
RADIATION LOSSES IN A PLANAR DIELECTRIC WAVEGUIDE WITH A ROUGH INTERFACE BETWEEN DIELECTRIC LAYERS

O.H. MATIEVOSOVA, A.V. KOVALENKO, V.N. KURASHOV

Taras Shevchenko National University of Kyiv

(64, Volodymyrs'ka Str., Kyiv 01033, Ukraine; e-mail: *matyevosova.lena@gmail.com*)

PACS 42.82.Et
©2012

Without the weakly guiding fiber approximation, a statistical model of light scattering by a rough surface in a planar dielectric waveguide has been developed. The dependences of radiation losses on the refractive index contrast, the waveguide core thickness, and the correlation characteristics of a scattering surface have been studied. The difference between the scattering of TE and TM modes has been analyzed, and the two-dimensional model was shown to be not suitable for quantitative estimates of losses in the case of TM modes.

1. Introduction

Light propagation in dielectric optical fibers with irregular interfaces between dielectric layers is accompanied by radiation losses caused by light scattering at a rough surface. This phenomenon has been known for a long time; however, it remained beyond the scope of interest of researchers working in fiber optics. This circumstance may probably be associated with the fact that the strength of this scattering is proportional to the refractive index contrast in the lightguide; therefore, it is negligibly small in weakly guiding fibers. Optical fibers of this type were the center of attention for a long time, because the single-mode regime of light propagation, which is required in the majority of applications, can be easily realized in them. An interest to surface scattering considerably grew owing to the miniaturization of optical elements and the creation of structures on submicronic and nanoscales. A reduction of the characteristic dimensions of optical waveguides is possible only provided that the difference between the refractive indices of dielectric layers in the waveguide structure increases adequately, which is accompanied by

an enhancement of the role of surface scattering. It was demonstrated theoretically and experimentally that the absolute values of such losses can be substantial, in particular, in photonic crystal fibers [1], nanofibers [2], and planar waveguide components of integrated optics [3, 4].

The expediency of studying the surface scattering in planar light-guiding structures follows from a number of reasons. First, simple boundary conditions allow analytical solutions of the corresponding electrodynamic problems to be found, which simplifies the analysis of the influence exerted by statistical structural defects. On the other hand, at the fabrication of planar waveguide layers, essentially different technological processes are used, which are characterized by different physical mechanisms of surface defect formation. This enables the influence of the statistics of heterogeneities on the scattering efficiency to be determined experimentally. At last, the planar geometry of the object under study practically excludes the influence of bends and noncontrollable changes in the thickness of a waveguide layer on light scattering, which is especially characteristic of cylindrical nano-sized fibers.

In the present work, a statistical model for light scattering by the rough surface of a planar optical fiber is developed. In so doing, we did not use the weakly guiding fiber approximation. The main feature of the proposed approach is the application of a nonlinear model for the formation of equivalent stimulated currents on a statistically non-uniform surface. This model allows the dependences of radiation losses on the refractive index contrast, the thickness of the optical fiber core, and the correlation characteristics of the the scattering surface to be analyzed in detail.

2. Radiation Losses in the Approximation of Weakly Perturbed Interface in the Optical Fiber

Consider a model of planar optical fiber, in which one boundary has a stochastic relief represented by a homogeneous Gaussian field $\xi(z)$, whose average value $\langle \xi \rangle = 0$, the correlation function $\langle \xi^*(z_1)\xi(z_2) \rangle = G_\xi(z_2 - z_1) \equiv G_\xi(\Delta z)$, and the mean-square deviation $\sigma_\xi \ll \rho$ (Fig. 1). The field magnitude $\xi(z)$ at a certain point z is equal to a deviation of the interface between the media with different refractive indices (in Fig. 1, the field $\xi(z)$ is reckoned from the coordinate ρ along the axis OX).

The refractive index is presented as a sum of two components,

$$n^2(x, z) = n_0^2(x) + n_1^2(x, z), \quad (1)$$

where $n_0(x)$ is the refractive index for the nonperturbed waveguide, and the second term depends on the field $\xi(z)$,

$$n_1^2(x, z) = \begin{cases} n_{co}^2 - n_{cl}^2, & \xi(z) > x - \rho; \\ n_{cl}^2 - n_{co}^2, & \xi(z) < x - \rho; \\ 0, & \text{otherwise,} \end{cases} \quad (2)$$

where n_{co} and n_{cl} are the refractive indices of the fiber core and cladding, respectively; and ρ is the half-width of the waveguide core (Fig. 1).

The refractive index is perturbed in a thin layer near the interface located at $x = \rho$. Within the layer limits, the fields can be considered constant. The tangential components $e_z(\rho)$ and $e_y(\rho)$ of the electric field are continuous across the interface, whereas the component e_x has a jump: $e_x(\rho + 0)$ at $x > \rho$ and $e_x(\rho - 0)$ at $x < \rho$.

In accordance with the standard procedure [5], let us change the perturbed waveguide to a nonperturbed one containing equivalent forced current sources. Since $\sigma_\xi \ll \rho$, the fields in the perturbed waveguide are little different from the fields in the nonperturbed one, so that the method of small perturbations can be applied. Namely, we may present the electric field as

$$\bar{E}_0(x, z) = \bar{E}_0^{(0)}(x, z) + \bar{E}_0^{(1)}(x, z), \quad (3)$$

where $\bar{E}_0^{(0)}(x, z) = a_0 \bar{e}_0(x) e^{j\beta_0 z}$ is the field in the nonperturbed waveguide, a_0 the amplitude of propagating mode, β_0 the corresponding propagation constant, and

$$\bar{E}_0^{(1)}(x, z) = \int_0^{Q_{\max}} a_{\text{ITE}}(Q, z) \bar{e}_{\text{ITE}}(x, Q) e^{j\beta_{\text{ITE}}(Q)z} dQ +$$

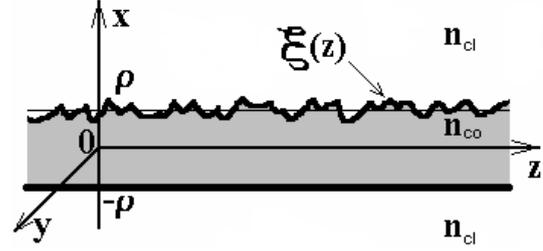


Fig. 1. Planar optical fiber with a perturbed interface core/cladding

$$+ \int_0^{Q_{\max}} a_{\text{ITM}}(Q, z) \bar{e}_{\text{ITM}}(x, Q) e^{j\beta_{\text{ITM}}(Q)z} dQ \quad (4)$$

is the total field of radiation modes. We assume that $\bar{E}_0^{(1)}(x, z) \ll \bar{E}_0^{(0)}(x, z)$. The following notations were introduced: a_{ITE} and a_{ITM} are the amplitude factors of radiation modes, $\bar{e}_{\text{ITE}}(x, Q)$ and $\bar{e}_{\text{ITM}}(x, Q)$ the electric fields of radiation modes (see Table), β_{ITE} and β_{ITM} the radiation mode propagation constants, and $Q_{\max} = \rho k n_{cl}$. The subscripts ITE and ITM correspond to the transverse magnetic and transverse, respectively, electric radiation modes. The expressions for magnetic field components are similar to expressions (3) and (4).

Substituting the expressions for fields (3) and (4) in the Maxwell equation and neglecting the terms, whose smallness order is higher than one, we obtain an extra term in the Maxwell equation, which can be interpreted as a forced current,

$$\bar{J}_0(x, z) = -j \sqrt{\frac{\epsilon_0}{\mu_0}} k n_1^2(x, z) a_0 \bar{e}_0(x) e^{j\beta_0 z}, \quad (5)$$

where k is the wave vector of a wave in vacuum. According to work [5], the amplitude factors of radiation modes for the current density \bar{J}_0 are

$$a_{\text{ITE}}(Q, z) = -\frac{1}{4N_{\text{ITE}}(Q)} \int_{-\infty}^z \int_{-\infty}^{\infty} \bar{e}_{\text{ITE}}^* \times \bar{J}_0(x, z') e^{-j\beta_{\text{ITE}}z'} dx dz', \quad (6)$$

where $N_{\text{ITE}}(Q)$ is the normalizing multiplier for the ITE mode (Table). The expression for the amplitudes a_{ITM} of ITM modes are similar with an accuracy to the substitutions $N_{\text{ITE}} \rightarrow N_{\text{ITM}}$, $\epsilon_{\text{ITE}} \rightarrow \epsilon_{\text{ITM}}$, and $\beta_{\text{ITE}} \rightarrow \beta_{\text{ITM}}$. The fields in the perturbed layer can be considered as constant. Therefore, after the corresponding simplifica-

ITM and ITE radiation modes of a planar optical fiber

Mode type	Electric field components	Parameters a, b	Normalization
odd ITM	$e_z(x) = \frac{j}{k\rho n^2(x)} \begin{cases} Q \cos Q(\frac{x}{\rho} - 1) + a, & x > \rho \\ bU \cos \frac{Ux}{\rho}, & -\rho < x < \rho \\ Q \cos Q(\frac{x}{\rho} + 1) - a, & x < \rho \end{cases}$ $e_x(x) = \frac{\beta}{kn^2(x)} \begin{cases} \sin Q(\frac{x}{\rho} - 1) + a, & x > \rho \\ b \sin \frac{Ux}{\rho}, & -\rho < x < \rho \\ \sin Q(\frac{x}{\rho} + 1) - a, & x < \rho \end{cases}$	$\tan a = \frac{n_{co}^2 Q}{n_{cl}^2 U} \tan U$ $b = \frac{\sin a}{\sin U}$	Normalization condition: $\frac{1}{2} \int_{-\infty}^{\infty} [\bar{e}_j(Q) \times \bar{h}_j^*(Q')] \hat{z} dx =$
even ITM	$e_z(x) = -\frac{j}{k\rho n^2(x)} \begin{cases} Q \sin Q(\frac{x}{\rho} - 1) + a, & x > \rho \\ bU \sin \frac{Ux}{\rho}, & -\rho < x < \rho \\ Q \sin Q(\frac{x}{\rho} + 1) - a, & x < \rho \end{cases}$ $e_x(x) = \frac{\beta}{kn^2(x)} \begin{cases} \cos Q(\frac{x}{\rho} - 1) + a, & x > \rho \\ b \cos \frac{Ux}{\rho}, & -\rho < x < \rho \\ \cos Q(\frac{x}{\rho} + 1) - a, & x < \rho \end{cases}$	$\tan a = \frac{n_{cl}^2 U}{n_{co}^2 Q} \tan U$ $b = \frac{\cos a}{\cos U}$	$= N_j(Q)\delta(Q - Q')$ $N_{ITM} = \frac{\pi\rho\beta}{2kn_{cl}^2} \sqrt{\frac{\varepsilon_0}{\mu_0}}$
odd ITE	$e_y(x) = \begin{cases} \sin Q(\frac{x}{\rho} - 1) + a, & x > \rho \\ b \sin \frac{Ux}{\rho}, & -\rho < x < \rho \\ \sin Q(\frac{x}{\rho} + 1) - a, & x < \rho \end{cases}$	$\tan a = \frac{Q}{U} \tan U, b = \frac{\sin a}{\sin U}$	$N_{ITE} = \frac{\pi\rho\beta}{2k} \sqrt{\frac{\varepsilon_0}{\mu_0}}$
even ITE	$e_y(x) = \begin{cases} \cos Q(\frac{x}{\rho} - 1) + a, & x > \rho \\ b \cos \frac{Ux}{\rho}, & -\rho < x < \rho \\ \cos Q(\frac{x}{\rho} + 1) - a, & x < \rho \end{cases}$	$\tan a = \frac{U}{Q} \tan U, b = \frac{\cos a}{\cos U}$	

tions, we obtain

$$a_{ITE}(Q, z) = \sqrt{\frac{\varepsilon_0}{\mu_0}} \frac{jka_0}{4N_{TE}(Q)} \int_0^z f_{ITE}(\xi(z)) \times$$

$$\times \exp(j(\beta_0 - \beta_{ITE}(Q))z') dz', \tag{7}$$

where the normalizing factor of the TE mode

$$N_{TE} = \frac{\rho\beta_0}{2k} \sqrt{\frac{\varepsilon_0}{\mu_0}} \frac{V^2}{U^2} \frac{1+W}{W},$$

$$f(\xi(z)) = e_{ITE}^*(x_0)\bar{e}_0(x_0)(n_{co}^2 - n_{cl}^2)\xi(z),$$

$$V = \rho k \sqrt{n_{co}^2 - n_{cl}^2}, \quad U^2 = \rho^2(k^2 n_{co}^2 - \beta_0^2),$$

$$W^2 = V^2 - U^2.$$

The power lost by the ITE mode on a length z on the rough surface is

$$p_{ITE}(Q) = \frac{N_{ITE}(Q)}{2} \langle |a_{ITE}|^2 \rangle. \tag{8}$$

Passing to the difference coordinates $\Delta z = z'' - z'$, $Z = \frac{z'+z''}{2}$ and bearing in mind that

$\langle f^*(\xi(z'))f(\xi(z'')) \rangle = G_f(\Delta z)$, we ultimately obtain

$$p_{ITE}(Q) = \frac{\varepsilon_0 k^2 a_0^2 z N_{ITE}(Q)}{32\mu_0 N_{TE}^2(Q)} \int_{-\infty}^{\infty} G_{f_{ITE}}(\Delta z) \times$$

$$\times e^{j(\beta_0 - \beta_{ITE}(Q))z} d(\Delta z). \tag{9}$$

A similar expression to within the substitutions $N_{ITE} \rightarrow N_{ITM}$, $N_{TE} \rightarrow N_{TM}$, $f_{ITE} \rightarrow f_{ITM}$, and $\beta_{ITE} \rightarrow \beta_{ITM}$ is obtained for the power losses by ITM modes.

3. Dependence of Power Losses on the Correlation of Surface Perturbations

According to Price's theorem, the relation between the correlation functions G_f and G_ξ can be presented in the form

$$\frac{d^2 G_f}{d\gamma_\xi^2} = G_f^2(0) \left\langle \frac{d^2 f^*}{d\xi^2(z')} \frac{d^2 f}{d\xi^2(z'')} \right\rangle, \tag{10}$$

$$\frac{d^2 G_f}{d\gamma_\xi^2} = |m_{cl} - m_{co}|^2 \frac{\sigma_\xi^2}{2\pi \sqrt{1 - \gamma_\xi^2(\Delta z)}}, \tag{11}$$

where the following notation was introduced for convenience:

$$m_{cl} = (n_{co}^2 - n_{cl}^2)\bar{e}_{ITE}^*(\rho + 0)\bar{e}_0(\rho + 0);$$

$$m_{co} = (n_{co}^2 - n_{cl}^2) \bar{\epsilon}_{\text{ITE}}^* (\rho - 0) \bar{\epsilon}_0 (\rho - 0). \quad (12)$$

Solving Eq. (11) with the initial conditions

$$G_f(\gamma_\xi = 0) = |\langle f \rangle|^2,$$

$$G_f(\gamma_\xi = 1) = |\langle f \rangle|^2, \quad (13)$$

we obtain the dependence of the averaged squared absolute value of the amplitude factor of ITE modes on the perturbation field correlation,

$$G_f(\gamma_\xi) = \frac{\sigma_\xi^2}{4} |m_{cl} + m_{co}|^2 \gamma_\xi + \frac{\sigma_\xi^2}{2\pi} |m_{cl} - m_{co}|^2 \left(\gamma_\xi \arcsin \gamma_\xi + \sqrt{1 - \gamma_\xi^2} \right). \quad (14)$$

The addend in Eq. (14),

$$f(\gamma_\xi) = \frac{2}{\pi - 2} \left(\gamma_\xi \arcsin \gamma_\xi + \sqrt{1 - \gamma_\xi^2} - 1 \right),$$

is well approximated by the square-law function γ_ξ^2 (Fig. 2).

Taking this circumstance into account, we obtain

$$G_f(\gamma_\xi) = \frac{G_\xi(\Delta z)}{4} |m_{cl} + m_{co}|^2 + \frac{\pi - 2}{4\pi} |m_{cl} - m_{co}|^2 \left(\frac{G_\xi^2(\Delta z)}{G_\xi(0)} + G_\xi(0) \right). \quad (15)$$

Since $S_\xi^{\text{ITE}}(\beta) = \int_{-\infty}^{\infty} G_\xi e^{j\beta_{\text{ITE}} \Delta z}$, the expression for scattered power looks like

$$p_{\text{ITE}}(Q) = A_1 S_\xi^{\text{ITE}}(\beta_0 - \beta_{\text{ITE}}(Q)) +$$

$$A_2 \int_{-\infty}^{\infty} S_\xi^{\text{ITE}}(\beta') S_\xi^{\text{ITE}}(\beta_0 - \beta_{\text{ITE}}(Q) - \beta') d\beta', \quad (16)$$

where

$$A_1 = C |m_{cl} + m_{co}|^2,$$

$$A_2 = C(\pi - 2) |m_{cl} - m_{co}|^2 / (\pi \sigma_\xi^2),$$

$$C = \varepsilon_0 k^2 a_0^2 z / (128 \mu_0 N_{\text{TE}}(Q)).$$

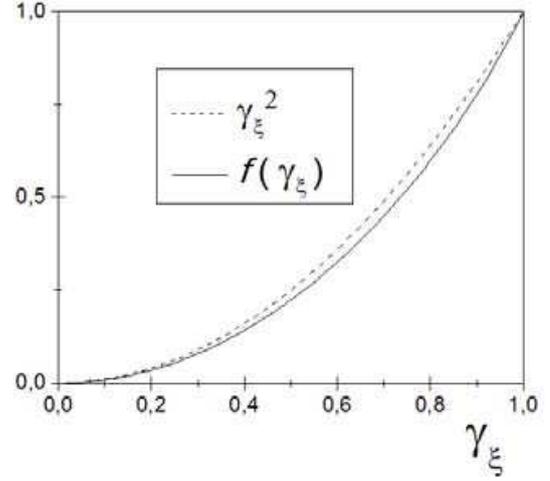


Fig. 2. Comparison of the dependences $f(\gamma_\xi)$ and γ_ξ^2 on the correlation coefficient γ_ξ

A similar expression is valid for ITM modes as well, with the corresponding change of notations.

The general radiation losses $P_{\text{rad}}(z)$ (z) are determined by

$$P_{\text{rad}} = \int_0^{Q_{\text{max}}} p_{\text{ITE}}(Q) dQ + \int_0^{Q_{\text{max}}} p_{\text{ITM}}(Q) dQ. \quad (17)$$

Formula (17) is valid, provided that $P_{\text{rad}} \ll P_{\text{tot}}$, i.e. if the lengths z are short. Therefore, to calculate power losses for arbitrary large z , the coefficient of damping per unit length has to be introduced,

$$\eta = -\frac{10}{z} \lg \left(1 - \frac{P_{\text{rad}}}{P_{\text{tot}}} \right), \quad (18)$$

where P_{tot} is the power of an incident wave.

Experimental researches of the surface of an optical fiber fabricated of quartz glass showed [7] that the spectrum of a surface relief has the Lorentzian shape, and the corresponding correlation function is determined by the expression

$$G_\xi(\Delta z) = G_\xi(0) e^{-\frac{|\Delta z|}{z_0}}. \quad (19)$$

Note that the correlation function presented in this form was used to calculate surface losses in photonic crystals [1] and nanofibers [2], where a surface relief is created by the thermodynamically equilibrium mechanism of formation of frozen capillary waves.

4. Numerical Results and Discussion

Numerical calculations of light losses in a planar waveguide with randomly perturbed interface were carried

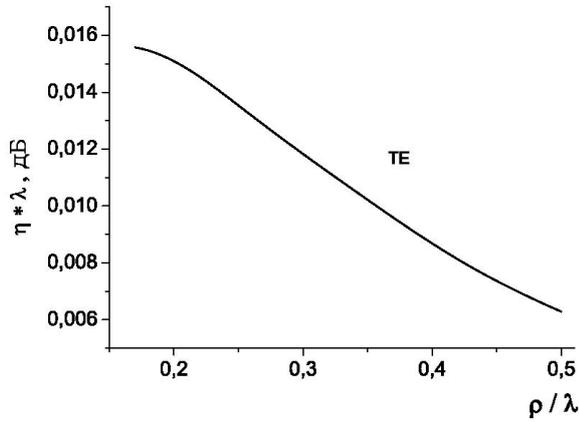


Fig. 3. Dependence of the power loss coefficient η for the TE mode on the waveguide thickness ρ . Both quantities are normalized to the wavelength λ

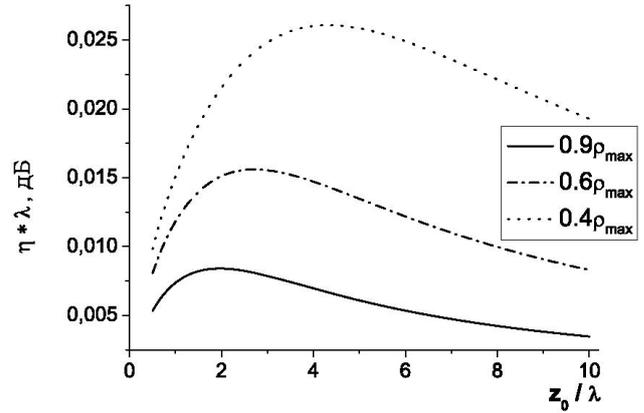


Fig. 5. Dependence of the power loss coefficient η for the TE mode on the interface relief correlation length z_0 . Both quantities are normalized to the wavelength λ

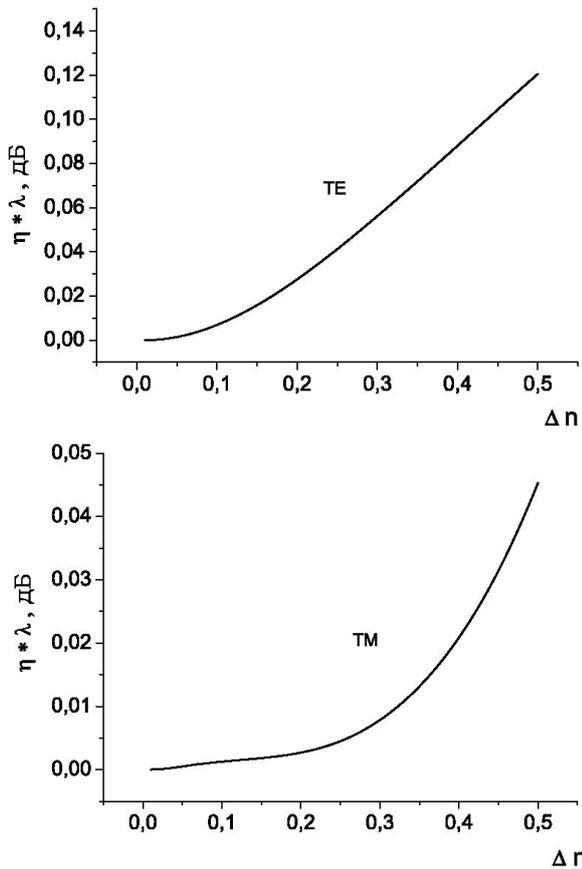


Fig. 4. Dependences of the power loss coefficient for the TM and TE modes on the refractive index contrast at the waveguide thickness $\rho = 0.9\rho_{\max}$, where $\rho_{\max} = \lambda/4\sqrt{n_{co}^2 - n_{cl}^2}$ is the maximal thickness of the core, at which the single-mode regime is realized

out for the following parameters: $n_{co} = 1.58$, $n_{cl} = 1.5$, the correlation length $z_0 = \lambda$, and the dispersion $\sigma_{\xi}^2 = 0.01\lambda^2$. The calculated dependence of the power loss coefficient on the waveguide thickness for the TE mode is exhibited in Fig. 3. If the waveguide becomes narrower, the distribution of the directed-mode field power over the waveguide cross-section changes; namely, the power density increases near the interfaces between the cladding and the core, i.e. in the perturbed layer. Therefore, if ρ is small, a large part of the directed mode interacts with the perturbation. On the other hand, a decrease of the thickness diminishes the directed-mode propagation constant β_0 . As a result, the difference $\Delta\beta = \beta_0 - \beta_{\text{ITE}}(Q)$ calculated at $\beta_{\text{ITE}} = kn_{cl}$ decreases, and, consequently, the contribution of low-frequency spectral components increases in the framework of the model selected for the spectrum of relief fluctuations.

The dependences of power loss coefficient on the refractive index contrast for TE and TM modes are depicted in Fig. 4. A growth of losses is expectedly observed at the growth of the refractive index contrast $\Delta n = n_{co} - n_{cl}$ for the modes of both types, because $P_{\text{rad}} \sim (n_{co}^2 - n_{cl}^2)^2$.

The dependences of radiation losses on the interface perturbation correlation length are depicted in Fig. 5. One can see that the obtained dependences have a quiresonance character, with the maximum value of loss coefficient being dependent on the waveguide parameters. This fact is explained by the opposite action of two factors, which govern the scattering efficiency. Scattering is effective at frequencies $\Delta\beta$. If z_0 is small, the spectral density of relief fluctuations has a low scattering

power at all frequencies, whereas, at large z_0 values, it has a maximum at low scattering frequencies. Therefore, in both cases, the power scattered at the frequency $\Delta\beta$ is low. When z_0 corresponds to the scattering frequency $\Delta\beta$, we obtain a maximum in the scattered power. In addition, a reduction of the waveguide thickness results in a decrease of $\Delta\beta$.

At last, we note that the contribution of the second term in expression (16), which is proportional to the squared correlation, to the general scattering of the TM mode is insignificant in a planar waveguide, in contrast to a fiber waveguide [2] (Fig. 6). For the TE mode, the second term in expression (16) equals zero, because it is proportional to the difference $m_{cl} - m_{co}$, and the electric field is not discontinuous at the core/cladding interface.

We see from Fig. 4 that the power losses for the TM mode are much lower than those for the TE one, which does not correspond to the data obtained earlier for multimode waveguides [3]. This can be explained as follows. The equivalent current (5) is codirectional with the electric field strength vector of the TM mode, which oscillates in the xz -plane and radiates a large portion of the power along the axis Oy . This was not taken into account by our two-dimensional model. At the same time, the equivalent current for the TE mode oscillates just along the axis Oy , so that its radiation was taken into consideration completely. Hence, the proposed two-dimensional model is not suitable for the quantitative analysis of power losses by TM modes.

5. Conclusions

To summarize, the conclusion can be drawn that the method proposed for the calculation of surface losses in optical waveguides adequately corresponds to the statistical character of the problem. The key feature is its independence of the refractive index contrast in the waveguide, which allows one to analyze strongly directed waveguide systems. Moreover, the account of nonlinear effects at the calculation of equivalent currents considerably affects the spectral characteristics of spatial relief frequencies, which are responsible for the scattering efficiency. It is worth noting that the considered approach is essentially based on the assumption about the normal character of the statistics of surface relief fluctuations. However, the corresponding experimental researches testify that this assumption is satisfied at least for thermodynamically equilibrium fluctuations, which

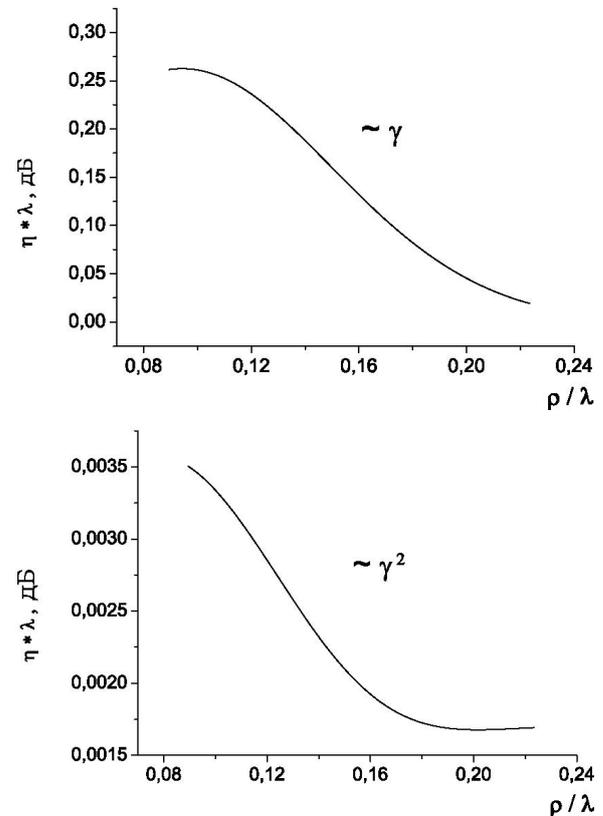


Fig. 6. Dependences of the power loss coefficient for the components of the TM mode that are proportional to the relief perturbation correlation γ and its square γ^2 on the waveguide thickness

arise in technological processes at the waveguide manufacturing.

This work was supported by the State Fund for Fundamental Researches of Ukraine (project No. F29.1/033).

1. P.J. Roberts, F. Couny, and H. Sabert, *Opt. Express* **13**, 7779 (2005).
2. A.V. Kovalenko, V.N. Kurashov, and A.V. Kisil, *Opt. Express* **16**, 5797 (2008).
3. J.M. Elson, *Opt. Express* **9**, 461 (2001).
4. J.P.R. Lacey and F.P. Payne, *IEE Proceedings-Optoelectronics* **137**, 282 (1990).
5. A.W. Snyder and J.D. Love, *Optical Waveguide Theory* (Chapman and Hall, London, 1983).
6. T.G. Theodoropoulos and I.G. Tigelis, *Int. J. Infr. Millim. Waves* **16**, 1811 (1995).
7. A. Goriachko, A. Kovalenko, V. Kurashov, and A. Shchyrba, *Visn. Kyiv. Univ. Ser. Radiofiz. Elektron.* **13**, 28 (2010).

Received 12.05.11.

Translated from Ukrainian by O.I. Voitenko

РАДІАЦІЙНІ ВТРАТИ У ПЛАНАРНОМУ СВІТЛОВODІ
ЗІ ШОРСТКОЮ МЕЖЕЮ ПОДІЛУ
ДІЕЛЕКТРИЧНИХ ШАРІВ

О.Г. Матевосова, А.В. Коваленко, В.Н. Курашов

Резюме

Розвинуто статистичну модель розсіювання світла шорсткою поверхнею планарного світловода, вільну від наближення сла-

бонаправленого світловода. Досліджено залежність величини втрат від висоти профілю показника заломлення, товщини серцевини світловода та кореляційних характеристик розсіюючої поверхні. Проаналізовано відмінності розсіювання ТЕ та ТМ мод, показано, що двовимірною моделлю не придатно для кількісних оцінок втрат у випадку ТМ мод.