beta e_\beta^2N_\beta^3}{6A_\perp m_\alpha e_\alpha^2N_\alpha^2N_e}\right]^3. \quad (43)$$

Let us look at a deuterium-tritium plasma as an example in the case of the strong electric field and in the regime of the two component heating without saturation. We will obtain the estimation of the existence time of the discussed strong field regime of the turbulent deuterium-tritium plasma heating in the case of plasma with the equal temperatures of ions and with $\left(N_d/N_t\right)=\sqrt{3/2}$. We have $g_e\cong 66$ and $\omega_{\rm L}\left(t_f-t_0\right)\cong 11.5$. During this time, the temperatures of electrons and ions increase in equal times

$$\frac{T_e\left(t_f\right)}{T_e\left(t_0\right)}=\frac{T_{\perp\alpha}\left(t_f\right)}{T_{\perp\alpha}\left(t_0\right)}=136\frac{E^2}{N_e\kappa_{\rm B}T_e\left(t_0\right)}. \quad (44)$$

As for such deuterium-tritium plasma, we have

$$K_{N2}\left(t_0\right)\cong 210\frac{E}{\sqrt{N_e\kappa_{\rm B}T\left(t_0\right)}}. \quad (45)$$

We can rewrite the previous relation as

$$\frac{T_e\left(t_f\right)}{T_e\left(t_0\right)}=\frac{T_{\perp\alpha}\left(t_f\right)}{T_{\perp\alpha}\left(t_0\right)}=\left(\frac{K_{N2}\left(t_0\right)}{18}\right)^2. \quad (46)$$

In particular, this relation allows one to see that, during the time of the turbulent heating, the temperatures of electrons and ions can grow by about a hundred times, when $K_{N2}\approx 180$.

The phenomenon presented here of the finite time existence of IAT is important for the understanding of IAT. It is necessary to underline that this phenomenon is discussed in the frame of the model of IAT that is related to plasmas with different ions with unequal charge-to-mass ratio. Therefore, there is a problem to understand the possibility of the analogous phenomenon in the frame of a simpler model of plasmas with one kind of ions. Moreover, this phenomenon is related to the theory developed in [9] and [10] only in the case of the neglect of the Cherenkov interaction of ions with turbulent pulsations. In addition, the numerical estimation of the existence time of IAT can be changed owing to the use of a more precise angular distribution in comparison with the asymptotic approximation of the strong field used in our discussion. So the aim of my discussion of the problem of the turbulent plasma heating is to attract a more attention to the question of the time duration of the IAT existence. At last, it is necessary to note that our estimation is near the lower bound of the IAT existence time, because we have neglected the long enough time of the slow heating of particles at the initial interval of the turbulence creation.

## 5. Time Dispersion of the Turbulent Conductivity

The other result of the plasma heating manifests itself in properties of the turbulent conductivity $\sigma$, which is usually presented in the theory of IAT as a nonlinear function of the electric field strength. Let us describe here the appropriate result for the case of the strong field when, according to (44) and (12), there is the explicit dependence $\sigma \approx E^{-1/2}$, which is usually discussed in the theory of IAT. However, besides this explicit dependence, the turbulent conductivity depends on the electron temperature, which is determined, due to the plasma heating, by the electric field. The obvious simple example is the plasma heating in the strong field, when ions are heated in the regime without saturation. The additional simplification arises under conditions of the strong plasma particle heating $T_e(t) \gg T_e(t_0)$, $T_{\perp\alpha}(t) \gg T_{\perp\alpha}(t_0)$. Then we can write down

$$\sigma = 0.30 g_e^{3/4} (N_e \kappa_B T_e(t)/E^2)^{1/4} \omega_L. \quad (47)$$

Using the previously discussed electron temperature time dependence, we have

$$\sigma = 0.16 g_e [\omega_L (t - t_0)]^{1/3} \omega_L. \quad (48)$$

In the particular case of the deuterium-tritium plasma discussed previously, we have $\sigma \cong 11 [\omega_L (t-t_0)]^{1/3} \omega_L < 24\omega_L$. Let us underline that, in accordance with this discussion, the turbulent conductivity is independent of the electric field strength. In this connection, we can speak about the reduction of the nonlinear dependence of the turbulent conductivity to the field-independent expression of the linear theory of electricity. But this linear theory describes the time-dependent electric field because of the time dependence of the turbulent conductivity.

Our model uses the time-independent electric field strength. In the case of the quasistationary electric field, we can see that the nonlinear turbulent conductivity can be described by the formula

$$\sigma(t) = 0.16 \omega_L g_e \left[ \omega_L \int_{t_0}^{t} dt' \frac{E^{3/2}(t')}{E^{3/2}(t)} \right]^{1/3}, \quad (49)$$

which corresponds to the nonlinear time dispersion of the electric conductivity. The last nonlinear field dependence is, in general, weaker than $\sigma \approx E^{-1/2}$ that we have without the influence of the plasma heating. In the case of $E = \mathrm{const}$, we have (48). This discussed effect is analogous to that discussed previously [13] in connection with the possible meaning of Demidov–Elagin–Fanchenko effect [14].

At last, we say that our inclusion of the heating of turbulent plasma particles into our kinetic description of plasma is the reason for the non-Markovian effect. Equations (21)–(23) and (32) give foundation for this general remark. Due to Eq. (22), we see that the diffusion coefficients $d_\alpha$ in the ion velocity space are functions of the temperatures of plasma particles. These temperatures are determined by Eqs. (22) and (23). Therefore, the kinetic equations (21) describe the evolution of the ion distribution which includes the memory effects. The result of the non-Markovian description can be seen directly in formula (49). In comparison with the statistical theory of turbulent transport [15, 16] related to non-Markovian effects, it is necessary to say the following. Our paper and papers [15, 16] have one trait in common, namely the plasma turbulence. Now let us look at the difference. The theory in [15] is devoted to the general non-Markovian Fokker–Planck approach to derive the diffusion coefficients. Work [16] considered the non-Markovian kinetics on the base of the Klimontovich equation which is connected with the electric field fluctuations. So, works [15, 16] have a vital importance for the way of the development of the non-Markovian kinetics of the fast processes. The first steps of such kinetics were based on the generalization of N.N. Bogolyubov's derivation of the Landau collision integral [17–19], The way of works [15, 16] can be named the fluctuation approach, which stands in need to describe the distribution of fluctuations or their correlations.

Instead of such general way, our approach is based on our known result on the turbulent fluctuation distribution and on the particular case of the nonlinear dependence of the Fokker–Planck ion kinetic equations, whose coefficients depend on the temperatures of plasma particles. The idea of the heating of quick particles allows us to use the time-dependent Fokker–Planck coefficients, which is the reason for a memory manifestation in such transport coefficient as the electric conductivity.

## 6. Conclusions

1. The coarsened description of the stimulated scattering of ion acoustic waves on ions is the base of the developed rough version of the IAT theory. It allowed us to analyze the heating conditions of plasma particles, when the two-particle collisions are not important.
2. The possibility to describe the strong heating of plasma particles allowed us to put the question about the duration of the existence of the IAT state and to obtain some estimation of such time interval.
3. The consideration of the turbulent plasma heating indicates how the nonlinear electric field-dependent turbulent conductivity turns into the field-independent conductivity. On the other hand, the time-dependent electric conductivity obtained in our discussion allows one to see the coarseness of the usual point of the IAT models for the case of a dense enough plasma. In this case, the coarseness is connected with the usual assumption of the IAT theory about the time-independent electric field strength, which creates IAT.
4. Our non-Markovian kinetic description of IAT is connected with the memory of the heating of plasma particles.

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ІОН-АКУСТИЧНА ТУРБУЛЕНТНІСТЬ У ПЛАЗМАХ

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Р е з ю м е

Обговорено нові властивості плазми з іонно-звуковою турбулентністю (ІЗТ), пов’язані з турбулентним нагрівом частинки. Викладено ідеї огрубленої теорії ІЗТ, яка дозволяє описати сильний нагрів частинок. Опис змагання електронного і іонного нагріву дозволяє відкрити обговорення кінцевого часу існування ІЗТ і дати деяку оцінку такого часу. Також показано, що звичайна модель ІЗТ зі сталою напруженістю електричного поля приводить до електричної провідності, що залежить від часу, а у випадку квазістаціонарного поля ми одержуємо нелінійну часову дисперсію провідності.