O.P. LELYAKOV, A.S. KARPENKO, R.-D.O. BABADZHAN

V.I. Vernadskyi Tavrida National University

(4, Vernadskyi Ave., Simferopil 95700, Ukraine; e-mail: lelyakov@crimea.edu)

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SCALAR-FIELD POTENTIAL DISTRIBUTION FOR A CLOSED "THICK" NULL STRING MOVING IN THE PLANE $z = 0$

A general form for the scalar-field potential distribution has been proposed for a closed "thick" null string either collapsing or expanding in the plane $z = 0$. Conditions, under which the energy-momentum tensor components for a scalar field that contracts into a one-dimensional object (a circle with a varying radius) asymptotically coincide with those for a closed null string moving along the same trajectory, have been found.

Keywords: scalar-field potential, "thick" null string, energy-momentum tensor.

1. Introduction

According to modern ideas, the space strings, which are one-dimensional regions where the energy density is concentrated, could arise in a natural way owing to a spontaneous symmetry violation at phase transitions in the course of the Universe evolution [1–7]. In the framework of various Grand Unified Theory models, the strings, together with domain walls and monopoles, are topological defects. As was shown in work [8], the presence of such objects in the Universe does not contradict the observation of the microwave relic radiation. Again, one cannot exclude that those objects could survive till now and, hence, can be observed [9, 10]. Null strings realize a zero-tension boundary in the string theory [5, 7]. In recent years, the possibilities of null string applications in the cosmology have been discussed. For instance, it was demonstrated in work [11] that, by considering a gas of null strings as a dominant source of the gravitation in D-dimensional Friedman–Robertson–Walker spaces with $k = 0$, the inflation mechanism typical of those spaces can be described. In a number of works, the gas of relic null strings is considered as a probable candidate for the role of a carrier of the so-called "dark" matter, whose existence of in the Universe can be regarded as the established fact. The research object in the cited works is not a separate null string, but a gas of null strings. Nevertheless, the properties of this gas still remain unclear. In our opinion, the problems concerning the gravitational field generated by a null string moving along various trajectories can be a first step to the understanding of properties of the gas of null strings.

The components of the energy-momentum tensor for a null string look like [11]

$$T^{mn}\sqrt{-g} = \gamma \int d\tau d\sigma \; x^m_{,\tau} x^n_{,\tau} \delta^4 \left(x^l - x^l \left(\tau, \sigma \right) \right), \quad (1)$$

where the superscripts m, n, and l can range from 0 to 3; the functions $x^m = x^m(\tau, \sigma)$ describe the null-string trajectory of motion; τ and σ are the parameters on the world surface of a null string; $x^m_{,\tau} = \partial x^m / \partial \tau$; $g = |g_{mn}|$, g_{mn} is the metric tensor of the outer space; and $\gamma = \text{const}$. In the cylindrical coordinate system,

$$x^0 = t, \quad x^1 = \rho, \quad x^2 = \theta, \quad x^3 = z,$$

the functions $x^m(\tau, \sigma)$ for the trajectories of a closed null string that are considered in this work are as follows:

$$t=\tau, \quad \rho=\mp\tau, \quad \theta=\sigma, \quad z=0, \tag{2}$$

where the sign "−" corresponds to the collapse of the null string in the plane $z=0$ (in this case, $\tau\in(-\infty,0]$), and the sign "+" to its radial expansion in the plane $z=0$ (in this case, $\tau\in[0,+\infty)$).

Since all directions on the hypersurfaces $z=\text{const}$ are equivalent for every trajectory (2), the metric functions are $g_{mn}=g_{mn}(t,\rho,z)$. Then, using the invariance of the quadratic form with respect to the coordinate transformation $\theta\to-\theta$, we obtain $g_{02}=g_{12}=g_{32}=0$. The quadratic form of the space-time should also be invariant with respect to the transformation $z\to-z$, so that

$$g_{mn}(t,\rho,z)=g_{mn}(t,\rho,-z) \tag{3}$$

and, as a consequence, $g_{03}=g_{31}=0$. At last, taking advantage of a freedom in choosing the coordinate systems in the general relativity theory, we make the reference frame partially fixed by the requirement $g_{01}=0$. Hence, the quadratic form for the problem concerned can be expressed in the form

$$dS^2=e^{2\nu}(dt)^2-A(d\rho)^2-B(d\theta)^2-e^{2\mu}(dz)^2, \tag{4}$$

where ν, μ, A, and B are some functions of the variables t, ρ, and z.

The components of the energy-momentum tensor for a massless field should satisfy the equality

$$T^\alpha_\alpha=0. \tag{5}$$

Taking Eqs. (1), (2), and (4) into account, equality (5) reads

$$T^0_0+T^1_1=\gamma\frac{e^{-(\nu+\mu)}}{\sqrt{AB}}\left\{e^{2\nu}-A\right\}\delta(z)\delta(\eta)=0, \tag{6}$$

where

$$\eta=t\pm\rho, \tag{7}$$

the sign "+" corresponds to the collapse of the null string in the plane $z=0$, and the sign "−" to its radial expansion in this plane. From equality (6), it follows that

$$e^{2\nu}\equiv A. \tag{8}$$

By analyzing the system of Einstein equations (1), (2), (4), and (8), we can determine the dependences of metric functions, namely,

$$\nu=\nu(\eta,z), \quad B=B(\eta,z), \quad \mu=\mu(\eta,z). \tag{9}$$

In this case, the Einstein system itself is reduced to the equations

$$\frac{B_{,\eta\eta}}{B}+2\mu_{,\eta\eta}-2\nu_{,\eta}\left(\frac{B_{,\eta}}{B}+2\mu_{,\eta}\right)-$$
$$-\frac{1}{2}\left(\frac{B_{,\eta}}{B}\right)+2\left(\mu_{,\eta}\right)^2=-2\chi T_{00}, \tag{10}$$

$$\left(\frac{B_{,z}}{B}\right)_{,z}+\frac{1}{2}\left(\frac{B_{,z}}{B}\right)^2+\frac{B_{,z}}{B}\left(2\nu_{,z}-\mu_{,z}\right)=0, \tag{11}$$

$$\frac{B_{,\eta z}}{B}+2\nu_{,\eta z}-\nu_{,z}\left(\frac{B_{,\eta}}{B}+2\mu_{,\eta}\right)-$$
$$-\frac{1}{2}\frac{B_{,z}}{B}\left(\frac{B_{,\eta}}{B}+2\mu_{,\eta}\right)=0, \tag{12}$$

$$2\nu_{,zz}+4(\nu_{,z})^2+\nu_{,z}\left(\frac{B_{,z}}{B}-2\mu_{,z}\right)=0, \tag{13}$$

$$(\nu_{,z})^2+\nu_{,z}\frac{B_{,z}}{B}=0, \tag{14}$$

where $T_{00}=\gamma\frac{e^{2\nu-\mu}}{\sqrt{B}}\delta(\eta)\delta(z)$.

Let us complement the system of Einstein equations (10)–(14) with the equations of motion for the null string. In a pseudo-Riemannian space-time, they are determined by the following system of equations:

$$x^m_{,\tau\tau}+\Gamma^m_{pq}x^p_{,\tau}x^q_{,\tau}=0, \tag{15}$$

$$g_{mn}x^m_{,\tau}x^n_{,\tau}=0, \quad g_{mn}x^m_{,\tau}x^n_{,\sigma}=0, \tag{16}$$

where Γ^m_{pq} are the Christoffel symbols. Substituting Eqs. (4), (8), and (9) into Eqs. (15) and (16), we can directly demonstrate that, for the functions determining the trajectories of motion (2), all equations of motion for the null string are satisfied identically, i.e. those trajectories are realized indeed, and the gravitational field of null strings does not change them.

With Eqs. (8) and (9), the quadratic form (4) looks like

$$dS^2=e^{2\nu}\left((dt)^2-(d\rho)^2\right)-B(d\theta)^2-e^{2\mu}(dz)^2, \tag{17}$$

where $\nu=\nu(\eta,z), B=B(\eta,z), \mu=\mu(\eta,z)$.

As follows from the system of equations (10)–(14), all components of the energy-momentum tensor of the string are identically equal to zero beyond the string, i.e. at $\eta\neq0$ and $z\neq0$, and differ from zero (tend to infinity) immediately on the string. This enables the system of Einstein equations to be studied in two directions: (i) to confine the analysis to the "outer" problem, i.e. to the region, where the components of energy-momentum tensor (the right-hand sides of the Einstein equations) equal zero; (ii) to consider the components of the energy-momentum tensor of a string as the boundary of a certain "smeared" distribution and carry out the analysis of the Einstein equations for this "smeared" distribution. It can be demonstrated that the analysis of the "outer" problem gives rise to a huge number of vacuum solutions for the Einstein equations, which satisfy the symmetry of the problem. For instance, it is easy to verify that the functions

$$e^{2\nu} = e^{2\mu} = 1, \quad B = z^2$$

or the functions

$$e^{2\nu} = |\beta_\eta|, \quad e^{2\mu} = (\beta(\eta))^2, \quad B = (\beta(\eta) \cdot z)^2,$$

where $\beta(\eta)$ is an arbitrary function, are the outer solutions for the system of equations (10)–(14). However, the criteria that would allow a unique solution describing the gravitational field of the null string to be chosen from this set of functions remain unclear.

On the other hand, while attempting to consider the components of the energy-momentum tensor of the string as a limiting case of a certain "smeared" distribution – e.g., a simple substitution of delta-functions in the energy-momentum tensor by corresponding peaked functions – there may arise errors associated with the fact that it is not clear how a possible emergence of terms (multipliers) tending to zero (constant) at the contraction of this "smeared" distribution into a one-dimensional object can be taken into account. Therefore, it is a simpler task to consider, from the very beginning, a "well-determined" "smeared" distribution, e.g., a real-valued massless scalar field (because we deal with a scalar zero object) and, afterward, to contract it into a string with a required configuration, provided that the corresponding components of the energy-momentum tensors of the scalar field and the null string should asymptotically coincide.

2. System of Einstein Equations for a "Smeared" Problem

The components of the energy-momentum tensor for a real-valued massless scalar field look like [2]

$$T_{\alpha\beta} = \varphi_{,\alpha}\varphi_{,\beta} - \frac{1}{2} g_{\alpha\beta} L, \tag{18}$$

where $L = g^{\omega\lambda}\varphi_{,\omega}\varphi_{,\lambda}$, $\varphi_{,\alpha} = \partial\varphi/\partial x^\alpha$, φ is the scalar field potential, and the subscripts α, β, ω, and λ vary from 0 to 3. In order to ensure the self-consistency of the Einstein equations (17) and (18), we require that

$$T_{\alpha\beta} = T_{\alpha\beta}(\eta, z) \to \varphi = \varphi(\eta, z). \tag{19}$$

Substituting Eqs. (17) and (19) into Eq. (18), we obtain

$$\begin{aligned} &T_{00} = (\varphi_{,\eta})^2 + \frac{e^{2(\nu-\mu)}}{2}(\varphi_{,z})^2, \\ &T_{03} = \pm T_{13} = \varphi_{,\eta}\varphi_{,z}, \\ &T_{11} = (\varphi_{,\eta})^2 - \frac{e^{2(\nu-\mu)}}{2}(\varphi_{,z})^2, \\ &T_{01} = \pm(\varphi_{,\eta})^2, \\ &T_{33} = \frac{1}{2}(\varphi_{,z})^2, \\ &T_{22} = -\frac{Be^{-2\mu}}{2}(\varphi_{,z})^2, \end{aligned} \tag{20}$$

where the sign "+" describes the collapse, and the sign "−" the expansion of the null string.

The system of Einstein equations (17) and (20) can be presented in the form

$$\frac{B_{,\eta\eta}}{B} + 2\mu_{,\eta\eta} - 2\nu_{,\eta}\left(\frac{B_{,\eta}}{B} + 2\mu_{,\eta}\right) - -\frac{1}{2}\left(\frac{B_{,\eta}}{B}\right)^2 + 2(\mu_{,\eta})^2 = -2\chi(\varphi_{,\eta})^2, \tag{21}$$

$$\left(\frac{B_{,z}}{B}\right)_{,z} + \frac{1}{2}\left(\frac{B_{,z}}{B}\right)^2 + \frac{B_{,z}}{B}(2\nu_{,z} - \mu_{,z}) = 0, \tag{22}$$

$$\frac{B_{,\eta z}}{B} + 2\nu_{,\eta z} - \nu_{,z}\left(\frac{B_{,\eta}}{B} + 2\mu_{,\eta}\right) - -\frac{1}{2}\frac{B_{,z}}{B}\left(\frac{B_{,\eta}}{B} + 2\mu_{,\eta}\right) = -2\chi\varphi_{,\eta}\varphi_{,z}, \tag{23}$$

$$2\nu_{,zz} + 4(\nu_{,z})^2 + \nu_{,z}\left(\frac{B_{,z}}{B} - 2\mu_{,z}\right) = 0, \tag{24}$$

$$(\nu_{,z})^2 + \nu_{,z}\frac{B_{,z}}{B} = \frac{\chi}{2}(\varphi_{,z})^2. \tag{25}$$

Let the scalar field distribution be initially concentrated in a "thin" ring, for which the variables η (see its definition in Eq. (7)) and z vary in the intervals

$$\eta \in [-\Delta\eta, \Delta\eta], z \in [-\Delta z, \Delta z], \tag{26}$$

where $\Delta\eta$ and Δz are small positive constants that determine the ring "thickness", i.e.

$$\Delta\eta \ll 1, \quad \Delta z \ll 1, \tag{27}$$

Then the ring is contracted into a one-dimensional object (a null string),

$$\Delta\eta \to 0, \quad \Delta z \to 0. \tag{28}$$

If the system of equations (21)–(25) is considered for this process, the space, in which this "smeared" null string is located and where the variables η and z vary in the intervals $\eta \in (-\infty, +\infty)$ and $z \in (-\infty, +\infty)$, can be conditionally divided into three regions: region *I*,

$$\eta \in (-\infty, -\Delta\eta) \cup (\Delta\eta, +\infty)\,, z \in (-\infty, +\infty)\,; \tag{29}$$

region *II*,

$$\eta \in [-\Delta\eta, +\Delta\eta]\,, z \in (-\infty, -\Delta z) \cup (\Delta z, +\infty)\,; \tag{30}$$

and region *III*,

$$\eta \in [-\Delta\eta, \Delta\eta]\,, z \in [-\Delta z, \Delta z]\,. \tag{31}$$

While contracting the scalar field into a string, the system of equations (21)–(25) for the scalar field should asymptotically coincide with the system of equations (10)–(14) for the closed null strings. Therefore, in regions *I* and *II* (Eqs. (29) and (30)),

$$\varphi \to 0, \quad \varphi_{,z} \to 0, \quad \varphi_{,\eta} \to 0, \tag{32}$$

and, in region (31)–inside the "thin" ring–in the general case,

$$\frac{\varphi_{I,II}}{\varphi_{III}} \le 1, \quad \frac{(\varphi_{,z})_{I,II}}{(\varphi_{,z})_{III}} \le 1, \quad \frac{(\varphi_{,\eta})_{I,II}}{(\varphi_{,\eta})_{III}} \le 1, \tag{33}$$

where $\varphi_{I,II}$ are the scalar field potentials in regions *I* and *II*, φ_{III} is the scalar field potential in region *III* (inside the "thin" ring), and the equalities take place at the region boundaries.

Comparing the system of equations (10)–(14) for the closed null string with the system of equations (21)–(25), a conclusion can be drawn that, when the scalar field is contracted into a string, i.e. at $\Delta\eta \to 0$ and $\Delta z \to 0$,

$$\begin{aligned} &(\varphi_{,z})^2\big|_{z\to 0,\eta\to 0} \to 0, \quad (\varphi_{,\eta})^2\big|_{z\to 0,\eta\to 0} \to \infty, \\ &(\varphi_{,z}\varphi_{,\eta})\big|_{z\to 0,\eta\to 0} \to 0. \end{aligned} \tag{34}$$

According to Eq. (32), in region *I*, the scalar field potential

$$\varphi(\eta_0, z) \to 0 \tag{35}$$

at any fixed $\eta = \eta_0 \in (-\infty, -\Delta\eta) \cup (+\Delta\eta, +\infty)$ and for all $z \in (-\infty, +\infty)$. However, if we consider the distribution of scalar field potential at any fixed $\eta = \eta_0 \in [-\Delta\eta, \Delta\eta]$ (regions *II* and *III*), then, in the case $z \in (-\infty, -\Delta z) \cup (\Delta z, +\infty)$ (region *II*), it must be

$$\varphi(\eta_0, z) \to 0, \tag{36}$$

and, in the case $z \in [-\Delta z, \Delta z]$ (region *III*),

$$\frac{\varphi(\eta_0, z)_{III}}{\varphi(\eta_0, z)_{II}} > 1. \tag{37}$$

3. Distribution of the Scalar Field Potential for a "Smeared" Null String

Under conditions (35)–(37), it is convenient to express the distribution of a scalar field potential in the form

$$\varphi(\eta, z) = \ln\left(\frac{1}{\alpha(\eta) + \lambda(\eta) f(z)}\right), \tag{38}$$

where the functions $\alpha(\eta)$ and $\lambda(\eta)$ are symmetric with respect to the inversion $\eta \to -\eta$, i.e.

$$\alpha(\eta) = \alpha(-\eta), \quad \lambda(\eta) = \lambda(-\eta), \tag{39}$$

the function $\alpha(\eta) + \lambda(\eta) f(z)$ is confined,

$$0 < \alpha(\eta) + \lambda(\eta) f(z) \le 1, \tag{40}$$

and the scalar field potential (38), according to Eq. (40), can change from

$$\varphi \to 0, \quad \text{при} \quad \alpha(\eta) + \lambda(\eta) f(z) \to 1, \tag{41}$$

to

$$\varphi \to \infty, \quad \text{при} \quad \alpha(\eta) + \lambda(\eta) f(z) \to 0. \tag{42}$$

In region *I*, according to Eqs. (35) and (41),

$$\alpha(\eta) \to 1, \quad \lambda(\eta) \to 0. \tag{43}$$

Since, according to Eq. (36), the scalar field potential in region *II* tends to zero, then, for $\eta \in [-\Delta\eta, \Delta\eta]$ and any fixed $z = z_0 \in (-\infty, -\Delta z) \cup (\Delta z, +\infty)$, there must be

$$\alpha(\eta) + \lambda(\eta) f(z_0) \to 1. \tag{44}$$

In region *III*, for the same values $\eta \in [-\Delta\eta, \Delta\eta]$ and $z = z_0 \in [-\Delta z, \Delta z]$,

$$0 < \alpha(\eta) + \lambda(\eta) f(z_0) < 1. \tag{45}$$

From Eq. (44), it follows that, for all $z \in$ $\in (-\infty, -\Delta z) \cup (\Delta z, +\infty)$, the function $f(z)$ tends to a certain non-zero constant,

$$f(z)|_{z\in(-\infty,-\Delta z)\cup(\Delta z,+\infty)} \to f_0 = \text{const} \neq 0, \tag{46}$$

and the functions $\alpha(\eta)$ and $\lambda(\eta)$ are related to each other,

$$\lambda(\eta) = \frac{1}{f_0}\left(1 - \alpha(\eta)\right). \tag{47}$$

Substituting Eq. (47) into Eq. (45), we obtain that, in region *III*,

$$0 < \alpha(\eta) + (1 - \alpha(\eta))\frac{f(z_0)}{f_0} < 1. \tag{48}$$

Then, from equalities (42) and (48), it follows that, at $\varphi \to \infty$,

$$\alpha(\eta) \to 0, \quad f(z) \to 0. \tag{49}$$

Hence, in expression (38) for the scalar field potential, the functions $\alpha(\eta)$ and $f(z)$ are bounded for all $z \in (-\infty, +\infty)$ and $\eta \in (-\infty, +\infty)$,

$$0 < \alpha(\eta) < 1, \quad 0 < f(z) < f_0. \tag{50}$$

Moreover, in region *I*, according to Eq. (43),

$$\alpha(\eta)|_{\eta\in(-\infty,-\Delta\eta)\cup(+\Delta\eta,+\infty)} \to 1. \tag{51}$$

At the same time, from Eq. (49) with regard for the symmetry of the function $\alpha(\eta)$ (see equality (39)), it follows that

$$\lim_{\eta\to 0} \alpha(\eta) \to 0. \tag{52}$$

The distribution for the function $f(z)$ is determined by equality (46) if $z \in (-\infty, -\Delta z) \cup (\Delta z, +\infty)$. Whereas,

$$f(z)|_{z\to 0} \to 0 \tag{53}$$

as $z \to 0$, according to Eq. (49).

Differentiating Eq. (38) and taking Eq. (47) into account, we obtain

$$\begin{aligned}\varphi_{,\eta} &= -\frac{\alpha_{,\eta}(1 - f(z)/f_0)}{\alpha(\eta) + (1 - \alpha(\eta))f(z)/f_0},\\ \varphi_{,z} &= -\frac{(1 - \alpha(\eta))f_{,z}/f_0}{\alpha(\eta) + (1 - \alpha(\eta))f(z)/f_0}.\end{aligned} \tag{54}$$

Substituting Eqs. (43), (44), and (46) into Eq. (54), we obtain that $\varphi_{,z} \to 0$ and $\varphi_{,\eta} \to 0$ in regions *I* and *II*, which coincides with Eq. (32). In region *III*, if $z \to 0$, the first of equalities (54) can be expressed in the following form in view of Eq. (53):

$$\varphi_{,\eta} = -\alpha_{,\eta}/\alpha(\eta). \tag{55}$$

From whence, in accordance with Eq. (34), we have

$$|\alpha_{,\eta}/\alpha(\eta)|_{\eta\to 0} \to \infty \tag{56}$$

as $\Delta\eta \to 0$ and $\Delta z \to 0$. Taking Eq. (53) into account, the second of equalities (54) can be expressed in the following form, as $z \to 0$:

$$\varphi_{,z} = -f_{,z}/f(z). \tag{57}$$

In accordance with Eq. (34), this yields

$$f_{,z}/f(z)|_{z\to 0} \to 0 \tag{58}$$

as $\Delta\eta \to 0$ and $\Delta z \to 0$.

On the other hand, considering equalities (54) in a small vicinity of the circle ($\eta = 0, z = 0$), i.e. in the region where the scalar field is concentrated and for which, according to Eqs. (52) and (53), $f(z)/f_0 \ll 1$ and $\alpha(\eta) \ll 1$, we can write down

$$\varphi_{,z}\varphi_{,\eta} = \frac{(\alpha_{,\eta}/\alpha(\eta))}{\left(1 + \frac{1}{f_0}\frac{f(z)}{\alpha(\eta)}\right)}\frac{(f_{,z}/f(z))}{\left(1 + f_0\frac{\alpha(\eta)}{f(z)}\right)}. \tag{59}$$

Then, according to Eq. (34), there must be

$$\left(\frac{\alpha_{,\eta}}{\alpha(\eta)}\right)\left(\frac{f_{,z}}{f(z)}\right)\Big|_{z\to 0,\eta\to 0} \to 0 \tag{60}$$

as $\Delta z \to 0$ and $\Delta\eta \to 0$.

As an example, the functions

$$\alpha(\eta) = \exp\left(\frac{-1}{\epsilon + (\xi\eta)^2}\right), \tag{61}$$

$$f(z) = f_0 \exp\left(-\gamma\left(1 - \exp\left(\frac{-1}{(\zeta z)^2}\right)\right)\right), \tag{62}$$

where the constants ξ and ζ describe the size (the “thickness”) of the ring, in which the scalar field is concentrated, satisfy the found conditions with respect to the variables η and z, respectively. Namely,

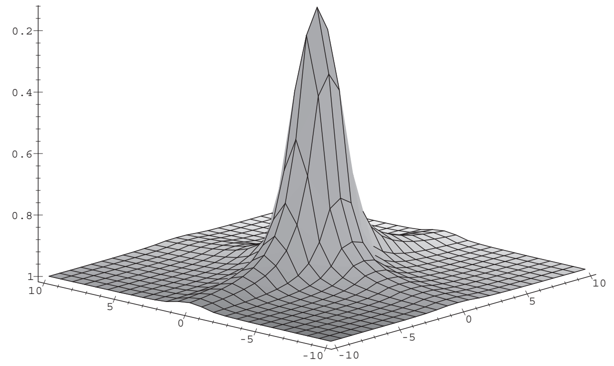

Fig. 1. Distributions of the function $\alpha(\eta)+(1-\alpha(\eta))f(z)/f_0$ for Eqs. (61) and (62) at $\epsilon=0.01$, $\xi=\zeta=1$, and $\gamma=4$ in the region $\eta\in[-10,10]$ and $z\in[-10,10]$

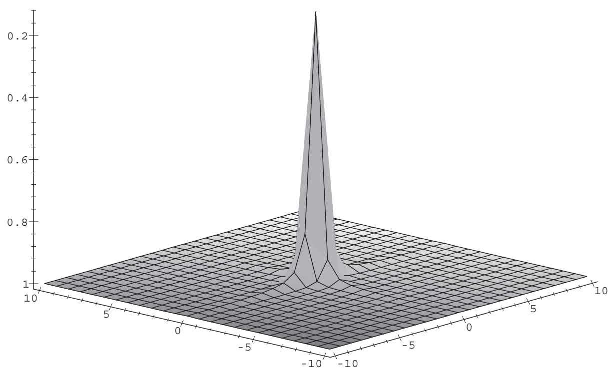

Fig. 2. The same as in Fig. 1, but for $\xi=\zeta=4$

as follows from Eqs. (61) and (62), if $\Delta\eta\to 0$ and $\Delta z\to 0$, we obtain

$$\xi\to\infty,\quad \zeta\to\infty. \tag{63}$$

The positive constants ϵ and γ provide the fulfillment of conditions (52), (53), (56), and (58) as $\Delta z\to 0$, $\Delta\eta\to 0$, $z\to 0$, and $\eta\to 0$; namely, at $\Delta\eta\ll 1$ and $\Delta z\ll 1$, we obtain

$$\epsilon\ll 1,\quad \gamma\gg 1, \tag{64}$$

and, at a further contraction into a one-dimensional object (a null string), i.e. as $\Delta z\to 0$ and $\Delta\eta\to 0$, we obtain

$$\epsilon\to 0,\quad \gamma\to\infty. \tag{65}$$

Substituting Eqs. (47), (61), and (62) into Eq. (38), we obtain an expression for one of the probable potential distributions for a real-valued massless scalar

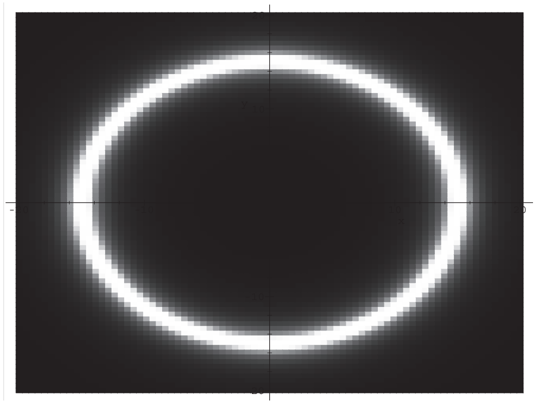

Fig. 3. Distribution of the scalar field potential given by Eqs. (38), (61), and (62) over the variable ρ ($\rho\in[0,20]$) at $\eta=t+\rho$, $z=0.01$, $\epsilon=0.01$, $\gamma=4$, $\xi=\zeta=0.6$, and the fixed $t=-15$

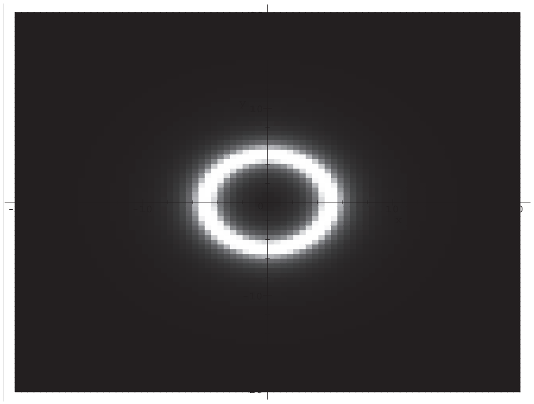

Fig. 4. The same as in Fig. 3, but for the fixed $t=-5$

field. During the contraction into a one-dimensional object, the components of its energy-momentum tensor asymptotically coincide with those of the energy-momentum tensor for a closed null strings that moves along trajectories (2) and (3).

In Figs. 1 and 2, the distributions of the function $\alpha(\eta)+(1-\alpha(\eta))f(z)/f_0$, where the functions $\alpha(\eta)$ and $f(z)$ are given by Eqs. (61) and (62), which were calculated for $\epsilon=0.01$ and $\gamma=4$, and corresponding to the constant values $\xi=\zeta=1$ (Fig. 1) and $\xi=\zeta=4$ (Fig. 2), are exhibited in the region $\eta\in[-10,10]$ and $z\in[-10,10]$. The figures demonstrate that, as the constants ξ and ζ increase, the region where the function concerned differs from unity (i.e. the region

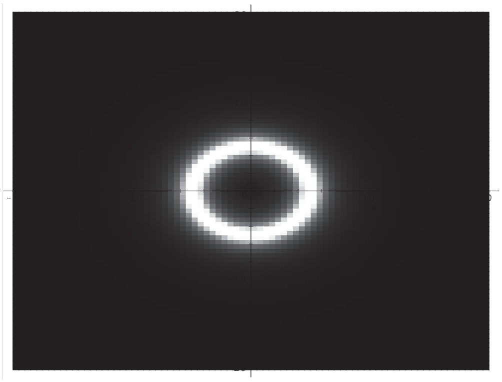

Fig. 5. The same as in Fig. 3, but at $\eta = t + \rho$ and for the fixed $t = 5$

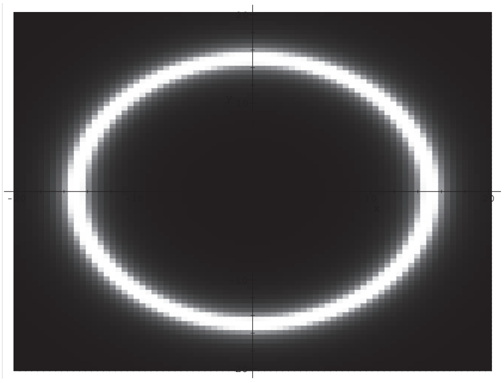

Fig. 6. The same as in Fig. 5, but for the fixed $t = 15$

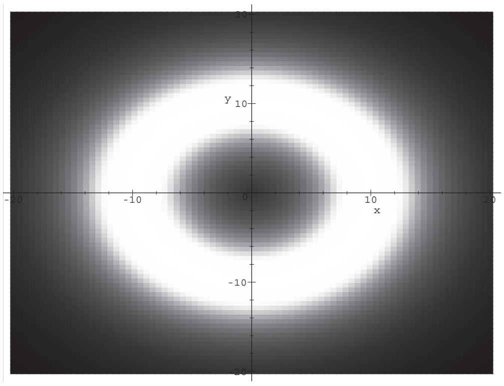

Fig. 7. Distributions of the scalar field potential given by Eqs. (38), (61), and (62) over the variable ρ ($\rho \in [0, 20]$) at $z = 0.01$, $\epsilon = 0.01$, $\gamma = 4$, $\xi = \zeta = 0.2$, and $t = -10$ ($\eta = t + \rho$) and 10 ($\eta = t - \rho$)

where the scalar field is concentrated, and the scalar field potential considerably differs from zero) contracts, and, respectively, the thickness of the ring, in which the scalar field is concentrated, diminishes.

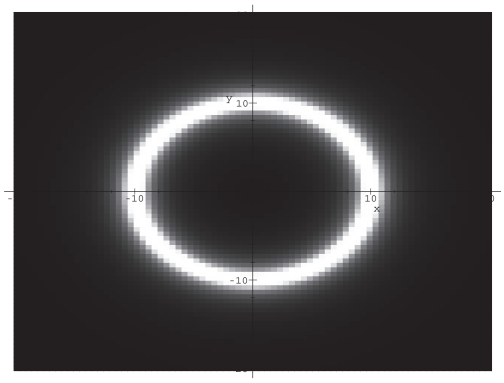

Fig. 8. The same as in Fig. 7, but at $\xi = \zeta = 0.6$

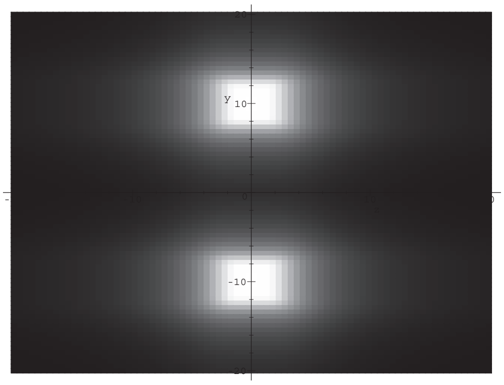

Fig. 9. Distribution of the scalar field potential given by Eqs. (38), (61), and (62) over the variable ρ ($\rho \in [0, 20]$) on the surface $\theta = \text{const}$ at $z \in [-10, 10]$, $\epsilon = 0.01$, $\gamma = 4$, and the fixed $\xi = \zeta = 0.2$, and for $t = -10$ ($\eta = t + \rho$) and 10 ($\eta = t - \rho$)

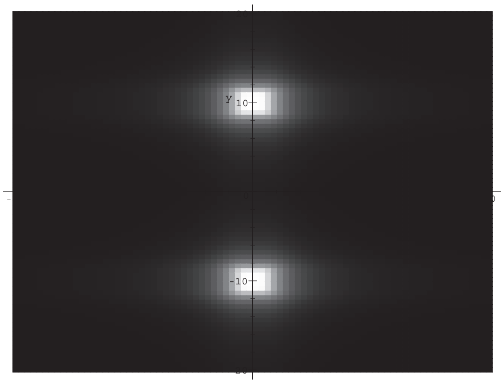

Fig. 10. The same as in Fig. 9, but for $\xi = \zeta = 0.6$

In Figs. 3 and 4, the distributions of the scalar field are shown for two fixed time values $t = -15$ and -5, respectively (here, the functions $\alpha(\eta)$ and $f(z)$ are given by Eqs. (61) and (62), and $\eta = t + \rho$).

From those figures, it follows immediately that, as the time t increases, the radius of a "smeared" null string decreases (the string collapses in the plane $z = 0$). The regions, in which $\varphi \to 0$, are marked by the dark color.

In Figs. 5 and 6, the distributions of the scalar field are shown for two fixed time values $t = 5$ and 15, respectively (here, $\eta = t - \rho$ and $t \in [0, +\infty)$). One can see that, as the time t increases, the radius of the "smeared" null string grows (the string radially expands in the plane $z = 0$).

In Figs. 7 and 8 (Figs. 9 and 10), the distributions of the scalar field potential (38) are shown at fixed values of variables t and z (t and θ), respectively (the functions $\alpha(\eta)$ and $f(z)$ are given by Eqs. (61) and (62)). The figures demonstrate that, as the constants ξ and ζ increase, the region where the scalar field potential substantially differs from zero becomes narrower, i.e. the thickness of the ring, in which the scalar field is concentrated, diminishes.

4. Conclusions

In this work, the systems of Einstein equations describing the distributions of a real-valued massless scalar field concentrated in a thin ring and in a closed null string that either collapses or contracts in the plane $z = 0$ are compared. The conditions imposed on the scalar field potential are obtained, under which the components of the energy-momentum tensors for a scalar field that contracts into a one-dimensional object (a circle with varying radius) and a closed null string asymptotically coincide. A general form of the potential distribution describing the motion of a scalar field concentrated in a thin ring is proposed in the cases where the ring either collapses or expands in the plane $z = 0$. An example is given for the scalar field potential distribution that satisfies those conditions.

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О.П. Леляков, А.С. Карпенко, Р.-Д.О. Бабаджан

РОЗПОДІЛ ПОТЕНЦІАЛУ СКАЛЯРНОГО ПОЛЯ ДЛЯ "РОЗМАЗАНОЇ" ЗАМКНЕНОЇ НУЛЬ-СТРУНИ, ЯКА ПРЯМУЄ В ПЛОЩИНІ $z = 0$

Р е з ю м е

У роботі запропоновано загальний вигляд розподілу потенціалу скалярного поля для "розмазаної" нуль-струни, яка колапсує в площині $z = 0$, а також для "розмазаної" нуль-струни, яка розширюється в площині $z = 0$. Знайдені умови, за яких компоненти тензора енергії-імпульсу скалярного поля, при стисканні поля в одновимірний об'єкт (коло змінного радіуса) асимптотично збігаються з компонентами тензора енергії-імпульсу замкненої нуль-струни, що прямує за тією самою траєкторією.