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INFLUENCE OF ANISOTROPIC SCATTERING MECHANISMS ON POLARIZATION DEPENDENCES OF TERAHERTZ RADIATION EMITTED BY HOT ELECTRONS

The influence of anisotropic scattering mechanisms on the polarization dependences of terahertz (THz) radiation emitted by hot electrons in multivalley semiconductors of the n-Ge type has been studied both theoretically and experimentally. The main attention is focused on a situation where the electric field applied to a multivalley semiconductor is directed asymmetrically with respect to the valleys. Changing from the anisotropic mechanism of electron scattering by ionized impurities to that by acoustic phonons is demonstrated to result in the transformation of maxima into minima in the periodic polarization angular dependence of the radiation intensity emitted by hot electrons. The substitution of one dominating mechanism of scattering by another one can result from a number of reasons: the lattice temperature variation or the variation in the concentration of ionized impurities, the change in the magnitude or the orientation of the electron-heating field, the application of a unidirectional pressure or the specimen illumination. All those factors are shown to affect the polarization dependences of spontaneous radiation emitted by hot electrons in the case where the temperatures of electrons in different valleys are also different.

Keywords: terahertz radiation, anisotropic scattering mechanisms, multivalley semiconductors.

1. Introduction

Free charge carriers in semiconductors can be involved in the processes of both absorption and emission of light. However, the conservation laws of energy and momentum cannot be satisfied simultaneously, when a free charge carrier interacts with a light quantum. Therefore, the processes concerned become possible only if "third bodies" take part in them. As such, these can be impurities and lattice vibrations in semiconductors, as well as the semiconductor surface. Which process (light absorption or emission) prevails depends on external conditions. If a system of charge carriers in the thermodynamically equilibrium state is irradiated with light, the absorption processes dominate. On the other hand, if external irradiation is absent but the charge carriers are heated up with the help of an electric field, radiation emission processes prevail. The latter have specific features in the case of multivalley semiconductors, which originate from the anisotropy of the electron dispersion law, the anisotropy of electron scattering mechanisms, and the very fact that there are several energetically equivalent valleys (minima) in the Brillouin zone.

Since the dispersion law for charge carriers is anisotropic in vicinities of those minima, the contribution made by electrons from a separate valley to the radiation spectrum has to be angular (polarization) dependent. However, some multivalley semiconductors, such as n-Ge and n-Si, have a cubic symmetry, so that the valleys are arranged symmetrically in the Brillouin zone. As a result, the total contribution made to radiation emission by all the valleys is averaged, and the angular dependence may disappear at all. This polarization dependence of the total (summed up over all valleys) radiation reveals itself only in the cases where the temperatures of electrons and their concentrations become different in the valleys owing to some reasons. Just this situation arises when the electron-heating field is oriented asymmetrically with respect to the valleys or when a unidirectional pressure is applied to the semiconductor.

The character of the polarization dependences is governed by both the dispersion law and the scattering mechanisms. The change of one scattering mechanism by another one (this change can be stimulated by the electron heating, the variation of the lattice temperature or the impurity concentration) results in that the polarization characteristics transform. In works [1]to [3], we studied the influence of the electron heating, lattice temperature, and pressure, respectively, on the polarization characteristics of radiation spontaneously emitted by hot electrons. In this work, we also consider a multivalley semiconductor of the n-Ge type. As an example of the asymmetric orientation of the electron-heating field, we chose the direction (1,1,1). In this case, there emerge three "hot" and one "cold" valleys. The electron temperatures in the valleys are determined, and their field dependences are plotted. We will demonstrate how the change of the dominating scattering mechanism affects the polarization characteristics. In particular, we show that the change of a scattering mechanism results in a smooth transformation of maxima in the periodic polarization angular dependence of radiation into minima. The field dependences of the coefficients describing the angular characteristics of radiation emission are obtained. The theoretical and experimental angular dependences of spontaneous radiation emission by hot electrons at various values of electric field intensity, electron and lattice temperatures, impurity concentration, and unidirectional pressure are plotted and explained.

2. Theory

In works [4, 5], the theory of spontaneous light emission in the THz frequency range by hot electrons in multivalley semiconductors of the n-Ge and n-Si types was developed. The dispersion law for the electron energy in such semiconductors near the minima in the Brillouin zone (i.e. in the "valleys") looks like

$$\varepsilon(\mathbf{p}) = \frac{p_x^2 + p_y^2}{2m_\perp} + \frac{p_z^2}{2m_\parallel}, \qquad (1)$$

where p_i are the components of the electron momentum, and $m_\parallel$ and $m_\perp$ are, respectively, the longitudinal and transverse components of the tensor of masses.

The dispersion law (1) was used in works [4, 5] to derive an expression for the energy emitted at the frequency ω and into the solid angle $d\Omega$ by electrons from every valley per unit time. Spontaneous emission by free electrons is possible only provided certain scattering mechanisms. Therefore, expressions for the angular dependence of radiation energy emitted by hot electrons were obtained in works [4,5] with regard for the anisotropy of the acoustic and impurity-based mechanisms of scattering.

2.1. Acoustic scattering

Let the orientation of the rotation axis of ellipsoid (1) for electrons in the k-th valley be given by the unit vector $\mathbf{i}_k$. Then, in accordance with the results of works [4, 5], the expression for the energy emitted by hot electrons of this valley per unit time at the frequency ω into the solid angle $d\Omega$, when they are scattered by acoustic phonons, reads

$$W_k^{(a)} = -\frac{2e^2}{3\,\pi^{5/2}c^3} n_k \theta_k \left\{ \frac{1}{m_\perp \tau_\perp^{(a)}(\theta_k)} + \right. \\ \left. + \left(\frac{1}{m_\parallel \tau_\parallel^{(a)}(\theta_k)} - \frac{1}{m_\perp \tau_\perp^{(a)}(\theta_k)} \right) (\mathbf{i}_k \mathbf{q}_0)^2 \right\} \times \\ \times (a_k)^3 e^{-a_k} \frac{d}{da_k} \left(K_1(a_k)/a_k \right) d\Omega. \qquad (2)$$

Here, n_k is the concentration of electrons from the k-th valley, θ_k is their temperature (in energy units), the unit vector $\mathbf{q}_0$ describes the polarization direction of an emitted electromagnetic wave, $a_k = \hbar\omega/\theta_k$, and $K_1(a_k)$ is the Bessel function of imaginary argument.

The anisotropic acoustic scattering of electrons in multivalley semiconductors of the n-Ge and n-Si types can be characterized by two components of the relaxation time tensor, $\tau_\perp^{(a)}(\varepsilon)$ and $\tau_\parallel^{(a)}(\varepsilon)$ (see, e.g., work [6]). The dependences of those components on the energy of an electron from the k-th valley look like

$$\tau_x^{(a)}(\varepsilon) = \tau_y^{(a)}(\varepsilon) \equiv \tau_\perp^{(a)}(\varepsilon) = \tau_\perp^{(0)} \left(\frac{\theta}{\varepsilon}\right)^{1/2}, \\ \tau_z^{(a)}(\varepsilon) \equiv \tau_\parallel^{(a)}(\varepsilon) = \tau_\parallel^{(0)} \left(\frac{\theta}{\varepsilon}\right)^{1/2}. \qquad (3)$$

From whence, we have

$$\tau_\perp^{(a)}(\theta) = \tau_\perp^{(0)}, \quad \tau_\parallel^{(a)}(\theta) = \tau_\parallel^{(0)}. \qquad (4)$$

In expressions (3) for $\tau_\perp^{(a)}(\varepsilon)$ and $\tau_\parallel^{(a)}(\varepsilon)$, the quantities $\tau_\perp^{(0)}$ and $\tau_\parallel^{(0)}$ do not depend on the temperature of hot electrons, but only on the lattice temperature and other parameters (the constants of deformation potential, the components of mass tensor, and so forth). The parameters $\tau_\perp^{(0)}$ and $\tau_\parallel^{(0)}$ can be expressed in terms of the components of the acoustic mobility tensor for cold (not heated up) electrons, proceeding from the relations

$$\mu_\perp^{(a)}(\theta) = \frac{4}{3\sqrt{\pi}}\,\frac{e\,\tau_\perp^{(0)}}{m_\perp}, \quad \mu_\parallel^{(a)}(\theta) = \frac{4}{3\sqrt{\pi}}\,\frac{e\,\tau_\parallel^{(0)}}{m_\parallel}, \tag{5}$$

where θ is the lattice temperature measured in energy units.

The general expression for spontaneous radiation emitted by hot electrons (2) can be substantially simplified in the limiting cases $\hbar\omega \ll \theta_e$ (the classical frequency interval) and $\hbar\omega \gg \theta_e$ (the quantum-mechanical interval). In particular, in the classical limit (at $\hbar\omega \ll \theta_e$), Eq. (2) yields

$$W_k^{(a)} = \frac{4e^2}{3\pi^{5/2}c^3} n_k\theta_k \times \times \left\{ \frac{\sin^2\varphi_k}{m_\perp\tau_\perp^{(a)}(\theta)} + \frac{\cos^2\varphi_k}{m_\parallel\tau_\parallel^{(a)}(\theta_k)} \right\} d\Omega, \tag{6}$$

Here, $\cos\varphi_k = \mathbf{i}_k\mathbf{q}_0$, and φ_k is the angle between the polarization direction and the orientation direction of the "valley" (the ellipsoid of revolution). Analogously, in the quantum-mechanical frequency limit ($\hbar\omega \gg \theta_e$), we obtain

$$W_k^{(a)} = \frac{e^2}{6\,\pi^2c^3}\,\frac{n_k}{\sqrt{\theta}}(\hbar\omega)^{3/2}e^{-\hbar\omega/\theta_k} \times \times \left\{ \frac{\sin^2\varphi_k}{m_\perp\tau_\perp^{(0)}} + \frac{\cos^2\varphi_k}{m_\parallel\tau_\parallel^{(0)}} \right\} d\Omega. \tag{7}$$

2.2. Impurity (Coulomb) scattering

In work [5], the theory of light absorption and emission by hot electrons in the case where the Coulomb scattering plays the dominating role was developed. The impurity potential was adopted in the form

$$\upsilon(r) = \frac{e^2}{\varepsilon_0 r}e^{-r/r_\mathrm{D}}, \tag{8}$$

where ε_0 is the static dielectric constant, and r_D is the Debye radius.

The general expression for spontaneous radiation emission by hot electrons at the dominant role of Coulomb scattering is presented in work [5]. Since this expression is cumbersome, only its forms in the limiting cases are presented here. In particular, in the classical frequency range ($\hbar\omega \ll \theta_e$), if the Coulomb scattering dominates, we obtain [5]

$$W_k^{(c)} \equiv \frac{3e^2}{16\,\pi^{3/2}c^3} n_k\theta_k \times \times \left\{ \frac{\sin^2\varphi_k}{m_\perp\tau_\perp^{(c)}(\theta_k)} + \frac{\cos^2\varphi_k}{m_\parallel\tau_\parallel^{(c)}(\theta_k)} \right\} d\Omega, \tag{9}$$

where $\tau_\perp^{(c)}(\theta_k)$ and $\tau_\parallel^{(c)}(\theta_k)$ are the transverse and longitudinal components of the relaxation time tensor at the Coulomb scattering of hot electrons,

$$\frac{1}{\tau_\perp^{(c)}(\theta_k)} = \frac{8}{3}\,\frac{e^4(2m_\parallel)^{1/2}}{\varepsilon_0^2\,m_\perp\theta_k^{3/2}} n_c\beta_\perp\,\ln(C_1\,x_\mathrm{min})^{-1}, \tag{10}$$

$$\frac{1}{\tau_\parallel^{(c)}(\theta_k)} = \frac{8}{3}\,\frac{e^4(2m_\parallel)^{1/2}}{\varepsilon_0^2\,m_\parallel\theta_k^{3/2}} n_c\beta_\parallel\,\ln(C_1\,x_\mathrm{min})^{-1}. \tag{11}$$

Here, n_c is the concentration of ions (according to the electroneutrality condition, $n_c = \sum_k n_k$), $\ln C_1 \approx \approx 0.577...$ is the Euler constant, and the notations

$$\beta_\perp = \frac{b_0}{2}\left[b_0 + (1-b_0^2)\,\mathrm{arctg}\,\frac{1}{b_0}\right], \tag{12}$$

$$\beta_\parallel = b_0\left[-b_0 + (1+b_0^2)\,\mathrm{arctg}\,\frac{1}{b_0}\right], \tag{13}$$

$$b_0^2 = \frac{m_\perp}{m_\parallel - m_\perp}; \quad x_\mathrm{min} = \frac{\hbar^2}{8\,m_\perp\theta_k r_\mathrm{D}^2}, \tag{14}$$

are introduced. (Unfortunately, the notations $\frac{1}{\tau_\perp^{(c)}(\theta_k)}$ and $\frac{1}{\tau_\parallel^{(c)}(\theta_k)}$ in work [5] were confused).

In the case of the impurity scattering (at potential (8)), the components of the relaxation time tensor (10) and (11) are related to the corresponding components of the mobility tensor for electrons from the k-th valley by the formulas

$$\mu_\perp^{(c)} = \frac{8}{\sqrt{\pi}}\,\frac{e\,\tau_\perp^{(c)}(\theta_k)}{m_\perp}, \quad \mu_\parallel^{(c)} = \frac{8}{\sqrt{\pi}}\,\frac{e\,\tau_\parallel^{(c)}(\theta_k)}{m_\parallel}. \tag{15}$$

In the quantum frequency limit ($\hbar\omega \gg \theta_e$) and in the case of the Coulomb scattering, we have

$$W_k^{(c)} = \frac{e^6\sqrt{2\,m_\parallel}}{\pi\,\varepsilon_0^2\,c^3}\,\frac{n_c\,e^{-\hbar\omega/\theta_k}}{\sqrt{\hbar\omega}}\,n_k \times \times \left\{ \frac{\beta_\perp}{m_\perp^2}\,\sin^2\varphi_k + \frac{\beta_\parallel}{m_\parallel^2}\,\cos^2\varphi_k \right\} d\Omega. \tag{16}$$

If the acoustic and impurity (Coulomb) mechanisms of scattering act simultaneously, the total energy emitted by hot electrons is equal to the sum of energies associated with separate scattering mechanisms.

The formulas for the "acoustic" [Eqs. (6) and (7)] and "impurity" [Eqs. (9) and (16)] radiation emission by hot electrons can be expressed in more convenient forms, if we use the relations

$$\sin^2\varphi_k = \frac{1}{2}\,(1-\cos\,2\varphi_k);\quad \cos^2\varphi_k = \frac{1}{2}\,(1+\cos\,2\varphi_k).$$

Then, e.g., if $\hbar\omega < \theta_k$, Eq. (6) yields

$$W_k^{(a)} = G_k^{(a)}(1 + g_a\cos 2\varphi_k)d\underline{0}, \tag{17}$$

where

$$G_k^{(a)} = \frac{2\,e^2 n_k\theta_k}{3\,\pi^{5/2}c^3}\left(\frac{1}{m_\parallel\tau_\parallel^{(a)}(\theta_k)} + \frac{1}{m_\perp\tau_\perp^{(a)}(\theta_k)}\right), \tag{18}$$

$$g_a = \left(\frac{1}{m_\parallel\tau_\parallel^{(0)}} - \frac{1}{m_\perp\tau_\perp^{(0)}}\right)\Bigg/\left(\frac{1}{m_\parallel\tau_\parallel^{(0)}} + \frac{1}{m_\perp\tau_\perp^{(0)}}\right). \tag{19}$$

An analogous expression can be written down for the "impurity" radiation emission,

$$W_k^{(c)} = G_k^{(c)}(1 + g_c\cos 2\varphi_k)d\Omega. \tag{20}$$

Using Eq. (10), we obtain

$$g_c = \left(\left(\frac{m_\perp}{m_\parallel}\right)^2\frac{\beta_\parallel}{\beta_\perp} - 1\right)\Bigg/\left(\left(\frac{m_\perp}{m_\parallel}\right)^2\frac{\beta_\parallel}{\beta_\perp}^2 + 1\right). \tag{21}$$

3. Manifestation of Multivalley Structure in Terahertz Radiation Emitted by Hot Electrons

In the previous section, we have analyzed the contribution made to spontaneous radiation emission by hot electrons belonging to the same valley. Now, we consider the total radiation emission of hot electrons from all valleys. As an example, we examine hot electrons in n-Ge. In this case, the orientations of "valleys" (the rotation axis of the mass tensor) are given by the unit vectors

$$\mathbf{i}_1 = \frac{1}{\sqrt{3}}(1,1,1),\ \mathbf{i}_2 = \frac{1}{\sqrt{3}}(-1,1,1),$$

$$\mathbf{i}_3 = \frac{1}{\sqrt{3}}(1,-1,1),\ \mathbf{i}_4 = \frac{1}{\sqrt{3}}(-1,-1,1). \tag{22}$$

The Joule power released in the k-th valley by the field $\mathbf{F}$ applied to the semiconductor looks like

$$W_{\rm D}(\theta_k) = n_k\Big\{\mu_\perp(\theta_k)F^2 + (\mu_\parallel(\theta_k) - \mu_\perp(\theta_k))(\mathbf{i}_k\mathbf{F})^2\Big\}, \tag{23}$$

where n_k and θ_k are the concentration and the temperature (in energy units), respectively, of electrons in the k-th valley. The electron temperature of the k-th valley is determined from the energy balance equation, i.e. from the equality between the Joule power released in the k-th valley and the energy transmitted by hot electrons in the k-th valley to lattice vibrations per unit time, $\left(\frac{dE_k}{dt}\right)_{st}$. In particular, for the scattering by longitudinal acoustic phonons, we have [6, 7]

$$\left(\frac{dE_k}{dt}\right)_{st}^{(a)} = n_k\,\frac{8\sqrt{2}\,\Sigma_0^2 m_\perp^2\sqrt{m_\parallel}}{\pi^{3/2}\,\hbar^4\,\rho}\theta_k^{3/2}\left(1 - \frac{\theta}{\theta_k}\right), \tag{24}$$

where ρ is the density, and Σ_0 is an energy constant. Analogously, for the scattering by optical lattice vibrations, we obtain

$$\left(\frac{dE_k}{dt}\right)_{st}^{(0)} = n_k\,\frac{\sqrt{2}\,D^2\,m_\perp\sqrt{m_\parallel}}{\pi^{3/2}\,\hbar^2\,\rho}\,n_k\sqrt{\theta_k}\,\frac{\exp\left(\frac{\hbar\,\omega_0}{\theta} - \frac{\hbar\,\omega_0}{\theta_k}\right) - 1}{\exp\left(\frac{\hbar\,\omega_0}{\theta}\right) - 1}\times \left(\frac{\hbar\,\omega_0}{2\,\theta_k}\right)\exp\left(\frac{\hbar\,\omega_0}{2\,\theta_k}\right)K_1\left(\frac{\hbar\,\omega_0}{2\,\theta_k}\right), \tag{25}$$

where ω_0 is the frequency of optical lattice vibrations, D the constant of the electron coupling with optical lattice vibrations, and $K_1(x)$ the Bessel function of the second kind of imaginary argument.

In the model of energetically independent valleys, the temperature of electrons in the k-th valley is determined by the energy balance equation

$$\left(\frac{dE_k}{dt}\right)_{st}^{(a)} + \left(\frac{dE_k}{dt}\right)_{st}^{(0)} = W_{\rm D}(\theta_k). \tag{26}$$

A more rigorous model must also involve the intervalley migration of electrons and the intervalley energy transmission from "hot" valleys to "cold" ones. Those issues were analyzed, e.g., in work [7] in detail, and they will not be considered here. Let us dwell in more details on the dependence of the electron heating on the heating field orientation.

From Eqs. (22) and (23), one can see that, if the electron-heating field is directed along a direction that is symmetric with respect to the valleys, i.e. along the direction (1,0,0), then $(\mathbf{i}_k\mathbf{F})^2 = \frac{1}{3}F^2$ for $k = 1, 2, 3, 4$. We obtain

$$W(\theta_k) = n_k \left(\frac{2}{3}\,\mu_\perp(\theta_k) + \frac{1}{3}\,\mu_\parallel(\theta_k)\right) F^2. \qquad (27)$$

Hence, if the electron-heating field is oriented along the direction (1,0,0), then, according to Eq. (27), the Joule power per one electron is identical for all valleys. This means that Eq. (26) predicts the identical temperatures θ_k for all valleys ($k = 1, 2, 3, 4$), with the electron concentrations n_k in all valleys being identical as well. If so, the polarization dependence of total spontaneous radiation emitted from all valleys disappears. Really, one can see from Eq. (6) or (9) that, provided identical concentrations and temperatures, the angular dependence remains to be contained only in the factors $\sin^2\varphi_k$ and $\cos^2\varphi_k$. Since, in accordance with Eq. (22),

$$\sum_{k=1}^{3}\cos^2\varphi_k = \sum_{k=1}^{4}(\mathbf{i}_k\,\mathbf{q}_0)^2 = \frac{4}{3}\,(q_{0x}^2 + q_{0y}^2 + q_{0z}^2) = \frac{4}{3},$$

we obtain from Eq. (6) that, in the case where the acoustic scattering dominates in the classical frequency limit,

$$\sum_{k=1}^{4} W_k = \frac{16}{9}\,\frac{e^2}{\pi^{5/2}c^3}\,n_e\,\theta_e \times$$

$$\times\left\{\frac{2}{m_\perp\tau_\perp^{(a)}(\theta_k)} + \frac{1}{m_\parallel\tau_\parallel^{(a)}(\theta_k)}\right\}d\Omega. \qquad (28)$$

Since the concentration of electrons and their temperature in this case, i.e. if the field $\mathbf{F}$ is directed along the axis (1,0,0), do not depend on the number of valley, we made the substitution of $n_k\theta_k$ by $n_e\theta_e$ in expression (28).

A formula similar to Eq. (28) can be obtained for the impurity scattering, by proceeding from expression (9). Hence, in spite of the angular dependence of spontaneous radiation emission by hot electrons from one valley, the total emission of all valleys does not depend on the angles, provided that the electron temperatures in the valleys are identical.

4. The Case of Different Electron Temperatures in the Valleys

Let the heating field be oriented along the direction (1,1,1), i.e.

$$\mathbf{F} = F\mathbf{i}_1. \qquad (29)$$

Then, in accordance with Eq. (22), we have

$$(\mathbf{i}_1\mathbf{F})^2 = F^2, \quad (\mathbf{i}_k\mathbf{F})^2 = \frac{1}{9}F^2; \quad k = 2, 3, 4. \qquad (30)$$

From Eq. (23) with regard for Eq. (30), we obtain

$$W_\mathrm{D}(\theta_1) = n_1\mu_\parallel(\theta_1)F^2, \qquad (31)$$

$$W_\mathrm{D}(\theta_k) = n_k\left\{\frac{8}{9}\mu_\perp(\theta_k) + \frac{1}{9}\mu_\parallel(\theta_k)\right\}F^2, \quad k = 2, 3, 4. \qquad (32)$$

The energy balance equation (26) and expressions (31) and (32) show that, if the field is oriented along the direction (1,1,1), three valleys ($k = 2, 3, 4$) are characterized by the same electron temperature and electron concentration,

$$\theta_2 = \theta_3 = \theta_4; \quad n_2 = n_3 = n_4. \qquad (33)$$

whereas one valley ($k = 1$) has the n_1- and θ_1-values different from those in Eq. (33). Since $\mu_\perp \gg \mu_\parallel$ for n-Ge, we may assert that there is one "cold" and three "hot" (with identical temperatures) valleys at this field orientation.

Radiation emission by hot electrons in all valleys is described by the formula

$$\sum_{k=1}^{4} W_k(\theta_k, n_k) = \sum_{k=1}^{4} W_k(\theta_k, n_k) +$$

$$+\,W_1(\theta_1, n_1) - W_1(\theta_2, n_2). \qquad (34)$$

On the right-hand side of expression (34), the same term $W_1(\theta_2, n_2)$–the contribution of the ($k = 1$)-valley, but with the same θ_2 and n_2 parameters as for other three hot valleys–is added and subtracted. As a result of this procedure, we obtain the sum over all valleys but with identical temperatures and concentrations of electrons, and two more terms, $W_1(\theta_{1'}n_1)$ and $-W_1(\theta_{2'}n_2)$. According to Eq. (28), the total radiation emission from all valleys with the same electron concentration and temperature has no angular (polarization) dependence. Hence, the entire angular dependence of spontaneous radiation emitted by hot electrons in n-Ge in the case where the electron-heating field $\mathbf{F}$ is oriented along the direction (1,1,1) is given by the difference $W_1(\theta_{1'}n_1)-W_1(\theta_{2'}n_2) \equiv \Delta W$.

In the classical frequency range, by supposing the dominant role of acoustic dispersion and taking Eq. (6) into account, we obtain

$$\Delta W^{(a)} = \frac{4e^2}{3\pi^{5/2}c^3}\left\{\left(\frac{n_1\theta_1}{m_\perp\tau_\perp^{(a)}(\theta_1)} - \frac{n_2\theta_2}{m_\perp\tau_\perp^{(a)}(\theta_2)}\right)\times \right.$$
$$\left.\times\sin^2\varphi_1 + \left(\frac{n_1\theta_1}{m_\parallel\tau_\parallel^{(a)}(\theta_1)} - \frac{n_2\theta_2}{m_\parallel\tau_\parallel^{(a)}(\theta_2)}\right)\cos^2\varphi_1\right\}d\Omega. \tag{35}$$

Expressing $\sin^2\varphi_1$ and $\cos^2\varphi_1$ in terms of $\cos 2\varphi_1$, formula (35) is transformed to the form analogous to expression (17) obtained earlier for the one-valley model,

$$\Delta W^{(a)} = \Delta G^{(a)}\left\{\perp + \Delta g_a\cos 2\varphi_1\right\}. \tag{36}$$

Here, the following notations are used:

$$\Delta g_a = \left\{n_1\theta_1\left[\frac{1}{m_\perp\tau_\perp^{(a)}(\theta_1)} - \frac{1}{m_\parallel\tau_\parallel^{(a)}(\theta_1)}\right] - \right.$$
$$\left. - n_2\theta_2\left[\frac{1}{m_\perp\tau_\perp^{(a)}(\theta_2)} - \frac{1}{m_\parallel\tau_\parallel^{(a)}(\theta_2)}\right]\right\}\times$$
$$\times\left\{n_1\theta_1\left[\frac{1}{m_\perp\tau_\perp^{(a)}(\theta_1)} - \frac{1}{m_\parallel\tau_\parallel^{(a)}(\theta_1)}\right] + \right.$$
$$\left. + n_2\theta_2\left[\frac{1}{m_\perp\tau_\perp^{(a)}(\theta_2)} - \frac{1}{m_\parallel\tau_\parallel^{(a)}(\theta_2)}\right]\right\}^{-1} \tag{37}$$

and

$$\Delta G^{(a)} = \frac{2e^2}{3\pi^{5/2}c^3}\left\{n_1\theta_1\left[\frac{1}{m_\perp\tau_\perp^{(a)}(\theta_1)} - \frac{1}{m_\parallel\tau_\parallel^{(a)}(\theta_1)}\right] + \right.$$
$$\left. + n_2\theta_2\left[\frac{1}{m_\perp\tau_\perp^{(a)}(\theta_2)} - \frac{1}{m_\parallel\tau_\parallel^{(a)}(\theta_2)}\right]\right\}d\Omega. \tag{38}$$

In the case of the acoustic scattering, according to Eq. (3), we have

$$\frac{1}{m_\perp\tau_\perp^{(a)}(\theta_1)} - \frac{1}{m_\parallel\tau_\parallel^{(a)}(\theta_1)} = \left(\frac{\theta_1}{\theta}\right)^{1/2}\times$$
$$\times\left\{\frac{1}{m_\perp\tau_\perp^{(0)}} - \frac{1}{m_\parallel\tau_\parallel^{(0)}}\right\}.$$

Therefore, Eqs. (37) and (38) can be rewritten as follows:

$$\Delta g_a = \left\{1 - \frac{n_2}{n_1}\left(\frac{\theta_2}{\theta_1}\right)^{3/2}\right\}\Big/\left\{1 + \frac{n_2}{n_1}\left(\frac{\theta_2}{\theta_1}\right)^{3/2}\right\}, \tag{39}$$

$$\Delta G^{(a)} = \frac{2e^2}{3\pi^{5/2}c^3}\left(\frac{1}{m_\perp\tau_\perp^{(0)}} - \frac{1}{m_\parallel\tau_\parallel^{(0)}}\right)\times$$
$$\times\left\{n_1\theta_1^{3/2} + n_2\theta_2^{3/2}\right\}\frac{d\Omega}{\sqrt{\theta}}. \tag{40}$$

Analogously, in the case of the impurity scattering in the classical frequency limit with regard for formulas (9)–(11), we obtain

$$\Delta W^{(c)} = \Delta G^{(c)}\left\{1 + \Delta g_c\cos 2\varphi_1\right\}, \tag{41}$$

where the notations

$$\Delta g_c = \left\{1 - \frac{n_2}{n_1}\left(\frac{\theta_2}{\theta_1}\right)^{1/2}\right\}\Big/\left\{1 + \frac{n_2}{n_1}\left(\frac{\theta_2}{\theta_1}\right)^{1/2}\right\} \tag{42}$$

and

$$\Delta G^{(c)} = \frac{3e^2}{32\pi^{3/2}c^3}\left(\frac{1}{m_\perp\tau_\perp^{(c)}(\theta_1)} - \frac{1}{m_\parallel\tau_\parallel^{(c)}(\theta_1)}\right)\theta_1^{3/2}\times$$
$$\times\left[\frac{n_1}{\sqrt{\theta_1}} + \frac{n_2}{\sqrt{\theta_2}}\right]d\Omega, \tag{43}$$

were introduced. Expressions for $\tau_{\perp,\parallel}^{(c)}(\theta_1)$ in Eq. (43) are given by formulas (10) and (11). Note that, if the weak (under the logarithm sign) dependences on the electron temperature and concentration are neglected, the expression

$$\left(\frac{1}{m_\perp\tau_\perp^{(c)}(\theta_1)} - \frac{1}{m_\parallel\tau_\parallel^{(c)}(\theta_1)}\right)\theta_1^{3/2}$$

can be regarded as a known parameter (a numerical value) for the given specimen.

We would like to attract attention to the fact that the quantities $\Delta W^{(a)}$ and $\Delta W^{(c)}$ depend differently on the electron and lattice temperatures. In addition, $\Delta W^{(c)}$ also depends on the concentration of ionized impurities. Therefore, the condition $\Delta W^{(a)} = \Delta W^{(c)}$ determines the concentration of ionized impurities, at which the acoustic and Coulomb contributions to radiation emission become equal to each other, provided that the temperature is fixed. Hence, it is easy to select such a concentration of ionized impurities, at which the transition from one dominating scattering mechanism to the other can be executed by varying the temperature of electrons (with the help of the applied electric field).

5. Discussion of the Results and Their Comparison with Experiment

Formulas (36) and (41) determine the angular dependence of spontaneous radiation emitted by hot electrons in the classical frequency range in semiconductors with the band structure of the n-Ge type in the case where the electron-heating field is oriented along the direction (1,1,1). In this case, we have one cold valley (with the electron concentration n_1 and temperature θ_1) and three hot ones (with the identical electron concentration n_2 and temperature θ_2). The temperatures θ_1 and θ_2 can be determined from the energy balance equation (see Eq. (26)). Let us discuss the electron concentrations (in our case, these are n_1 and n_2).

Our calculations were based on the results of experiments carried out at low temperatures (the lattice temperature was equal to 4 K). The concentrations of electrons in the valleys were determined from the concentration balance equation. As a rule, this equation is derived with the use of the electron distribution functions in the valleys taken as the Maxwellian function with the corresponding effective electron temperature. The electron migration between the valleys occurs owing to high-energy "tails" in the electron distribution functions, i.e. with the participation of electrons with the energy $\varepsilon > \hbar\omega_\mu$, where ω_μ is the frequency of intervalley phonons. Therefore, the effective temperature approximation in the framework of this energy balance automatically means that the electron–electron scattering dominates at energies $\varepsilon > \hbar\omega_\mu$ as well. However, the $e - e$ scattering intensity is known to be proportional to the squared electron concentration, whereas the corresponding scattering cross-section is reciprocal to the relative velocity raised to a power of four. Hence, when the electron concentration is not high enough, the $e - e$ scattering can thrust the Maxwellian distribution with a certain effective temperature on a group of electrons with energies close to the average one ($\varepsilon \sim \theta_e$), but it ceases to remain the dominating mechanism of scattering for electrons in the energy "tail" with energies $\varepsilon > \hbar\omega_\mu$. As a result, the electron migration between the valleys becomes insignificant. Just this situation is realized in our case (relatively high-resistance specimens and low temperatures). Therefore, we consider below that $n_1 \approx n_2$. Nevertheless, it is worth noting that, besides the different heating of electrons in the valleys that occurs at definite field orientations and acts as a driving force, the electron migration between the valleys can also be a result of the unidirectional deformation in the specimen. In this case, the concentrations of electrons in the valleys are also the functions of applied unidirectional pressure.

Hence, if the field is oriented along the direction (1,1,1), we obtain one "cold" valley with the electron temperature θ_1 and three "hot" ones with the temperature θ_2. In this case,

$$\theta_2 > \theta_1; \quad n_2 \approx n_1. \tag{44}$$

From Eq. (39), one can see that, in the case where the acoustic scattering dominates and conditions (44) are satisfied, the value of Δg_a is negative, whereas, at the impurity scattering in accordance with Eq. (39), the value of Δg_c is positive. So, a conclusion can be drawn that, if, owing to the temperature growth, the mechanism of scattering transforms from the impurity to the acoustic one, the angular dependences of the radiation intensity would undergo the following modification: minima of intensity should be observed at those angles where it had maxima.

If the concentration of ionized impurities is known, the equality $\Delta W^{(a)} = \Delta W^{(c)}$ allows one to determine the temperature, at which the transition takes place (as the temperature grows) from the impurity mechanism of scattering to the acoustic one. In Fig. 1, the theoretically calculated temperatures in the "cold" and "hot" valleys as the functions of the applied electric field are plotted. In the corresponding calculations, we considered that, at the parameter values quoted in the figure caption, the energy losses of elec-

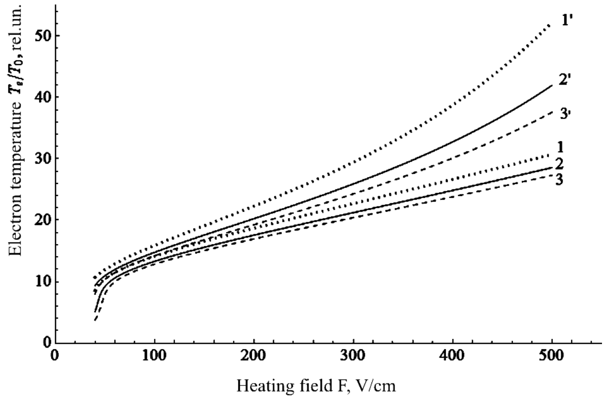

Fig. 1. Dependences of the electron temperatures in "cold" (numbers) and "hot" (primed numbers) valleys of n-Ge on the heating field at various electron concentrations $n = 0.8 \times 10^{14}$ (curves *1* and *1′*), 1.2×10^{14} (curves *2* and *2′*), and 1.5×10^{14} cm^{-3} (curves *3* and *3′*). The lattice temperature $T_0 = 4.2$ K

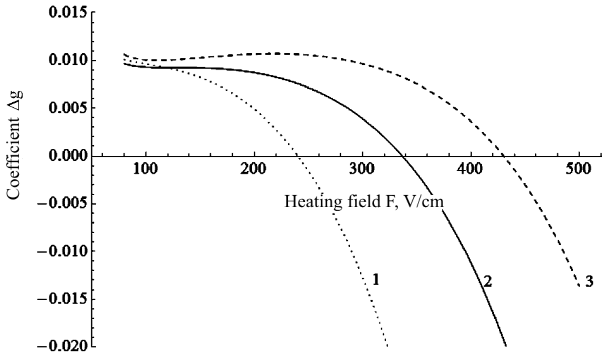

Fig. 2. Dependences of the coefficient Δg on the heating field F for various concentrations of charge carriers in n-Ge $n = 0.8\times10^{14}$ (*1*), 1.2×10^{14} (*2*), and 1.5×10^{14} cm^{-3} (*3*). The lattice temperature $T_0 = 4.2$ K

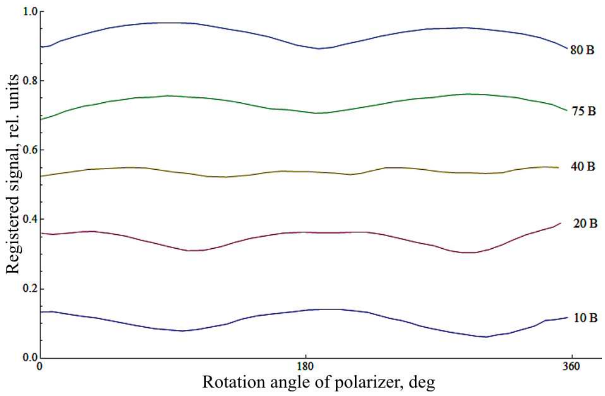

Fig. 3. Polarization angular dependences of THz radiation emitted by n-Ge ($n = 2.5 \times 10^{15}$ cm^{-3}) under the action of electric field

trons stem from their interaction with lattice acoustic vibrations, and the momentum relaxation results from the electron scattering by impurities and acoustic phonons. We also considered the electron momentum relaxation at the spontaneous emission of acoustic phonons, which is a substantial factor at low temperatures [9]. In calculations, the minimum temperature of electrons was selected to equal 60 K, because the validity of the Born approximation used to derive the expressions for the electron mobility components becomes problematic at lower temperatures.

Note that, if either of the scattering mechanisms dominates in the momentum relaxation (the momentum is scattered by either acoustic phonons or ionized impurities) and the interaction of electrons with acoustic phonons prevails in the energy relaxation, the expressions for the temperatures of "cold" and "hot" electrons can be obtained from formulas (23), (24), and (26) in the general analytical form. In particular, if the relaxation of both the electron momentum and energy takes place on acoustic phonons, we obtain the following expression for the temperature of electrons in the k-th valley, the orientation of which is given, in accordance with Eq. (27), by the unit vector $\mathbf{i}_k$:

$$\theta_k = \frac{\theta}{2} + \left\{\left(\frac{\theta}{2}\right)^2 + \frac{\sqrt{\theta}}{c_a}\left[\mu_\perp^{(a)}(\theta)\,F^2 + \right.\right.$$
$$\left.\left. + \left(\mu_\parallel^{(a)}(\theta) - \mu_\perp^{(a)}(\theta)\right)(\mathbf{i}_k\mathbf{F})^2\right]\right\}^{1/2}, \qquad (45)$$

where

$$c_a \equiv \frac{8\sqrt{2}m_\perp^2\sqrt{m_\parallel}}{\pi^{3/2}\hbar^4\rho}. \qquad (46)$$

The corresponding expressions for the acoustic mobility components are given by formulas (5).

In the case where the energy relaxation of hot electrons is related to their interaction with acoustic phonons, and the momentum relaxation to the scattering by ionized impurities, we obtain from Eq. (26) that

$$\theta_k = \theta\left\{1 - \frac{1}{c_a\theta^{3/2}}\left[\mu_\perp^{(c)}(\theta)\,F^2 + \right.\right.$$
$$\left.\left. + \left(\mu_\parallel^{(c)}(\theta) - \mu_\perp^{(c)}(\theta)\right)(\mathbf{i}_k\mathbf{F})^2\right]\right\}^{-1}. \qquad (47)$$

The corresponding expressions for the impurity mobility components are given by formulas (15). (While deriving Eq. (47), we used the approximation

$$\mu_{\perp,\parallel}^{(c)}(\theta_k) \approx \left(\frac{\theta_k}{\theta}\right)^{3/2} \mu_{\perp,\parallel}^{(c)}(\theta), \tag{48}$$

i.e. we considered the temperature dependence of the impurity scattering in the form $\theta_k^{3/2}$ and neglected the weak dependence on θ_k under the logarithm sign.)

From Eqs. (45) and (47) with the use of formula (27), it is easy to obtain the temperatures of electrons in the "cold" and "hot" valleys.

Note that, in the case where the momentum relaxation is associated with the electron scattering by both acoustic phonons and ionized impurities, the expression for Joule energy (23) must include the components of the resulting mobility, $\mu_{\perp,\parallel}(\theta)$. Approximately, those components can be expressed in terms of the components of the acoustic and impurity mobilities as follows:

$$\frac{1}{\mu_{\perp,\parallel}(\theta)} \approx \frac{1}{\mu_{\perp,\parallel}^{(a)}(\theta)} + \frac{1}{\mu_{\perp,\parallel}^{(c)}(\theta)}. \tag{49}$$

Formula (49) testifies that the resulting mobility is mainly governed by the mechanism that results in a lower mobility. Therefore, when the impurity mobility "becomes higher" than the acoustic one, as the temperature grows, formula (47) for the electron temperature has to be substituted by formula (45).

In Fig. 2, the field dependences of the coefficient

$$\Delta g = \frac{\Delta G^{(a)}\Delta g_a + \Delta G^{(c)}\Delta g_c}{\Delta G^{(a)} + \Delta G^{(c)}} \tag{50}$$

that characterizes the resulting angular dependence of types (36) and (41) at the simultaneous action of the acoustic and impurity mechanisms of scattering are exhibited. The figure shows that the coefficient Δg changes its sign at different values of the electron-heating field, depending on the concentration of ionized impurities. This regularity is associated with the fact that the substitution of the impurity mechanism of scattering by the acoustic one occurs at different electron temperatures, depending on the impurity concentration. The change of the Δg-sign means that the maxima in the periodic angular dependence of spontaneous radiation emitted by hot electrons gradually transform into minima, as the field varies.

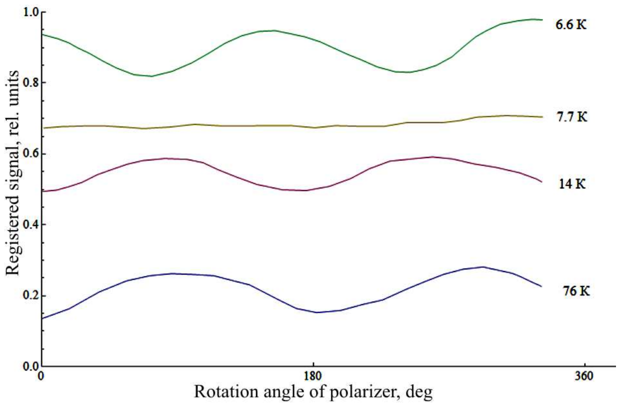

Fig. 4. Polarization angular dependences of THz radiation emitted by n-Ge ($n = 6 \times 10^{14}$ cm^{-3}) at various temperatures (temperature of the environment of the emitting specimen is indicated). The applied electric field $F = 140$ V/cm

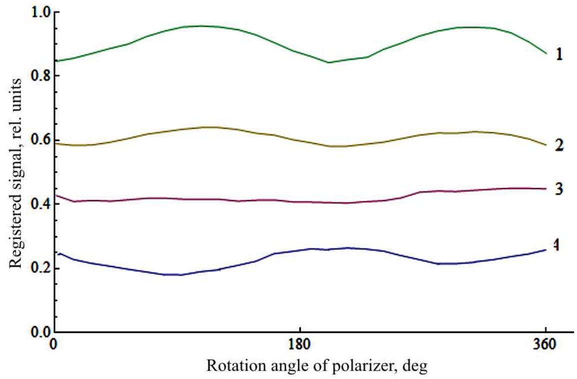

Fig. 5. Polarization angular dependences of THz radiation emitted by n-Ge ($n = 1.5 \times 10^{14}$ cm^{-3}) at various pressures $P = 0$ (*1*), 1 (*2*), 3 (*3*), and 5 kbar (*4*). $F = 200$ V/cm

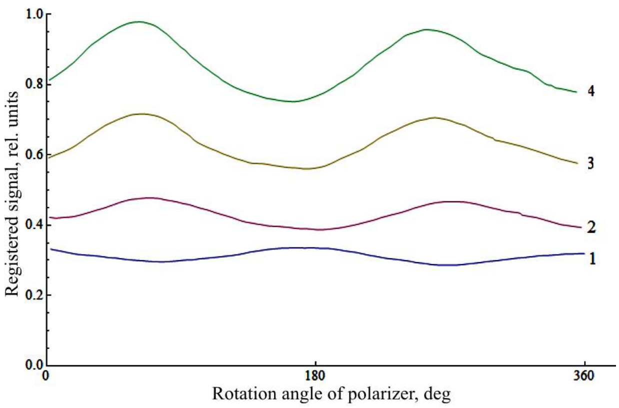

Fig. 6. Polarization angular dependences of THz radiation emitted by n-Ge ($n = 2.5 \times 10^{15}$ cm^{-3}) in the absence of illumination (*1*) and when the illumination intensity grows from minimum (*2*) to maximum (*4*). The electric field $F = 32$ V/cm

Figures 3 to 5 exhibit the polarization dependences experimentally measured for n-Ge. The measurement technique was described in work [8] in detail. The electron-heating field was oriented in the direction (1,1,1). Figures 3 and 4 demonstrate the influence of the electric field magnitude and the lattice temperature, respectively, on the angular dependences concerned. As one can see from Eqs. (39) and (42), the sign of coefficient Δg can be inverted by changing the concentration of electrons in the "hot" valleys by means of a unidirectional pressure (as a result, the electrons are moved into the "cold" valley). Figure 5 illustrates how this action affects the polarization dependences.

It is worth noting that the acoustic mechanism of scattering can be substituted by the Coulomb one by illuminating the specimen with visible light. Such illumination increases the concentration of Coulomb scattering centers and can change the lattice scattering to the Coulomb one. As is seen from Fig. 6, the illumination considerably changes the polarization characteristics. However, for this effect to be interpreted unequivocally, the influence of illumination on the electron temperature should be estimated.

Hence, which of the electron scattering mechanisms dominates under specific conditions depends on the lattice temperature, electron heating (which, in turn, depends on the field magnitude and orientation), impurity concentration, unidirectional pressure, and illumination. Therefore, all those factors affect the polarization characteristics of spontaneous radiation emitted by hot electrons.

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ВПЛИВ АНІЗОТРОПНИХ МЕХАНІЗМІВ РОЗСІЯННЯ НА ПОЛЯРИЗАЦІЙНІ ЗАЛЕЖНОСТІ ТЕРАГЕРЦОВОГО ВИПРОМІНЮВАННЯ ГАРЯЧИХ ЕЛЕКТРОНІВ

Р е з ю м е

Теоретично і експериментально досліджено вплив анізотропних механізмів розсіяння на поляризаційні залежності терагерцового (ТГЦ) випромінювання гарячих електронів у багатодолинних напівпровідниках типу n-Ge. Основну увагу приділено ситуації, коли прикладене до багатодолинного напівпровідника електричне поле направлено у напрямку, несиметричному відносно долин. Показано, що заміна анізотропного механізму розсіяння електронів іонізованими домішками на анізотропний механізм розсіяння їх акустичними фононами веде до заміни максимумів на мінімуми на періодичній поляризаційній кутовій залежності інтенсивності випромінювання гарячих електронів. Заміна одного домінуючого механізму розсіяння іншим може бути зумовлена низкою причин: зміною температури ґратки або концентрації іонізованих домішок, зміною величини або орієнтації розігріваючого електрони поля, прикладенням однонаправленого тиску або підсвіткою зразка. Показано, що всі ці причини впливають на поляризаційні залежності спонтанного випромінювання гарячих електронів у випадку, коли температури електронів різні в різних долинах.