OPTICS, LASERS, AND QUANTUM ELECTRONICS

REGISTRATION OF THE ROTATION ANGLE OF LIGHT-BEAM POLARIZATION PLANE WITH THE USE OF OPTICALLY TRANSPARENT FERRIMAGNETIC CRYSTALS

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We have analyzed a modulation photopolarimeter, in which the active element is a ferrimagnetic crystal transparent in the infrared range. The Faraday effect is 3 to 4 times stronger than that for applied earlier paramagnetic crystals. The method developed for the registration of a polarization-plane rotation angle allows one to work in the range of the optimum amplitudes of polarization-plane angular vibrations, i.e. when the signal-to-noise ratio is maximal. It has been shown that, in this case, the sensitivity of the photopolarimeter decreases insignificantly, if the quality of the optical channel gets worse sharply, i.e. if the passing light beam is strongly depolarized.

1. Introduction

The modulation method for registering the rotation angle of light-beam polarization plane was proposed for the first time by V.I. Kudryavtsev in work [1]. Afterward, V.I. Kudryavtsev and R.Ya. Keimakh described a photopolarimeter, in which this registration technique was implemented [2]. The device included an automatic monitoring system, the sensitivity of which reached 0.01°. Then, the photopolarimeter layout underwent various modifications; however, the principle applied to the registration of the rotation angle of a light-beam polarization plane remained invariant. For instance, in work [3], a He–Ne laser was used as a source of radiation, which allowed the measurement sensitivity to be raised up to 0.0001°. In work [4], a computer-controlled photopolarimeter with automatic feedback and with the measurement accuracy amounting to 0.001° was proposed. The authors of work [5] introduced a second Faraday cell into the optical channel and suggested an improved measurement technique, which allowed them

to reach a sensitivity of 10^{-6} deg. In spite of their high sensitivity, the modulation photopolarimeters are characterized by a lower absolute accuracy of measurement of the polarization-plane rotation angle in comparison with classical and optimized classical photopolarimeters [6–8].

Paramagnetic crystals, which are used in modulation polarimeters, have a small rotation angle of the polarization plane (of about 1°) at applied magnetic fields of about 80×10^3 A/m. The latter are created with the use of powerful magnetically biased solenoids with a large time constant. As a result, those photopolarimeters, although having a high sensitivity, are inertial, power-consuming, and big. Therefore, they can mainly be used in laboratory-like environments. Moreover, their high sensitivity is attained only in the optical channel with an insignificant depolarization of the transmitting beam.

It is of interest to study the parameters of a modulation photopolarimeter with an optically transparent ferrimagnet as an active element in the Faraday cell, for which the rotation angle of the polarization plane reaches 100° at applied magnetic fields of about 80 A/m. Being implemented as a device, such a photopolarimeter is space-saving, low-power, and characterized by a high performance.

In works [9, 10], the results of earlier studies concerning the characteristics of a modulation photopolarimeter with an optically transparent ferrimagnetic crystal were reported. It was indicated that the large rotation angle of the polarization plane provided by this crystal allows working with poor-quality polarization prisms, without substantially reducing the sensitivity of a photopolarimeter, which is important at its use under in-
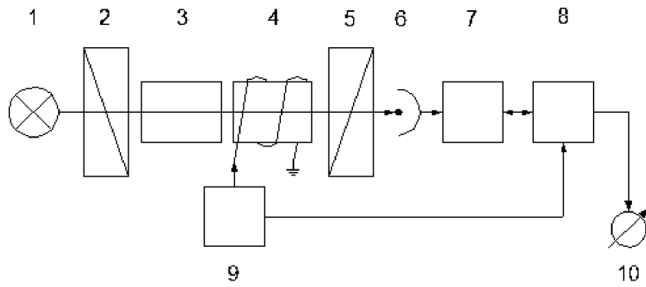
Fig. 1. Block diagram of photopolarimeter: light source (*1*), polarizer (*2*), turbid active medium (*3*), Faraday cell (*4*), analyzer (*5*), photodetector (*6*), amplifier (*7*), synchronous detector (*8*), sound generator (*9*), microammeter (indicator) (*10*)

dustrial and "actual-field" conditions. It this work, we continue the researches dealing with the properties of a photopolarimeter with an optically transparent ferrimagnetic crystal. Here, we consider the influence of not only polarization prisms, but also the examined turbid medium and the Faraday cell, on the optical channel quality.

2. Calculation of Polarimeter Parameters

The block diagram of a photopolarimeter (Fig. 1) is analogous to those described in works [9, 10]. The photopolarimeter was so tuned previously (without an active specimen) that the output signal disappeared. The specimen rotates the polarization plane, which gives rise to the emergence of a signal, which is compensated by additionally rotating the analyzer by a certain angle. The angle, by which the specimen rotates the polarization plane of the beam, is the desired angle φ.

The parameters were calculated using the Stokes vector method and the Mueller matrices. For a light beam that passed through a photopolarimeter, the Stokes vector is defined by the expression

$$(Ve)_{\text{PA}} = [P_{\text{A}}][P_{\text{FC}}][P_{\text{TS}}][P_{\text{POL}}](V_i), \qquad (1)$$

where $(V_i) = (\; I_0 \;\; 0 \;\; 0 \;\; 0 \;)$ is the Stokes vector of the incident beam; I_0 the light flux intensity at the system input; and $[P_{\text{A}}]$, $[P_{\text{FC}}]$, $[P_{\text{TS}}]$, and $[P_{\text{POL}}]$ are matrices that describe the properties of analyzer [11], Faraday cell [12], turbid specimen [8], and polarizer [11], respectively.

The magnetization direction for every domain in a ferrimagnet is oriented arbitrarily (Fig. 2,*a*). Therefore, the polarization over a certain transverse cross-section is constant only for a region, the dimensions of which are comparable with the domain size. Actually, the polarization state changes at every point, because the light
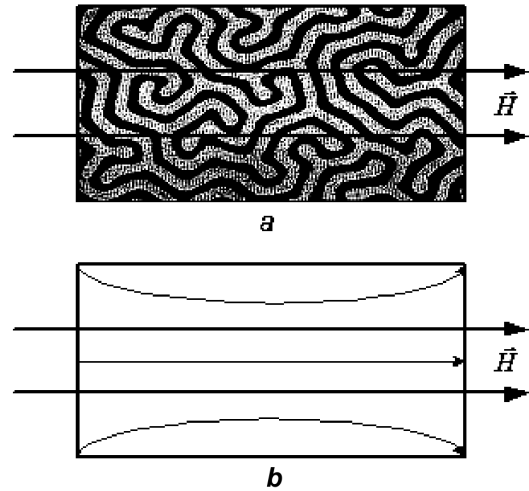

Fig. 2. Domain structures of the ferrimagnet crystal at magnetic fields (*a*) much lower than the saturation one [14] and (*b*) comparable with and exceeding the saturation field

beam passes through several domains in the ferrimagnet crystal. If the light flux is summed up over the whole cross-section at the ferrimagnet output, the beam becomes depolarized. However, the light is polarized at every point of the transverse cross-section. The beam scattering by domains can be neglected, because the variation of the refractive index, Δn, at their interfaces does not exceed 10^{-5}, i.e. the geometrical shape of the beam keeps unchanged [13].

If the external magnetic field increases, the size of domains with the magnetization directed along the field grows. When having achieved the saturation fields, the domain structure disappears; however, the magnetization of the crystal remains non-uniform (Fig. 2,*b*). Only spherical and revolution-ellipsoidal crystals become uniformly magnetized in the saturation fields, so that the beam depolarization for them will be minimal.

In order to study the influence of a domain structure in the crystal on the transmitting light beam, let us introduce – by analogy with the principal transmittances of polarization prisms k_1 and k_2 [11] – the generalized parameters, k_1' and k_2', i.e. the largest and the lowest, respectively, principal transmittances of the optical channel. The parameters k_1' and k_2' were measured, when the analyzer was tuned for the maximum and minimum, respectively, light transmittances. The expression for the light polarization degree in the optical channel looks like [11]

$$p' = \frac{k_1' - k_2'}{k_1' + k_2'} = 1 - 2Gd',$$

where $Gd' = k_2'/(k_1' + k_2')$ is the polarization defect of the optical channel in a photopolarimeter (by analogy with the same parameter for polarization prisms [11]).

In Fig. 3, the dependences of the polarization plane rotation angle, φ', the largest, k_1', and the lowest, k_2', principal transmittances, and the light polarization degree in the optical channel, p', on the controlling magnetic field applied to an yttrium-gallium ferrite-garnet crystal are depicted. While carrying out measurements, we used high-quality polarization prisms, which enabled us to estimate the depolarizing influence of the ferrimagnet domain structure only.

From Fig. 3, one can see that, when the controlling magnetic field grows, k_1' increases and k_2' decreases, because the magnetization in the ferrimagnet becomes uniform, and the depolarization of light in it diminishes. The polarization degree p' grows with increase of the controlling magnetic field and equals 0.8 at fields close to saturation ones. However, the value of 1 is not reached, because the crystal magnetization is non-uniform over the cross-section even in saturation fields.

By analogy with the matrix of an optically active scattering medium, the matrix of a Faraday cell with an optically transparent ferrimagnetic crystal as an active element can be written down in the form [8]

$$[P_{\rm FC}] = (1 - R_1)^2 e^{-\gamma z} \times$$

$$\times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & p'\cos 2\,(\theta_= + \theta) & -p'\sin 2\,(\theta_= + \theta) & 0 \\ 0 & p'\sin 2\,(\theta_= + \theta) & p'\cos 2\,(\theta_= + \theta) & 0 \\ 0 & 0 & 0 & p' \end{bmatrix},$$

where R_1 and γ are the coefficients of light reflection and absorption, respectively, by the Faraday cell; z is the cell thickness; $\theta_=$ an additional rotation angle of the light polarization plane associated with geometrical defects and the residual magnetization in the ferrimagnet; and θ the polarization-plane rotation angle induced by a modulator. The angle θ has a periodic character, $\theta = \theta_0 \Phi(t)$, where $\Phi(t)$ is an arbitrary periodic function, which changes in time with the frequency ω.

From Eq. (1), we can determine the intensity of a light beam (the first component of the Stokes vector), which has passed through the system,

$$I = \frac{I_0}{4}(1 - R_1)^2 e^{-\gamma z} \times$$

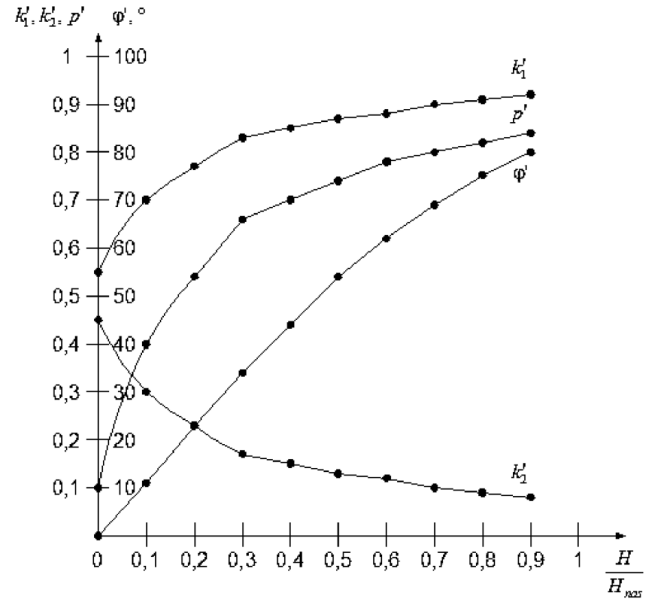$$\times [(k_1 + k_2)^2 - (k_1 - k_2)^2 pp' \cos 2(\beta - \varphi + \theta + \theta_=)], \quad (2)$$

Fig. 3. Plots of the dependences $\varphi'\left(\frac{H}{H_{\rm nas}}\right), k_1'\left(\frac{H}{H_{\rm nas}}\right), k_2'\left(\frac{H}{H_{\rm nas}}\right),$ and $p'\left(\frac{H}{H_{\rm nas}}\right)$

where p is the degree of light polarization in the examined substance, k_1 and k_2 are the principal transmittance values of polarization prisms, and β is the azimuth of the plane of maximum transmittance of the analyzer.

At measurements, the plane of maximum transmittance of the analyzer and the polarization plane of incident light that passed through the investigated specimen were mutually perpendicular to within the accuracy of the experimental sensitivity Δ [9, 10],

$$\beta - \varphi + \theta_= = \frac{\pi}{2} + \Delta[9, 10].$$

Then expression 2 reads

$$I = \frac{I_0}{4}(1 - R_1)^2 e^{-\gamma z}(k_1 + k_2)^2 \times$$

$$\times [1 - (1 - 2Gd)^2 pp' \cos 2\theta + 2\Delta(1 - 2Gd)^2 pp' \sin 2\theta],$$

where $Gd = k_2/(k_1 + k_2)$ is the polarization defect of the prisms [8]. The obtained expression for the intensity at the photopolarimeter output coincides with that derived in work [9], provided that the Faraday cell has no domain structure, its magnetization is uniform, and the examined specimen does not depolarize light, i.e. $p = p' = 1$.

Let us determine the polarization degree, P, of light that passed through the optical system [6],

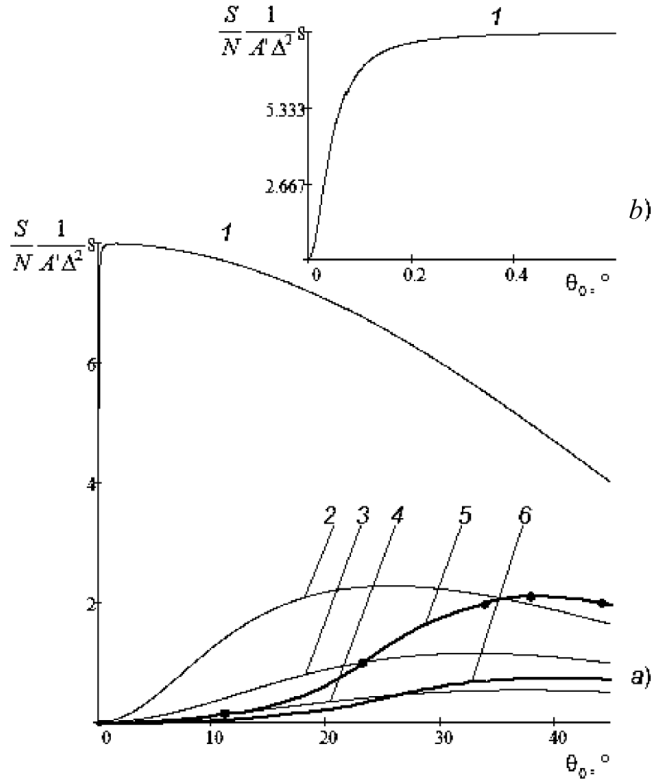$$P = \frac{I_{II} - I_\perp}{I_{II} + I_\perp} = (1 - 2Gd)^2 pp', \quad (3)$$

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Fig. 4. Dependence $\frac{1}{A'\Delta^2} \times \frac{S}{N}(\theta_0)$ at $P = 1$ (*1*), 0.9 (*2*), 0.7 (*3*), and 0.5 (*4*)

where $I_{\parallel}$ and $I_{\perp}$ are the intensities of light that passed through a pair of polarizers at their mutually parallel and perpendicular, respectively, polarization directions. According to formula (3), the parameter P depends in our case on the properties of polarizers (the parameter Gd), the Faraday cell (the degree of light polarization in an optically transparent ferrimagnetic crystal, p'), and a turbid medium (the degree of light polarization in the examined substance, p), i.e. on the properties of the optical system in whole. It should be noted that formula (3) is valid only for partially linearly polarized radiation.

Now, let us calculate the signal-to-noise ratio for the system concerned by analogy with work [9]. We suppose that the modulation signals that are supplied to the Faraday cell have a rectangular shape. In this case, p' is maximal within the modulation period, and

$$\frac{S}{N} = \frac{U_S^2}{U_T^2 + U_{SH}^2} =$$

$$= \frac{A_1 A^2 \Delta^2 4 (k_1 + k_2)^4 (1 - 2Gd)^4 p^2 p'^2 \sin^2 2\theta_0}{U_T^2 + A_2 A (k_1 + k_2)^2 \left[1 - (1 - 2Gd)^2 pp' \cos 2\theta_0\right]},$$

where U_S, U_{SH}, and U_T are the voltages created by the signal and the shot and thermal noises, respectively; $A_1 = N^2 R^2 \mu$; $A_2 = 2e'\Delta f N k R^2$; $A = \frac{e'\eta}{h\nu}(1 - R_1)^2 e^{-\gamma z} \frac{I_0}{4}$; N is the amplification coefficient of a photomultiplier; R the output load resistance; μ the coefficient of signal depression between the photocathode and the first dynode; $e' = 1.6 \times 10^{-19}$ C is the elementary charge; Δf the frequency interval; k an additional multiplier dependent on random oscillations of the current at the dynodes ($k = 2.5$); η the quantum yield; and $h\nu$ the photon energy [9, 15]. After transformations, we have

$$\frac{S}{N} = A'\Delta^2 (k_1 + k_2)^2 \times$$

$$\times \frac{4(1 - 2Gd)^4 p^2 p'^2 \sin^2 2\theta_0}{\frac{U_T^2}{A_2 A(k_1 + k_2)^2} + 1 - (1 - 2Gd)^2 pp' \cos 2\theta_0}, \qquad (4)$$

where $A' = \frac{A_1}{A_2} A$. Substituting expression (3) for P into the signal-to-noise ratio (4), we obtain

$$\frac{S}{N} = A'\Delta^2 (k_1 + k_2)^2 \frac{4P^2 \sin^2 2\theta_0}{\frac{U_T^2}{A_2 A(k_1 + k_2)^2} + 1 - P \cos 2\theta_0}. \qquad (5)$$

In Fig. 4, the dependences of the signal-to-noise ratio on the amplitude θ_0 of polarization-plane angular vibrations are plotted for various degrees of light polarization in the optical channel. Here and below, the following values of relevant parameters were adopted: $A = 5 \times 10^{-3}$, $A_1 = 2 \times 10^{13}$, $A_2 = 2 \times 10^{-6}$, $I_0 = 20$ mW, and $U_T = 0.2$ μV [15]. Curve *1* was plotted for a perfect nondepolarizing optical channel with $P = 1$. Curves *2* to *4* were calculated with regard for the influence of polarization prisms and the turbid active medium; the calculation parameters were $k_1 = 0.7$, $k_2 = 0.01$, and $p' = 1$. Curve *5* was calculated without regard for the influence of polarization prisms and the turbid active medium, with the polarization degree P at every point changing with the variation of the amplitude of polarization-plane angular vibrations and, accordingly, p'. Curve *6* was calculated in view of the influence of all factors in the optical channel for $k_1 = 0.7$, $k_2 = 0.01$, and $p = 0.9$; with the polarization degree P at every point changing with the variation of the amplitude θ_0 of polarization-plane angular vibrations and, accordingly, p'.

From Fig. 4, one can see that the dependence of the signal-to-noise ratio on θ_0 has a maximum. The θ_0-values that correspond to the maximum of the S/N-ratio are called optimum. Figure 4,b exhibits the dependence of the signal-to-noise ratio on the amplitude

of polarization-plane angular vibrations at $P = 1$ in the interval of small θ_0's. At $P = 1$, the curve of the $S/N(\theta_0)$ dependence starts from the coordinate origin and passes through a maximum, which has not been noticed in the previous research [9]. This circumstance is connected with the fact that, in this work, we consider the thermal noise. The latter, although being small, affects the value of the signal-to-noise ratio in the interval of small θ_0-angles, whereas the shot noise almost vanishes, because the depolarization of a light beam is absent. From Fig. 4,a, one can see that, when the parameter P diminishes, i.e. the quality of the optical channel worsens, the optimum θ_0-values increase, whereas the corresponding maximum S/N-values decrease.

Figure 4,a also testifies that the Faraday cell based on yttrium-gallium ferrite-garnet substantially depolarizes the light beam that passes through it. However, the signal-to-noise ratio and, accordingly, the sensitivity decrease by a factor of only four in comparison with the corresponding parameters for the completely nondepolarizing optical system (curve 5).

Curves 2 and 5 in Fig. 4 have almost identical values at the maximum of the dependence $S/N(\theta_0)$. However, the optimum θ_0 in curve 5 is twice as large as that in curve 2. This fact results from the influence of the domain structure in the optically transparent ferrimagnet. This structure considerably depolarizes the transmitting light beam at low magnetic fields. Hence, the influence of the domain structure is similar to the light scattering in the optical channel with $P = 0.9$.

2.1. Optimum amplitude of polarization-plane angular vibrations

Let us determine the optimum θ_0, $\theta_{0\mathrm{OPT}}$, at which the signal-to-noise ratio is maximum. For this purpose, we take the derivative of expression (5) with respect to θ_0 and equate it to zero. We obtain

$$\cos 2\theta_{0\mathrm{OPT}} = \frac{\frac{U_\mathrm{T}^2}{A_2 A (k_1 + k_2)^2} + 1}{P} -$$

$$- \sqrt{\left(\frac{\frac{U_\mathrm{T}^2}{A_2 A (k_1 + k_2)^2} + 1}{P} \right)^2 - 1}.$$

Substituting $\theta_{0\mathrm{OPT}}$ into the expression for the signal-to-noise ratio, we obtain the maximum S/N-value. In Fig. 5,a, the dependence of the optimum amplitude $\theta_{0\mathrm{OPT}}$ on the degree of light polarization in the optical channel is depicted. In our calculations, we took the
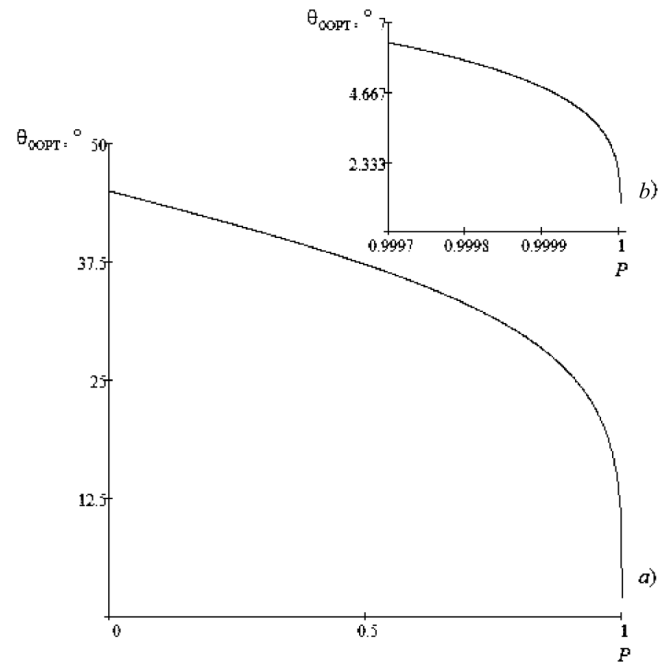

Fig. 5. Dependence $\theta_{0\mathrm{OPT}}(P)$

following values of parameters: $k_1 = 0.7$ and $k_2 = 0.01$ (it is worth noting that, at $P = 1$, the equalities $k_1 = 1$ and $k_2 = 0$ are obeyed). The plot demonstrates that the improvement of the optical channel quality gives rise to a reduction of the optimum θ_0-values, and, at P close to 1, the $\theta_{0\mathrm{OPT}}$ values drastically decrease. However, $\theta_{0\mathrm{OPT}} \neq 0$ at $P = 1$ (Fig. 5,b), because, as was mentioned above, the influence of the thermal noise has to be taken into account.

2.2. Imbalance angle

The minimum of the imbalance angle is determined from the condition $S/N = 1$, and, according to expression (5) (by analogy with work [10]), it equals

$$\Delta = \sqrt{\frac{1}{A' (k_1 + k_2)^2} \frac{\frac{U_\mathrm{T}^2}{A_2 A (k_1 + k_2)^2} + 1 - P \cos 2\theta_0}{4 P^2 \sin^2 2\theta_0}}.$$

The theoretical curves in Fig. 6 illustrate the dependence of the imbalance angle – in other words, the sensitivity of the system – on the degree of light polarization in the optical channel, P, at various amplitudes of polarization-plane angular vibrations, including the optimum ones. The following parameters were adopted at calculations: $k_1 = 0.7$ and $k_2 = 0.01$ (it is worth noting that, at $P = 1$, the equalities $k_1 = 1$ and $k_2 = 0$
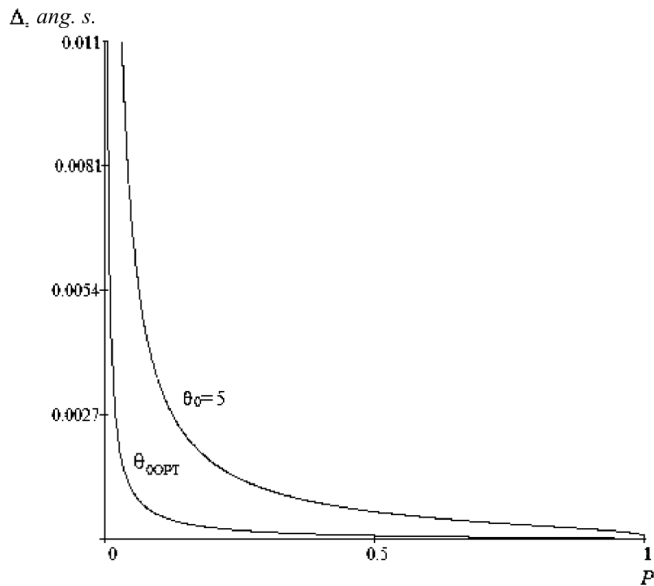
Fig. 6. Dependence of the system sensitivity $\Delta(P)$

hold true). It is evident that, at low P, the sensitivity of the system drastically depends on the parameter θ_0. A reduction of θ_0 worsens the system sensitivity, and its maximum shifts toward larger P-values. At the optimum θ_0, the sensitivity of a photopolarimeter is maximal for the given P. The optimum value of the amplitude of polarization-plane angular vibrations is specific for every value of the parameter P.

3. Conclusions

1) The application of a transparent ferrimagnetic crystal in the modulation photopolarimeter, owing to a large amplitude of polarization-plane angular vibrations, allows a high value of signal-to-noise ratio and, respectively, the sensitivity to be preserved even at a high illumination of a photodetector, which arises due to the depolarization and the scattering of a light beam in the examined turbid medium, the domain structure of ferrite, and so on.

2) The Faraday cell with a transparent ferrimagnet should be modulated with rectangular signals in order to minimize the influence of the illumination associated with the light beam depolarization by the domain structure, because, in this case, the degree of crystal polarization is maximum within the modulation period.

3) We have generalized the concept of light beam polarization degree in the optical channel. We hope for that the application of this concept in calculations will allow a complete analysis of the photopolarimeter functioning

and the results obtained to be carried out, because the introduced parameter depends on the properties of the optical channel as a whole.

4) The described procedure for the registration of a polarization-plane rotation angle can be used to measure the concentration of optically active substances in solutions [16], change the azimuthal direction [17], determine the parameters of polarized radiation [18], and so forth.

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СПОСІБ РЕЄСТРАЦІЇ КУТА
ОБЕРТАННЯ ПЛОЩИНИ ПОЛЯРИЗАЦІЇ
СВІТЛОВОГО ПРОМЕНЯ З ВИКОРИСТАННЯМ
ОПТИЧНО ПРОЗОРИХ ФЕРИМАГНІТНИХ КРИСТАЛІВ

А.В. Скрипець, В.Д. Тронько, М.М. Асанов

Р е з ю м е

Дана робота присвячена аналізу модуляційного фотополяри-
метра, в якому в ролі активного елемента комірки Фарадея
використовується прозорий в інфрачервоному діапазоні фери-
магнітний кристал, який має на 3-4 порядки більший ефект
Фарадея, ніж застосовані раніше парамагнітні кристали. Опи-
саний метод реєстрації кута обертання площини поляризації
дозволяє працювати в області оптимальних кутів розгойдува-
ння, тобто при максимальному відношенні сигнал–шум. Пока-
зано, що при оптимальному куті розгойдування чутливість фо-
тополяриметра зменшується незначно у випадку, коли якість
оптичного каналу різко погіршується, тобто при сильній депо-
ляризації світлового променя.