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TWO SYSTEMS OF MAXWELL’S EQUATIONS AND TWO CORRESPONDING SYSTEMS OF WAVE EQUATIONS FOR ELECTROMAGNETIC FIELD VECTORS E AND B IN A ROTATING FRAME OF REFERENCE: A LINEAR APPROXIMATION

*On the base of two systems of Maxwell’s equations for the electromagnetic field vectors **E** and **B** in a uniformly rotating frame of reference, which were first proposed in the works by L.I. Schiff [Proc. Natl. Acad. Sci. USA **25**, 391 (1939)] and W. Irvine [Physica **30**, 1160 (1964)], two corresponding systems of wave equations are derived (to the first order in* Ω*). The analysis of these systems implies that: 1) the factor of rotation causes the arising of longitudinal **E**- and **B**-components of electromagnetic waves, which interact with the transversal ones; 2) the wave equations for the vector **E** in both systems of equations have the same form, while the equations for the vector **B** have different form; 3) the structure of equations for vectors **E** and **B** in the first case is asymmetric. Therefore, the propagation of the **E**- and **B**-components of electromagnetic waves in a rotating frame of reference will be governed by qualitatively different laws; 4) the structure of the wave equations in the second case is symmetric. Hence, the propagation of these field components will be governed by similar laws. It is also shown that, in the approximation of transversal electromagnetic waves, the distinction between the two systems of wave equations for the vectors **E** and **B** vanishes: both are transformed into two identical sets of separate (independent of one another) wave equations for the vector **E** and the vector **B** of a simpler (and already known from the literature) form.*

K e y w o r d s: Maxwell’s equations, wave equations, Sagnac effect, ring laser gyro.

1. Introduction

The analysis of the literature shows that there are mainly two basic systems of Maxwell’s equations for the electromagnetic field vectors **E** and **B** written in a frame of reference uniformly rotating with angular velocity Ω. Both systems are based on the Galilean description of a rotation, and both utilize the Newton (absolute) time t.

In the absence of free charges and currents, the first system (proposed in work [1]) has the form (we keep the terms only up to the first order in Ω)

$$\begin{aligned} &\nabla\times\mathbf{E}+\frac{\partial\mathbf{B}}{\partial t}=0,\\ &\nabla\,\mathbf{B}=0,\\ &\nabla\times\Big(\mathbf{B}-\frac{1}{c^2}\mathbf{v}\times\mathbf{E}\Big)-\frac{1}{c^2}\frac{\partial}{\partial t}(\mathbf{E}-\mathbf{v}\times\mathbf{B})=0,\\ &\nabla\,(\mathbf{E}-\mathbf{v}\times\mathbf{B})=0,\end{aligned}\tag{1}$$

and the second one (proposed in work [2]) is

$$\nabla\times\mathbf{E}+\frac{\partial}{\partial t}\Big(\mathbf{B}+\frac{1}{c^2}\mathbf{v}\times\mathbf{E}\Big)=0,$$
$$\nabla\,\Big(\mathbf{B}+\frac{1}{c^2}\mathbf{v}\times\mathbf{E}\Big)=0,$$

$$\begin{aligned}&\boldsymbol{\nabla}\times\mathbf{B}-\frac{1}{c^2}\frac{\partial}{\partial t}(\mathbf{E}-\mathbf{v}\times\mathbf{B})=0,\\&\boldsymbol{\nabla}(\mathbf{E}-\mathbf{v}\times\mathbf{B})=0.\end{aligned}\tag{2}$$

Both systems (and their derivation) are discussed in works [3, 4].

In expressions (1) and (2), all the quantities are specified by the formulas

$$\begin{aligned}&\boldsymbol{\nabla}=\hat{x}\,(\partial/\partial x)+\hat{y}\,(\partial/\partial y)+\hat{z}\,(\partial/\partial z),\\&\mathbf{E}=E_x\,\hat{x}+E_y\,\hat{y}+E_z\,\hat{z},\\&\mathbf{B}=B_x\,\hat{x}+B_y\,\hat{y}+B_z\,\hat{z},\\&\boldsymbol{\Omega}=\Omega_x\,\hat{x}+\Omega_y\,\hat{y}+\Omega_z\,\hat{z},\\&\mathbf{r}=x\,\hat{x}+y\,\hat{y}+z\,\hat{z},\\&\mathbf{v}=\boldsymbol{\Omega}\times\mathbf{r}=v_x\,\hat{x}+v_y\,\hat{y}+v_z\,\hat{z},\\&v_x=\Omega_y\,z-\Omega_z\,y,\\&v_y=\Omega_z\,x-\Omega_x\,z,\\&v_z=\Omega_x\,y-\Omega_y\,x.\end{aligned}\tag{3}$$

Here, $\hat{x}$, $\hat{y}$, and $\hat{z}$ are the unit vectors that form an orthogonal coordinate basis $\{\hat{x}\hat{y}\hat{z}\}$ of a rotating frame; E_x, E_y, E_z and B_x, B_y, B_z are components of the vectors $\mathbf{E}$ and $\mathbf{B}$ in this basis; $\boldsymbol{\Omega}$ is the vector of angular velocity with which the basis $\{\hat{x}\hat{y}\hat{z}\}$ rotates in an inertial frame; Ω_x, Ω_y, Ω_z are components of the vector $\boldsymbol{\Omega}$; $\mathbf{r}$ is the radius-vector of a given observation point in the basis $\{\hat{x}\hat{y}\hat{z}\}$; x, y, z are components of the vector $\mathbf{r}$; $\mathbf{v}$ is the vector of linear tangential velocity of the observation point calculated in the inertial frame; and v_x, v_y, v_z are components of the vector $\mathbf{v}$.

As we can see, the above two systems of Maxwell's equations (1) and (2) for the electromagnetic field vectors are not identical: system (1) has an asymmetric structure with respect to Ω in the sense that the rotation manifests itself only in the third and fourth equations, but not in the first and second ones; system (2) has a symmetric structure with respect to Ω, because the rotation manifests itself in all four equations. In this situation, we may ask the question: what will the form of the corresponding wave equations be for the indicated vectors in the first and second cases? The answer to this question is not given in the literature. So, the purpose of this paper is to derive the wave equations for the vectors $\mathbf{E}$ and $\mathbf{B}$: at first, on the base of the system of Maxwell's equations (1) and then on the base of system (2). All calculations must be performed with accuracy to the first order in v ($v=|\mathbf{v}|$) or, equivalently, Ω ($\Omega=|\boldsymbol{\Omega}|$).

2. Auxiliary Relations

In this section, we are going to present some useful formulas for the quantities $\boldsymbol{\nabla}\times(\boldsymbol{\nabla}\times\mathbf{G})$, $\boldsymbol{\nabla}\times(\mathbf{v}\times\mathbf{G})$, $\boldsymbol{\nabla}(\mathbf{v}\times\mathbf{G})$, $\boldsymbol{\nabla}(\mathbf{v}\,\mathbf{G})$, $\boldsymbol{\nabla}^2(\mathbf{v}\times\mathbf{G})$, which involve the vectors $\mathbf{v}=\boldsymbol{\Omega}\times\mathbf{r}$ and $\mathbf{G}$ ($\mathbf{G}=\mathbf{E},\mathbf{B}$).

A. Consider the term $\boldsymbol{\nabla}\times(\boldsymbol{\nabla}\times\mathbf{G})$. It is known (see, e.g., [5]) that

$$\boldsymbol{\nabla}\times(\boldsymbol{\nabla}\times\mathbf{G})=-\boldsymbol{\nabla}^2\mathbf{G}+\boldsymbol{\nabla}(\boldsymbol{\nabla}\,\mathbf{G}).\tag{4}$$

B. Consider the identity $\boldsymbol{\nabla}\times(\mathbf{v}\times\mathbf{G})=(\mathbf{G}\,\boldsymbol{\nabla})\mathbf{v}-(\mathbf{v}\,\boldsymbol{\nabla})\mathbf{G}+\mathbf{v}(\boldsymbol{\nabla}\,\mathbf{G})-\mathbf{G}(\boldsymbol{\nabla}\,\mathbf{v})$. In the case where $\mathbf{v}=\boldsymbol{\Omega}\times\mathbf{r}$, we have $(\mathbf{G}\,\boldsymbol{\nabla})\mathbf{v}=\boldsymbol{\Omega}\times\mathbf{G}$, $\boldsymbol{\nabla}\,\mathbf{v}=0$, and, to the first order in Ω, $\mathbf{v}(\boldsymbol{\nabla}\,\mathbf{G})=0$. Therefore,

$$\boldsymbol{\nabla}\times(\mathbf{v}\times\mathbf{G})=-(\mathbf{v}\,\boldsymbol{\nabla})\mathbf{G}+\boldsymbol{\Omega}\times\mathbf{G}.\tag{5}$$

C. Consider the identity $\boldsymbol{\nabla}(\mathbf{v}\times\mathbf{G})=\mathbf{G}\,(\boldsymbol{\nabla}\times\mathbf{v})-\mathbf{v}\,(\boldsymbol{\nabla}\times\mathbf{G})$. Since $\boldsymbol{\nabla}\times\mathbf{v}=2\boldsymbol{\Omega}$,

$$\boldsymbol{\nabla}(\mathbf{v}\times\mathbf{G})=2\boldsymbol{\Omega}\,\mathbf{G}-\mathbf{v}\,(\boldsymbol{\nabla}\times\mathbf{G}).\tag{6}$$

D. Consider the identity $\boldsymbol{\nabla}(\mathbf{v}\,\mathbf{G})=(\mathbf{v}\,\boldsymbol{\nabla})\mathbf{G}+(\mathbf{G}\,\boldsymbol{\nabla})\mathbf{v}+\mathbf{v}\times(\boldsymbol{\nabla}\times\mathbf{G})+\mathbf{G}\times(\boldsymbol{\nabla}\times\mathbf{v})$. Since $(\mathbf{G}\,\boldsymbol{\nabla})\mathbf{v}=\boldsymbol{\Omega}\times\mathbf{G}$, $\boldsymbol{\nabla}\times\mathbf{v}=2\boldsymbol{\Omega}$, and $\mathbf{G}\times(\boldsymbol{\nabla}\times\mathbf{v})=-2(\boldsymbol{\Omega}\times\mathbf{G})$,

$$\boldsymbol{\nabla}(\mathbf{v}\,\mathbf{G})=(\mathbf{v}\,\boldsymbol{\nabla})\mathbf{G}-\boldsymbol{\Omega}\times\mathbf{G}+\mathbf{v}\times(\boldsymbol{\nabla}\times\mathbf{G}).\tag{7}$$

E. Consider the vector $\boldsymbol{\nabla}^2(\mathbf{v}\times\mathbf{G})$. We have

$$\begin{aligned}&\boldsymbol{\nabla}^2(\mathbf{v}\times\mathbf{G})=\boldsymbol{\nabla}^2(\mathbf{v}\times\mathbf{G})_x\hat{x}+\\&+\boldsymbol{\nabla}^2(\mathbf{v}\times\mathbf{G})_y\hat{y}+\boldsymbol{\nabla}^2(\mathbf{v}\times\mathbf{G})_z\hat{z}.\end{aligned}\tag{8}$$

First, let us calculate the projection $\boldsymbol{\nabla}^2(\mathbf{v}\times\mathbf{G})_x$ of this vector onto the axis $\hat{x}$. In view of (3), we have

$$\boldsymbol{\nabla}^2(\mathbf{v}\times\mathbf{G})_x=\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)(v_yG_z-v_zG_y)\tag{9}$$

or

$$\begin{aligned}&\boldsymbol{\nabla}^2(\mathbf{v}\times\mathbf{G})_x=\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\times\\&\times[(\Omega_zx-\Omega_xz)G_z-(\Omega_xy-\Omega_yx)G_y].\end{aligned}\tag{10}$$

After the calculation, we get

$$\boldsymbol{\nabla}^2(\mathbf{v}\times\mathbf{G})_x=v_y\left(\frac{\partial^2G_z}{\partial x^2}+\frac{\partial^2G_z}{\partial y^2}+\frac{\partial^2G_z}{\partial z^2}\right)-$$

$$-v_z\left(\frac{\partial^2 G_y}{\partial x^2}+\frac{\partial^2 G_y}{\partial y^2}+\frac{\partial^2 G_y}{\partial z^2}\right)-$$
$$-2\Omega_x\left(\frac{\partial G_y}{\partial y}+\frac{\partial G_z}{\partial z}\right)+2\left(\Omega_y\frac{\partial G_y}{\partial x}+\Omega_z\frac{\partial G_z}{\partial x}\right). \quad (11)$$

Let us add the following terms to the right-hand side of (11): $-2\Omega_x(\partial G_x/\partial x)$ and $+2\Omega_x(\partial G_x/\partial x)$. Then

$$\boldsymbol{\nabla}^2(\mathbf{v}\times\mathbf{G})_x = v_y\left(\frac{\partial^2 G_z}{\partial x^2}+\frac{\partial^2 G_z}{\partial y^2}+\frac{\partial^2 G_z}{\partial z^2}\right)-$$
$$-v_z\left(\frac{\partial^2 G_y}{\partial x^2}+\frac{\partial^2 G_y}{\partial y^2}+\frac{\partial^2 G_y}{\partial z^2}\right)-$$
$$-2\Omega_x\left(\frac{\partial G_x}{\partial x}+\frac{\partial G_y}{\partial y}+\frac{\partial G_z}{\partial z}\right)+$$
$$+2\left(\Omega_x\frac{\partial G_x}{\partial x}+\Omega_y\frac{\partial G_y}{\partial x}+\Omega_z\frac{\partial G_z}{\partial x}\right) \quad (12)$$

or

$$\boldsymbol{\nabla}^2(\mathbf{v}\times\mathbf{G})_x = [\mathbf{v}\times(\boldsymbol{\nabla}^2\mathbf{G})]_x-$$
$$-2\Omega_x(\boldsymbol{\nabla}\,\mathbf{G})+2\frac{\partial}{\partial x}(\boldsymbol{\Omega}\,\mathbf{G}). \quad (13)$$

Similarly, we obtain

$$\boldsymbol{\nabla}^2(\mathbf{v}\times\mathbf{G})_y = [\mathbf{v}\times(\boldsymbol{\nabla}^2\mathbf{G})]_y-$$
$$-2\Omega_y(\boldsymbol{\nabla}\,\mathbf{G})+2\frac{\partial}{\partial y}(\boldsymbol{\Omega}\,\mathbf{G}), \quad (14)$$
$$\boldsymbol{\nabla}^2(\mathbf{v}\times\mathbf{G})_z = [\mathbf{v}\times(\boldsymbol{\nabla}^2\mathbf{G})]_z-$$
$$-2\Omega_z(\boldsymbol{\nabla}\,\mathbf{G})+2\frac{\partial}{\partial z}(\boldsymbol{\Omega}\,\mathbf{G}). \quad (15)$$

Therefore,

$$\boldsymbol{\nabla}^2(\mathbf{v}\times\mathbf{G}) = \mathbf{v}\times(\boldsymbol{\nabla}^2\mathbf{G})-2\boldsymbol{\Omega}(\boldsymbol{\nabla}\,\mathbf{G})+2\boldsymbol{\nabla}(\boldsymbol{\Omega}\,\mathbf{G}). \quad (16)$$

Finally, taking into account that, to the first order in Ω, $\boldsymbol{\Omega}(\boldsymbol{\nabla}\,\mathbf{G})=0$, we get

$$\boldsymbol{\nabla}^2(\mathbf{v}\times\mathbf{G}) = \mathbf{v}\times(\boldsymbol{\nabla}^2\mathbf{G})+2\boldsymbol{\nabla}(\boldsymbol{\Omega}\,\mathbf{G}). \quad (17)$$

Formulas (4)–(7) and (17) will be used in the next sections.

3. First System of Wave Equations for the Vectors E and B in a Rotating Frame of Reference

In this section, we are going to derive the first system of wave equations for the electromagnetic field vectors **E** and **B**, which will correspond to the system of Maxwell's equations (1). The first, second, third, and fourth equations of the system will be mentioned in the text as (1a), (1b), (1c), and (1d), respectively.

3.1. Equation for the vector E

To derive the wave equation for the vector **E**, we apply the operator $\boldsymbol{\nabla}\times$ to expression (1a):

$$\boldsymbol{\nabla}\times(\boldsymbol{\nabla}\times\mathbf{E})+\frac{\partial}{\partial t}(\boldsymbol{\nabla}\times\mathbf{B})=0. \quad (18)$$

Taking (4) and (1c) into account, we rewrite (18) in the form

$$\boldsymbol{\nabla}^2\mathbf{E}-\frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2}+\frac{1}{c^2}\frac{\partial}{\partial t}\left(\mathbf{v}\times\frac{\partial\mathbf{B}}{\partial t}\right)-\frac{1}{c^2}\frac{\partial}{\partial t}\times$$
$$\times[\boldsymbol{\nabla}\times(\mathbf{v}\times\mathbf{E})]-\boldsymbol{\nabla}(\boldsymbol{\nabla}\,\mathbf{E})=0 \quad (19)$$

or, using (1a),

$$\boldsymbol{\nabla}^2\mathbf{E}-\frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2}-\frac{1}{c^2}\frac{\partial}{\partial t}[\boldsymbol{\nabla}\times(\mathbf{v}\times\mathbf{E})+$$
$$+\mathbf{v}\times(\boldsymbol{\nabla}\times\mathbf{E})]-\boldsymbol{\nabla}(\boldsymbol{\nabla}\,\mathbf{E})=0. \quad (20)$$

With the help of (5), we get

$$\boldsymbol{\nabla}^2\mathbf{E}-\frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2}+\frac{1}{c^2}\frac{\partial}{\partial t}[(\mathbf{v}\,\boldsymbol{\nabla})\mathbf{E}-\boldsymbol{\Omega}\times\mathbf{E}-$$
$$-\mathbf{v}\times(\boldsymbol{\nabla}\times\mathbf{E})]-\boldsymbol{\nabla}(\boldsymbol{\nabla}\,\mathbf{E})=0. \quad (21)$$

Consider the term $\boldsymbol{\nabla}(\boldsymbol{\nabla}\,\mathbf{E})$ in (21). According to (1d), $\boldsymbol{\nabla}\,\mathbf{E}=\boldsymbol{\nabla}\,(\mathbf{v}\times\mathbf{B})$. Taking (6) and (1c) into account, we have $\boldsymbol{\nabla}\,(\mathbf{v}\times\mathbf{B}) = 2\boldsymbol{\Omega}\,\mathbf{B}-(1/c^2)\times$ $\times(\partial/\partial t)(\mathbf{v}\,\mathbf{E})$, so

$$\boldsymbol{\nabla}(\boldsymbol{\nabla}\,\mathbf{E})=2\boldsymbol{\nabla}(\boldsymbol{\Omega}\,\mathbf{B})-\frac{1}{c^2}\frac{\partial}{\partial t}\boldsymbol{\nabla}(\mathbf{v}\,\mathbf{E}). \quad (22)$$

Consider the last term in (22). In accordance with (7),

$$\boldsymbol{\nabla}(\mathbf{v}\,\mathbf{E})=(\mathbf{v}\,\boldsymbol{\nabla})\mathbf{E}-\boldsymbol{\Omega}\times\mathbf{E}+\mathbf{v}\times(\boldsymbol{\nabla}\times\mathbf{E}). \quad (23)$$

Therefore,

$$\boldsymbol{\nabla}(\boldsymbol{\nabla}\,\mathbf{E})=2\boldsymbol{\nabla}(\boldsymbol{\Omega}\,\mathbf{B})-$$
$$-\frac{1}{c^2}\frac{\partial}{\partial t}[(\mathbf{v}\,\boldsymbol{\nabla})\mathbf{E}-\boldsymbol{\Omega}\times\mathbf{E}+\mathbf{v}\times(\boldsymbol{\nabla}\times\mathbf{E})]. \quad (24)$$

Finally, substituting (24) into (21), we obtain the desired wave equation for the vector **E**:

$$\boldsymbol{\nabla}^2\mathbf{E}-\frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2}+\frac{2}{c^2}\frac{\partial}{\partial t}[(\mathbf{v}\,\boldsymbol{\nabla})\mathbf{E}-\boldsymbol{\Omega}\times\mathbf{E}]-$$
$$-2\boldsymbol{\nabla}(\boldsymbol{\Omega}\,\mathbf{B})=0. \quad (25)$$

3.2. Equation for the vector B

To derive the wave equation for the vector $\mathbf{B}$, we apply the operator $\boldsymbol{\nabla}\times$ to expression (1c):

$$\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \mathbf{B}) - \frac{1}{c^2}\boldsymbol{\nabla} \times [\boldsymbol{\nabla} \times (\mathbf{v} \times E)] - \\ - \frac{1}{c^2}\frac{\partial}{\partial t}(\boldsymbol{\nabla} \times \mathbf{E}) + \frac{1}{c^2}\frac{\partial}{\partial t}[\boldsymbol{\nabla} \times (\mathbf{v} \times \mathbf{B})] = 0. \quad (26)$$

In view for (4), (1a), and (1b), we rewrite (26) as

$$\boldsymbol{\nabla}^2\mathbf{B} - \frac{1}{c^2}\frac{\partial^2\mathbf{B}}{\partial t^2} - \frac{1}{c^2}\frac{\partial}{\partial t}[\boldsymbol{\nabla} \times (\mathbf{v} \times \mathbf{B})] + \\ + \frac{1}{c^2}\boldsymbol{\nabla} \times [\boldsymbol{\nabla} \times (\mathbf{v} \times \mathbf{E})] = 0. \quad (27)$$

Since $\boldsymbol{\nabla}\times[\boldsymbol{\nabla}\times(\mathbf{v}\times\mathbf{E})] = -\boldsymbol{\nabla}^2(\mathbf{v}\times\mathbf{E})+\boldsymbol{\nabla}[\boldsymbol{\nabla}\,(\mathbf{v}\times\mathbf{E})]$, we have

$$\boldsymbol{\nabla}^2\mathbf{B} - \frac{1}{c^2}\frac{\partial^2\mathbf{B}}{\partial t^2} - \frac{1}{c^2}\frac{\partial}{\partial t}[\boldsymbol{\nabla} \times (\mathbf{v} \times \mathbf{B})] - \\ - \frac{1}{c^2}\boldsymbol{\nabla}^2(\mathbf{v} \times \mathbf{E}) + \frac{1}{c^2}\boldsymbol{\nabla}[\boldsymbol{\nabla}\,(\mathbf{v} \times \mathbf{E})] = 0. \quad (28)$$

Using (5), we get

$$\boldsymbol{\nabla}^2\mathbf{B} - \frac{1}{c^2}\frac{\partial^2\mathbf{B}}{\partial t^2} + \frac{1}{c^2}\frac{\partial}{\partial t}[(\mathbf{v}\,\boldsymbol{\nabla})\mathbf{B} - \boldsymbol{\Omega} \times \mathbf{B}] - \\ - \frac{1}{c^2}\boldsymbol{\nabla}^2(\mathbf{v} \times \mathbf{E}) + \frac{1}{c^2}\boldsymbol{\nabla}[\boldsymbol{\nabla}\,(\mathbf{v} \times \mathbf{E})] = 0. \quad (29)$$

Consider the term $\boldsymbol{\nabla}[\boldsymbol{\nabla}\,(\mathbf{v} \times \mathbf{E})]$ in (29). In accordance with (6), $\boldsymbol{\nabla}\,(\mathbf{v} \times \mathbf{E}) = 2\boldsymbol{\Omega}\,\mathbf{E} - \mathbf{v}\,(\boldsymbol{\nabla} \times \mathbf{E})$ or, with (1a), $\boldsymbol{\nabla}\,(\mathbf{v} \times \mathbf{E}) = 2\boldsymbol{\Omega}\,\mathbf{E} + (\partial/\partial t)(\mathbf{v}\,\mathbf{B})$. Then

$$\boldsymbol{\nabla}[\boldsymbol{\nabla}\,(\mathbf{v} \times \mathbf{E})] = 2\boldsymbol{\nabla}(\boldsymbol{\Omega}\,\mathbf{E}) + \frac{\partial}{\partial t}\boldsymbol{\nabla}(\mathbf{v}\,\mathbf{B}) \quad (30)$$

or, taking (7) into account,

$$\boldsymbol{\nabla}[\boldsymbol{\nabla}\,(\mathbf{v} \times \mathbf{E})] = 2\boldsymbol{\nabla}(\boldsymbol{\Omega}\,\mathbf{E}) + \frac{\partial}{\partial t}\Big[(\mathbf{v}\,\boldsymbol{\nabla})\mathbf{B} - \boldsymbol{\Omega} \times \mathbf{B}] + \\ + \frac{\partial}{\partial t}[\mathbf{v} \times (\boldsymbol{\nabla} \times \mathbf{B})\Big]. \quad (31)$$

With the help of (1c), we find

$$\boldsymbol{\nabla}[\boldsymbol{\nabla}\,(\mathbf{v} \times \mathbf{E})] = 2\boldsymbol{\nabla}(\boldsymbol{\Omega}\,\mathbf{E}) + \frac{\partial}{\partial t}[(\mathbf{v}\,\boldsymbol{\nabla})\mathbf{B} - \boldsymbol{\Omega} \times \mathbf{B}] + \\ + \mathbf{v} \times \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2}. \quad (32)$$

Substituting (32) into (29), we obtain

$$\boldsymbol{\nabla}^2\mathbf{B} - \frac{1}{c^2}\frac{\partial^2\mathbf{B}}{\partial t^2} + \frac{2}{c^2}\frac{\partial}{\partial t}[(\mathbf{v}\,\boldsymbol{\nabla})\mathbf{B} - \boldsymbol{\Omega} \times \mathbf{B}] + \\ + \frac{2}{c^2}\boldsymbol{\nabla}(\boldsymbol{\Omega}\,\mathbf{E}) - \frac{1}{c^2}\left[\boldsymbol{\nabla}^2(\mathbf{v} \times \mathbf{E}) - \mathbf{v} \times \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2}\right] = 0. \quad (33)$$

Consider the last term in (33). In accordance with (17), $\boldsymbol{\nabla}^2(\mathbf{v} \times \mathbf{E}) = \mathbf{v} \times (\boldsymbol{\nabla}^2\mathbf{E}) + 2\boldsymbol{\nabla}(\boldsymbol{\Omega}\,\mathbf{E})$. Hence,

$$\boldsymbol{\nabla}^2(\mathbf{v} \times \mathbf{E}) - \mathbf{v} \times \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} = \\ = \mathbf{v} \times \left(\boldsymbol{\nabla}^2\mathbf{E} - \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2}\right) + 2\boldsymbol{\nabla}(\boldsymbol{\Omega}\,\mathbf{E}). \quad (34)$$

It is clear that, to the first order in Ω,

$$\mathbf{v} \times \left(\boldsymbol{\nabla}^2\mathbf{E} - \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2}\right) = 0. \quad (35)$$

So, expression (34) may be rewritten as

$$\boldsymbol{\nabla}^2(\mathbf{v} \times \mathbf{E}) - \mathbf{v} \times \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} = 2\boldsymbol{\nabla}(\boldsymbol{\Omega}\,\mathbf{E}). \quad (36)$$

By inserting (36) into (33), we obtain the desired wave equation for the vector $\mathbf{B}$:

$$\boldsymbol{\nabla}^2\mathbf{B} - \frac{1}{c^2}\frac{\partial^2\mathbf{B}}{\partial t^2} + \frac{2}{c^2}\frac{\partial}{\partial t}[(\mathbf{v}\,\boldsymbol{\nabla})\mathbf{B} - \boldsymbol{\Omega} \times \mathbf{B}] = 0. \quad (37)$$

3.3. Result of Section 3

According to (25) and (37), the first system of wave equations for the electromagnetic field vectors $\mathbf{E}$ and $\mathbf{B}$ in a rotating frame of reference has the following form:

$$\begin{aligned} &\boldsymbol{\nabla}^2\mathbf{E} - \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} + \frac{2}{c^2}\frac{\partial}{\partial t}[(\mathbf{v}\,\boldsymbol{\nabla})\mathbf{E} - \boldsymbol{\Omega} \times \mathbf{E}] - \\ &- 2\boldsymbol{\nabla}(\boldsymbol{\Omega}\,\mathbf{B}) = 0, \\ &\boldsymbol{\nabla}^2\mathbf{B} - \frac{1}{c^2}\frac{\partial^2\mathbf{B}}{\partial t^2} + \frac{2}{c^2}\frac{\partial}{\partial t}[(\mathbf{v}\,\boldsymbol{\nabla})\mathbf{B} - \boldsymbol{\Omega} \times \mathbf{B}] = 0. \end{aligned} \quad (38)$$

4. Second System of Wave Equations for the Vectors E and B in a Rotating Frame of Reference

In this section, we are going to obtain the second system of wave equations for the electromagnetic field vectors $\mathbf{E}$ and $\mathbf{B}$, which will correspond to the system of Maxwell's equations (2). The first, second, third, and fourth equations of the system will be mentioned in the text as (2a), (2b), (2c), and (2d), respectively.

4.1. Equation for the vector E

To derive the wave equation for the vector $\mathbf{E}$, we apply the operator $\nabla\times$ to expression (2a):

$$\nabla\times(\nabla\times\mathbf{E})+\frac{\partial}{\partial t}(\nabla\times\mathbf{B})+ +\frac{1}{c^2}\frac{\partial}{\partial t}[\nabla\times(\mathbf{v}\times\mathbf{E})]=0. \tag{39}$$

Taking (2c) and (4) into consideration, we rewrite (39) in the form

$$\nabla^2\mathbf{E}-\frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2}+\frac{1}{c^2}\frac{\partial}{\partial t}\left(\mathbf{v}\times\frac{\partial\mathbf{B}}{\partial t}\right)- -\frac{1}{c^2}\frac{\partial}{\partial t}[\nabla\times(\mathbf{v}\times\mathbf{E})]-\nabla(\nabla\,\mathbf{E})=0 \tag{40}$$

or, using (2a),

$$\nabla^2\mathbf{E}-\frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2}-\frac{1}{c^2}\frac{\partial}{\partial t}[\nabla\times(\mathbf{v}\times\mathbf{E})+ +\mathbf{v}\times(\nabla\times\mathbf{E})]-\nabla(\nabla\,\mathbf{E})=0. \tag{41}$$

In view of (5), we have

$$\nabla^2\mathbf{E}-\frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2}+\frac{1}{c^2}\frac{\partial}{\partial t}[(\mathbf{v}\,\nabla)\mathbf{E}-\boldsymbol{\Omega}\times\mathbf{E}- -\mathbf{v}\times(\nabla\times\mathbf{E})]-\nabla(\nabla\,\mathbf{E})=0. \tag{42}$$

Consider the term $\nabla(\nabla\,\mathbf{E})$ in (42). According to (2d), $\nabla\,\mathbf{E}=\nabla\,(\mathbf{v}\times\mathbf{B})$, where, taking (6) into account,

$$\nabla\,(\mathbf{v}\times\mathbf{B})=2\boldsymbol{\Omega}\,\mathbf{B}-\mathbf{v}\,(\nabla\times\mathbf{B}). \tag{43}$$

With the help of (2c), we find $\nabla\,(\mathbf{v}\times\mathbf{B})=2\boldsymbol{\Omega}\,\mathbf{B}-(1/c^2)(\partial/\partial t)(\mathbf{v}\,\mathbf{E})$. Hence,

$$\nabla(\nabla\,\mathbf{E})=2\nabla(\boldsymbol{\Omega}\,\mathbf{B})-\frac{1}{c^2}\frac{\partial}{\partial t}\nabla(\mathbf{v}\,\mathbf{E}). \tag{44}$$

Consider the term $\nabla(\mathbf{v}\,\mathbf{E})$ in (44). In accordance with (7),

$$\nabla(\mathbf{v}\,\mathbf{E})=(\mathbf{v}\,\nabla)\mathbf{E}-\boldsymbol{\Omega}\times\mathbf{E}+\mathbf{v}\times(\nabla\times\mathbf{E}). \tag{45}$$

Then

$$\nabla(\nabla\,\mathbf{E})=2\nabla(\boldsymbol{\Omega}\,\mathbf{B})- -\frac{1}{c^2}\frac{\partial}{\partial t}[(\mathbf{v}\,\nabla)\mathbf{E}-\boldsymbol{\Omega}\times\mathbf{E}+\mathbf{v}\times(\nabla\times\mathbf{E})]. \tag{46}$$

Substituting (46) into (42), we obtain the desired wave equation for the vector $\mathbf{E}$:

$$\nabla^2\mathbf{E}-\frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2}+\frac{2}{c^2}\frac{\partial}{\partial t}[(\mathbf{v}\,\nabla)\mathbf{E}-\boldsymbol{\Omega}\times\mathbf{E}]- -2\nabla(\boldsymbol{\Omega}\,\mathbf{B})=0. \tag{47}$$

4.2. Equation for the vector B

To derive the wave equation for the vector $\mathbf{B}$, we apply the operator $\nabla\times$ to expression (2c):

$$\nabla\times(\nabla\times\mathbf{B})-\frac{1}{c^2}\frac{\partial}{\partial t}(\nabla\times\mathbf{E})+ +\frac{1}{c^2}\frac{\partial}{\partial t}[\nabla\times(\mathbf{v}\times\mathbf{B})]=0. \tag{48}$$

Taking (2a) and (4) into account, we rewrite (48) in the form

$$\nabla^2\mathbf{B}-\frac{1}{c^2}\frac{\partial^2\mathbf{B}}{\partial t^2}-\frac{1}{c^2}\frac{\partial}{\partial t}\left(\mathbf{v}\times\frac{1}{c^2}\frac{\partial\mathbf{E}}{\partial t}\right)- -\frac{1}{c^2}\frac{\partial}{\partial t}[\nabla\times(\mathbf{v}\times\mathbf{B})]-\nabla(\nabla\,\mathbf{B})=0 \tag{49}$$

or, using (2c),

$$\nabla^2\mathbf{B}-\frac{1}{c^2}\frac{\partial^2\mathbf{B}}{\partial t^2}-\frac{1}{c^2}\frac{\partial}{\partial t}[\nabla\times(\mathbf{v}\times\mathbf{B})+ +\mathbf{v}\times(\nabla\times\mathbf{B})]-\nabla(\nabla\,\mathbf{B})=0. \tag{50}$$

Consider the term $\nabla\times(\mathbf{v}\times\mathbf{B})$ in (50). According to (5), $\nabla\times(\mathbf{v}\times\mathbf{B})=-(\mathbf{v}\,\nabla)\mathbf{B}+\boldsymbol{\Omega}\times\mathbf{B}$. Thus,

$$\nabla^2\mathbf{B}-\frac{1}{c^2}\frac{\partial^2\mathbf{B}}{\partial t^2}+\frac{1}{c^2}\frac{\partial}{\partial t}[(\mathbf{v}\,\nabla)\mathbf{B}-\boldsymbol{\Omega}\times\mathbf{B}- -\mathbf{v}\times(\nabla\times\mathbf{B})]-\nabla(\nabla\,\mathbf{B})=0. \tag{51}$$

Consider the term $\nabla(\nabla\,\mathbf{B})$ in (51). In accordance with (2b),

$$\nabla\,\mathbf{B}=-(1/c^2)\nabla\,(\mathbf{v}\times\mathbf{E}). \tag{52}$$

Taking (6) into account, we have

$$\nabla\,(\mathbf{v}\times\mathbf{E})=2\boldsymbol{\Omega}\,\mathbf{E}-\mathbf{v}\,(\nabla\times\mathbf{E}) \tag{53}$$

or, using (2a),

$$\nabla\,(\mathbf{v}\times\mathbf{E})=2\boldsymbol{\Omega}\,\mathbf{E}+\frac{\partial}{\partial t}(\mathbf{v}\,\mathbf{B}). \tag{54}$$

So,

$$\nabla\,\mathbf{B}=-\frac{2}{c^2}\boldsymbol{\Omega}\,\mathbf{E}-\frac{1}{c^2}\frac{\partial}{\partial t}(\mathbf{v}\,\mathbf{B}). \tag{55}$$

Hence,

$$\nabla(\nabla\,\mathbf{B})=-\frac{2}{c^2}\nabla(\boldsymbol{\Omega}\,\mathbf{E})-\frac{1}{c^2}\frac{\partial}{\partial t}\nabla(\mathbf{v}\,\mathbf{B}). \tag{56}$$

Consider the term $\nabla(\mathbf{v}\,\mathbf{B})$ in (56). In accordance with (7),

$$\nabla(\mathbf{v}\,\mathbf{B}) = (\mathbf{v}\,\nabla)\mathbf{B} - \boldsymbol{\Omega}\times\mathbf{B} + \mathbf{v}\times(\nabla\times\mathbf{B}). \quad (57)$$

Then

$$\nabla(\nabla\,\mathbf{B}) = -\frac{2}{c^2}\nabla(\boldsymbol{\Omega}\,\mathbf{E}) - \\ -\frac{1}{c^2}\frac{\partial}{\partial t}[(\mathbf{v}\,\nabla)\mathbf{B} - \boldsymbol{\Omega}\times\mathbf{B} + \mathbf{v}\times(\nabla\times\mathbf{B})]. \quad (58)$$

Substituting (58) into (51), we obtain the desired wave equation for the vector **B**:

$$\nabla^2\mathbf{B} - \frac{1}{c^2}\frac{\partial^2\mathbf{B}}{\partial t^2} + \frac{2}{c^2}\frac{\partial}{\partial t}[(\mathbf{v}\,\nabla)\mathbf{B} - \boldsymbol{\Omega}\times\mathbf{B}] + \\ +\frac{2}{c^2}\nabla(\boldsymbol{\Omega}\,\mathbf{E}) = 0. \quad (59)$$

4.3. Result of Section 4

According to (47) and (59), the second system of wave equations for the electromagnetic field vectors **E** and **B** in a rotating frame of reference has the following form:

$$\begin{aligned}&\nabla^2\mathbf{E} - \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} + \frac{2}{c^2}\frac{\partial}{\partial t}[(\mathbf{v}\,\nabla)\mathbf{E} - \boldsymbol{\Omega}\times\mathbf{E}] - \\ &-2\nabla(\boldsymbol{\Omega}\,\mathbf{B}) = 0, \\ &\nabla^2\mathbf{B} - \frac{1}{c^2}\frac{\partial^2\mathbf{B}}{\partial t^2} + \frac{2}{c^2}\frac{\partial}{\partial t}[(\mathbf{v}\,\nabla)\mathbf{B} - \\ &-\boldsymbol{\Omega}\times\mathbf{B}] + \frac{2}{c^2}\nabla(\boldsymbol{\Omega}\,\mathbf{E}) = 0.\end{aligned} \quad (60)$$

5. Two Systems of Wave Equations for the Vectors E and B in a Rotating Frame of Reference: a Comparative Analysis

The first system of wave equations for the electromagnetic field vectors **E** and **B** [which was derived from system of Maxwell's equations (1)] has the form

$$\begin{aligned}&\nabla^2\mathbf{E} - \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} + \frac{2}{c^2}\frac{\partial}{\partial t}[(\mathbf{v}\,\nabla)\mathbf{E} - \boldsymbol{\Omega}\times\mathbf{E}] - \\ &-2\nabla(\boldsymbol{\Omega}\,\mathbf{B}) = 0, \\ &\nabla^2\mathbf{B} - \frac{1}{c^2}\frac{\partial^2\mathbf{B}}{\partial t^2} + \frac{2}{c^2}\frac{\partial}{\partial t}[(\mathbf{v}\,\nabla)\mathbf{B} - \boldsymbol{\Omega}\times\mathbf{B}] = 0.\end{aligned} \quad (61)$$

The second system of wave equations [which was obtained from system of Maxwell's equations (2)] is

$$\begin{aligned}&\nabla^2\mathbf{E} - \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} + \frac{2}{c^2}\frac{\partial}{\partial t}[(\mathbf{v}\,\nabla)\mathbf{E} - \boldsymbol{\Omega}\times\mathbf{E}] - \\ &-2\nabla(\boldsymbol{\Omega}\,\mathbf{B}) = 0, \\ &\nabla^2\mathbf{B} - \frac{1}{c^2}\frac{\partial^2\mathbf{B}}{\partial t^2} + \frac{2}{c^2}\frac{\partial}{\partial t}[(\mathbf{v}\,\nabla)\mathbf{B} - \boldsymbol{\Omega}\times\mathbf{B}] + \\ &+\frac{2}{c^2}\nabla(\boldsymbol{\Omega}\,\mathbf{E}) = 0.\end{aligned} \quad (62)$$

Expressions (61) and (62) represent the two different systems of wave equations for the vectors **E** and **B** in a uniformly rotating frame of reference. From the analysis of these systems, it follows:

1) the factor of rotation causes [via the quantities $\boldsymbol{\Omega}\times\mathbf{E}$, $\boldsymbol{\Omega}\times\mathbf{B}$, $\nabla(\boldsymbol{\Omega}\,\mathbf{E})$, $\nabla(\boldsymbol{\Omega}\,\mathbf{B})$] the arising of the longitudinal **E**- and **B**-components of electromagnetic waves, which interact with the transversal ones (this agrees with the statement of work [6]);

2) the wave equations for the vector **E** in both systems have the same form (this confirms the statement of work [3]), while the equations for the vector **B** have different form;

3) the structure of equations for the vectors **E** and **B** in the first system is asymmetric (with respect to Ω). Therefore, the propagation of the **E**- and **B**-components of electromagnetic waves in a rotating frame of reference will be governed by qualitatively different laws;

4) the structure of the wave equations in the second system is symmetric. Hence, the propagation of the indicated field components will be governed by similar laws.

The systems of wave equations (61) and (62) may serve as a theoretical basis for the detailed study of the propagation of electromagnetic waves in a rotating frame of reference. But, before the beginning of such study, the researcher must first solve the problem of choosing between the mentioned systems (because the final results will be different).

6. Simplified Wave Equations for the Vectors E and B in a Rotating Frame of Reference

As we can see from two above systems of wave equations (61) and (62), the factor of rotation causes the arising of the longitudinal **E**- and **B**-components of electromagnetic waves, which interact with the transversal ones. To find the analytic solutions of such systems of equations is a difficult task (see, e.g., calculations in work [6]). But if it is acceptable to ignore these longitudinal components to make the wave equations more simple for analysis, then the quantities $\boldsymbol{\Omega}\times\mathbf{E}$, $\boldsymbol{\Omega}\times\mathbf{B}$, $\nabla(\boldsymbol{\Omega}\,\mathbf{E})$, $\nabla(\boldsymbol{\Omega}\,\mathbf{B})$ in (61) and (62) may be dropped. As a result, both systems of wave equations are transformed into two identical sets of separate (independent of one another) wave equations for the vector **E** and the vector **B** of a simplified form:

$$\begin{aligned}&\nabla^2\mathbf{E}-\frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2}+\frac{2}{c^2}\frac{\partial}{\partial t}[(\mathbf{v}\,\nabla)\mathbf{E}]=0,\\&\nabla^2\mathbf{B}-\frac{1}{c^2}\frac{\partial^2\mathbf{B}}{\partial t^2}+\frac{2}{c^2}\frac{\partial}{\partial t}[(\mathbf{v}\,\nabla)\mathbf{B}]=0.\end{aligned}\qquad(63)$$

The equation for the vector **E** in (63) and its analytic solution are well known in the theory of ring laser gyro (see, e.g., works [7, 8]). It was obtained in the mentioned works by approximate methods (some terms have been neglected in the process of calculation) on the base of the system of Maxwell's equations (1).

From the analysis of the simplified wave equations (63), it follows that the propagation of the **E**- and **B**-components of electromagnetic waves in a rotating frame of reference will be governed by identical laws.

Thus, in the approximation of transversal electromagnetic waves, the distinction between two above systems of wave equations (61) and (62) vanishes: both of them take the form (63).

7. Conclusions

On the base of two systems of Maxwell's equations (1) and (2) for the electromagnetic field vectors **E** and **B** in a uniformly rotating frame of reference, which were first proposed in works [1] and [2], respectively, two corresponding systems of wave equations (61) and (62) have been derived (to the first order in Ω). From the analysis of these systems, it follows:

1) the factor of rotation causes the arising of the longitudinal **E**- and **B**-components of electromagnetic waves, which interact with the transversal ones;

2) the structure of the wave equations for the vectors **E** and **B** in system (61) is asymmetric. Therefore, the propagation of the **E**- and **B**-components of electromagnetic waves in a rotating frame of reference will be governed by qualitatively different laws;

3) the structure of the wave equations for the vectors **E** and **B** in system (62) is symmetric. Hence, the propagation of the indicated field components will be governed by similar laws.

It is also shown that, in the approximation of transversal electromagnetic waves, the distinction between two systems of wave equations (61) and (62) vanishes: both are transformed into the two identical sets of separate (independent of one another) wave equations for the vector **E** and vector **B** of the simpler (and already known from the literature) form (63).

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ДВІ СИСТЕМИ РІВНЯНЬ МАКСВЕЛА ТА ДВІ ВІДПОВІДНІ СИСТЕМИ ХВИЛЬОВИХ РІВНЯНЬ ДЛЯ ВЕКТОРІВ ЕЛЕКТРОМАГНІТНОГО ПОЛЯ **E** І **B** В ОБЕРТОВІЙ СИСТЕМІ ВІДЛІКУ: ЛІНІЙНЕ НАБЛИЖЕННЯ

Р е з ю м е

На основі двох систем рівнянь Максвела для векторів електромагнітного поля **E** і **B** в рівномірно обертовій системі відліку, що були вперше запропоновані в працях [L.I. Schiff, Proc. Natl. Acad. Sci. USA **25**, 391 (1939)] та [W. Irvine, Physica **30**, 1160 (1964)], отримано дві відповідні системи хвильових рівнянь (в першому порядку по Ω). З аналізу цих систем випливає: 1) фактор обертання зумовлює виникнення поздовжніх **E**- і **B**-компонентів електромагнітних хвиль, що взаємодіють з їх поперечними компонентами; 2) хвильові рівняння для вектора **E** для обох систем рівнянь мають однаковий вигляд, тоді як рівняння для вектора **B** мають різний вигляд; 3) структура хвильових рівнянь для векторів **E** і **B** в першій системі є асиметричною. Через це поширення **E**- і **B**-компонентів електромагнітних хвиль в обертовій системі відліку буде підпорядковуватись якісно різним законам; 4) структура хвильових рівнянь в другому випадку є симетричною. Тому поширення вказаних компонентів хвиль буде підпорядковуватись схожим законам. У статті також показано, що у наближенні поперечних електромагнітних хвиль – відмінність між двома системами хвильових рівнянь для векторів **E** і **B** зникає: обидві системи перетворюються на два ідентичних набори окремих (незалежних одне від одного) хвильових рівнянь для вектора **E** та вектора **B** більш простої (і вже відомої з літератури) форми.