SOFT MATTER

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INFLUENCE OF THE THERMODYNAMIC PROPERTIES OF A SPRAYED LIQUID ON THE DROPLET-AIR MIXTURE PARAMETERS IN THE FRAMEWORK OF THE MAXIMUM ENTROPY MODEL

The influence of the thermodynamic properties of a sprayed liquid on the size and velocity distribution functions for droplets in the droplet-air mixture has been analyzed, by using the maximum entropy method. The results obtained numerically are compared with the experimental data concerning the spraying of diesel fuel and its mixtures with rapeseed oil. The thermodynamic properties of a sprayed liquid are found to affect the final state of the droplet-air system.

K e y w o r d s: maximum entropy method, droplet-air and disperse systems, droplet distribution functions, biofuel.

1. Introduction

Quantitative information concerning the size and velocity distributions of droplets at their motion in a sprayed liquid comprises a required set of physical parameters that makes it possible to estimate the fundamental properties of the whole droplet-air system. The data of this kind are needed both at the purposeful design of technical devices for the generation of droplet-air systems with given parameters and while searching for optimal conditions of their functioning. It is natural that those data also allow one to seek for effective application regimes for fuels with various compositions.

Difficulties arising, while the direct experiments aimed at the determination of the state parameters of sprayed liquids are carried out, are well known. First of all, they are associated with difficulties of applying the straightforward measurement procedures, because the price of a corresponding precision equipment is very high. In this situation, various theoreti-

cal models come to the foreground, which allows the stated problems to be resolved to some extent at the theoretical level.

Among a considerable body of publications that describe processes occurring in droplet-air mixtures, two groups of works can be clearly distinguished. One group deals with the development and solution of the most general kinetic models describing the dynamics of evolution in the droplet-air systems. Among the recent works that are the most interesting from the viewpoint of their application to the solution of the problem devoted to the creation of droplet-air mixtures with prescribed properties on the basis of liquids of various kinds, we would like to mark, e.g., works [1–3]. The cited authors considered the behavior of the solutions of a system of differential kinetic equations describing the natural conservation laws that are satisfied under the conditions of sprayed fuel formation. This approach allowed them to obtain data on the size and velocity distributions of droplets, as well as their time evolution, i.e. practically the exhaustive information about the state of the system.

At the same time, the full solution of such problems requires that rather a sophisticated mathematical apparatus and the corresponding software should be used. As a rule, this gives rise to significant expenses of the computational time in order to obtain results. Furthermore, the implementation of the models of this kind includes the determination of a solution of a multiparametric problem, in which not every initial thermodynamic characteristic of the system is known with a required accuracy. Among the shortcomings of such models, we should point to a large number of initial parameters and the uncertainty of their initial values.

The problem concerning the thermodynamic description of a state created by sprayed fuel droplets can be solved in the framework of simpler equilibrium models of their formation [4–6]. The application of the equilibrium approximation to the estimation of a droplet-air system state is also favored by the fact that its usage takes advantage of a mathematical model with a smaller number of parameters in comparison with its kinetic analogs. A smaller number of input model parameters should facilitate the interpretation of the data obtained and reveal the main thermodynamic parameters of fuel that determine the final state of a system of droplets.

The indicated equilibrium models make it possible to draw the following conclusion, which is important for practice. Since the basic tenets of this approach do not require one to possess information on how the engineering system creates the droplet-air mixture, the obtained results can be regarded as a probable theoretical limit for the formation of droplet system properties. In this case, the final state parameters depend only on the thermodynamic properties of interacting media. In essence, this is a limit, which we should aim at when developing engineering systems for the generation of fuel-air mixtures with prescribed parameters.

Equilibrium models describing the formation of a droplet-air mixture include, first of all, the models that are based on the maximum entropy principle [4–6]. The latter is based on the ideas of the second law of thermodynamics; i.e. during its evolution, the system of sprayed liquid droplets should develop in the direction that provides its largest entropy.

This is the condition of maximum entropy for the droplet-air system that allows conditions for the determination of the distribution function for sprayed liquid droplets to be formulated. The role of restrictions that provide the existence of a local maximum for the droplet system entropy is played by conditions that follow from the conservation laws of mass, momentum, and energy. In other words, the maximum entropy model is based on the search for the distributions of droplet sizes and velocities that would satisfy the conservation laws, whereas the entropy of the system reaches its maximum. Therefore, the aim of this work consisted in analyzing how the thermodynamic fuel parameters affect the size and motion parameters of droplets in fuel-air mixtures in the framework of the maximum entropy model.

2. Maximum Entropy Method

As a model of spray, let us consider a set of droplets, each of them being characterized by a specific diameter D and a specific motion velocity u. To describe the state of this system, we can use the differential distribution function over the droplet sizes and velocities, $f(D, u)$, which was determined with the help of the maximum entropy method.

Recall that, according to the main principle of the method, a state is realized of all probable spray states, in which the entropy of the droplet-air mixture reaches its maximum. Therefore, the process of droplet formation, which is described by a variation of the distribution function in the given space volume, is considered as a transition from one state into another final one, which corresponds to a new equilibrium state. According to the thermodynamic laws, the mass, momentum, and energy have to obey the conservation laws during the transition between those states, whereas the entropy of the whole system after its spraying has to reach a maximum. It is evident that the state of a liquid at the injector output is selected as the initial one, and the state obtained, when the ultimately formed droplets of a sprayed liquid fill the entire volume is taken as the final one.

The equations that describe the transition between the indicated states of the system were considered in work [5] in more details. In particular, the account for the main processes taking place during the droplet system formation gives rise to a system of integral equations for the distribution function parameters. For the problems that are dealt in this work, it is enough to consider the motion of the droplet system along a single coordinate axis only. Then, in the

framework of the chosen approximation, the conservation laws of mass, momentum, and energy can be written in terms of the droplet distribution function over the sizes and velocities in the following dimensionless form [5, 6]:

$$\iint f \bar{D}^3 d\bar{u} d\bar{D} = 1 + \bar{G}_m, \tag{1}$$

$$\iint f \bar{D}^3 \bar{u} d\bar{u} d\bar{D} = 1 + \bar{G}_{mv}, \tag{2}$$

$$\iint f \left(\bar{D}^3 \bar{u}^2 + B \bar{D}^2 \right) d\bar{u} d\bar{D} = 1 + \bar{G}_e, \tag{3}$$

$$\iint f d\bar{u} d\bar{D} = 1, \tag{4}$$

where $\bar{G}_m = G_m/m_0$, $\bar{G}_{mv} = G_{mv}/J_0$, $\bar{G}_e = G_e/E_0$, $E_0 = m_0 U_0^2$, $\bar{V} = V/V_m$, $\bar{u} = u/U_0$, $V_m = m_0/\rho n$, $We = \rho U_0^2 D_{30}/\sigma$, $B = 12/We$, and We is the Weber number for the droplet system. The injector operating conditions taking place during the air-droplet mixture formation determine the rate of liquid droplet formation $\dot{n}$, as well as the rates of mass, $\dot{m}_0$, momentum, $\dot{J}_0$, and energy, $\dot{E}_0$, supplies into the system. The parameters $\bar{G}_m$, $\bar{G}_{mu}$, and $\bar{G}_e$ are the powers of mass, momentum, and energy, respectively, sources in the sprayed fuel; σ is the surface tension of the liquid and ρ its density; $\bar{V}$ and $\bar{u}$ are the dimensionless volume and velocity, respectively, of a droplet obtained by normalizing the corresponding values to the average droplet volume V_m and the average liquid injection rate U_0, respectively. If we adopt that all droplets have a spherical shape, the average droplet volume V_m can be related to the average droplet diameter by the formula $V_m = \frac{\pi}{6} D_{30}^3$. In this formula, the diameter D_{30} averaged over the droplet volume and introduced by formulas from work [5] is introduced. Then, the corresponding dimensionless droplet diameter equals $\bar{D} = D/D_{30}$. The ultimate mathematical formulation of the problem includes the normalization condition for the probability density distribution, Eq. (4). In the presented expressions, the integration is carried out over the whole droplet volume and the whole interval of the droplet motion velocity.

Equation (2) corresponds to the conservation law of momentum during the droplet system formation and is expressed in the scalar form. The latter corresponds to the one-dimensional character of droplet motion indicated above, when all droplets move along the same axis and do not form a spray cone. This assumption confines the body of initial information about the droplet state, but the physical essence of the model allows us to hope for that the quantitative data obtained in its framework will adequately reflect the main regularities of the influence of sprayed liquid properties on the final state of a droplet-air mixture.

Equation (3) takes into account that, besides the kinetic energy, the droplet also has a surface energy required for its formation. This energy component, in accordance with work [6], is calculated as the direct product of the surface area of the droplet with the current diameter and the surface tension coefficient of the liquid.

The representation of model equations [5, 6] in terms of dimensionless quantities is useful not only for the development of a mathematical apparatus for the solution of the problem, but it also allows a number of physical features to be revealed in the behavior of key parameters of the droplet-air system. This analysis will be performed on the basis of calculations presented below.

According to the maximum entropy method, the entropy S of the sprayed droplet system in its final state should reach the maximum value [5, 6]. According to classical interrelations between the entropy and the probability density for the filling of possible states in the system [5, 6], we may write down that

$$S = -k_{\mathrm{B}} \iint f \ln f d\bar{V} d\bar{u}, \tag{5}$$

where k_{B} is the Boltzmann constant.

The system of equations forms a mathematical problem for the search for the entropy maximum for a system of droplets under the restricting conditions (1)–(4). As a rule, such a classical problem of finding the function extremum is solved, by using the method of Lagrange multipliers. The application of this method to the system of equations (1)–(4) was carried out in works [5, 6] devoted to the development of the maximum entropy method. In the cited works, it was shown that, in the continuous approximation used for the variations of droplet volumes and motion velocities, the specific probability distribution function can be written in the form

$$f \left(\bar{D}, \bar{u} \right) = 3 \bar{D}^2 \exp \big[-\alpha_0 - \alpha_1 \bar{D}^3 - \\ - \alpha_2 \bar{D}^3 \bar{u} - \alpha_3 \left(\bar{D}^3 \bar{u}^2 + B \bar{D}^2 \right) \big], \tag{6}$$

where α_i are Lagrange multipliers ($i = 0, 1, 2, 3$).

Hence, ultimately, the mathematical model for the evolution of a system of moving droplets is described by the system of equations (1)–(4), where the function $f(D, u)$ is defined by expression (6) and satisfies the principle of maximum entropy (5). Thus, the solution of the problem formulated in this work is reduced to the analysis of the solutions of the system of integral algebraic equations (1)–(6). The input data for them include the sets of thermodynamic parameters for liquids with various compositions and the operating conditions for the spraying system.

3. Specific Features of Model Equations. Thermodynamic Parameters of Liquids

Before proceeding to the solution of the obtained equations, it is necessary to analyze some of their features and substantiate the input thermodynamic parameters of the fuel, whose spraying is modeled. Really, the application of our model to the description of real processes that take place at the fuel spraying makes it possible to substantiate the following assumptions and to perform certain simplifications in the initial equations.

It is quite reasonable to neglect variations in the liquid mass at the liquid spraying, when the latter is associated, e.g., with the liquid evaporation (with or without a change in the charge states of ions) or liquid distillation. In this case, we may assume with a high probability that the component $\bar{G}_m = 0$ in Eq. (1). A similar situation takes place for the functioning of additional energy sources in the sprayed liquid. It is hard to imagine that a considerable amount of energy can be released during a real process of fuel spraying in addition to that taken into account in Eq. (3). Recall that those equations make allowance for the kinetic energy of droplet motion and the surface energy of droplet formation.

As for a variation in the momentum of the system in the course of droplet motion within the given volume, the losses of the momentum should certainly be observed, at least owing to the presence of the forces of resistance to the motion of droplets in air. Those momentum losses have to be taken into consideration, when calculating the final state of a sprayed liquid in the general case.

The same pattern is observed, when we examine the energy balance for droplets that move in the air environment. The friction forces arising at that and the work that they do will inevitably result in a reduction of the total energy of the system.

In view of the complex character of the problem dealing with the calculation of momentum and energy losses in the system, it is no wonder that there are a number of mathematical expressions allowing those losses at various stages of the liquid jet destruction and droplet formation processes to be estimated [5, 6]. At the same time, one can see from expressions (1)–(3) that, at the quantitative level in the framework of this model, the mechanism of liquid jet destruction is not so important as the energy and momentum fractions that are lost in the course of this process. This situation is associated with the application of the following approach to the solution of the problem devoted to the analysis of the influence of fuel parameters on the final state of the droplet system.

In the framework of this approach, the droplet distribution was modeled for various fractions of scattered momentum and various fractions of energy that was spent to do work against the friction forces. For those data to be obtained, various values of the parameters $\bar{G}_{mv}$ and $\bar{G}_e$, which involve the work done by the momentum and energy sinks, respectively, played the role of input parameters for Eqs. (1)–(6). In the framework of the selected model, the indicated parameters were considered as fitting ones, and their values were determined from the best correspondence to the experiment.

At the same time, the values, around which the parameters were varied to achieve the best correspondence with experimental data, were maintained to be located in the intervals that were equal or close by magnitude to the values found in work [6], in which this model was applied to describe the process of water spraying. According to the cited work, the momentum fraction scattered, when the friction forces are overcome at the transition from the initial state into the state of isoentropic dynamic process characterized by the largest possible entropy amounted to $\bar{G}_{mv} = -0.0175$.

Concerning the energy losses for the work of friction forces done during the same time interval of the liquid jet destruction, the following approach was used in order to reduce the number of model fitting parameters and simplify the interpretation of the results obtained. The fraction of energy losses spent to do work against the friction forces at the motion of droplets

Thermodynamic parameters of liquids and conditions of their spraying

Fuel	Density, kg/m^3	Surface tension, N/m	Average liquid velocity, m/s	D_{30}, μm	Sources	Weber number
Diesel	837.0	0.0298	43.7	24*	[8, 9]	1280
Biodiesel	883.0	0.0332	28.3	28*	[8, 9]	610
Rapeseed oil	917.0	0.0330	–	–	[8]	–
Water	998.2	0.0736	40.8	13.7	[6, 7]	311

* Recalculated from the data of work [9] using formula (7).

in air and the internal friction forces arising at the motion and the destruction of a liquid jet was taken equal to the corresponding value of the scattered momentum fraction. In other words, we assumed that $\bar{G}_{mv} = \bar{G}_e$ in the majority of performed calculations.

A specific feature of system (1)–(5) is the fact that, besides the parameters that characterize the momentum and energy losses in the system and which were considered above, the only parameter governing the shape of the sought distribution function is the Weber number We. The thermodynamic parameters needed for its calculation are quoted in Table. The calculations of the Weber number on the basis of the presented data demonstrate that, provided a constant average liquid velocity at the output of the spraying injector, the required values fall within an interval from 220 to 500.

There is a possibility to vary them by changing the pressure, at which the fuel is injected. The growth of the pressure in the system increases the average velocity of liquid jet motion at the initial stage of spraying. At the same time, it should be noted that, under almost all important conditions required for the formation of fuel droplet-air mixtures with various compositions, the magnitudes of the Weber number do not go beyond the limits of the 220 to 1500 interval.

4. Mathematical Apparatus of the Problem Solution

In order to find the distribution of droplets over their sizes and velocities in the droplet-air system on the basis of the maximum entropy principle, it is necessary to maximize function (5), in which the differential distribution function f looks like expression (6) with the Lagrange multipliers α_i that satisfy a certain system of equations. The solution of this system

is a difficult task [5, 6], because the corresponding Jacobi matrix is ill-conditioned at the initial stage of the iterative process, which is connected with a considerable aprioristic uncertainty of the initial values chosen for sought variables.

In order to overcome the indicated difficulty, we used a little modified approach to the solution of the system of equations, which is based on the Newton method (features of the corresponding iterative procedure were discussed in work [6]). Here, we used the possibilities to analytically integrate the distribution function f and to confine the integration limits to finite values. This can be done due to a rapid recession of subintegral exponential functions, with the resulting calculation errors being acceptable.

In the course of numerical simulation, we evaluated the calculation error inserted by the accepted assumptions to the final result. The results of calculations testified that the corrections of integration limits gave rise to a final error less than 0.2%.

At the same time, our experience of exploiting the developed algorithm showed that the integration limits $\bar{D}, \bar{U} \in (0, 3.5)$, which were proposed in work [6], do not always provide the stable work of the computation program. This circumstance manifested itself in the appearance of solutions that did not correspond to the physical sense of the problem or gave rise to the divergence of the iteration process. In those cases, the convergence of the algorithm was managed to be restored, by expanding the integration interval to $\bar{D}, \bar{U} \in (0, 8)$.

The applicability of the developed mathematical apparatus and the corresponding software was proved by confirming the results of numerical calculations obtained in work [6], when simulating the process of water spraying in the framework of the maximum entropy approach. The test result obtained for the case $We = 311$ (see Table) is depicted in Fig. 1 (curve *3*)
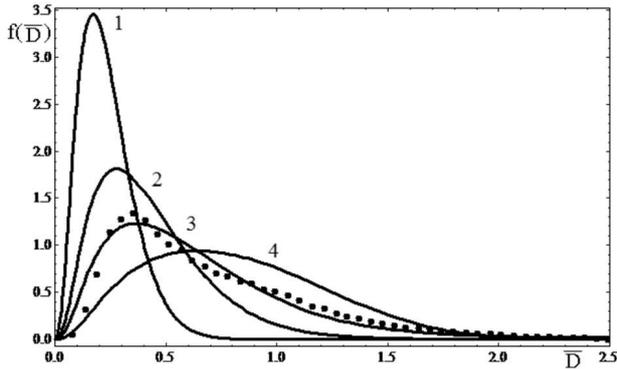
Fig. 1. Densities of droplet diameter distributions at the liquid spraying for various Weber numbers $We = 150$ (*1*), 220 (*2*), 311 (*3*), and 1000 (*4*). Experimental data for the water spraying (•) were taken from work [6]

together with the results of our calculations and experimental data taken from work [6]. The Lagrange multipliers obtained at those test calculations also coincide with the data of work [6].

The result obtained confirms the applicability of the developed software programs for the simulation of the influence of the thermodynamic properties on the final state of a fuel droplet-air mixture.

5. Results and Their Discussion

The main simulation results obtained for the processes of fuel spraying are exhibited in Figs. 1 to 3. In particular, Fig. 1 illustrates the results of calculations for the dependence of the probability density to detect droplets with a given size on the droplet normalized diameter carried out for various Weber number values. The calculations were performed in the case where the fraction of momentum losses needed to overcome friction forces at the liquid spraying amounts to $\bar{G}_{mv} = -0.0175$, whereas the energy losses equal zero, $\bar{G}_e = 0$. Just this evaluation of the indicated parameters was made in work [6] in order to describe the formation of a water droplet-air mixture under various conditions. From Fig. 1, it follows that the process of fuel spraying under conditions that provide the growth of the Weber number from 100 to 1000 brings about a displacement of the analyzed distributions toward the region, where the size of formed droplets is larger.

By considering the expression determining the Weber number, we may assert that, provided the given thermodynamic parameters of the fuel (see Table),
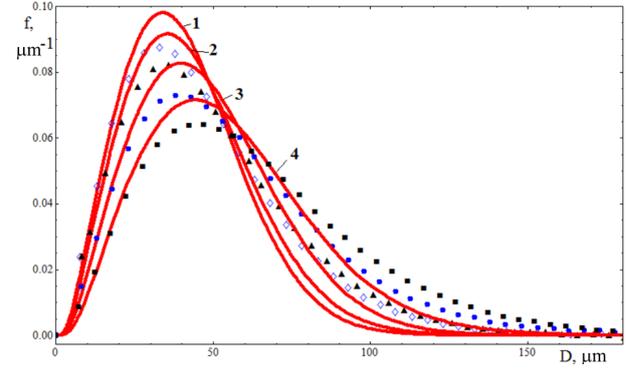

Fig. 2. Densities of droplet diameter distributions for spraying fuels with various compositions: (1,•) diesel fuel, $We = 900$, $\bar{G}_{mv} = \bar{G}_e = -0.017$; (2,◊) 20% rapeseed oil + 80% diesel fuel, $We = 825$, $\bar{G}_{mv} = \bar{G}_e = -0.018$; (3,▲) 30% rapeseed oil + 70% diesel fuel, $We = 750$, $\bar{G}_{mv} = \bar{G}_e = -0.020$; and (4,■) 40% rapeseed oil + 60% diesel fuel, $We = 450$, $We = 450$, $\bar{G}_{mv} = \bar{G}_e = -0.032$. Symbols correspond to experimental data taken from work [9]
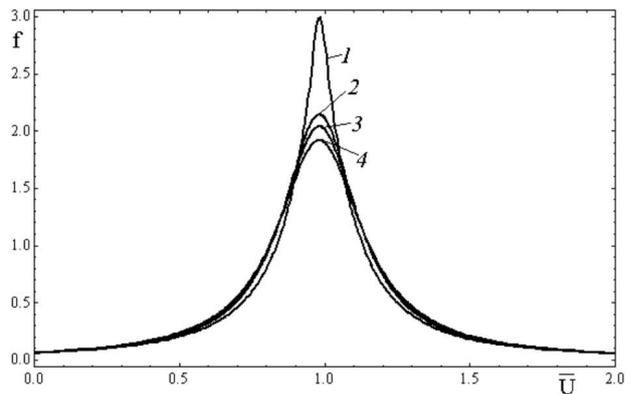

Fig. 3. Densities of normalized droplet velocity distributions at the relative momentum, $\bar{G}_{mv} = -0.017$, and energy, $\bar{G}_e = 0$, losses in the system for various Weber numbers $We = 311$ (*1*), 811 (*2*), 1011 (*3*), and 1511 (*4*)

the geometrical parameters of droplets in the sprayed state can be varied by changing the average spraying rate; namely, we can form a required distribution of fuel droplets over their sizes. Note that this phenomenon should reveal itself for any fuel with any parameters and for various values of mean droplet diameter D_{30}. Recall that the experimental results for the parameter D_{30} are different depending on both the spraying conditions and the fuel type (Table).

The results of calculations in the case $We > 1000$ testify that the peculiarities in the behavior of the

droplet distribution function over the droplet size are not less informative. In particular, it turned out that the growth of the Weber number above 1000 practically does not affect the shape of the distribution function over droplet sizes. In practice, this means that the growth of the average liquid velocity at the injector output influences the geometrical parameters of the droplet system only up to a certain limit (according to Fig. 1, at $We < 800$), so that our hopes for a substantial variation in droplet sizes by increasing the fuel spraying rate in the interval $We > 1000$ may fail.

From Fig. 1, it follows that the location of the distribution function maximum depends considerably on the Weber number value. In particular, the distribution maximum shifts toward larger droplet diameters, as the Weber number grows. Note that the parameter $\bar{D}_{30}$ cannot be used to describe this effect, because $\bar{D}_{30} = 1$ in all cases. Therefore, we may assert that a variation in the Weber number changes the shape of the distribution curve, whereas the third momentum ($\bar{D}_{30}$ is a parameter determined from the calculation of the volumetric droplet characteristics [5]) remains constant for the distributions. Furthermore, at $We > 1500$, the dependence of the most probable droplet diameter on the Weber number becomes rather weak, which reflects a tendency for the shape of the distribution function plot to be less sensitive to the We magnitude.

The results of direct calculations of fuel parameters (Table) testify that, under typical spraying conditions, the Weber number falls within the interval from 220 to 500 for various fuels from classical diesel to promising biodiesel ones. Nevertheless, this interval of the We number variation turns out too narrow for the behavior of the distribution curves to change substantially. At the same time, the influence of thermodynamic fuel properties on the droplet parameters in the mixture can be observed in Fig. 1 rather pronouncedly.

Figure 1 demonstrates the growth of the most probable dimensionless droplet diameter from 0.25 to 0.5 if the classical diesel fuel is changed to the biodiesel one. Provided that the biodiesel fuel is sprayed in the same engineering system, the average droplet size is known [9] to exceed the size obtained, if the classical diesel fuel is sprayed. In this case, the obtained theoretical result can be considered as such that found its experimental confirmation.

The most sound evidence that the maximum entropy method can be applied to describe the spraying of fuels with various chemical compositions can be obtained by studying the experimental data reported in work [9]. The principal specific feature of those data consists in that they were obtained for fuel mixtures with various compositions, but following the same procedure and on the same equipment. Those experimental results for the droplet size distribution obtained in the cases where the mixtures of diesel fuel and rapeseed oil taken in various ratios were sprayed, as well as the calculation data, are exhibited in Fig. 2.

Before proceeding to the discussion of the results illustrated in this figure, we should consider the features of the procedure used to obtain the presented theoretical dependences. Really, in the experiments of work [9], the volumetric-surface droplet diameter D_{32} was measured, whereas the corresponding distribution functions were calculated, by taking the diameter D_{30} as an independent variable ($D_{30} = 1 \div 4$). The integral formula to recalculate the diameter D_{30} into D_{32} is well known [5] and looks like

$$\bar{D}_{32} = \bar{D}_{30} \int\limits_{0}^{\infty} \bar{D} f(\bar{D}) d\bar{D}. \tag{7}$$

From whence, it follows that the integral value, i.e. the coefficient of recalculation between differently averaged diameters depends on the distribution function together with its parameters. At the same time, the calculations carried out in the framework of the maximum entropy method provide the exhaustive quantitative information about those distributions.

In this work, the required integration in Eq. (7) was carried out numerically. The results of calculations showed that if expression (6) was chosen for the distribution function, the recalculation coefficient turned out almost constant and equal to about 1.19 in the interval from 220 to 1000 for the Weber number. This fact means that, when plotting the curves in Fig. 2, the recalculation from the diameter D_{30} to the diameter D_{32} was made, by using the formula $D_{32} = 1.19 D_{30}$. Note also that the magnitude of indicated coefficient, when being calculated with the use of the gamma-function (as was recommended in work [5]), turned out to equal 1.29. On the whole, this value correlates well with the value of 1.19 calculated in this work. However, the result of work [5] was

obtained in the case where a generalized theoretical shape was used for the distribution function, whereas the value of 1.19 is associated with the parameters of the specific distribution that was obtained in this work as a solution of the specific problem. Therefore, just the value of 1.19 was used to compare our numerical results with the experimental data of work [9].

The magnitudes of the Weber number and the fractions of momentum and energy losses, which were required for the numerical processing of the data of work [9], were evaluated on the basis of the following considerations. Concerning the density and the surface tension of fuel mixtures with various compositions, they were evaluated, by using the interpolation method on the basis of the data on corresponding parameters of the initial fuel components. This approach can be regarded as substantiated for the evaluation of the indicated thermodynamic parameters of the mixture, at least because the variation intervals of the indicated quantities are narrow, so that the linear approximation should not result in a large error for the sought values.

A considerable uncertainty is inserted to the final Weber number value by the average fuel injection rate, which should be measured at the injector output. It is natural that the sought rate depends on the constructional injector features, the parameters of fuel supply (first of all, the injection pressure), and the properties of a sprayed fuel. Such a considerable number of the factors that govern the fuel injection rate make a reliable theoretical estimation of the required parameter for the experimental installation in work [9] rather a hard task.

We would also like to draw attention to other factors that affect the variation of this parameter, when changing from one composition of the mixture of diesel fuel with rapeseed oil to another. The viscosity is a key parameter that determines the motion velocity of real liquids. The experimental data on the viscosity of diesel fuel, biodiesel fuel, and rapeseed oil are quoted in Table. The presented data demonstrate a substantial difference between the viscosities of diesel fuels and rapeseed oil. It is natural that the addition of rapeseed oil to the diesel fuel gives rise to a higher viscosity of the mixture. Furthermore, this growth is most likely nonlinear, because linear variations of the solution parameters with the change in the solution composition are usually observed, when the solution is formed from the substances that are members of the same homologous series, i.e. when the substances close by their thermodynamic properties are mixed up.

In the case where the diesel fuel is mixed with oil, and the equiatomic composition of the mixture is exceeded, the solvent in the system changes from the low-viscosity diesel fuel to the viscous oil. Therefore, the viscosity of the fuel mixture should grow with an increase of the rapeseed oil fraction in it, although this growth could be not very noticeable at low oil fractions. It is natural that, in turn, the fuel velocity at the sprayer output will decrease substantially, but not catastrophically rapidly, with the growth of the oil fraction in the system. Those considerations are qualitatively confirmed by the experimental fact [9] that, provided a fixed injection pressure, it is impossible to spray a fuel mixture, if the corresponding oil fraction in it exceeds 40%. On the basis of the presented speculations about the dependence of the fuel mixture viscosity on the mixture composition, a hypothesis can be put forward that the Weber number decreases, if the oil fraction in a mixture increases. Those speculations formed a basis for the performed calculations.

The growth of the fuel viscosity in the system with an increase of the dissolved oil fraction in it should also manifest itself in the magnitude of total losses spent for the internal friction in a sprayed liquid. The above consideration means that the parameters of momentum and energy losses, $\bar{G}_{mv}$ and $\bar{G}_e$, have also to be considered in this case as depending on the fuel composition. In so doing, we may assert that the growth of the oil fraction in a fuel mixture corresponds to an increase of the momentum and energy losses in the system.

The presented considerations allow us to substantiate the theoretical description of experimental dependences from work [9], in particular, the plotting of the droplet distribution functions over droplet sizes for the sprayed fuel mixtures with various compositions (Fig. 2). Here, the Weber number and the loss fractions $\bar{G}_{mv}$ and $\bar{G}_e$, provided that their variations satisfy the conditions presented above, composed a set of fitting parameters.

The analysis of the results of preliminary calculations showed that the quantity $\bar{G}_{mv}$ has the greatest influence on the distribution function plots. The variations of the indicated parameters have the same qualitative influence on the position and shape of

the distribution curve. In other words, the increase of either parameter results in the expanding of the distribution curve and the shift of its maximum toward larger droplet diameters. The accumulated calculation experience gave us ground to assume that $\bar{G}_{mv} = \bar{G}_e$. Further calculations showed that this assumption does not reduce the general reliability of the results obtained, but diminishes the number of fitting model parameters and simplifies the interpretation of final results.

The results of calculations for the droplet distributions over droplet sizes are shown in Fig. 2 together with the experimental data of work [9]. Rather a satisfactory correspondence between the results obtained under our assumptions in the framework of the maximum entropy model and the experimental data is observed. Note that the values of the Weber number We and the losses $\bar{G}_{mv}$ and $\bar{G}_e$ that were determined from the best agreement of the calculation results with the experiment correspond to the requirements indicated above.

The attained correspondence between the theoretical and experimental results made it possible to apply the developed model ideas in order to analyze features in the behavior of droplets during the evolution of the fuel-air system. For instance, Fig. 3 demonstrates plots for the distribution functions over droplet velocities under the conditions similar to that used to calculate the data in Fig. 1. As was in the case of Fig. 1, the parameters describing the momentum and energy losses in the sprayed system were maintained at the levels $\bar{G}_{mv} = -0.017$ and $\bar{G}_e = 0$, i.e. the same as in work [6].

From the results of calculations, it follows that the growth of the Weber number does not affect the average velocity of droplet motion, but diminishes the height of the plot maximum and expands the differential distribution function along the horizontal coordinate axis. If the velocities of droplet motion remain invariable, when the Weber number is varied, which corresponds to a modification of the thermodynamic fuel properties, it is possible to assert that the system reacts to new values of the parameters governing the droplet formation (change in the specific surface energy), by changing the sizes of formed droplets. This conclusion corresponds to the data quoted in Fig. 1, which demonstrates that the droplet diameters change together with the Weber number.

Our assumption that the droplets in a sprayed fuel move at the velocities that are only a little different from the average value was applied in work [10], when analyzing multifractal spectra registered from the impact marks left by fuel droplets on a plate covered with a black coating. In the relevant calculations [10], the shape of impact marks was considered to reproduce the shape of droplets themselves. Really, this assumption is substantiated, if the velocities of droplet motion are almost identical, and the corresponding force action of the droplets on the registering layer surface is determined only by the droplet mass only. From this viewpoint, the presence of sharp maxima in the droplet velocity distributions shown in Fig. 3 can be considered as a quantitative confirmation of the assumption made in work [10].

6. Conclusions

The maximum entropy method was used to evaluate the influence of the thermodynamic parameters of a sprayed fuel on the final state of the droplet-air mixture. Quantitative data were obtained for the influence of the thermodynamic fuel characteristics on the droplet sizes and the velocities of droplet motion. The droplet-air mixture was demonstrated to response to a variation in the thermodynamic parameters of a sprayed fuel, first of all, by changing the droplet diameters, whereas the kinetic characteristics of droplet motion change only slightly at that. The obtained results make it possible to develop recommendations concerning the choice of conditions for the drop mixture formation from fuels with different thermodynamic properties.

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ВПЛИВ ТЕРМОДИНАМІЧНИХ
ВЛАСТИВОСТЕЙ РОЗПИЛЮВАНОЇ РІДИНИ
НА ПАРАМЕТРИ КРАПЕЛЬНО-ПОВІТРЯНИХ
СУМІШЕЙ В РАМКАХ МОДЕЛІ
МАКСИМУМУ ЕНТРОПІЇ

Р е з ю м е

У межах методу максимуму ентропії проаналізовано вплив термодинамічних властивостей рідини, що розпилюється, на функції розподілу крапель за їхніми розмірами та швидкостями рухів у крапельно-повітряній системі. Розрахунки співставляються з експериментами з розпиленням дизельного палива та його сумішей з ріпаковою олією. Встановлено вплив термодинамічних властивостей розпиленої рідини на кінцевий стан крапельно-повітряної суміші.