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MICROWAVES IN STRUCTURED METAMATERIALS: SUPELUMINAL, SLOW, AND BACKWARD WAVES

The dispersion properties of structured metamaterials consisting of strips of a copper wire (electron subsystem) and square copper split-ring resonators (magnetic subsystem) with different and coinciding resonant frequencies are studied. In a narrow frequency band above the resonant frequency of the electron subsystem, the structured metamaterial is described by a negative refractive index. In addition to this, there are some peculiar properties observed in these metamaterials. Among these properties is the nonanalytic behavior of the real part of the refractive index as a function of the frequency with a discontinuity of its derivative in the metamaterial with two resonances. It is also shown that the superluminal, slow, and backward microwaves can exist in the structured metamaterials. However, in the absence of gain components, only the slow microwaves can propagate considerably.

Keywords: structured metamaterials, supeluminal waves, slow waves, backward waves.

1. Introduction

Metamaterials have controllable electrodynamic properties. Some of these properties have been utilized to demonstrate a negative refractive index. Such types of materials were theoretically predicted long ago by Veselago [1]. Those materials with negative refractive index in a certain frequency domain are referred as left-handed media (LHM) and has only been manufactured in laboratories recently. In a narrow frequency band, these materials possess a negative refractive index in one [2] and two polarization directions [3]. Moreover, the three-dimensional left-handed metamaterials based on a fully symmetric multigap single-ring split-ring resonator design and crossing continuous wires was shown to possess isotropic transmission properties [4]. The scattering data at microwave frequencies on structured metamaterials (SMMs) exhibiting a microwave frequency band with negative refractive index are reported in Ref. [5]. These SMMs consist of copper wire strips and square copper split ring resonators that corresponds to the electron and magnetic subsystems, respectively. Their resonant frequencies are in the microwave frequency range of the order of 10 GHz. In a narrow frequency band, where the permittivity $\varepsilon(\omega)$ and permeability $\mu(\omega)$ are simultaneously negative (ω is the frequency), the propagation of electromagnetic waves (EMWs) can be described with the help of the negative refractive index.

In this paper, we show that the electrodynamic properties of the above-mentioned SMMs have additional peculiar characteristics in the microwave frequency band. Our numerical study of the refractive index and the group velocity of EMWs in these systems shows that the real part of the refractive index can be a nonanalytic function of the frequency with a discontinuous first derivative at two frequencies. This results in a jump of the group velocity. Moreover, at these particular frequencies, the real part of the re-

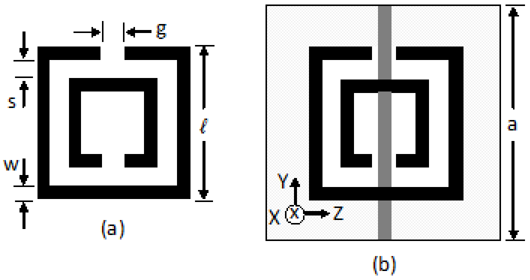

Fig. 1. Schematic of a single square split-ring resonator (SRR) (*a*). A schematic showing one possible arrangement of an SRR and a wire strip printed on a square dielectric board of side length a (*b*). The wire strip is centered on the SRR and is on the opposite side of the board from the SRR (Drawings not to scale)

fractive index is practically zero, whereas its imaginary part is nonzero. In a vicinity of the higher frequency nonanalytic point, narrow wave packets can be observed experimentally. Moreover, in the frequency bands of strong dispersion of the refractive index, the propagation of EMWs is possible with a group velocity that can be smaller and larger than the speed of light in vacuum (c). Such waves are intensively studied at optical frequencies and referred as the slow and superluminal light (see review [6] and references therein).

It is worth noting that the existence of superluminal light with a group velocity greater than c does not contradict the relativity theory. The group velocity characterizes the propagation of the "bulk" and the peak of a pulse, but not the point of discontinuity that starts from zero at some instant and describes the wave front. Physically, a positive group velocity that exceeds c means that the peak of a pulse emerging from the medium occurs at the same time as the peak of the pulse entering the medium; whereas a negative group velocity means that the peak of the emerging pulse occurs at an earlier time than the peak of the incident pulse [7]. Superluminal, as well as slow, light waves have been experimentally observed in different systems possessing a strong frequency dispersion [8, 9]. Recently, the existence of slow and superluminal lights have been theoretically analyzed for composite materials [10]. However, to our best knowledge, such waves have not been discussed in the microwave frequency range yet.

The paper is devoted to the analysis of these exotic waves in the microwave frequency range in SMMs. It is designed as follows. In Section 2, we analyze the refractive index of SMMs proposed in [5]. The character of nonanalyticity of the real part of the refractive index of SMMs with two resonances is considered in Section 3. Section 4 is devoted to the analysis of the group velocity of different types of microwaves propagating in this metamaterial. In Section 5, EMWs propagating in SMM with coinciding frequencies of the electron and magnetic subsystems are discussed. The conclusion summarizes the results of the paper.

2. Refractive Index of SMM with Two Resonances

The structured metamaterial under consideration consists of square copper split ring resonators and copper wire strips on a dielectric substrate; with the rings and wires placed on opposite sides of the substrate. Shelby *et al.* [3] have analyzed a similar system by introducing a unit cell consisting of six copper SRRs and two metal strips; the parameter values (referred to Fig. 1) were $w = 0.25$ mm, $s = 0.30$ mm, $g = 0.46$ mm, $\ell = 2.62$ mm, each wire strip was $3a = 1$ cm in length, and the resulting SMM was shown to possess LHM properties in the 10.3–11.1-GHz frequency domain, which lies in the microwave frequency region. It is worth noting that the effective permittivity and permeability of SMMs are practically obtained by employing a retrieval procedure from measured data of the complex transmission and reflection amplitudes of a metamaterial of a finite length [11].

Suppose a plane electromagnetic field is incident on the SMM in such a way that the electric field is polarized parallel to the strips, along the y-axis, the magnetic field is directed along the ring axis, x-axis, and its wave vector is directed along the z-axis, as depicted in Fig. 1. For such polarization, the permittivity and permeability of the SMMs can be described by [5]

$$\begin{aligned} \varepsilon(\omega) &= 1 - \frac{\omega_{ep}^2 - \omega_{eo}^2}{\omega^2 - \omega_{eo}^2 + i\gamma\omega}, \\ \mu(\omega) &= 1 - \frac{\omega_{mp}^2 - \omega_{mo}^2}{\omega^2 - \omega_{mo}^2 + i\gamma\omega}, \end{aligned} \qquad (1)$$

where ω_{eo} and ω_{mo} are the resonant frequencies of the electron and magnetic subsystems, respectively; γ is the damping constant of these subsystems (for the sake of simplicity, it is chosen to be the same for both subsystems), ω_{ep} and ω_{mp} are parameters related to the electron and magnetic subsystems, respectively. We note that all known models of LHM in the SHF range have a similar frequency dependence of the permittivity and permeability as (1), but with different values of parameters [12–14]. Moreover, when the metallic strips maintain the electrical continuity, $\omega_{eo} = 0$ [3], so that the permittivity reduces to the Drude expression. In our case, $\omega_{eo} \neq 0$.

Further, it will be convenient to measure all frequencies in terms of ω_{eo} and to introduce the dimensionless frequency $z = \omega/\omega_{eo}$. Separating the real and imaginary parts of the permittivity and permeability in (1) as $\varepsilon(\omega) = \varepsilon_1(\omega)+i\varepsilon_2(\omega)$, $\mu(\omega) = \mu_1(\omega)+i\mu_2(\omega)$, we obtain

$$\begin{aligned} \varepsilon_1(z) &= 1 - \frac{\alpha(z^2-1)}{(z^2-1)^2+\nu^2z^2}, \\ \varepsilon_2(z) &= \frac{\alpha\nu z}{(z^2-1)^2+\nu^2z^2}, \\ \mu_1(z) &= 1 - \frac{\beta(z^2-z_m^2)}{(z^2-z_m^2)^2+\nu^2z^2}, \\ \mu_2(z) &= \frac{\beta\nu z}{(z^2-z_m^2)^2+\nu^2z^2}. \end{aligned} \qquad (2)$$

Here, the resonant frequencies of the electron and magnetic subsystems are $z_{re} = 1$ and $z_{rm} \equiv z_m = \omega_{mo}/\omega_{eo}$, respectively, $\nu = \gamma/\omega_{eo}$ is the dimensionless decay constant, and $\alpha = z_e^2 - 1$, $\beta = z_{mp}^2 - z_m^2$ are dimensionless parameters with $z_e = \omega_{ep}/\omega_{eo}$, $z_{mp} = \omega_{mp}/\omega_{eo}$.

The complex refractive index of the metamaterial $n = n_1 + in_2$ is obtained from the relation

$$n(\omega) = \sqrt{\varepsilon(\omega)\mu(\omega)}. \qquad (3)$$

Substituting (1) into (3) with account of (2) and equating the real and imaginary parts, we get the following system of equations for the real n_1 and imaginary n_2 parts of the refractive index:

$$n_1^2 - n_2^2 = A, \quad 2n_1n_2 = B, \qquad (4)$$

where

$$A = \varepsilon_1\mu_1 - \varepsilon_2\mu_2, \quad B = \varepsilon_1\mu_2 + \varepsilon_2\mu_1. \qquad (5)$$

Solving the system of equations (4), we get

$$n_1(z) = \pm\sqrt{\frac{\sqrt{A^2+B^2}+A}{2}},$$

$$n_2(z) = \sqrt{\frac{\sqrt{A^2+B^2}-A}{2}}. \qquad (6)$$

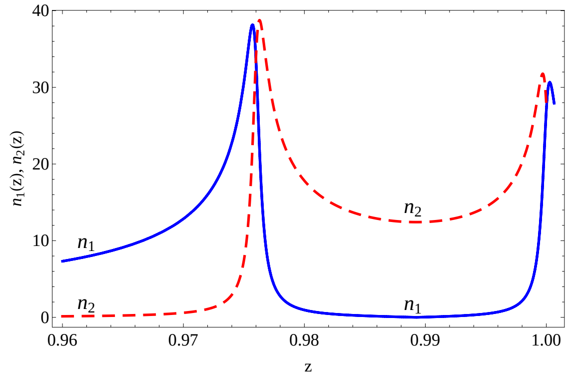

Fig. 2. Real n_1 (solid line) and imaginary n_2 (dashed line) parts of the refractive index versus z in a frequency range $0.96 < z \leq 1$. The peaks correspond to the resonances $z_m = 0.976$ and $z_e = 1$. Parameters of SMM are given in the Table shown below

It is clear that the imaginary part of the refractive index n_2 of equilibrium systems (no external gain) must be positive, which corresponds to the decay of EMWs. The sign of n_1 is "–" for $\varepsilon_1 < 0$, $\mu_1 < 0$ (LHM) and "+" otherwise.

We calculated n_1 and n_2 as functions of the dimensionless frequency z in a wide frequency range according to (6) with the parameters of SMMs given in the Table shown below. Because of the large scale of $\omega_{e0} \approx 10^{11}$ rad/s and the necessity to show domains of RHM and LHM, we present the graphs of n_1 and n_2 in the following successive frequency bands.

Figure 2 depicts $n_1(z)$ and $n_2(z)$ in a frequency range $0.96 < z \leq 1$. In the frequency band $z < z_m$, the SMM behaves as RHM, with $\varepsilon_1 > 0$, $\mu_1 > 0$, and $n_1(z) \gg n_2(z)$. On the other hand, for the frequency band between the resonances, SMM corresponds to RHM with $\varepsilon_1 > 0$ and $\mu_1 < 0$. Here, $n_2 \approx 15 \gg n_1$, which results in a very strong absorption of microwaves.

In the next frequency band $1 < z < 1.063$, the system possesses LHM properties. Figure 3 shows $n_1(z)$

Parameters of the structured metamaterial (frequencies in GHz) [5]

f_{eo}	f_{ep}	f_{mp}	f_{mo}	ν	z_m	α	β
10.30	12.80	10.95	10.05	10^{-3}	0.976	0.545	0.178

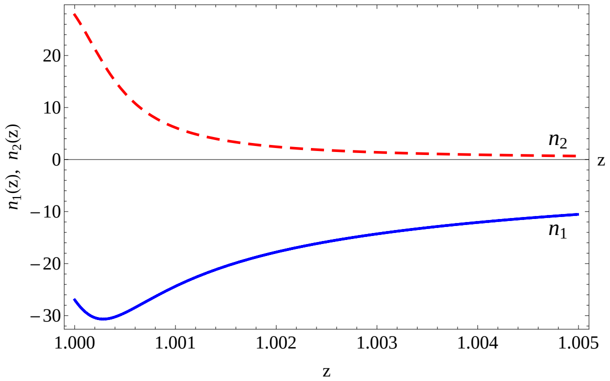

Fig. 3. Real n_1 (solid line) and imaginary n_2 (dashed line) parts of the refractive index versus z at the beginning of the LHM domain $1 < z < 1.005$, where $\varepsilon_1 < 0$, $\mu < 0$. Parameters of SMM are the same as those in Fig. 2

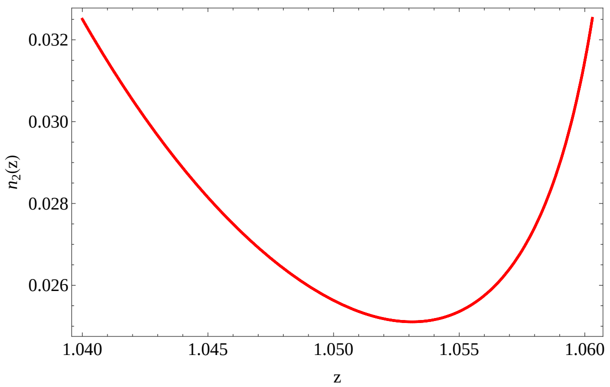

Fig. 4. Imaginary part n_2 of the refractive index versus z in the frequency band $1.04 < z < 1.06$ of the LHM domain. Parameters of SMM are the same as those in Fig. 2

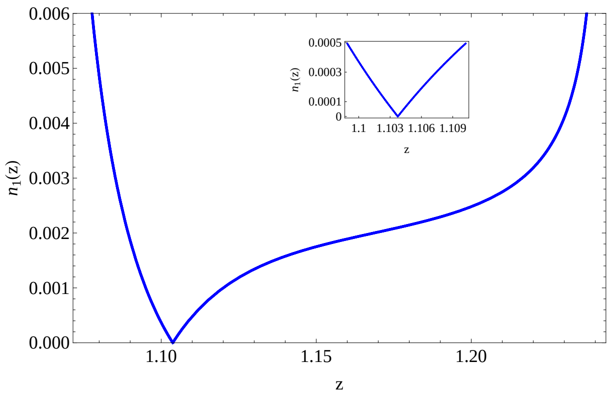

Fig. 5. Real part n_1 of the refractive index versus z. The inset shows enlarged $n_1(z)$ in a vicinity of the nonanalytic point $z = 1.1037$. Parameters of SMM are the same as those in Fig. 2

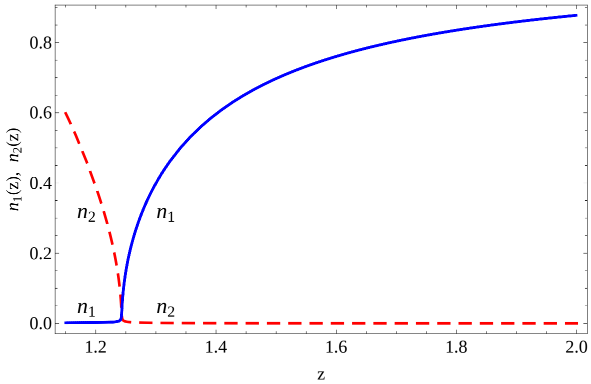

Fig. 6. Real n_1 (solid line) and imaginary n_2 (dashed line) parts of the refractive index n_1 versus z in the frequency range $1.15 < z < 2$. Parameters of SMM are the same as those in Fig. 2

and $n_2(z)$ for $1 < z < 1.005$, which corresponds to the beginning of the LHM domain. The real part n_1 of the refractive index is negative with $n_1 \approx -30$ at its minimum with $n_2 \approx 15$. Moreover, very near to the resonance $z = 1$, the value of n_2 is about 30 (Fig. 2). Therefore, at the beginning of the LHM domain, EMWs are strongly absorbed. However, the values of $|n_1(z)|$ and $n_2(z)$ decrease with z, so that $n_2(z)$ is of the order of 0.03 at the end of the LHM frequency band (see Fig. 4), while $n_1 \approx -2$. We note that the observed LHM frequency domain agrees with that obtained in experiments [5]. When the frequency passes the resonance $z = 1$, $n_1(z)$ rapidly changes from +30 to −30 (see Figs. 2 and 3).

In the frequency band $1.063 < z < 1.243$, where $\varepsilon_1 < 0$ and $\mu_1 > 0$, the system possesses the RHM properties and the system is expected to strongly absorb EMWs. The detailed numerical analysis shows an interesting behavior of n_1. Figure 5 shows $n_1(z)$ versus z in this frequency band. It is a continuous function of z, but its derivative dn_1/dz is discontinuous at $z = 1.1037$. The inset of Fig. 5 shows an enlarged fragment of $n_1(z)$ in a vicinity of the nonanalytic point. It means that $n_1(z)$ is a nonanalytic function of z. We also note that a similar type of the nonanalytic behavior of $n_1(z)$ takes place at the frequency $z = 0.989$ between the resonances with $n_2 \approx 30$.

For frequencies $z > 1.243$, the SMM possesses RHM properties. Figure 6 shows $n_1(z)$ and $n_2(z)$ in the frequency band $1.15 < z < 2$. It is seen that, immediately after $z = 1.243$, $n_1(z)$ increases rapidly, by approaching 1 for $z > 2$, whereas $n_2(z)$ quickly decreases, which results in a small decay of microwaves.

The numerical analysis of the refractive index of SMM discussed above shows that it is possible to specify the narrow frequency bands, where $n_1(z)$ has a strong frequency dispersion with small enough $n_2(z)$, which allows one to specify considerably propagating microwaves. In these frequency bands, the group velocity of narrow wave packets demonstrates some interesting peculiarities that we will discuss below. First, we consider analytically the behavior of the real part of the refractive index in a vicinity of the frequencies, where it is equal to zero.

3. Nonanalyticity of Refractive Index of SMM with Two Resonances

The conventional solution of the system of equation (4) requires the inequality $\varepsilon_2\mu_2 \ll \varepsilon_1\mu_1$. For $\nu \ll 1$, it provides $n_2 \ll n_1$ and results in $n_1 = \pm\sqrt{\varepsilon_1\mu_1}$ with a positive quantity under the square root. But this system has one more solution, when $\varepsilon_1\mu_1 < 0$ with a large modulus. Again neglecting ε_2 and μ_2, we get

$$n_1(z) = \frac{|\varepsilon_1\mu_2 + \varepsilon_2\mu_1|}{2\sqrt{|\varepsilon_1\mu_1|}}, \quad n_2(z) = \sqrt{|\varepsilon_1\mu_1|}. \tag{7}$$

It is clear that, in this case,

$$n_2 \gg n_1. \tag{8}$$

For $|n_1| \approx 1$, the inequality $n_2 \gg n_1$ results in a strong absorption of EMWs. In [15], this case is referred as a nearly perfect absorption. However, for $n_1(z) = 0$, inequality (8) can be true even for comparatively small n_2 that reduce the absorption of EMWs.

Let us check when the equation $n_1(z) = 0$ has real roots. According to the first relation of (7), it reduces to

$$\varepsilon_1\mu_2 + \varepsilon_2\mu_1 = 0. \tag{9}$$

For a small damping constant $\nu \ll 1$ and far from the resonances, it is possible to neglect the term $\nu^2 z^2$ in the denominators of (2) and to obtain a biquadratic equation for z. The substitution $z^2 = 1+x$ transforms (9) to

$$x^2 - 2ax - b = 0, \tag{10}$$

where $a = \alpha(\beta - \delta)/(\alpha + \beta)$, $b = \delta a$, and $\delta = 1 - z_m^2$. The solutions of (10) can be written in the form

$$x_{1,2} = a\{1 \mp \sqrt{1 + \delta/a}\}. \tag{11}$$

Formula (11) shows that $n_1(z) = 0$ at two points. For close resonant frequencies $\delta = 1 - z_m^2 \ll 1$, it is possible to expand (11) in the small parameter δ. With accuracy of leading terms, we obtain the following roots of (10):

$$z_1 = \sqrt{(1 + z_m^2)/2}, \quad z_2 = \sqrt{(1 + 2\alpha\beta)/(\alpha + \beta)}. \tag{12}$$

The root z_1 lies between the resonant frequencies, and the root $z_2 > 1$ is located above the resonant frequency of the electron subsystem. As was mentioned in Section 2, $n_2(z_1) \approx 15$, and EMWs strongly decay. In a vicinity of z_2, $n_2 \approx 0.7$, which makes the propagation of narrow microwave packets possible.

It is interesting to compare the approximate numerical values of $z_1 = 0.988$ and $z_2 = 1.126$ obtained with the help of (12) with the parameters of SMM from Table and the exact roots of z_1 and z_2 obtained from the equation $n_1(z) = 0$ (first relation of (6) with $A < 0$): $z_1 = 0.989$ and $z_2 = 1.104$. We find a very good agreement for the corresponding z_1 and a satisfactory agreement for z_2.

It follows from Fig. 5 that, in a vicinity of z_2, n_1 can be expressed as

$$n_1(z) = a(z_2)|z - z_2|. \tag{13}$$

Our numerical evaluations give $a(z_2) \approx 0.1$. Just for the reference, $n_1(z) \approx 60|z - z_1|$. Increasing the separation between the resonances, the above-described nonanalytic behavior of $n_1(z)$ disappears.

In conclusion of this section, we note that the propagation of EMWs in composite media with a zero index of refraction has some peculiarities. Similar cases with $n_1 = n_2 = 0$ were considered in [16]. The propagation of narrow wave packets centered at the frequency, where $n_1(z) = 0$, in a magnetized plasma with ferrite grains and with negligible losses was studied in [17].

4. Microwaves in SMM with Two Resonances

It is known that the group velocity V_g is a physical quantity that properly describes the propagation of narrow wave packets in media. It is given by the relation [18]

$$V_g = \frac{c}{n_1(z) + z\frac{dn_1(z)}{dz}}, \tag{14}$$

where c is the speed of light in vacuum, and n_1 is the real part of the refractive index. It is worth to recall that V_g comes from the Taylor expansion of

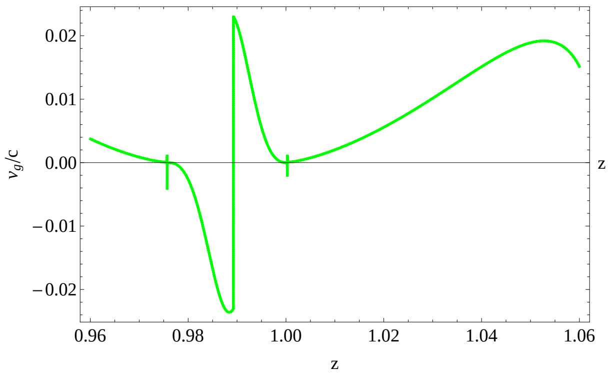

Fig. 7. Normalized group velocity V_g/c versus z in the frequency band $0.96 < z < 1.06$. Notice the jump of V_g/c which is associated with the nonanalyticity of $n_1(z)$ at the first singular point $z_1 = 0.989$. The small peaks at $z_m = 0.976$ and $z_e = 1$ correspond to V_g/c at the resonances. Parameters of SMM are the same as those in Fig. 2

the frequency $\omega(k)$ in a vicinity of the wave vector k_0, which corresponds to the center of the wave packet, i.e.,

$$\omega(k)=\omega(k_0)+\omega'(k)|_{k_0}(k-k_0)+\frac{1}{2}\omega''(k)|_{k_0}(k-k_0)^2\ldots. \quad (15)$$

Here, $V_g(k_0) = d\omega/dk|_{k_0} \equiv \omega'(k)|_{k_0}$ describes the velocity of the maximum of a wave packet provided that we ignore the second and higher derivatives terms in (15). From this requirement, it follows that V_g has the physical meaning for the narrow wave packets provided that

$$\omega^{(n)}(k_0)(k-k_0)^{n-1} \ll \omega'(k_0) \equiv V_g(k_0),\ n = 2, 3, ..., \quad (16)$$

where $\omega^{(n)}$ is the n^{th}-derivative of $\omega(k)$. These inequalities allow one to check whether V_g is a "good" physical quantity or not. As usual, the set of inequalities is limited by $n = 2$.

Below, we will calculate the normalized group velocity $V_g(z)/c$ according to (14) with the help of $n_1(z)$ given by the first formula of (6) with the parameters of SMM given in the Table, by focusing on the frequency bands, where n_2 is small enough to be ignored. The absorption coefficient α of EMWs is given by [19]

$$\alpha = \frac{2n_2(z)\omega}{c}, \quad (17)$$

and the typical decay length l for EMWs in a medium can be evaluated with $l \approx 1/\alpha$. The considerably propagating waves can be specified by imposing the condition $l \gg \lambda$, where $\lambda = c/f$ is the wavelength. For $f = 10$ GHz, a typical value is $\lambda = 3$ mm, and we obtain the inequality $n_2 \ll 1$. Below, we assume that $n_2 \leq 0.1$ provides the considerably propagating microwaves.

Figure 7 presents the general picture of $V_g(z)/c$ in the frequency range $0.96 \leq z \leq 1.06$. In the frequency band $z \leq z_m$ close to a resonance of the magnetic subsystem, $V_g/c < 1$ is positive and describes slow microwaves. Particularly, for $0.8 < z < 0.92$, the group velocity decreases from $0.18c$ to $0.04c$. The imaginary part of the refractive index in this band is $2.5 \times 10^{-3} < n_2 < 1.75 \times 10^{-2}$ and allows the considerably propagating slow microwaves. It is seen that V_g/c is discontinuous between the resonances of the magnetic and electron subsystems at $z_1 = 0.989$. However, there are no propagating waves in this band since $n_2 \gg 1$. Next, for the frequency band $1 < z \leq 1.063$, where the system is LHM, we have $0 < V_g/c \ll 1$. This corresponds to the slow microwaves having its maximum $V_g = 0.02c$ at $z = 1.05$, and n_2 is of the order of 0.025 (see Fig. 4). The small peak (left side) in Fig. 7 corresponds to V_g/c at the resonant point $z = z_m$. The second small peak (right side in Fig. 7) at $z = 1$ is not pronounced clearly on the graph because of the sign change of n_1 (see Figs. 2 and 3) from positive to negative in the LHM domain.

Figure 8 shows $V_g(z)/c$ in the most interesting frequency range $1.065 < z < 1.25$. One can see the jump of the group velocity $\triangle(V_g/c) \approx 20$ resulting from the nonanalyticity of $n_1(z)$ at the second singular point $z_2 = 1.104$, where $n_1 = 0$. In the frequency band $1.063 < z < z_2$, the group velocity is negative and lies between $-10c$ to $-0.01c$. For $z_2 < z < 1.25$, V_g firstly increases from $10c$ to $60c$ at $z = 1.17$ and, after this, decreases to $0.025c$. Figure 9 shows the maximum value of $n_2 \approx 0.7$ and its minimum ≈ 0.02.

Our numerical analysis of $V_g(z)/c$ and $n_2(z)$ in the frequency range $1.065 < z < 1.25$ allows us to claim that the SMMs can support the considerably propagating slow microwaves (including backward waves). The most favorable situation for the propagation of superluminal microwaves is $V_g \approx 2c$ with $n_2 \approx 0.15$. By varying the parameters of the SMMs, it is possible to slightly decrease n_2.

Next, we briefly discuss the effect of the nonanalytic behavior of n_1 on the group velocity and the propagation of narrow wave packets of microwaves. A rather interesting peculiarity of the group velocity is that it is not a continuous function of z at $z_1 = 0.989$. At $z_2 = 1.1037$, it has a jump, which is associated with the jump of the derivative dn_1/dz. As was mentioned above, there are no propagating waves in a vicinity of the first singular point. In a vicinity of the second singular point, $z_2 = 1.104$, n_2 is on the level of 0.7, which still supports the propagation of microwaves. With the help of gain components, it is possible to decrease n_2 and to provide a propagating wave in a vicinity of the singular frequencies [20, 21]. In principle, the gain may be achieved, by placing active inclusions designed from diode arrays across the gaps in the SRR units of the structured metamaterials [22,23]. In this case, for narrow wave packets centered at z_2, one can expect splitting of this wave packet into two packets moving with negative and positive group velocities.

According to [10], the addition of a small negative part to ε_2 does not affect practically n_1 and V_g in the optical frequency range, but it can considerably decrease n_2. Hence, the problem of the gain of microwaves in the SMMs requires a further study.

5. SMM with One Resonance

Let us consider a hypothetical SMM with equal (overlapping) resonant frequencies of the magnetic and electron subsystems with $z_m = 1$ and $\beta = \alpha$. In this case, we have to set $\mu_1 = \varepsilon_1$ and $\mu_2 = \varepsilon_2$ in (5). Equations (6) give the following simple expressions of the real and imaginary parts of the refractive index:

$$\begin{aligned} n_1 &= \varepsilon_1 = 1 - \frac{\alpha(z^2-1)}{(z^2-1)^2 + z^2\nu^2}, \\ n_2 &= \varepsilon_2 = \frac{\alpha\nu z}{(z^2-1)^2 + z^2\nu^2}. \end{aligned} \tag{18}$$

Now, the sign of n_1 automatically specifies the RHM and LHM domains.

Next, we have calculated $n_1(z)$ and $n_2(z)$, by using (18) with $\alpha = 0.545$ and $\nu = 10^{-2}$, 10^{-3}. Figure 10 shows $n_1(z)$ and $n_2(z)$ near the resonant frequency $z = 1$ for $\nu = 10^{-2}$. It is seen that, at $z = 0.9945$ and $z = 1.0045$, $n_1(z)$ has maximum and minimum ± 25, respectively. The real part of the refractive index is negative in the frequency band $1 < z \le 1.243$, where the SMM with one resonant frequency corresponds to LHM. Beyond this band, $n_1 > 0$, and this corresponds to RHM. The imaginary part of the refractive index is of the Lorentz type with a maximum

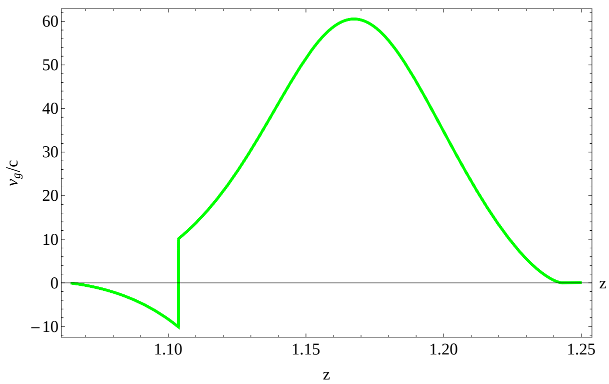

Fig. 8. Normalized group velocity $V_{g/c}$ versus z in the frequency band $1.065 < z < 1.25$. Notice the jump of $V_{g/c}$ at the second singular point $z_2 = 1.104$ of $n_1(z)$. Parameters of SMM are the same as those in Fig. 2

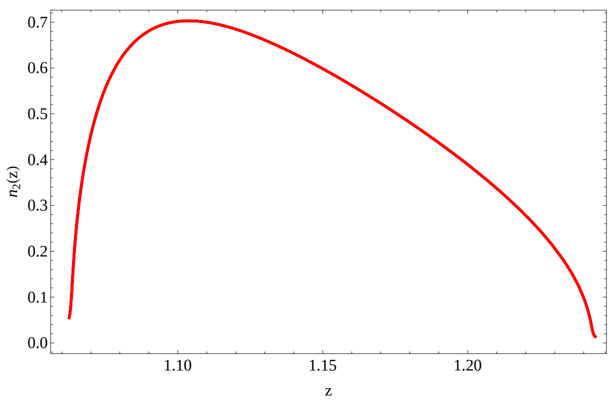

Fig. 9. Imaginary part of the refractive index n_2 versus z in the frequency band $1.062 \le z \le 1.245$ with a maximum value around 0.7. Parameters of SMM are the same as those in Fig. 2

of about 55. For the decay constant $\nu = 10^{-3}$, $n_1(z)$ and $n_2(z)$ have the same pattern as in Fig. 10, but with the extremum points $n_1 \approx \pm 230$ and $n_2 \approx 600$ (not plotted here).

The considerably propagating microwaves are possible only for the frequencies far from the resonance $z = 1$, where $n_2 < 0.1$, according to the criterion assumed in the previous section. The general picture of $V_g(z)/c$ can be understood with the help of Fig. 10 and the group refractive index

$$n_g(z) = n_1(z) + z\frac{dn_1(z)}{dz}, \tag{19}$$

which is the denominator of (14). Figure 11 shows $n_g(z)$ calculated by (19) using (18) for the frequency

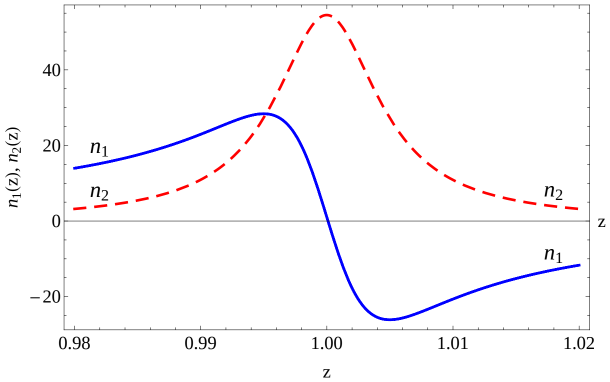

Fig. 10. Real n_1 (solid line) and imaginary n_2 (dashed line) parts of the refractive index versus z of SMM with one resonance in the frequency band $0.98 < z < 1.02$. The parameters of SMM $\alpha = 0.545$ and $\gamma = 10^{-2}$

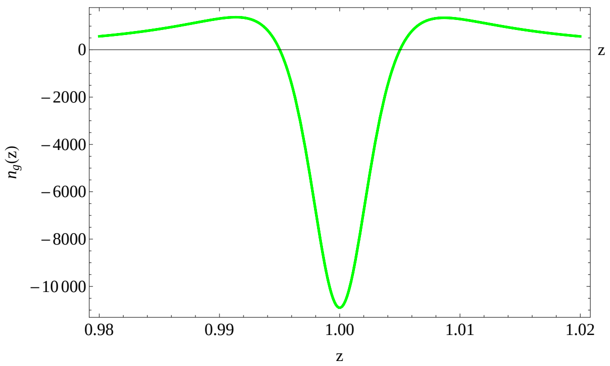

Fig. 11. Group refractive index n_g of SMM with one resonance versus z in the frequency band $0.98 < z < 1.02$. Parameters of SMM are the same as those in Fig. 10

range $0.98 < z < 1.2$. The positive and negative superluminal values of V_g/c correspond to the frequency bands, where $n_g(z)$ is close to zero. In a vicinity of $z = 1$, $V_g/c \approx -10^{-4}$, which corresponds to extremely slow microwaves. According to Fig. 10, $n_2 \gg 1$ in this frequency band, which results in a strong absorption of the corresponding microwaves.

Our numerical calculations show that the slow microwaves with $n_2 < 0.1$ exist in some frequency bands to the left of the resonance for $z < 0.85$ and to the right of it for $z > 1.175$. Particularly, in the frequency band $0.5 < z < 0.85$, the group velocity can be interpolated by the linear dependence $V_g = c(0.95-z)$. In the frequency band $1.2 < z < 2.4$, it is possible to determine the weakly damping microwaves with the group velocity lying in the interval $0.1c < V_g < 0.8c$. We note that a narrow frequency band $1.2 < z < 1.243$ is located at the end of the LHM domain of the SMM with one resonant frequency.

6. Conclusions

We have considered the refractive index and the group velocity of microwaves in structured metamaterials (SMMs) consisting of the strips of copper wires and square copper split-ring resonators with different and coinciding resonant frequencies. Similar types of SMM have been used for the experimental verification of a negative refractive index. We claim that SMMs can be considered as a "laboratory" for studying the slow, superluminal, and backward microwaves. However, only the slow microwaves can be considerably propagating in both types of SMMs. The superluminal microwaves in the SMMs with two resonances are decaying, but can be studied experimentally. The experimental study of the superluminal microwaves can be an additional source for the clarification of their physical nature and whether the conventional expression of the group velocity can be relevant to their description.

We have also shown that the real part of the refractive index of the SMMs with close resonance frequencies of the magnetic and electron subsystems could be a nonanalytic function of the frequency with a discontinuous first derivative. This happens at the frequencies, where $n_1 = 0$, and this results in a jump of the group velocity V_g of microwaves. The jump of V_g above the resonant frequency of the electron subsystem supports the weakly decaying superluminal microwaves (negative and positive) and can be checked experimentally.

Moreover, varying the parameters of SMMs has no significant effect in decreasing the imaginary part of the refractive index and the associated absorption. Obtaining the considerably propagating superluminal microwaves requires the application of gain components in SMMs. Formally, it can be done by including a small negative part in the permittivity of the SMM. But this requires to consider how it affects the real part of the refractive index.

This work is dedicated to the late Professor V.N. Mal'nev who was our teacher, mentor, and col-

league. He departed suddenly while this manuscript was in its final stage. We greatly acknowledge his invaluable contributions. Let his soul rest in peace.

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МІКРОХВИЛІ У СТРУКТУРОВАНИХ МЕТАМАТЕРІАЛІВ: ПОНАДСВІТОВІ, ПОВІЛЬНІ, І ЗВОРОТНІ ХВИЛІ

Р е з ю м е

Досліджуються дисперсійні властивості структурованих метаматеріалів, що складаються зі смужок з мідних дротиків (електронна підсистема) і квадратних резонаторів з мідним розрізаним кільцем (магнітна підсистема) з різними і збігаючими резонансними частотами. У вузькій смузі частот вище резонансної частоти електронної підсистеми структуровані метаматеріали характеризуються негативним індексом заломлення. В цих метаматеріалів виявлено також таку особливу властивість, як неаналітичність дійсної компоненти індексу заломлення як функції частоти з особливою точкою її похідної в метаматеріалі з двома резонансами. Показано, що в структурованих метаматеріалах можуть поширюватися суперсвітові, повільні і зворотні мікрохвилі. Однак за відсутності підживлючих компонент, істотно тільки поширення повільних мікрохвиль.