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# POSSIBILITY OF NORMAL SPHERICAL DETONATION IN A HYDROGEN-OXYGEN GAS MIXTURE: ALLOWABLE TEMPERATURE, MACH NUMBER, AND HYDROGEN CONTENT

*In the framework of the classical theory of detonation with the use of the previously obtained relations for spherical waves, the ranges of the allowable values of temperature, Mach number, and hydrogen content in a gas mixture, where the normal spherical detonation is possible, are determined. The critical values of parameters associated with the kinetics of chemical reactions at the blast wave front and the parameters responsible for the shock transition intensity (the minimum and the maximum of the Mach number) are calculated for the reacting medium. By analyzing the interaction between $H_2$ and $O_2$, the intervals of the critical temperature, the temperature of detonation in a stationary medium, and the hydrogen content in the mixture, at which the spherical detonation is possible, are determined graphically.*

*Keywords*: spherical detonation, Chapman–Jouguet regime, hydrogen-oxygen mixture, Lewis scheme, chain reaction, critical temperature, Mach number, Jouguet point, hydrodynamic theory of detonation.

## 1. Introduction

Explosions are widely used in many areas of science and engineering, and their models are applied to elucidate various physical phenomena. Moreover, the unexpected explosions in industry and everyday life often result in catastrophes with numerous human losses, which invokes the intensive study of a supersonic burning nowadays. Those researches are carried out using both analytical methods [1] and numerical simulations [2, 3]. This work aims at studying the range of parameters needed for a normal spherical detonation in a gas mixture to take place. It is the kind of detonation that precedes the plane (classical) detonation, but emerges at lower shock wave velocities [4]. The spherical wave produced by a strong point explosion corresponds to the initial stage of the whole detonation process and transforms gradually into the classical variant. In the earlier work [4], the model for the transition of an explosion spherical wave to the Chapman–Jouguet regime was proposed. In the other work [5], the concept of the critical temperature at the wave front was introduced as a basic criterion for the transformation of a shock wave to the detonation one. In this work, using the hydrogen-oxygen gas mixture as an example, an attempt was made to graphically determine the ranges of physical parameters, at which the spherical detonation is probable.

## 2. Critical Values of Parameters Related to the Chemical Reaction Kinetics

The classical theory considers detonation waves with sharp front edge. In its framework, the chemical transformations are assumed to begin right after a jump-like increase of the pressure. Actually, the process develops somewhat differently [6]. The temperature and pressure profiles behind the shock front of a detonation wave are schematically shown in Fig. 1. After the shock transition (1-2), the vibrational and rotational degrees of freedom of gas molecules become excited (2-3), which is accompanied by a temperature reduction. Then the induction period (3-4) takes place, the duration of which can be equal to more than 90% of the whole chemical reaction time (3-5), if the activation energy of the process is sufficiently high ($E = 20{\div}40$ kcal/mol). In the stationary detonation regime (the Chapman–Jouguet regime), profile 1-5 does not change in time. The reaction zone adjoins the region of non-stationary flow, rarefaction wave (5-6), the profile of which can change.

It turns out that, in the case of a hydrogen-oxygen mixture compressed by a shock wave, a lot of free radicals emerge in section 2-4 [7,8], with their concentration reaching $10^{12}{\div}10^{14}$ cm$^{-3}$. Rapid chain transformations start just from those initial centers [7] and run following the Lewis scheme. In this case, we have

$$\mathrm{OH} + \mathrm{H_2} = \mathrm{H_2O} + \mathrm{H}, \tag{1}$$

$$\mathrm{H} + \mathrm{O_2} = \mathrm{OH} + \mathrm{O}. \tag{2}$$

Note also that the temperature $T_2$, at which the branching probability $\delta$ equals unity,

$$\delta = 1, \tag{3}$$

is critical: the process becomes considerably accelerated, and the rapid chain reaction takes place. Under the indicated conditions, according to the results of work [5], the equality

$$T_2 = T_x \tag{4}$$

plays the role of a criterion for qualitative variations in the kinetics of the interaction between hydrogen and oxygen. The Lewis scheme ignites the detonation mechanism, although the process itself can run in a certain different way, following a different scenario, in which the reaction rate is higher by an order of magnitude. From the chemical viewpoint, we have already stated the fact that, in order to obtain the supersonic burning at the shock wave front, it is necessary to reach the temperature $T_x$ in the medium, at which the branching probability $d$ equals unity. How can $T_x$ be determined?

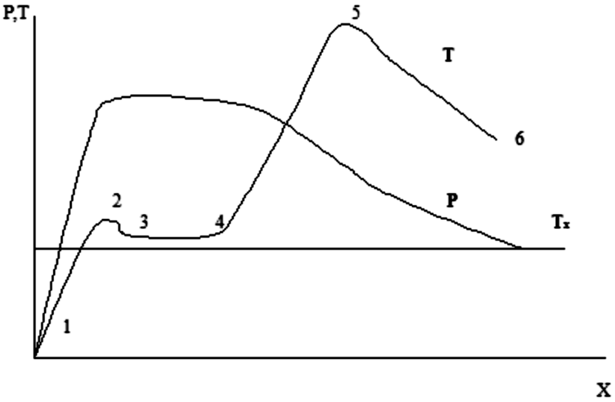

***Fig. 1.*** Schematic diagrams of the pressure, $P$, and gas temperature, $T$, profiles behind the shock wave front [6] under the condition $T_2 \geq T_x$, where $T_2$ is the temperature at point 2

In work [5], the relation between the key parameters of a chemical reaction at the shock wave front, on the one hand, and the physical quantities that characterize the process of shock transition, on the other hand, was obtained,

$$T_x^2 = \frac{2.5 \times 10^5 Q T_0 (\gamma - 1)}{4\gamma^2(\gamma + 1) K^* P_0} \times \times \frac{(2\gamma M^2 - \gamma + 1)(2 + (\gamma - 1)M^2)^2}{M^6} \times \times \exp\left(-\frac{E_2}{K^* T_x}\right), \tag{5}$$

where $M$ is the Mach number (it reflects the shock transition intensity); $P_0$ the initial, before the explosion ignition (at 293 K), pressure of the gas mixture reckoned in mm Hg units; $E_2$ the activation energy of the branching reaction (2); $K^*$ the gas constant; $Q$ the combustion energy per gas mole; and $\gamma$ the adiabatic index for the given gas mixture. For the hydrogen-oxygen mixture, the corresponding numerical values are [9]: $\gamma = 1.4$, $Q = 286.5$ kJ/mol, $K^* = 8.31$ J/mol/K, $E_2 = 16 \times 10^3 \times 4.19$ J/mol, and $T_0 = 293$ K. Then Eq. (5) reads

$$T_x^2 = \frac{5.38 \times 10^{10} (2.8M^2 - 0.4)(2 + 0.4M^2)^2}{P_0 M^6} \times \times e^{-8067/T_x}. \tag{6}$$

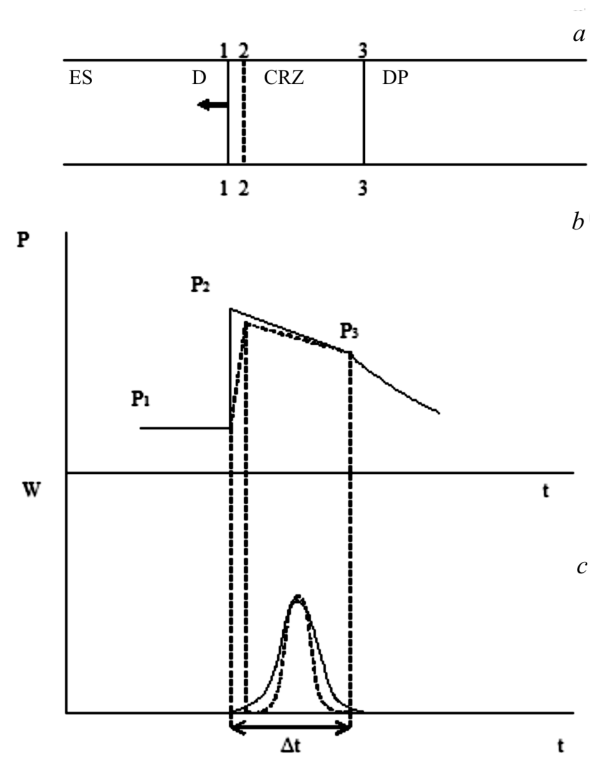

***Fig. 2.*** Schematic structure of a plane detonation wave: explosive substance (ES), detonation products (DP), and chemical reaction zone (CRZ) (*a*). The pressure changing in time: in front of the wave front ($P_1$), at the wave front ($P_2$), and in the chemical reaction zone (the Jouguet point, $P_3$) (*b*). Reaction rates (*c*)

It is evident that the temperature $T_x$ is different for different Mach numbers. Formula (6) describes the functional dependence of the critical temperature $T_x$ on the Mach number $M$ for the given initial pressure $P_0$ and allows one to compare its value with the real temperature

$$T_2 = \frac{(2\gamma M^2 - \gamma + 1)(2 + (\gamma - 1)M^2)}{(\gamma + 1)^2 M^2} T_1, \tag{7}$$

where $T_1$ is the temperature of the medium in front of the wave front. Hence, in our case, the important criterion,

$$T_2 \geq T_x, \tag{8}$$

has to be satisfied for the detonation to take place as a real process.

## 3. Elements of the Hydrodynamic Theory of Detonation. Limiting Parameters Dependent on the Mach Number Minimum and Maximum

The detonation process of explosive materials is considered as a cumulative action of the shock wave and the chemical reaction, when the shock compression initiates the reaction, and the reaction energy maintains the detonation process afterward. The hydrodynamic theory [10] enables one to evaluate the size of a chemical reaction zone and the values of medium parameters in the chemical reaction zone (at the interface with the detonation products). The classical theory considers a plane detonation front,

$$d = \Delta t(D - v_g), \tag{9}$$

where $d$ is the chemical reaction zone width, $\Delta t$ the reaction duration, $D$ the shock wave velocity, and $v_g$ the gas velocity behind the reaction front (the Jouguet point). To be exact, in a real situation (Fig. 2), there exists some shock transition interval (1-2) before the temperature $T_2$ is achieved, which is not taken into account in this case. One can see in Fig. 2 that front (3-3) separates the chemical reaction zone from detonation products. This means that the substance being suddenly compressed by the shock wave burns out completely within the time interval $\Delta t$.

The theory is based on two important postulates: 1) the whole substance compressed by the shock wave burns out, and 2) the combustion energy is enough to maintain the shock wave velocity to be constant ($D = \text{const}$). According to the theory, the pressure $P$ and the density $\rho$ in the chemical reaction zone and at the interface with detonation products (the Jouguet point) are connected with each other by the following relations [10]:

$$P_3 = \frac{P_1 + \rho_1 D^2}{\gamma + 1}, \tag{10}$$

$$\frac{\rho_3}{\rho_1} = \frac{D^2(\gamma + 1)}{\gamma D^2 + c_1^2}, \tag{11}$$

where $P_3$ is the pressure at front (3-3) separating the reaction zone from the reaction products, $P_1$ the pressure in front of the shock wave front, $\rho_1$ the gas density in front of the wave front, $D$ the wave velocity, $g$ the adiabatic index, $\rho_3$ the medium density at the wave front (3-3), and $c$ the sound velocity in the motionless medium in front of the front. From the Mendeleev–Clapeyron equation

$$PV = \frac{m}{\mu} K^* T \Rightarrow T = \frac{P\mu}{\rho K^*}, \tag{12}$$

substituting the values of $P_3$ (10) and $\rho_3$ (11), we determine the temperature $T_3$ at the Jouguet point,

$$T_3 = \frac{\rho_1 D^2}{\gamma + 1} \frac{\mu}{K^*} \frac{\gamma D^2 + c_1^2}{\rho_1 D^2 (\gamma + 1)} = \frac{\mu(\gamma D^2 + c_1^2)}{K^*(\gamma + 1)^2} =$$
$$= \frac{\mu c_1^2}{K^*} \frac{\gamma M^2 + 1}{(\gamma + 1)^2} = T_1 \gamma \frac{\gamma M^2 + 1}{(\gamma + 1)^2}, \tag{13}$$

where $D = c_1 M$. Here, we took into account that

$$P_3 \approx \frac{\rho_1 D^2}{\gamma + 1}, \tag{14}$$

when $\frac{P_3}{P_1} \gg 1$, and that

$$c_1^2 = \gamma \frac{K^* T_1}{\mu}, \tag{15}$$

where $\mu$ is the molar mass. It is evident that if we consider the detonation and the support of a chemical reaction by the shock wave, the following condition has to be satisfied:

$$T_3 > T_2; \tag{16}$$

or, in a wider sense (Fig. 3, *a*),

$$T_3 > T_2 > T_x. \tag{17}$$

Let us analyze inequality (16) in detail. From the theory of shock waves [10], it is known that the temperature at point 2 in Fig. 3 is determined by relation (7). Therefore, inequality (16) can be transformed as follows:

$$T_1 \gamma \frac{\gamma M^2 + 1}{(\gamma + 1)^2} >$$
$$> T_1 \frac{(2\gamma M^2 - \gamma + 1)(2 + (\gamma - 1)M^2)}{(\gamma + 1)^2 M^2} \tag{18}$$

or

$$\gamma(\gamma M^2 + 1) -$$
$$- \frac{(2\gamma M^2 - \gamma + 1)(2 + (\gamma - 1)M^2)}{M^2} > 0, \tag{19}$$

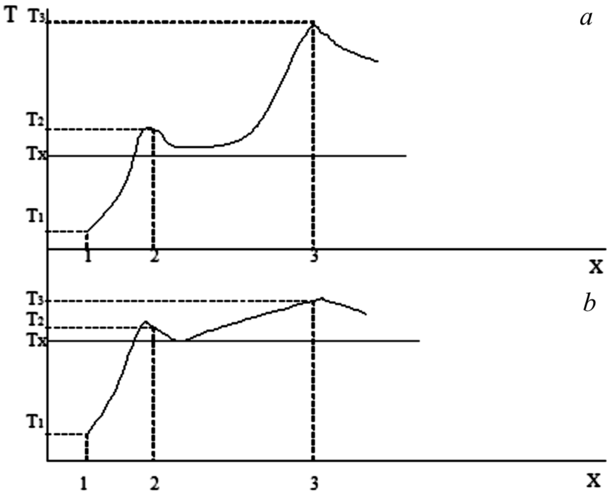

***Fig. 3.*** Schematic diagrams for the temperature profiles behind the wave front: the general case (*a*) and the case where $T_3 \approx T_2 \approx T_x$ (*b*) corresponding to the limiting detonation process

since $T_1 > 0$ and $\gamma > 0$. By solving the equation

$$\gamma(\gamma M^2 + 1) -$$
$$- \frac{(2\gamma M^2 - \gamma + 1)(2 + (\gamma - 1)M^2)}{M^2} = 0 \tag{20}$$

with respect to $M$ (keeping in mind that $M > 0$), we obtain

$$M^4(2\gamma - \gamma^2) + M^2(\gamma^2 - 5\gamma + 1) + 2\gamma - 2 = 0 \tag{21}$$

or, substituting the corresponding $\gamma$-value,

$$0.84M^4 - 4.04M^2 + 0.8 = 0. \tag{22}$$

The positive roots of this equation are $M_1 = 2.145$ and $M_2 = 0.455$. For shock transitions, the most interesting is the first root, $M_1 = 2.145 \approx 2.2$. On the basis of inequality (18), we may assert that the detonation process is not possible for all shock waves, but only for those with the Mach number $M > 2.2$. Owing to hydrodynamic reasons, there is no detonation for waves with $M < 2.2$.

While analyzing Eq. (20), it is expedient to admit that the temperature equality

$$T_3 \approx T_2 \approx T_x \tag{23}$$

describes the lower temperature limit of the detonation (Fig. 3, *b*). In so doing, we took into account

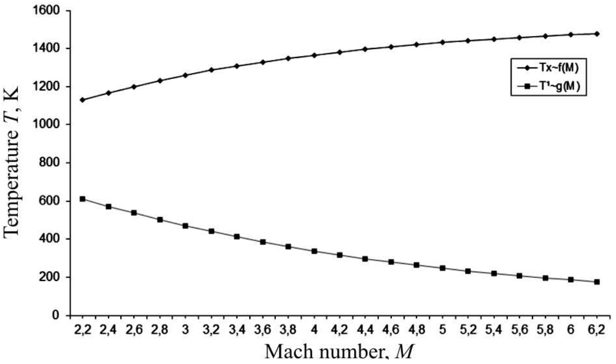

***Fig. 4.*** Dependences of the critical temperature $T_x$ and the detonation temperature in the motionless medium, $T^1$, on the Mach number $M$ at the fixed pressure $P = 60$ mm Hg

that the rate of the chemical reaction decreases together with the temperature in the chemical reaction zone. At the same time, according to the hydrodynamic theory, the amount of the substance that was compressed by the shock wave and interacted under its action has to remain at the previous level. This circumstance inevitably results in the time growth for the active reaction phase, and, as a consequence of the process continuity, gives rise to a considerable reduction of the induction period (interval 3-4 in Fig. 1). In this connection, there emerges a possibility for the detonation wave to create a gas layer with an approximately identical temperature, and the Mach number $M \approx 2.2$ should be considered as the lower detonation limit.

In order to determine the upper limit of the detonation wave emergence by initiating an explosion in reacting gas media, let us use the model describing the continuous transition of a spherical explosion wave into the Chapman–Jouguet regime [4]. For the normal spherical detonation, it can be determined from the formula

$$M = \left[\frac{(\gamma+1)^2(\gamma-1)}{4\gamma^2}\,\frac{Qc}{K^*T_1}\right]^{1/2} \qquad (24)$$

derived in work [4]. All quantities in this formula are known, except for the parameter $c$, the specific content of the burned out gas (hydrogen). The intensity of a detonation wave can be controlled by changing, mainly, two parameters: $c$ (in the numerator) and $T_1$ (in the denominator). In our case, all hydrogen compressed by the shock wave burns out. The values of coefficient $c$ are confined within the interval $0.66 \geq c > 0$. Let we have the stoichiometric mixture of hydrogen with oxygen ($c = 0.66 = \max$), and the medium temperature $T_1 = -100$ °C $\approx 173$ K $=$ $= \min$. We suppose that a further temperature decrease will result in changes of the adiabatic index $\gamma$ and the physical properties of the reacting mixture [11], i.e. let formula (24) be valid for real gases at $T_1 \geq 173$ K. In this case, we obtain a rough estimate for the Mach number maximum, $M_{\max} = 6.2$. Note again that a strong explosion takes place in a cooled down medium. In this case, $M_{\max} = 6.2$. Hence, we estimated the interval of possible Mach numbers for the normal spherical detonation of the hydrogen-oxygen gas mixture under real conditions:

$$6.2 \geq M \geq 2.2. \qquad (25)$$

In view of relation (6), let us plot the dependence of the critical temperature on the Mach number, $T_x(M)$ (Fig. 4). Since the Mach number range was found, we will calculate the critical temperature $T_x$ for every $M$ from the indicated interval with an increment of 0.2 and the fixed initial pressure $P_0$ (the procedure was described in work [5]). The larger the Mach number, the higher is the critical temperature. However, at $M \geq 5$, the critical temperature growth becomes a little slower. At the lower limit $M = 2.2$, $T_x = 1130$ K, and, at the upper limit $M = 6.2$, $T_x = 1479$ K. Hence, in a hydrogen-oxygen mixture, the critical temperature $T_x$ for the allowable values of Mach number $M$ accepts values from the following interval:

$$1479\ \text{K} \geq T_x \geq 1130\ \text{K}. \qquad (26)$$

Figure 4 also exhibits the dependence of the detonation temperature $T^1$ on the Mach number $M$, which can easily be obtained by substituting the critical temperature $T_x$ [5] into relation (7):

$$T^1 = \frac{(\gamma+1)^2M^2}{(2\gamma M^2-\gamma+1)(2+(\gamma-1)M^2)}T_x. \qquad (27)$$

From Eqs. (25) and (26), it follows that the detonation temperature for a motionless medium falls within the interval

$$609\ \text{K} \geq T^1 \geq 176\ \text{K}. \qquad (28)$$

This is the minimum temperature in front of the shock wave that makes the detonation possible.

## 4. Results of Calculations and Their Discussion

Using the intervals obtained above for some physical quantities, let us graphically determine the region of existence for the normal spherical detonation. The upper limit of the hydrogen content in the mixture is confined in our case by the value $c = 0.66$. Above this value, the chemical reactions resulting from the interaction between hydrogen and oxygen in the mixture, which were considered in work [5], become more complicated, and this circumstance may result in different values of critical temperature. Further researches of this issue are required. Below, the choice $T_1 = 800$ K for the upper limit of the medium temperature is explained in detail.

With regard for dependence (24) of the Mach number $M$ on the temperature of a motionless medium $T_1$ and the hydrogen content $c$, let us plot the dependences $M(c)$, $T_x(c)$, and $T^1(c)$ at fixed $T_1$ and $P_0$. We proceed from the plots of the dependence $M(c)$ at $T_1 = \mathrm{const}$ exhibited in Fig. 5 for $T_1$-temperatures in the interval 800 K $\geq T_1 \geq$ 173 K. The lower curve corresponds to the gas mixture temperature $T_1 = 800$ K, and the upper one to $T_1 = 173$ K. According to expression (24), this family of curves has a power dependence on the hydrogen content in the mixture, $c$, with a power exponent of 0.5. Let us fix the maximum content of burned out hydrogen, $c = 0.66$, which corresponds to the stoichiometric composition of hydrogen-oxygen mixture, and draw a vertical line. The Mach number corresponding to its intersection with the mentioned family of curves changes from $M = 2.2$ at point 4 to $M = 6.2$ at point 5. Another important detail should be emphasized. Four dashed lines are drawn in Fig. 5. Two horizontal ones confine the region of allowable Mach numbers corresponding to the normal spherical detonation. The first line corresponds to the minimum $M_{\min} = 2.2$, and the second one to the maximum $M_{\max} = 6.2$. The third dashed vertical line corresponds to the stoichiometric composition of the hydrogen-oxygen mixture and is the optimal variant for the detonation. The fourth line will be discussed below.

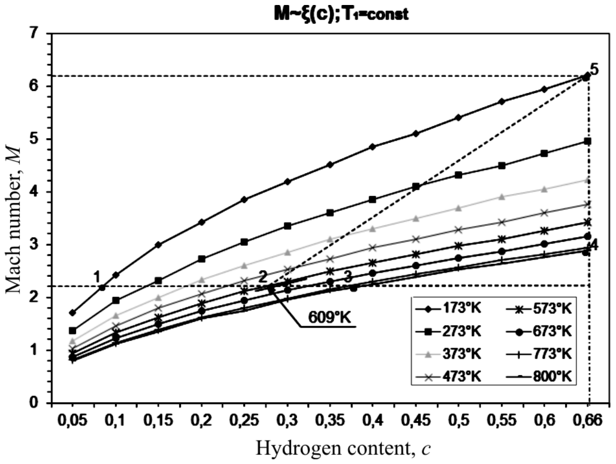

***Fig. 5.*** Dependence of the Mach number $M$ on the hydrogen content $c$ in the gas mixture ($P_0 = 60$ mm Hg) for various temperatures in the motionless medium, $T_1$

Let us consider points 1 to 5 in Fig. 5 separately. Segment 1-2 corresponds to the lower limit of the shock wave velocity $M_{\min} = 2.2$, but the medium temperature for the segment points turns out lower than the detonation one (Fig. 4). Therefore, the detonation is impossible in this case. The region of the probable detonation for this Mach number is restricted to segment 2-3, because the temperature of motionless medium reaches the detonation temperature values here. On the basis of Fig. 4, it is also possible to draw a conclusion that, for the medium temperature $T_1 = 173$ K, the detonation is possible if $M = M_{\max} = 6.2$ (point 5 in Fig. 5). In other words, for the chosen temperature, 800 K $\geq T_1 \geq$ 173 K, and hydrogen content, $0.66 \geq c \geq 0.075$, intervals, the region of the probable detonation is bounded by segments 2-3, 3-4, 4-5, and 5-2. Segment 5-2 is presented in Fig. 5 schematically by a straight line. In the general case, in view of the nonlinear dependence $M(c, T_1)$, this segment is curvilinear.

Let us derive the functional dependence $T_x(c)$ by substituting the $M(c)$ dependence (Eq. (24)) into Eq. (5). To make transformations simpler, let us rewrite Eq. (24) in a slightly different form,

$$M = [\eta c]^{1/2}, \tag{29}$$

where

$$\eta = \frac{(\gamma+1)^2(\gamma-1)}{4\gamma^2}\frac{Q}{K^* T_1}. \tag{30}$$

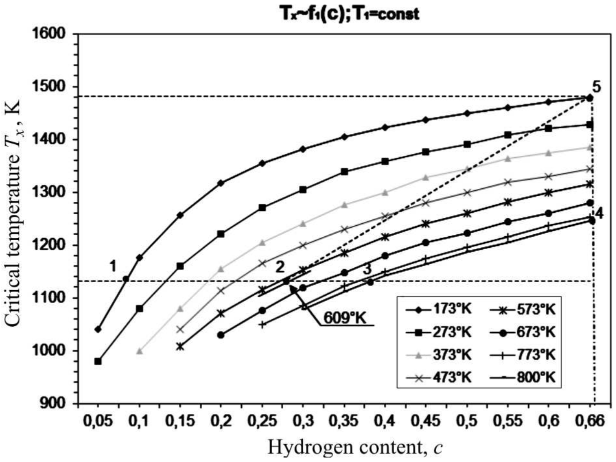

***Fig. 6.*** Dependences of the critical temperature $T_x$ on the hydrogen content $c$ in the gas mixture ($P = 60$ mm Hg) for various temperatures in the motionless medium, $T_1$

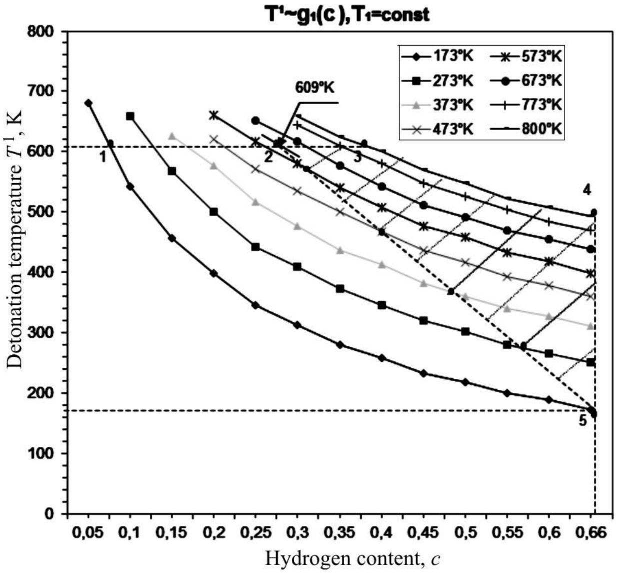

***Fig. 7.*** Dependences of the detonation temperature $T^1$ on the hydrogen content $c$ in the explosive gas mixture $H_2 + O_2$ ($P = 60$ mm Hg) for various temperatures in the motionless medium, $T_1$

Then we obtain the following transcendental equation for the critical temperature $T_x$:

$$T_x^2 = \frac{2.5 \times 10^5 Q T_0(\gamma - 1)}{4\gamma^2(\gamma+1)K^* P_0} \times$$
$$\times \frac{(2\gamma\eta c - \gamma + 1)(2 + (\gamma - 1)\eta c)^2}{\eta^3 c^3} \exp\left(-\frac{E_2}{K^* T_x}\right) \quad (31)$$

or, taking Eq. (30) into account,

$$T_x^2 = \frac{2.5 \times 10^5 T_0 T_1}{(\gamma+1)^3 P_0} \times$$
$$\times \frac{(2\gamma\eta c - \gamma + 1)(2 + (\gamma - 1)\eta c)^2}{\eta^2 c^3} \exp\left(-\frac{E_2}{K^* T_x}\right). \quad (32)$$

In Fig. 6, using expression (32), we plotted the dependences $T_c(c)$ at $P_0 = \mathrm{const}$ and $T_1 = \mathrm{const}$. By the form, they are similar to the previous plots (Fig. 5) and confirm the conclusions made for points 1 to 5.

More interesting is the dependence of the detonation temperature in the motionless medium on the hydrogen content, $T_1(c)$, at $P_0 = \mathrm{const}$ and $T_1 = \mathrm{const}$. It can be determined from relation (5) with regard for Eq. (7):

$$(T^1)^2 = \frac{2.5 \times 10^5 Q T_0(\gamma - 1)(\gamma+1)^3}{4\gamma^2 K^* P_0(2\gamma M^2 - \gamma + 1)M^2} \times$$
$$\times \exp\left(-\frac{E_2(\gamma+1)^2 M^2}{K^* T^1(2\gamma M^2 - \gamma + 1)(2 + (\gamma - 1)M^2)}\right). \quad (33)$$

Making allowance for Eqs. (29) and (30), relation (33) can be simplified to the following form:

$$(T^1)^2 = \frac{2.5 \times 10^5 T_0 T_1(\gamma+1)}{P_0 c(2\gamma\eta c - \gamma + 1)} \times$$
$$\times \exp\left(-\frac{E_2(\gamma+1)^2 \eta c}{K^* T^1(2\gamma\eta c - \gamma + 1)(2 + (\gamma - 1)\eta c)}\right). \quad (34)$$

The corresponding family of curves is shown in Fig. 7. While analyzing the plots, the attention should be drawn to the following facts. (i) Every temperature $T_1$ of the gas mixture is associated with a specific dependence $T^1(c)$. (ii) As the hydrogen content in the mixture grows, the temperature of the detonation $T^1$ in the motionless medium drastically decreases, which is especially appreciable at low temperatures. (iii) Let us draw a horizontal line that intersects the family of curves (for example, let it be the dashed line $T^1 = 173$ K). At the point of its intersection with the curve corresponding to the same temperature of the motionless medium (in our case, this is $T_1 = 173$ K), the detonation condition $T_2 = T_x$ (point 5) is satisfied. Detonation becomes probable, because the current temperature of motionless medium reaches the detonation temperature for this medium ($T_1 = T^1$). (iv) The intersection points of any horizontal line (see item iii) correspond to the critical hydrogen contents in the mixture, below which the detonation is impossible. The dashed line connecting points 2 and 5 in Fig. 7 corresponds to the condition $T_2 \geq T_x$ for the whole family of curves.

Let the hydrogen content in the mixture change from 0.075 (point 1) to 0.66 (point 4). Then, on the basis of the plots shown in Fig. 7, one may assert the following.

1. As was indicated above, a temperature lower than $T^1 \approx 173 \div 176$ K can give rise to a variation in the physical properties of the reacting mixture. Then the proposed formulas will produce erroneous results. The horizontal dashed line that passes through point 5 corresponds to this temperature, and point 5 testifies to the explosion with the maximum Mach number $M_{\max} = 6.2$.

2. According to the plot of the functional dependence $T^1(c)$ at $T_1 = 273$ K (see Eq. (33) and Fig. 7), the detonation is possible if the hydrogen content in the mixture is not lower than 0.57.

3. Physical restrictions imposed by the minimum Mach number $M_{\min} = 2.2$ bring about the existence of the upper limit for the detonation temperature, $T^1 = 609$ K. Points of both segments 1-2 and 2-3 correspond to the allowable values of Mach number. However, the detonation is possible only for the points on segment 2-3, because the main condition $T_2 \geq T_x$ is satisfied at $T_1 \geq 609$ K. Whence it follows that $c = 0.27$ is the minimum hydrogen content in the mixture, below which the detonation is impossible even at very high temperatures.

4. Experimental results testify that, if the temperature of a gas mixture is higher than $T_1 = 800$ K, the spontaneous ignition takes place, which can transform into the detonation, if the hydrogen content in the mixture is not lower than 0.37. Therefore, this temperature is a kind of upper limit, to which the hydrogen-oxygen mixture can be heated up without ignition.

From the reasons given above, it follows that the region of spherical supersonic burning is bounded by segments 2-3, 3-4, 4-5, and 5-2 in Fig. 7. For the illustrative purpose, it was hatched.

## 5. Conclusions

The dependences between the temperature, Mach number, the hydrogen content in the hydrogen-oxygen mixture as the main parameters characterizing the process of transformation of a shock wave into a detonation one and affecting the chemical reactions between reacting components are studied. On the basis of relations obtained earlier [5], the conditions are found, under which the probability of a chain branching reaches unity ($\delta = 1$), and a fast chain reaction is started. The existence of the critical temperature $T_x$ at the front of a shock wave, above which the detonation takes place, is substantiated, as well as the functional dependence (5) of the critical temperature on the Mach number. In author's opinion, the latter should be taken as a basis, while studying the processes of spherical detonation. Summarizing the results of work [5], the condition $T_2 \geq T_x$ is found, which connects the kinetics of a chemical reaction with the detonation in a gas mixture. On the basis of the relations of the hydrodynamic theory of detonation, the region of possible values for the temperatures at the shock wave front, $T_2$, and in the chemical reaction zone, $T_3$, is determined. The equality

$$T_x \approx T_2 \approx T_3,$$

which couples them, corresponds to the lower limit, at which the detonation is possible. The minimum and maximum values of Mach number in reacting gas media are also determined, which enables the process of supersonic burning to be analyzed in more details and the region of physical parameters (the critical temperature, the temperature of detonation in the motionless medium, and the hydrogen content in the mixture), at which the spherical detonation is probable, to be indicated. The latter is illustrated, by using the hydrogen-oxygen mixture as an example.

To summarize, it should be noted that this paper is final in a cycle of works devoted to the study of the whole process of normal spherical detonation.

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## ОБЛАСТЬ ДОПУСТИМИХ ЗНАЧЕНЬ ТЕМПЕРАТУРИ, ЧИСЕЛ МАХА І ПИТОМОГО ВМІСТУ ВОДНЮ В ГАЗОВІЙ СУМІШІ МОЖЛИВОГО ІСНУВАННЯ НОРМАЛЬНОЇ СФЕРИЧНОЇ ДЕТОНАЦІЇ

Резюме

Використовуючи елементи теорії класичної детонації і раніше отримані співвідношення для сферичних хвиль, автор спробував встановити область допустимих значень температури, чисел Маха і питомого вмісту водню в газовій суміші, що уможливлюють існування нормальної сферичної детонації. У роботі враховувалися критичні значення параметрів, пов'язані з кінетикою хімічної реакції на фронті ударної хвилі і параметри, що визначають інтенсивність ударного переходу (мінімальне і максимальне число Маха) для даного реагуючого середовища. На прикладі взаємодії $H_2$ і $O_2$ вдалося графічно визначити область значень критичної температури, температури детонації нерухомого середовища та питомого вмісту водню в суміші, за яких можлива сферична детонація.