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APPLICATION OF MICROWAVE RAY REFRACTION IN INHOMOGENEOUS PLASMA INTERFEROMETRY

The applicability of plasma interferometry with inclined microwave rays for measuring the plasma density in different layers has been studied both analytically and numerically. Plasma density profiles consisting of a single peak or several peaks with the variable number and radial positions are proposed to verify the method. By considering the specific features of the refraction, a possibility of using the inclined-ray interferometry of plasma is analyzed.

Keywords: interferometry, plasma, radial distribution function, refraction, plasma density.

1. Introduction

The development of researches in plasma physics, including those in controlled nuclear fusion, and the progress attained in this domain are determined to a great extent by the development of methods for the measurement of plasma parameters. Modern researches in plasma physics are carried out in rather wide intervals of plasma density ($10^9 \div 10^{15}$ cm^{-3} and higher) and temperatures (from $T \leq 1$ eV to $T \geq$ ≥ 10 keV). Various methods of plasma researches are used at that [1–3]: probe, spectrometric, microwave, laser, and corpuscular ones. Among the methods of plasma diagnostics, microwave methods occupy an important place [4–8]. They are divided into active and passive ones. In passive methods, microwave radiation emitted by plasma is measured. The active microwave methods are based on measuring the result of the interaction of electromagnetic waves with plasma. They include, in particular, plasma interferometry and reflectometry. In those methods, plasma is probed with the help of microwave rays, and the result of the plasma interaction is used to determine certain plasma parameters. The application of plasma reflectometry makes it possible, e.g., to determine the spatial position of a plasma layer with the critical density and, by probing plasma at several frequencies, to determine the plasma density profile. With the help of interferometric methods at the transmission probing, it is possible, e.g., to determine the average density of plasma and, in some cases, by applying the multichord probing, one can also determine the plasma density profile.

In plasma diagnostics, the methods that are based on the refraction of microwave rays in plasma [9–18] are also applied; they are classed to active microwave ones. In those methods, the dependences of the refraction angle on the plasma parameters at the plasma probing by an ordinary wave and on the magnetic field strength in the case of extraordinary wave are analyzed [17, 18]. The application of the refraction allows the distribution of the plasma density to be determined. An important condition for this method to be applicable is the usage of narrow microwave rays that satisfy the geometrical optics criteria. On the other hand, the rays propagating from the horn diverge and cross various plasma layers, so that they can be used for the diagnostics of the latter. This is especially useful if the transmission probing is impossible, when the plasma density exceeds the critical value or the spatial density distribution is non-uniform, so that the maximum or several maxima of the density are located at some distance from the center (axis) of a plasma formation. This kind of the plasma density profile is observed in devices for separating a substance into mass groups and elements [19, 20]. It is worth noting that if the interferometric measurements are carried out under conditions, when the dimensions of plasma and the diameter of a probing microwave flux are comparable, the measured phase shift obtains an additional error [21–24].

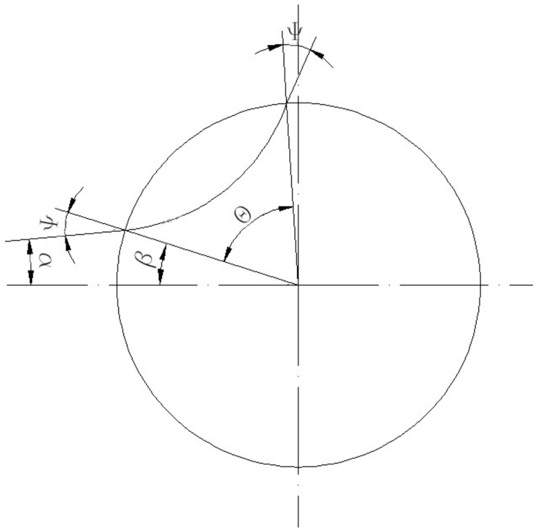

Fig. 1. Trajectory of a microwave ray in a plasma cylinder: α is the incidence angle of a ray on the plasma cylinder [4], and β is the angle between the normal and the inclined ray

Therefore, this research is aimed at studying the refraction features and a possibility to use plasma interferometry by inclined microwave rays in the case where the density profile has a maximum or several maxima, the number and the positions of which along the ray radius can change. The method is especially convenient at plasma interferometry, when plasma dimensions are larger than the diameter of a probing microwave flux.

2. Refraction of Microwaves in an Inhomogeneous Plasma Cylinder

2.1. Basic relations describing the trajectories of microwave rays in the plasma cylinder

Let the plasma probing be performed in a plane that is normal to the cylinder axis. Using the microwave rays that are inclined with respect to the plasma boundary, the deviations of the rays from the initial direction and their path lengths in plasma depend on the entrance angle and the plasma parameters. In the geometrical optics approximation, the differential equation for the trajectory of a microwave ray in a plasma cylinder (see Fig. 1) looks like [12, 17]

$$\frac{dr}{d\Theta} = \frac{r^2}{R\sin\Psi}\sqrt{n_{\rm O}^2(r) - \frac{R^2}{r^2}\sin^2\Psi}, \quad (1)$$

where R is the cylinder radius, r the current coordinate, Ψ the angle between the ray propagation direction and the cylinder radius at the point, where the ray strikes the plasma cylinder ($\Psi = \alpha + \beta$), Θ the deviation angle of the radius vector from its initial position, and $n_{\rm O}$ the refractive index for the ordinary wave. The deviation angle of the ray radius vector from its orientation at the ray entrance into plasma can be determined by the formula

$$\Theta(\Psi) = 2R\sin\Psi \int\limits_{r_0}^{R} \frac{dr}{r^2\sqrt{n_{\rm O}^2(r) - \frac{R^2}{r^2}\sin^2\Psi}}, \quad (2)$$

where r_0 is the turn point of a ray trajectory, which is determined from the condition:

$$\frac{R^2}{r^2}\sin^2\Psi = n_{\rm O}^2(r). \quad (3)$$

In the case where the ratio between the effective collision frequency $\nu_{\rm eff}$ and the probing frequency ω is very small, $\nu_{\rm eff}/\omega \ll 1$, the refractive index for the ordinary wave in plasma amounts to [4]

$$n_{\rm O}(r) = \left(1 - \frac{\omega_p^2(r)}{\omega^2}\right)^{1/2}, \quad (4)$$

where $\omega_p(r) = \left(N_p(r)\, e^2/\varepsilon_0 m_e\right)^{1/2}$ is the plasma frequency, e the elementary charge, m_e the electron mass, ε_0 the dielectric constant, and $N_p(r)$ the concentration of electrons in plasma. In the case $\omega > \omega_p$, i.e. $N_{\rm cr} > N_p$, where $N_{\rm cr} = \varepsilon_0 m_e \omega^2/e^2$ is the critical density of plasma, the plasma is transparent for the ordinary wave, whereas, at $\omega \leqslant \omega_p$, i.e. at $N_{\rm cr} \leqslant N_p$, it is opaque and completely reflects the microwave wave [4]. As one can see from Eqs. (1)–(4), the trajectories of microwave rays depend on the radial distribution of the plasma density.

2.2. Trajectories of inclined microwave rays in inhomogeneous plasma

In a number of plasma experimental researches carried out in devices for separating a substance into mass groups and elements [19, 20], a non-uniform spatial distribution of the density was observed, namely, there was a maximum or some maxima of the density located at a certain distance(s) from the center (axis) of a plasma formation. Therefore, the development of plasma diagnostics methods for the revealing and determination of a non-uniform profile of the plasma density is of importance. Here, the contactless methods, which include microwave ones, are the most convenient.

Let us consider the trajectories of microwave rays in plasma, whose density profile has a maximum or several maxima, the number and the positions of which along the radius can change. Such profile of the plasma density can be given in the form

$$N_p(r) = N_p(0) \times$$
$$\times \left[1 - \left(\frac{r}{R}\right)^{\gamma} + \left| a \left(\sin\left(k\pi \left(\frac{r}{R}\right)^{b}\right)\right)^{c} \right| \right], \tag{5}$$

where $N_p(0)$ is the electron concentration in plasma at the plasma cylinder center; and γ, a, b, c, and k are constants. The constant a determines the maximum amplitude, b the radial maximum position, c the maximum width, and k the periodicity. In the case where a or k equals zero, function (5) looks like

$$N_p(r) = N_p(0) \left[1 - \left(\frac{r}{R}\right)^{\gamma}\right]. \tag{6}$$

In Fig. 2, the radial distributions of the plasma density given by formulas (5) and (6) are depicted. The radial position of the density maximum for function (5) can be found from the relation

$$\frac{\gamma \left(\frac{r}{R}\right)^{\gamma-1}}{\pi abck \left(\frac{r}{R}\right)^{b-1}} = \cos\left[\pi k \left(\frac{r}{R}\right)^{b}\right] \times$$
$$\times \sin\left[\pi k \left(\frac{r}{R}\right)^{b}\right]^{c-1}. \tag{7}$$

In the case where $\gamma = 2$ and $a = b = c = k = 1$ (curve *2* in Fig. 2), relation (7) reads

$$r = \frac{R\pi \cos\left(\pi \frac{r}{R}\right)}{2}. \tag{8}$$

Figures 3 and 4 demonstrate the trajectories of microwave rays that pass through the plasma cylinder when the plasma density distribution is given by formula (6) or (5), respectively, for various values of the angle of ray incidence on the plasma cylinder and for various ratios $N_{\max}/N_{\rm cr}$. As is seen from Fig. 3, if the plasma density has a parabolic profile (see Fig. 2, curve *1*), the trajectory of a microwave ray deviates from the linear path. As $N_{\max}$ increases and approaches $N_{\rm cr}$, the ray deviation from the linear path becomes more and more substantial (see Fig. 3, panels from a to c). If the profile is given by function (5), the radial dependence of the refraction angle at various entrance angles considerably depends on the position and value of a maximum. If the angle of ray incidence on the plasma cylinder is small, the presence of a density maximum shifted from the plasma formation axis results in that the sign of the ray trajectory curvature changes, and the ray deviates in the opposite side with respect to the initial direction. At the same time, at large incidence angles, the curvature sign does not change (see Fig. 4). As $N_{\max}$ approaches $N_{\rm cr}$, this difference between the ray trajectories becomes more appreciable (see Figs. 4, b and c). A comparison of the microwave ray trajectories for two density distributions considered above at $N_{\max} = N_{\rm cr}$ shows (see Fig. 5) that, in both cases, the ray is reflected from the plasma layer with the density $N_{\max}/N_{\rm cr}$, or the microwave ray trajectory turns without changing the curvature sign. In this case, the value of Θ depends on the density distribution function for certain angles of ray incidence on the plasma layer. Therefore, the registration of ray exit angles at various incidence angles will allow a conclusion to be drawn concerning the presence of a maximum in the plasma density distribution.

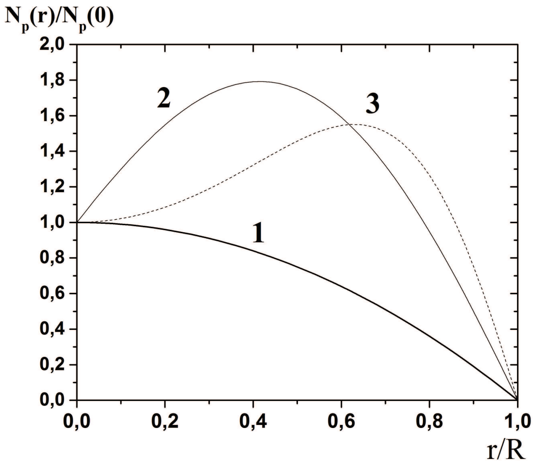

Fig. 2. Radial distributions of the plasma density: (*1*) $\gamma = 2$ in formula (6), (*2*) $\gamma = 2$ and $a = b = c = k = 1$ in formula (5), (*3*) $\gamma = b = 2$ and $a = c = k = 1$ in formula (5)

3. Specific Features of Microwave Interferometry with Inclined Rays

The phase shift between the waves passed through the reference and measuring channels of an interferometer

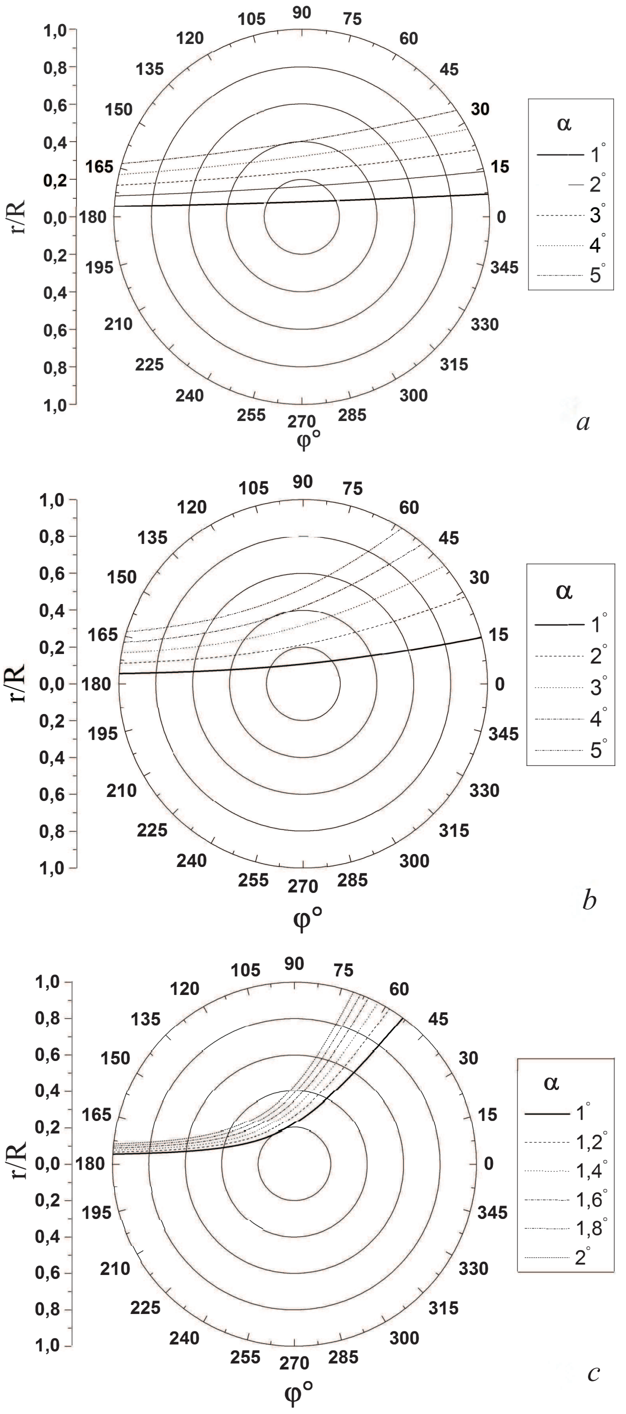

Fig. 3. Trajectories of microwave rays passing through the plasma cylinder with the density distribution (6), where $\gamma = 2$ and $N_{\max} = N_p\,(0)$, at $N_{\max}/N_{\rm cr} = 0.18$ (a), 0.54 (b), and 0.9 (c); α is the angle of ray incidence on the plasma cylinder

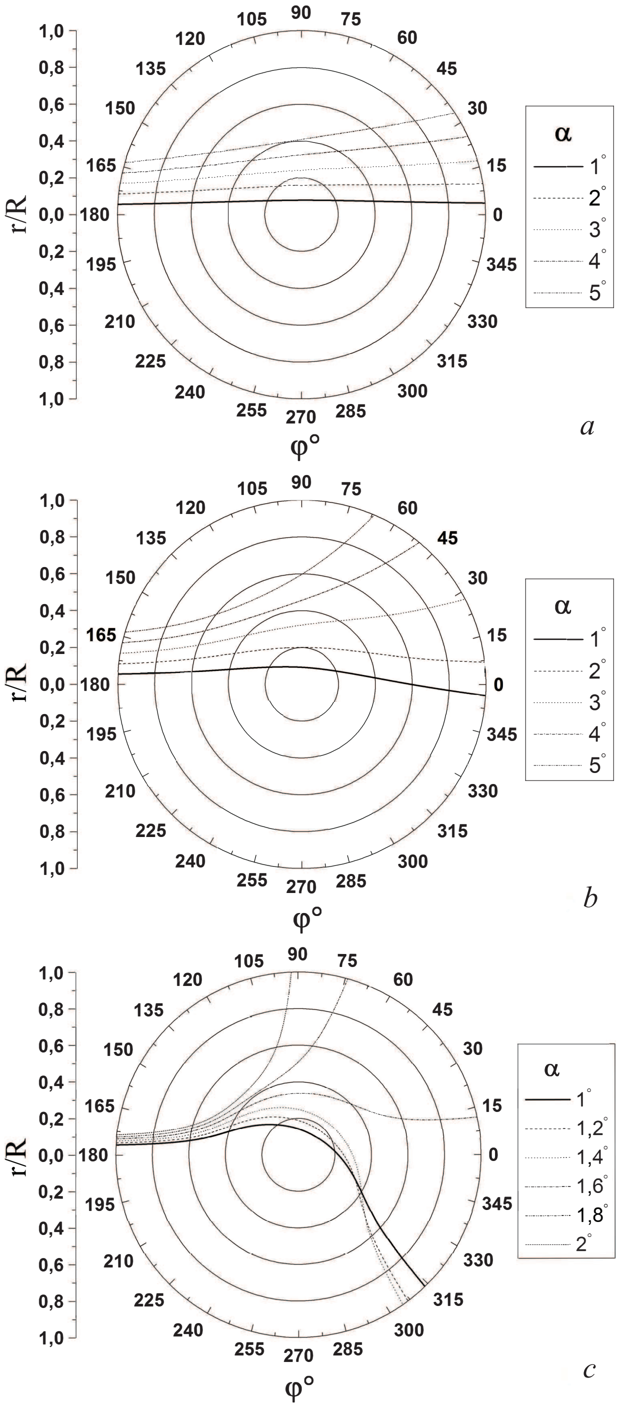

Fig. 4. Trajectories of microwave rays passing through the plasma cylinder with the density distribution (5), where $\gamma = 2$, $a = b = c = k = 1$, and $N_{\max} > N_p\,(0)$, at $N_{\max}/N_{\rm cr} = 0.18$ (a), 0.54 (b), and 0.9 (c). α is the angle of ray incidence on the plasma cylinder

equals

$$\Delta\Phi(t) = \varphi_0 - \varphi_p(t), \quad (9)$$

where φ_0 is the phase in the reference channel, and $\varphi_p(t)$ the phase in the measuring one. The phase in the reference channel always remains constant and is equal to

$$\varphi_0 = \frac{\omega}{c} L = \frac{2\pi}{\lambda_0} L. \quad (10)$$

In the measuring channel, the phase changes depend on the plasma density:

$$\varphi_p(t) = \frac{2\pi}{\lambda_0} \int_0^L n_\mathrm{O}(l,t)\, dl, \quad (11)$$

where λ is the length of a probing wave in vacuum, L the size of the researched plasma (the optical path length), and $n_\mathrm{O}(l,t)$ the index of wave refraction at the given point l of plasma. Substituting Eqs. (10) and (11) into Eq. (9), we obtain the phase shift

$$\Delta\Phi(t) = \frac{2\pi}{\lambda_0} \left(L - \int_0^L n_\mathrm{O}(l,t)\, dl \right). \quad (12)$$

The ray trajectory length in plasma depends on the entrance angle and the plasma parameters. In the case of cylindrical symmetry, the element of a ray path is determined as [12, 17]

$$ds = dr \sqrt{1 + r^2 \left(\frac{d\Theta}{dr} \right)^2}. \quad (13)$$

Accordingly, the optical path length of the ray in plasma, taking formulas (2) and (3) into account, equals [22]:

$$L_\mathrm{opt} = \int_s n_\mathrm{O} ds =$$

$$= 2 \int_{r_0}^{R} \frac{n_\mathrm{O}^2(r)\, r dr}{\sqrt{n_\mathrm{O}^2(r)\, r^2 - R^2 \sin^2 \Psi}}. \quad (14)$$

Using Eqs. (11) and (14), the phase change in the measuring channel can be presented in the form

$$\varphi_p(t) = \frac{2\pi}{\lambda_0} 2 \int_{r_0}^{R} \frac{n_\mathrm{O}^2(r)\, r dr}{\sqrt{n_\mathrm{O}^2(r)\, r^2 - R^2 \sin^2 \Psi}}. \quad (15)$$

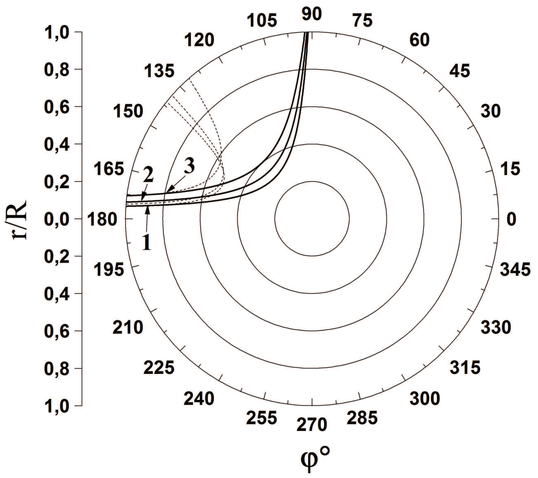

Fig. 5. Trajectories of microwave rays at $N_\mathrm{max}/N_\mathrm{cr} = 1$. Solid curves correspond to density distribution (6) with $\gamma = 2$, and dash-dotted ones to the density distribution (5) with $\gamma = 2$ and $a = b = c = k = 1$. $\alpha = 1.2°$ (*1*), 1.6° (*2*), and 2.2° (*3*)

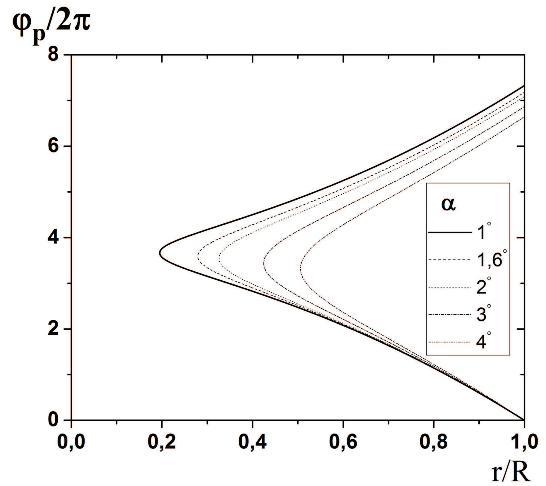

Fig. 6. Dependence of the phase change along the inclined ray trajectory at the parabolic density profile (6) with $\gamma = 2$ for various α

The radial dependences of the phase change along the inclined ray trajectory at a parabolic density profile ($\gamma = 2$ in formula (6)) and for various angles of ray incidence on plasma are depicted in Fig. 6. The analysis of this figure shows that the phase in the measuring channel and the wave penetration depth into plasma decrease, as the incidence angle increases.

Similar dependences in the case where the plasma density maximum is located at some distance from the center (axis) of a plasma formation are exhibited

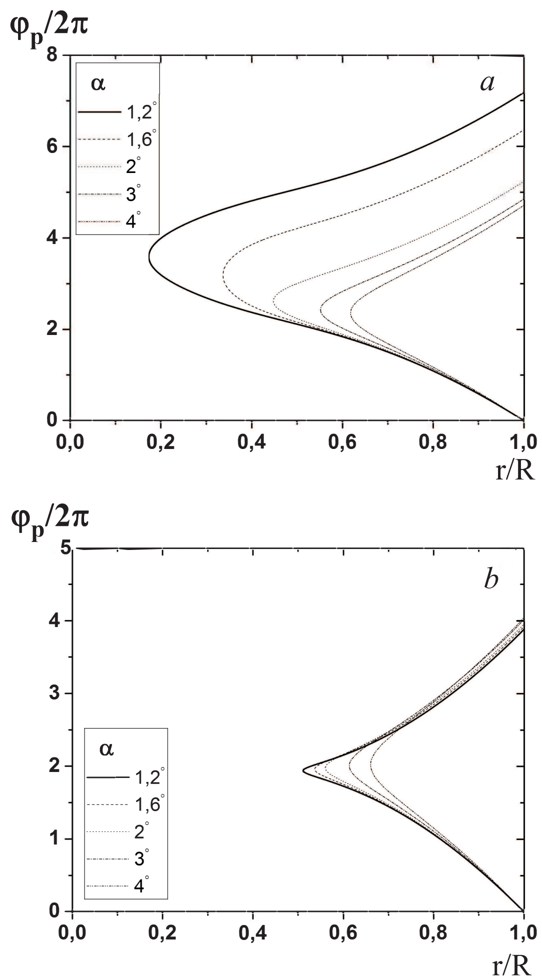

Fig. 7. Dependence of the phase change along the inclined ray trajectory at the density profile (5) with $\gamma = 2$, $a = b = c = k = 1$, $N_{\rm max}/N_{\rm cr} = 0.9$ (*a*) and 1 (*b*) for various α

in Fig. 7. In this case, the phase change along the inclined ray trajectory substantially depends on the maximum density. Figure 7, *a* demonstrates that if the maximum density is lower than the critical one, $N_{\rm max}/N_{\rm cr} = 0.9$, the phase shift and the wave penetration depth are considerably different for the angles $\alpha < 2°$ and $\alpha > 2°$. If the maximum density is equal to the critical one (see Fig. 7, *b*), the phase change and the wave penetration depth are less different, and the mentioned dependence on the incidence angle is violated.

The application of inclined microwave rays for plasma interferometry has a number of specific features. One of them consists in that the measuring system should be so tuned that the ray could hit the fixed horn, when the density changes in time. The receiving horns should be arranged at the angles $\angle\varphi = \Theta + \beta$ with respect to the transmitting one, where β is the central angle between the normal and inclined rays, which is connected with the angle α (see Fig. 1). Another feature is the fact that, owing to the variation of plasma parameters in time, the refraction angle will change as well. For this reason, the time of a phase shift measurement should be shorter than the time of the ray exit from the horn zone. This circumstance puts forward a requirement to the method that would be suitable for measuring the phase shift in this case. For this purpose, the plasma probing by frequency-modulated waves can be used. This method was applied in works [25, 26] to diagnose quasistationary plasma.

4. Conclusions

1. Trajectories of microwave rays are calculated for various profiles of the plasma density, its maximum values, and angles of ray incidence on the plasma cylinder. It is shown that if the angle of ray incidence on a plasma cylinder is small, the presence of a maximum in the plasma density shifted from the axis of a plasma formation results in a change of the ray trajectory curvature sign, and the ray deviates to the opposite side with respect to the initial direction.

2. Since the plasma density changes in time, the phase shift should be measured using the methods that are suitable for measurements in quasistationary plasma, e.g., probing with frequency-modulated waves.

3. It is shown that the application of plasma interferometry with inclined rays will allow the plasma density to be measured in separate plasma layers, if the density profile contains a maximum that is shifted from the axis, or the transmission probing of a plasma formation is impossible.

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ВИКОРИСТАННЯ РЕФРАКЦІЇ НВЧ ПРОМЕНІВ
ПРИ ІНТЕРФЕРОМЕТРІЇ НЕОДНОРІДНОЇ ПЛАЗМИ

Р е з ю м е

Аналітично і чисельно вивчається можливість використання інтерферометрії плазми похилими мікрохвильовими променями для вимірювання густини в різних шарах плазми. Для моделювання методу запропонований вид профілю густини з максимумом або декількома максимумами, кількість і положення яких можуть змінюватися по радіусу. Для такого профілю розглянуті особливості рефракції і можливість використання інтерферометрії плазми похилими мікрохвильовими променями.