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V.G. BAR'YAKHTAR,[1] A.G. DANILEVICH[1,2]

[1] Institute of Magnetism, National Academy of Sciences of Ukraine
and Ministry of Education and Science, Youth, and Sport of Ukraine
(36, Academician Vernadsky Blvd., Kyiv 03142, Ukraine; e-mail: victor.baryakhtar@gmail.com)
[2] National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute"
(37, Peremoga Ave., Kyiv 03056, Ukraine; e-mail: alek_tony@ukr.net)

MAGNETOELASTIC WAVES IN FERROMAGNETS IN THE VICINITY OF LATTICE STRUCTURAL PHASE TRANSITIONS

The dispersion laws for coupled magnetoelastic waves in ferromagnets with uniaxial or cubic symmetry have been calculated. The features of obtained dispersion laws in the vicinity of spin-reorientation phase transitions are analyzed. The interaction between elastic and spin waves is shown to depend on the direction of the ferromagnet magnetic moment. The influence of the magnetoelastic interaction on the dispersion law of quasispin waves in the degenerate ground state of a uniaxial "easy plane" ferromagnet is studied. The results of calculations show that the magnetoelastic interaction eliminates the degeneration and leads to the appearance of a magnetoacoustic gap in the ferromagnet spectrum. The behavior of the spectra of coupled magnetoelastic waves in the vicinity of lattice phase transitions, namely, in the vicinity of martensitic phase transformations in materials with the shape memory effect, is analyzed. The obtained results are used to interpret experimental data obtained for the Ni–Mn–Ga alloy. The phenomenon of a drastic decrease of the elastic moduli for this alloy, when approaching the martensitic phase transition point is explained theoretically. It is shown that the inhomogeneous magnetostriction is the main factor affecting the elastic characteristics of the material concerned. A model dissipative function describing the relaxation processes associated with a damping of coupled magnetoelastic waves in ferromagnets with cubic or uniaxial symmetry is developed. It takes the symmetry of a ferromagnet into account and describes both the exchange and relativistic interactions in the crystal.

Keywords: magnetoelastic interaction, dispersion law, ferromagnet, elastic modulus.

1. Introduction

The magnetoelastic interaction leads to the "coupling" of spin waves, which propagate in magnetically ordered crystals, with acoustic (elastic) waves. Such magnetoelastic oscillations have been studied for many years [1, 2], and their phenomenological model has been developed rather well [3–5]. But nowadays, the study of the interaction between the magnetic and elastic subsystems becomes topical again. This is connected with numerous experiments [6–9] that are performed with magnetically ordered systems, in which the interaction concerned can be rather large.

The magnetoelastic interaction manifests itself substantially, when the frequencies of spin and acoustic waves approach each other. In the case of magnetoacoustic resonance, the "repulsion" of the quasispin and quasiacoustic branches of the wave spectrum takes place [3, 4]. It is also well known that the magnetoelastic interaction increases, as a magnetically ordered system approaches the spin-reorientation phase transitions [4, 10]. This occurs because the energy gap in the spin wave spectrum decreases in this case. When the energy gap becomes comparable by magnitude with the "repulsion" between the quasispin and quasiacoustic branches of the spectrum, the magnetoelastic interaction brings about a significant reduction of the quasiacoustic wave velocity. Such a behavior of magnetically ordered systems in the vicinity of spin-reorientation phase transitions stimulated active studies of coupled magnetoelastic waves at phase transformations of other types.

Lately, structural phase transitions have been an object of intensive researches, because of their decisive role in such phenomena as superelasticity and shape memory effect. Of special interest are the so-called "martensitic transformations", structural phase transitions of the first kind from a high-symmetry structure to a distorted low-symmetry one, which take place at low temperatures [6–9]. A giant magnetostriction phenomenon was discovered in materials, for which this phase transition is possible. The phenomenon originates from a drastic decrease of the elastic energy in the vicinity of martensitic transformations [11]. Such systems are called "ferromagnetic alloys with the shape memory effect". The martensitic transformation in alloys with the shape memory effect is accompanied by a spontaneous deformation of the crystalline lattice and a considerable softening (reduction) of the shear elastic modulus [7, 8, 12, 13]. In particular, the Ni–Mn–Ga alloys, in which a martensitic transformation from the cubic phase to the tetragonal one takes place, are intensively studied. The most interesting feature of those materials is a giant (more than 5%) magnetically induced deformation. This deformation is caused by the transformation in the twin structure of a single crystal of the alloy under the action of an external magnetic field [14–16].

The phenomena of magnetostriction growth and shear elastic modulus softening, which were experimentally revealed in a single crystal of the Ni–Mn–Ga alloy in the vicinity of the martensitic transformation temperature, bring us to an idea about the strong influence of the magnetoelastic interaction on the spectra of collective vibrations in such materials. However, the specific features of the magnetoelastic interaction in the vicinity of phase transitions of this type, namely, the phase transitions in the lattice, were not considered in the classical works [3–5, 10].

In work [17], we have already calculated the influence of the interaction concerned on one of the elastic moduli in the cubic ferromagnet with the shape memory effect. But the experimental data [6, 8] demonstrate that the corresponding theoretical calculations are extremely urgent for other elastic moduli as well, because they also undergo appreciable variations at martensitic phase transitions. Hence, the main aim of this work is to estimate the influence of the magnetoelastic interaction for all possible acoustic modes and for main magnetic phases in ferromagnets with cubic and uniaxial symmetry.

It is also worth mentioning that a complete description of collective magnetoelastic oscillations cannot be done making no allowance for their damping. The theory describing the dissipation of elastic waves is well developed. It is based on general principles expounded in the works by Landau [18, 19] and Gilbert [20]. However, our further researches showed that the indicated classical models describing the damping of magnetization fluctuations have very significant shortcomings [21–23]. Therefore, in the present work, we also focused attention on the development of a model describing the dissipation of coupled magnetoelastic waves. We will also present the mechanism of construction of a general dissipative function for such oscillations.

2. Coupled Magnetoelastic Waves in the Ferromagnet with Cubic Symmetry

2.1. Spectra of coupled magnetoelastic waves in a ferromagnet with cubic symmetry

Let us consider a ferromagnet with the cubic symmetry of its lattice in an external magnetic field. When describing the interaction between spin and elastic waves, the total energy density for the crystal with cubic symmetry can be represented in the form

$$F = F_m + F_e + F_{me}. \tag{2.1}$$

The first term on the right-hand side of expression (2.1) corresponds to the magnetic component of the energy density. In the case of cubic symmetry, it looks like [3]

$$F_m = \frac{\alpha}{2}\frac{\partial\mu}{\partial x_i}\frac{\partial\mu}{\partial x_k} + K_1\left(\mu_x^2\mu_y^2 + \mu_x^2\mu_z^2 + \mu_y^2\mu_z^2\right) + + K_2\mu_x^2\mu_y^2\mu_z^2 - \mathbf{MH}, \tag{2.2}$$

where α is the constant of the inhomogeneous exchange interaction; K_1 and K_2 are the magnetic anisotropy constants of a cubic-symmetry ferromagnet; $\mathbf{M}$ and $\mathbf{H}$ are the vectors of magnetization and external magnetic fields, respectively; $\boldsymbol{\mu} = \frac{\mathbf{M}}{M_0}$ is the normalized magnetization vector, since the constants in expression (2.2) are the energy-dimensional quantities; and M_0 is the saturation magnetization value. The energy of demagnetizing fields in Eq. (2.2) is neglected, because we do not consider a specific form of the ferromagnetic specimen.

The term F_e describing the density of the elastic deformation energy looks like [24]

$$F_e = \frac{3}{2}(C_{11} + 2C_{12})u_1^2 + \frac{1}{6}C'(u_2^2 + u_3^2) + \\ + 2C_{44}(u_4^2 + u_5^2 + u_6^2). \quad (2.3)$$

The quantities C_{11}, C_{12}, C_{44}, and $C' = (C_{11} - C_{12})/2$ are elastic moduli of the second order for the crystal with cubic symmetry [18]. The variables $u_1 = \frac{1}{3}(\mathrm{E}_{xx} + \mathrm{E}_{yy} + \mathrm{E}_{zz})$, $u_2 = \sqrt{3}(\mathrm{E}_{xx} - \mathrm{E}_{yy})$, $u_3 = (2\mathrm{E}_{zz} - \mathrm{E}_{xx} - \mathrm{E}_{yy})$, $u_4 = \frac{1}{2}(\mathrm{E}_{yz} + \mathrm{E}_{zy})$, $u_5 = \frac{1}{2}(\mathrm{E}_{xz} + \mathrm{E}_{zx})$, and $u_6 = \frac{1}{2}(\mathrm{E}_{xy} + \mathrm{E}_{yx})$ are linear combinations of the strain tensor components. They are transformed according to the one- (u_1), two- (u_2 and u_3), and three-dimensional (u_4, u_5, and u_6) irreducible representations of the crystal symmetry group.

Finally, the third term on the right-hand side of expression (2.1) stands for the density of the interaction energy between the magnetic and elastic subsystems [24],

$$F_{me} = -\delta_0 u_1(\mu_x^2 + \mu_y^2 + \mu_z^2) - \delta_1\{\sqrt{3}u_2(\mu_x^2 - \mu_y^2) + \\ + u_3(2\mu_z^2 - \mu_x^2 - \mu_y^2)\} - \\ - \delta_2(u_4\mu_y\mu_z + u_5\mu_x\mu_z + u_6\mu_x\mu_y), \quad (2.4)$$

where the constants δ_0, δ_1, and δ_2 are parameters of the magnetoelastic interaction.

By minimizing the magnetic component of the energy, it is easy to show that there are three basic states for the magnetization vector in a cubic ferromagnet in the absence of an external magnetic field ($\mathbf{H} = 0$):

- along the fourth-order axis, $\mathbf{M} \parallel \langle 001 \rangle$ ("phase 1"),
- along the diagonal of one of the cube faces, $\mathbf{M} \parallel \langle 101 \rangle$ ("phase 2"),
- along the cube diagonal, $\mathbf{M} \parallel \langle 111 \rangle$ ("phase 3").

All other possible directions of the magnetic moment are equivalent to one of the indicated above. In the real experiments devoted to the study of the elastic and magnetic properties of materials [6–9], the direction of the external magnetic field coincides with one of the indicated directions of the magnetic moment, and the magnitude of $\mathbf{H}$ is sufficiently large (about 1000 Oe). Therefore, we may assume that the equilibrium value of $\mathbf{M}$ is directed along one of those directions.

Below, we consider small adiabatic oscillations of the magnetic moment density $\boldsymbol{\mu}$ in a ferromagnet [3]. Accordingly, this parameter can be written in the form

$$\boldsymbol{\mu}(\mathbf{r}, t) = \boldsymbol{\mu}_0 + \mathbf{m}(\mathbf{r}, t), \quad (2.5)$$

where $\mathbf{m}(\mathbf{r}, t)$ are small fluctuation-induced deviations from the equilibrium value $\boldsymbol{\mu}_0$, and the magnetization vector in the the equilibrium state has the components $\boldsymbol{\mu}_0 = (0, 0, 1)$ in "phase 1", $\boldsymbol{\mu}_0 = \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$ in "phase 2", and $\boldsymbol{\mu}_0 = (\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$ in "phase 3".

From the condition $\partial F/\partial \mathrm{E}_{ik} = 0$, the equilibrium values E_{ik}^0 of the strain tensor components can be obtained for each ground state of a cubic ferromagnet (they will be quoted below). Hence, each strain tensor component can also be written as the sum of a homogeneous part and a small deviation from it,

$$\mathrm{E}_{ik} = \mathrm{E}_{ik}^0 + \varepsilon_{ik}. \quad (2.6)$$

The inhomogeneous part of the elastic strain tensor can be expressed in terms of the particle displacement vector $\mathbf{U}$, by using the formula [4]

$$\varepsilon_{ik} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_k} + \frac{\partial U_k}{\partial x_i}\right). \quad (2.7)$$

In order to find the dispersion laws for coupled magnetoelastic waves in all ground states of a cubic ferromagnet, let us use the dynamical equations [3, 4] for the magnetization vector $\boldsymbol{\mu}$ (the Landau–Lifshitz equation),

$$\frac{\partial \mathbf{m}}{\partial t} = -\gamma \boldsymbol{\mu} \times \mathbf{H}_{\text{eff}}, \quad (2.8)$$

and the particle displacement vector $\mathbf{U}$,

$$\rho \ddot{\mathbf{U}} = -\frac{\delta F}{\delta \mathbf{U}}, \quad (2.9)$$

where $\mathbf{H}_{\rm eff} = -\frac{\delta F}{\delta \mathbf{m}}$ is an effective magnetic field, γ the gyromagnetic ratio, and ρ the density.

From Eq. (2.9), it is easy to obtain the dispersion laws for free acoustic waves, if only the elastic energy is taken into account [18]. As follows from the obtained formulas, the following elastic waves can propagate in a crystal with cubic symmetry: the first ($s_{l1}^2 = C_{11}/\rho$), second ($s_{l2}^2 = (C_{11} + C_{12} + 2C_{44})/2\rho$), and third ($s_{l3}^2 = (C_{11} + 2C_{12} + 4C_{44})/3\rho$) longitudinal sound waves, and the first ($s_{t1}^2 = C_{44}/\rho$), second ($s_{t2}^2 = C'/\rho$), and third ($s_{t3}^2 = (C_{11} - C_{12} + C_{44})/3\rho$) transverse sound waves [18]. In the case of magnetoelastic interaction, these elastic waves cannot be considered separately. Each of them, under certain conditions, interacts with the fluctuations of the crystal magnetic moment.

For further calculations, let us expand the total energy density (2.1) in a power series in small deviations m_i and ε_{ik}. Substituting them into the dynamical equations (2.8) and (2.9), we linearize the latter. If we change in the obtained equations to the Fourier components of the small deviations $\mathbf{m} = \mathbf{m}_0 \exp\{i(\mathbf{kr} - \omega t)\}$ and $\mathbf{U} = \mathbf{U}_0 \exp\{i(\mathbf{kr} - \omega t)\}$, where ω is the frequency and $\mathbf{k}$ the wave vector of collective waves, with respect to the time t and the coordinates $\mathbf{r}$, Eqs. (2.8) and (2.9) bring us to a system of six equations for the components of the vectors $\mathbf{m}_0$ and $\mathbf{U}_0$. The resulting systems of equations for each ground state of a cubic ferromagnet are given in Appendix A. By equating the determinant of the system of dynamical equations to zero, we obtain the dispersion laws for coupled magnetoelastic waves in the ground states of a cubic ferromagnet.

Let us consider three directions of the wave vector of elastic waves. This procedure will allow us to describe all possible elastic waves that can propagate in a ferromagnet with cubic symmetry.

Phase 1: H ∥ m ∥ ⟨001⟩

The equilibrium values of the strain tensor components in this ground state look like

$$\mathrm{E}^0_{xx} = \mathrm{E}^0_{yy} = \frac{\delta_0}{3(C_{11} + 2C_{12})} - \frac{2\delta_1}{C_{11} - C_{12}},$$

$$\mathrm{E}^0_{zz} = \frac{\delta_0}{3(C_{11} + 2C_{12})} + \frac{4\delta_1}{C_{11} - C_{12}},$$

$$\mathrm{E}^0_{xz} = \mathrm{E}^0_{zx} = \mathrm{E}^0_{yz} = \mathrm{E}^0_{zy} = \mathrm{E}^0_{xy} = \mathrm{E}^0_{yx} = 0.$$

The dispersion laws are as follows. If $\mathbf{k} \parallel \langle 100 \rangle$,

$$(\omega^2 - s_{t1}^2 k^2)(\omega^2 - s_{l1}^2 k^2)\Big[(\omega^2 - s_{t1}^2 k^2)(\omega^2 - \gamma^2 M_0^2 \omega_{m1}^2) - \delta_2^2 \Big\{\frac{\omega_{m1}\gamma^2 k^2}{4\rho}\Big\}\Big] = 0. \quad (2.10)$$

If $\mathbf{k} \parallel \langle 110 \rangle$,

$$(\omega^2 - s_{t2}^2 k^2)(\omega^2 - s_{l2}^2 k^2)\Big[(\omega^2 - s_{t1}^2 k^2)(\omega^2 - \gamma^2 M_0^2 \omega_{m1}^2) - \delta_2^2 \Big\{\frac{\omega_{m1}\gamma^2 k^2}{4\rho}\Big\}\Big] = 0. \quad (2.11)$$

If $\mathbf{k} \parallel \langle 111 \rangle$,

$$(\omega^2 - s_{t3}^2 k^2)\Big[(\omega^2 - s_{t3}^2 k^2)(\omega^2 - s_{l3}^2 k^2)(\omega^2 - \gamma^2 M_0^2 \omega_{m1}^2) - \delta_2^2 \Big\{\frac{\omega_{m1}\gamma^2 k^2}{3\rho}(\omega^2 - (s_{l3}^2 + 2s_{t3}^2)k^2/3)\Big\}\Big] = 0. \quad (2.12)$$

In expressions (2.10)–(2.12), the following notation was introduced:

$$\omega_{m1} = \frac{\alpha k^2}{M_0^2} + \frac{H}{M_0} + \frac{2K_1}{M_0^2} + \frac{72\delta_1^2}{M_0^2(C_{11} - C_{12})}. \quad (2.13)$$

Phase 2: H ∥ m ∥ ⟨101⟩

The equilibrium values of the strain tensor components in this ground state look like

$$\mathrm{E}^0_{xx} = \mathrm{E}^0_{zz} = \frac{\delta_0}{3(C_{11} + 2C_{12})} + \frac{\delta_1}{C_{11} - C_{12}},$$

$$\mathrm{E}^0_{yy} = \frac{\delta_0}{3(C_{11} + 2C_{12})} - \frac{2\delta_1}{C_{11} - C_{12}},$$

$$\mathrm{E}^0_{xz} = \mathrm{E}^0_{zx} = \frac{\delta_2}{8C_{44}}, \quad \mathrm{E}^0_{yz} = \mathrm{E}^0_{zy} = \mathrm{E}^0_{xy} = \mathrm{E}^0_{yx} = 0.$$

The dispersion laws are as follows. If $\mathbf{k} \parallel \langle 100 \rangle$,

$$(\omega^2 - s_{t1}^2 k^2)\Big[(\omega^2 - s_{t1}^2 k^2)(\omega^2 - s_{l1}^2 k^2) \times (\omega^2 - \gamma^2 M_0^2 \omega_{m2}\omega_{m3}) - \delta_1^2 \Big\{\frac{36\omega_{m2}\gamma^2 k^2}{\rho} \times (\omega^2 - s_{t1}^2 k^2)\Big\} - \delta_2^2 \Big\{\frac{\omega_{m3}\gamma^2 k^2}{8\rho}(\omega^2 - s_{l1}^2 k^2)\Big\}\Big] = 0. \quad (2.14)$$

If $\mathbf{k} \parallel \langle 110 \rangle$,

$$(\omega^2 - s_{t1}^2 k^2)(\omega^2 - s_{t2}^2 k^2)(\omega^2 - s_{l2}^2 k^2) \times (\omega^2 - \gamma^2 M_0^2 \omega_{m2}\omega_{m3}) -$$

$$-\delta_1^2\left\{\frac{18\omega_{m2}\gamma^2k^2}{\rho}(\omega^2-s_{t1}^2k^2)(\omega^2-\frac{(s_{l2}^2+s_{t2}^2)}{2}k^2)\right\}-$$
$$-\delta_2^2\left\{\frac{3\omega_{m3}\gamma^2k^2}{16\rho}(\omega^2-s_{t2}^2k^2)(\omega^2-\frac{(s_{l2}^2+2s_{t1}^2)}{3}k^2)\right\}=0. \quad (2.15)$$

If $\mathbf{k}\parallel\langle111\rangle$,

$$(\omega^2-s_{t3}^2k^2)\Big[(\omega^2-s_{t3}^2k^2)(\omega^2-s_{l3}^2k^2)\times$$
$$\times(\omega^2-\gamma^2M_0^2\omega_{m2}\omega_{m3})-\delta_1^2\left\{\frac{24\omega_{m2}\gamma^2k^2}{\rho}(\omega^2-s_{l3}^2k^2)\right\}-$$
$$-\delta_2^2\left\{\frac{\omega_{m3}\gamma^2k^2}{4\rho}(\omega^2-(s_{l3}^2+8s_{t3}^2)k^2/9)\right\}\Big]=0. \quad (2.16)$$

In expressions (2.14)–(2.16), the following notations were introduced:

$$\omega_{m2}=\frac{\alpha k^2}{M_0^2}+\frac{H}{M_0}+\frac{K_1}{M_0^2}+\frac{K_2}{2M_0^2}+$$
$$+\frac{36\delta_1^2}{M_0^2(C_{11}-C_{12})}+\frac{\delta_2^2}{8M_0^2C_{44}}, \quad (2.17)$$
$$\omega_{m3}=\frac{\alpha k^2}{M_0^2}+\frac{H}{M_0}-\frac{2K_1}{M_0^2}+\frac{\delta_2^2}{4M_0^2C_{44}}.$$

Phase 3: $\mathbf{H}\parallel\mathbf{m}\parallel\langle111\rangle$

The equilibrium values of the strain tensor components in this ground state look like

$$\mathrm{E}_{xx}^0=\mathrm{E}_{yy}^0=\mathrm{E}_{zz}^0=\frac{\delta_0}{3(C_{11}+2C_{12})},$$
$$\mathrm{E}_{xz}^0=\mathrm{E}_{zx}^0=\mathrm{E}_{yz}^0=\mathrm{E}_{zy}^0=\mathrm{E}_{xy}^0=\mathrm{E}_{yx}^0=\frac{\delta_2}{12C_{44}}.$$

The dispersion laws are as follows. If $\mathbf{k}\parallel\langle100\rangle$,

$$(\omega^2-s_{t1}^2k^2)\Big[(\omega^2-s_{t1}^2k^2)(\omega^2-s_{l1}^2k^2)\times$$
$$\times(\omega^2-\gamma^2M_0^2\omega_{m4}^2)-\delta_1^2\left\{\frac{32\omega_{m4}\gamma^2k^2}{\rho}(\omega^2-s_{t1}^2k^2)\right\}-$$
$$-\delta_2^2\left\{\frac{\omega_{m4}\gamma^2k^2}{9\rho}(\omega^2-s_{l1}^2k^2)\right\}\Big]=0. \quad (2.18)$$

If $\mathbf{k}\parallel\langle110\rangle$,

$$(\omega^2-s_{t1}^2k^2)(\omega^2-s_{t2}^2k^2)\times$$
$$\times(\omega^2-s_{l2}^2k^2)(\omega^2-\gamma^2M_0^2\omega_{m4}^2)-\delta_1^2\left\{\frac{32\omega_{m4}\gamma^2k^2}{\rho}\times\right.$$
$$\times(\omega^2-s_{t1}^2k^2)\left(\omega^2-\frac{(3s_{l2}^2+s_{t2}^2)}{4}k^2\right)\Big\}-$$
$$-\delta_2^2\left\{\frac{\omega_{m4}\gamma^2k^2}{12\rho}(\omega^2-s_{t2}^2k^2)\left(\omega^2-\frac{(s_{l2}^2+2s_{t1}^2)}{3}k^2\right)\right\}-$$
$$-\delta_1\delta_2\left\{\frac{4\omega_{m4}\gamma^2k^2}{3\rho}(\omega^2-s_{t1}^2k^2)(\omega^2-s_{t2}^2k^2)\right\}=0. \quad (2.19)$$

If $\mathbf{k}\parallel\langle111\rangle$,

$$(\omega^2-s_{t3}^2k^2)(\omega^2-s_{l3}^2k^2)\Big[(\omega^2-s_{t3}^2k^2)\times$$
$$\times(\omega^2-\gamma^2M_0^2\omega_{m4}^2)-\delta_1^2\left\{\frac{32\omega_{m4}\gamma^2k^2}{\rho}\right\}-$$
$$-\delta_2^2\left\{\frac{\omega_{m4}\gamma^2k^2}{18\rho}\right\}-\delta_1\delta_2\left\{\frac{8\omega_{m4}\gamma^2k^2}{3\rho}\right\}\Big]=0. \quad (2.20)$$

In expressions (2.18)–(2.20), the following notation was introduced:

$$\omega_{m4}=\frac{\alpha k^2}{M_0^2}+\frac{H}{M_0}-\frac{4K_1}{3M_0^2}-\frac{4K_2}{9M_0^2}+\frac{\delta_2^2}{4M_0^2C_{44}}. \quad (2.21)$$

Thus, expressions (2.10)–(2.20) are the dispersion laws written in the general form for coupled magnetoelastic waves in a ferromagnet with cubic symmetry. These dispersion equations have the standard structure [3, 4]. If the magnetoelastic interaction is neglected ($\delta_i\to0$), they become split into classical dispersion laws for spin waves [3] and elastic waves in cubic crystals [18].

The dispersion laws (2.10)–(2.20) calculated for coupled magnetoelastic waves in a ferromagnet with cubic symmetry allow the influence of the magnetic subsystem on the elastic properties of a crystal, more specifically, on the corresponding elastic moduli, to be estimated. For example, from the dispersion laws obtained for a cubic ferromagnet, it is easy to see that the magnetoelastic interaction with the first and third transverse sounds takes place for all equilibrium directions of the magnetic moment in the cubic ferromagnet, unlike other sound modes. For the illustrative purpose, the possibility of the magnetoelastic interaction depending on the magnetic moment direction in a ferromagnet is indicated in Table 2.1 for each acoustic mode. It is important to note that the application of the magnetoelastic energy in the form (2.4) makes it possible to distinctly identify the part of this energy (i.e. the constant δ_i) that is responsible for the interaction with a definite acoustic mode, in contrast to the classical expression that was used, e.g., in work [4]. This circumstance is also convenient to be displayed in Table 2.1. Thus, if the magnetoelastic interaction is possible in a certain acoustic mode, Table 2.1 demonstrates the magnetoelastic constant characterizing this interaction.

The analysis of the dispersion laws (2.10)–(2.20) reveals that they do not contain the constant δ_0. As a result, the influence of the equilibrium part of the magnetoelastic energy is not taken into account. Really, if dynamical phenomena (e.g., the magnetoelastic resonance) are considered, the influence of this term cannot be taken into account. A theoretical model making allowance for the influence of the equilibrium part of the magnetoelastic energy was proposed in work [25].

The constant δ_1 characterizes the influence of the magnetic subsystem on the second transverse sound and, accordingly, on the elastic modulus C'. From the obtained dispersion laws, one can easily see that, as was shown earlier [17], the interaction with this acoustic mode cannot be described in phase 1. The constant δ_2, in its turn, characterizes the influence of the magnetic subsystem on the first transverse sound and the modulus C_{44}.

2.2. Magnetoelastic interaction with the first transverse sound in an alloy with the shape memory effect

As shown in Table 2.1, the first transverse sound can be described for two directions of the wave vector of elastic vibrations: along the fourth-order axis and along the diagonal of the cube face. So let us consider below the directions $\mathbf{k} \parallel \langle 100\rangle$ and $\mathbf{k} \parallel \langle 110\rangle$. The influence of the magnetic subsystem on the first transverse sound and, accordingly, on the elastic modulus C_{44} can be described, by analyzing the magnetoacoustic resonance at the frequency $\omega_{\mathrm{ph}} = (C_{44}/\rho)^{1/2}k$. In this case, the dispersion laws given above transform into the following dispersion equation, which has a single general form for all directions of the crystal magnetic moment:

$$(\omega^2 - \omega_{\mathrm{ph}}^2)(\omega^2 - \omega_{\mathrm{sw}}^2) - \delta_2^2\xi = 0, \quad (2.22)$$

where ω_{sw} is the frequency of uncoupled spin waves, and ξ the coefficient of the magnetoelastic interaction. The values of those parameters depend on the direction of the magnetic moment in a ferromagnet and the direction of the wave vector of collective waves. They are quoted in Table 2.2.

The solution of Eq. (2.22) looks like

$$\omega_{\pm}^2 = \frac{1}{2}\left\{\omega_{\mathrm{ph}}^2 + \omega_{\mathrm{sw}}^2 \pm [4\xi\delta_2^2 + (\omega_{\mathrm{ph}}^2 - \omega_{\mathrm{sw}}^2)^2]^{1/2}\right\}. \quad (2.23)$$

The corresponding dispersion curve consists of two branches: a quasimagnon and quasiphonon ones (see Fig. 2.1). From Eq. (2.23), one can easily see that, when the system approaches the magnetoacoustic resonance, $\omega_{\mathrm{sw}} \to \omega_{\mathrm{ph}}$, these are the quantities ξ and δ_2 that govern the "repulsion" between the quasimagnon and quasiphonon branches.

Let us evaluate the obtained dispersion law (2.23) in various cases and use a material with the shape memory effect as an example (Fig. 2.1). The values of the constants that enter Eq. (2.23) are taken those for the Ni–Mn–Ga alloy, because nowadays this alloy is one of the most interesting representatives of the materials with the shape memory effect. This alloy undergoes a martensitic phase transformation, namely, a transition from the cubic phase into the tetragonal one, in the vicinity of room temperature [26].

While making calculations for the Ni–Mn–Ga alloy, the known experimental values of its anisotropy constants in the cubic phase (austenite) [27], which

Table 2.1. **Interaction of acoustic modes with spin waves in a ferromagnet with cubic symmetry**

Acoustic mode and wave vector direction	Phase 1: $\mathbf{H}\parallel\mathbf{M}\parallel\langle 001\rangle$	Phase 2: $\mathbf{H}\parallel\mathbf{M}\parallel\langle 101\rangle$	Phase 3: $\mathbf{H}\parallel\mathbf{M}\parallel\langle 111\rangle$
s_{l1} $\mathbf{k}\parallel\langle 100\rangle$	No interaction	δ_1	δ_1
s_{t1} $\mathbf{k}\parallel\langle 100\rangle$	δ_2	δ_2	δ_2
s_{t1} $\mathbf{k}\parallel\langle 110\rangle$	δ_2	δ_2	δ_2
s_{l2} $\mathbf{k}\parallel\langle 110\rangle$	No interaction	δ_1, δ_2	δ_1, δ_2
s_{t2} $\mathbf{k}\parallel\langle 110\rangle$	No interaction	δ_1	δ_1
s_{l3} $\mathbf{k}\parallel\langle 111\rangle$	δ_2	δ_2	No interaction
s_{t3} $\mathbf{k}\parallel\langle 111\rangle$	δ_2	δ_1, δ_2	δ_1, δ_2

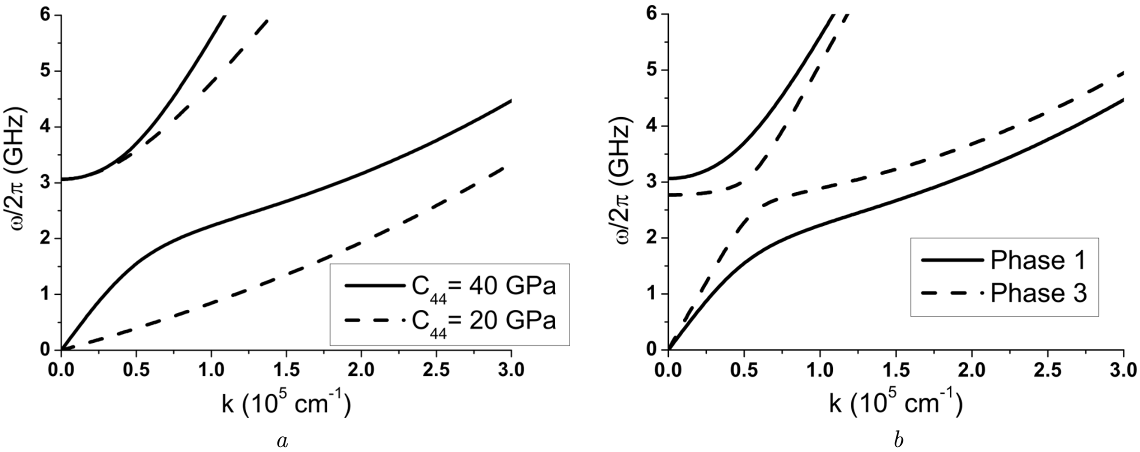

Fig. 2.1. Dispersion laws for magnetoelastic waves in the ground state **H** ∥ **M** ∥ ⟨001⟩ for two values of the modulus (a) and in the ground states **H** ∥ **M** ∥ ⟨001⟩ and **H** ∥ **M** ∥ ⟨111⟩ for C_{44} = 40 GPa (b)

Table 2.2. **Coefficient of magnetoelastic interaction with the first transverse sound for various ground states of a cubic ferromagnet**

Wave vector direction	Phase 1: **H** ∥ **M** ∥ ⟨001⟩ $\omega_{\rm sw} = \gamma M_0\omega_{m1}$	Phase 2: **H** ∥ **M** ∥ ⟨101⟩ $\omega_{\rm sw} = \gamma M_0(\omega_{m2}\omega_{m3})^{1/2}$	Phase 3: **H** ∥ **M** ∥ ⟨111⟩ $\omega_{\rm sw} = \gamma M_0\omega_{m4}$
k ∥ ⟨100⟩	$\zeta = \frac{\omega_{m1}\gamma^2k^2}{4\rho}$	$\zeta = \frac{\omega_{m3}\gamma^2k^2}{8\rho}$	$\zeta = \frac{\omega_{m4}\gamma^2k^2}{9\rho}$
k ∥ ⟨110⟩	$\zeta = \frac{\omega_{m1}\gamma^2k^2}{4\rho}$	$\zeta = \frac{\omega_{m3}\gamma^2k^2}{16\rho}$	$\zeta = \frac{\omega_{m4}\gamma^2k^2}{36\rho}$

correspond to phase 1, $K_1 = 2.7 \times 10^4$ erg/cm^3 and $K_2 = -6.1 \times 10^4$ erg/cm^3, as well as the saturation magnetization M_0 = 600 Gs and the density $\rho \approx 8$ g/cm^3, were used. The value of the inhomogeneous exchange interaction constant can be estimated from the expression $\alpha \cong (k_{\rm B}T_{\rm C}A^2M_0)/\mu_{\rm B}$ [3], where $T_{\rm C}$ = 360 K is the Curie temperature [27], $A = 0.41 \times 10^{-8}$ cm is the distance between the magnetic atoms [27], $\mu_{\rm B}$ is the Bohr magneton, and $k_{\rm B}$ the Boltzmann constant. The external magnetic field must be sufficiently large to satisfy the conditions for the ground states to exist ($\omega_{mi} \geqslant 0$, where $i = 1, 2, 3, 4$) and to correspond to the conditions of experimental researches that are usually performed with such materials. Therefore, we selected H = 1000 Oe. The elastic moduli were also taken in the austenite case: C_{44} = 40 GPa and C' = 14 GPa [28]. The magnetoelastic interaction constant δ_2 has not been evaluated till now. Proceeding from the fact that it must be not less than δ_1 and $\delta_1 \sim 10^7$ erg/cm^3 [24], we took $\delta_2 \sim 10^9$ erg/cm^3 for the results to be more illustrative.

The coefficient ξ of magnetoelastic interaction between the spin waves and the first transverse sound depends on the magnetic moment direction in the ferromagnet (see Table 2.2 and Fig. 2.1, b). This interaction reveals itself most strongly in the ground state at **H** ∥ **M** ∥ ⟨001⟩. It turns out that the coefficient of magnetoelastic interaction in the ground states at **H** ∥ **M** ∥ ⟨101⟩ and **H** ∥ **M** ∥ ⟨111⟩ also depends on the direction of the wave vector of collective oscillations (Table 2.2).

The collective oscillations of spin waves and the first transverse sound are described by the dispersion equation (2.23), which has the same behavior for each direction of the ferromagnet magnetic moment. From Eq. (2.23), it follows that if the elastic modulus C_{44} drastically decreases, the magnetoelastic interaction considerably increases. In Fig. 2.1, a, using the Ni–Mn–Ga alloy as an example, it is shown that the reduction of the elastic modulus C_{44} by a factor of only two gives rise to a significant "repulsion" between the quasimagnon and quasiphonon branches of the dispersion curve. Such a behavior of the quasiphonon mode results in that resonance measurement methods can give even more underestimated values of the elastic modulus C_{44}.

2.3. Magnetoelastic interaction with the second transverse sound in an alloy with the shape memory effect

As was marked above, an inherent property of ferromagnetic alloys with the shape memory effect belonging to the Ni–Mn–Ga family is the martensitic transformation accompanied by a spontaneous deformation of the crystal lattice and a pronounced softening (reduction) of the shear elastic modulus C' [7, 8, 12, 13]. That is why the acoustic mode characterized by this elastic modulus, $s_{t2}^2 = C'/\rho$, is also called the "soft" mode.

The second transverse sound can be described, if the wave vector of elastic vibrations is directed along the diagonal of the cube face. Therefore, let us consider the case $\mathbf{k}\| \langle 110\rangle$ below. The influence of the magnetic subsystem on the second transverse sound and, accordingly, on the elastic modulus C' can be described, by analyzing the magnetoacoustic resonance at the frequency $\omega_{\rm ph} = (C'/\rho)^{1/2}k$. In this case, dispersion laws (2.20) and (2.23) transform into a dispersion equation, which has the following general form for two directions of the crystal magnetic moment:

$$(\omega^2 - \omega_{\rm ph}^2)(\omega^2 - \omega_{\rm sw}^2) - \delta_1^2\xi = 0, \qquad (2.24)$$

where $\omega_{\rm sw}$ is the frequency of uncoupled spin waves, and ξ is the coefficient of magnetoelastic interaction. The values of those parameters depend on the magnetic moment direction in a ferromagnet and are quoted in Table 2.3.

The solution of Eq. (2.24) is analogous to expression (2.23) and looks like [17]

$$\omega_\pm^2 = \frac{1}{2}\Big\{\omega_{\rm ph}^2 + \omega_{\rm sw}^2 \pm [4\xi\delta_1^2 + (\omega_{\rm ph}^2 - \omega_{\rm sw}^2)^2]^{1/2}\Big\}. \qquad (2.25)$$

Proceeding from the available experimental data for the magnetic anisotropy constants of the Ni–Mn–Ga alloy [27], which were quoted above, the equilibrium direction of the magnetization vector in a Ni–Mn–Ga single crystal in the vicinity of the martensitic transformation temperature is parallel to direction [100]. But, in this case, there is no interaction between the soft mode and magnetic oscillations. Modern experimental facilities can generate external magnetic fields that are strong enough for the magnetization vector to be reoriented in their direction. Therefore, let us consider the case where the magnetic field is parallel to the crystallographic direction [101]. According to the magnetic energy minimization condition (2.2), the magnetic field stabilizes the magnetic moment in the direction $\mathbf{M}\,\|[101]$, if the inequality $H > H_1 \equiv 2K_1/M_0$ is satisfied. For the experimental values given above, the value of the characteristic field H_1 is about 90 Oe.

Solution (2.25) describes the dispersion of quasiacoustic (ω_-) and quasispin (ω_-) waves in a crystal [17]. The quasiacoustic mode is gapless, whereas the quasispin wave spectrum has a gap $\omega_0 = \gamma(H - H_1)^{1/2}(H+H_2)^{1/2}$, where $H_2 \equiv (K_1+K_2/2)/M_0 \approx \approx 100$ Oe. The both spectra are depicted in Fig. 2.2. Due to a large discrepancy of the values for the shear elastic modulus in various Ni–Mn–Ga alloys (the $C'_{\rm min}$ values within an interval of 1–60 GPa were measured at the temperature of the martensitic transformation for quasistoichiometric alloys [7, 8, 12, 13]), the spectra shown in Fig. 2.2, *a* were calculated for three different values of the shear elastic modulus. The field value $H = 3300$ Oe corresponds to the frequency $\omega_0/2\pi = 9.1$ GHz.

Let us consider the case of strong external magnetic field (Fig. 2.2, *a*). Then the magnetoelastic interaction in a crystal with the 60-GPa shear modulus manifests itself in a narrow interval of the wave vector values, which includes the resonance value $k_0 \approx 2.4\times10^5$ cm^{-1}. The magnetoelastic interaction does not change significantly the dispersion curves of the acoustic and spin waves far from the resonance. In a crystal with the 35-GPa shear modulus, the interaction between the acoustic and spin waves results in

Table 2.3. **Coefficient of magnetoelastic interaction with the second transverse sound for various ground states of a cubic ferromagnet**

Wave vector direction	Phase 2: $\mathbf{H}\parallel\mathbf{M}\parallel\langle101\rangle$ $\omega_{\rm sw} = \gamma M_0(\omega_{m2}\omega_{m3})^{1/2}$	Phase 3: $\mathbf{H}\parallel\mathbf{M}\parallel\langle111\rangle$ $\omega_{\rm sw} = \gamma M_0\omega_{m4}$
$\mathbf{k}\parallel\langle110\rangle$	$\xi = \frac{9\omega_{m2}\gamma^2k^2}{\rho}$	$\xi = \frac{24\omega_{m4}\gamma^2k^2}{\rho}$

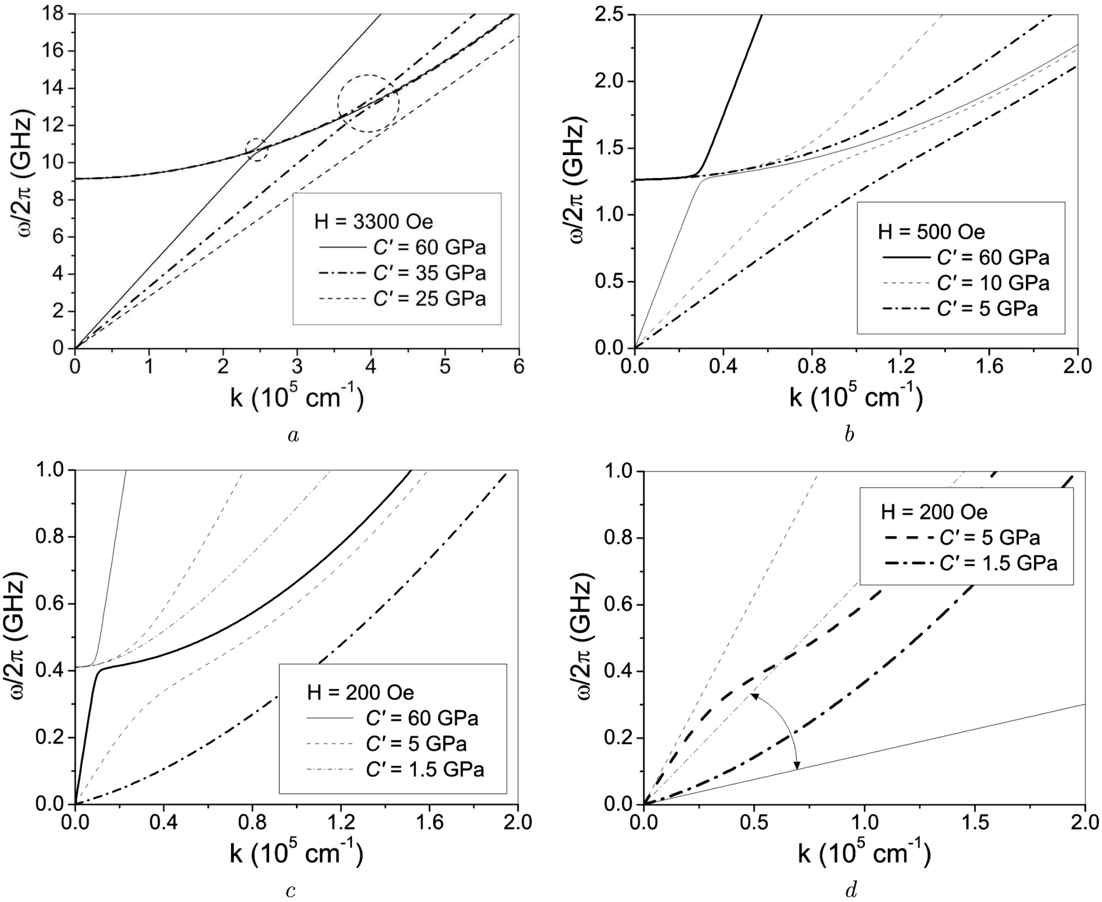

Fig. 2.2. Dispersion laws for coupled magnetoelastic waves calculated for various values of the elastic shear modulus and an external magnetic field ($\mathbf{H}\,\|[101]$) (a, b, c), and in the cases $\mathbf{H}\,\|[001]$ (thin curves) and $\mathbf{H}\,\|[101]$ (thick curves) (d). The slope of the thin solid line corresponds to the sound velocity that decreases due to the magnetoelastic interaction [17]

a strong "repulsion"[1] between the quasiacoustic and quasispin spectral branches. Therefore, it can manifest itself in a wider interval of wave vector values, $\Delta k \sim 10^5$ cm^{-1}. In this Δk-interval, an appreciable nonlinearity of the quasiacoustic branch of the dispersion curve is observed. In a crystal with the 25-GPa shear modulus, the magnetoelastic interaction does not change the dispersion law of collective oscillations significantly, because the quasiacoustic branch passes far from the quasispin one [17]. The shear moduli for the family of Ni–Mn–Ga alloys diminish from a few tens of gigapascals to about one gigapascal, when the alloy temperature approaches the martensitic transformation temperature. Thus, the influence of the magnetoelastic interaction on the wave spectra is the most pronounced within a certain temperature interval located above the martensitic transformation temperature. In this interval, the value of the shear elastic modulus is rather close to 35 GPa.

Standard experimental methods allow magnetoelastic resonance oscillations to be observed in various frequency intervals and, hence, at various external magnetic fields. The reduction of the resonance field to the value $H = 500$ Oe results in a displacement of the resonance interval Δk to lower wave vector values (see Fig. 2.2, b). In this case, a large "repulsion" between the quasiacoustic and quasispin branches of the spectrum is appreciable even at very

[1] The "repulsion" is formally defined as $\Delta\omega \equiv \omega_+(k_0) - \omega_-(k_0)$, where k_0 satisfies the equation $\omega_{\rm sw}(k_0) = \omega_{\rm ph}(k_0)$.

low values of the shear modulus ($C' = 5$ GPa). Thus, the lower section of the temperature interval, where the magnetoelastic interaction of oscillations is substantial, can reach the martensitic transformation temperature.

Another manifestation of the magnetoelastic interaction is shown in Fig. 2.2, *c*, which illustrates a strong nonlinearity in the initial section of the quasiacoustic dispersion curve. This feature is well observable for crystals with $C' < 5$ GPa, if the frequency gap ω_0 is comparable with the repulsion magnitude $\Delta\omega$. As was already mentioned above, the change of the dispersion law for the quasiacoustic mode was revealed long ago in the closest vicinity of spin-orientation phase transitions [29, 30]. As far as we know, a possibility to observe this effect when approaching martensitic phase transformations has not been discussed yet.

Figure 2.2, *d* is a good illustration of the difference between the dispersion laws for quasiacoustic waves associated with the reorientation of an external magnetic field applied to a single-crystalline specimen of the Ni–Mn–Ga alloy. The bold dispersion curves correspond to the field direction $\mathbf{H}\,\|[101]$ (see also Fig. 2.2, *c*), and the thin curves to the case $\mathbf{H}\,\|[001]$. As was shown above, the change of the magnetic field direction from [101] to [001] (or [100]) "switches off" the influence of the magnetoelastic interaction on the soft elastic modulus. Consequently, the thin dotted and dot-dashed lines demonstrate linear dispersion laws calculated in the cases where "free" sound waves propagate in the crystals with the shear elastic moduli $C' = 5$ and 1.5 GPa, respectively. The latter case is the most interesting one, because it reveals a possibility of a drastic change of the dispersion dependence induced by the reorientation of an external magnetic field. If the field is directed along the direction [001], the dispersion law looks like $\omega_{\rm ph} = (C'/\rho)^{1/2}k$. However, if the field is parallel to the direction [101], the dispersion law for the quasiacoustic mode can be expressed as $\tilde{\omega}_{\rm ph} = c_1k + c_2k^2 + \dots$. The coefficient c_1 is the tangent of the slope angle of the solid line shown in Fig. 2.2, *d*. The "effective" shear modulus, which can be determined from ultrasonic or Dynamic Mechanical Analysis (DMA) experiments, can be expressed in terms of this coefficient as follows: $C^{\rm eff} = \rho c_1^2$. A careful consideration of the initial section of the dispersion curve testifies that the dispersion of a quasiacoustic wave with a wavelength of about 1 cm is practically linear and is characterized by the effective modulus $C^{\rm eff} \approx 70$ MPa. Such a value calculated for the effective modulus is evidently too small to be measured experimentally. Moreover, under real experimental conditions, the mixing of waves of various types cannot be avoided completely. Nevertheless, the obtained calculation results can explain abnormally low experimental values of the elastic moduli reported for the Ni–Mn–Ga alloys in works [7, 8, 12].

If the magnetic field vector is parallel to the direction [101], a strong influence of the magnetoelastic interaction on the soft elastic mode can be observed even at the magnetic field $H = 500$ Oe. To illustrate this statement, let us consider the interesting case of an abnormally soft shear modulus that varies from 5 to 0.5 GPa when the temperature approaches the martensitic transition region.

If the shear modulus exceeds 4 GPa, the dispersion curves of the free acoustic and spin waves twice intersect each other (see Fig. 2.3, *a*). In this case, the repulsion between the dispersion curves of quasiacoustic and quasispin waves is well pronounced in a wide wave vector interval $\Delta k \sim 3 \times 10^5$ cm^{-1}. If the shear modulus equals 4 GPa, the dispersion curves of the free acoustic and spin waves touch each other at a certain point, but the dispersion curves still repulse each other (see Fig. 2.3, *b*). The reduction of the shear modulus to 1.5 GPa gives rise to a significant approaching of the quasispin and purely spin dispersion curves. The dispersion law for the quasiacoustic wave is practically linear, but the speed of quasiacoustic wave is lower than that of the free acoustic wave (see Fig. 2.3, *c*). The shear elastic modulus is proportional to the square of the sound speed. Therefore, even a minor reduction in the sound speed brings about a significant difference between the real and effective shear moduli: $C^{\rm eff}/C' \approx 0.75$. Furthermore, the softening of the shear elastic modulus leads to a further reduction of the quasiacoustic wave velocity (see Fig. 2.3, *d*) and the restoration of the shear elastic modulus nonlinearity.

The theoretical analysis of the spectra of coupled waves in ferromagnetic alloys with the shape memory effect testifies that there can be an abnormally strong coupling of spin waves with the soft elastic vibration mode, when approaching the martensitic transformation temperature. The main effect obtained owing to this analysis consists in a substantial reduction of the

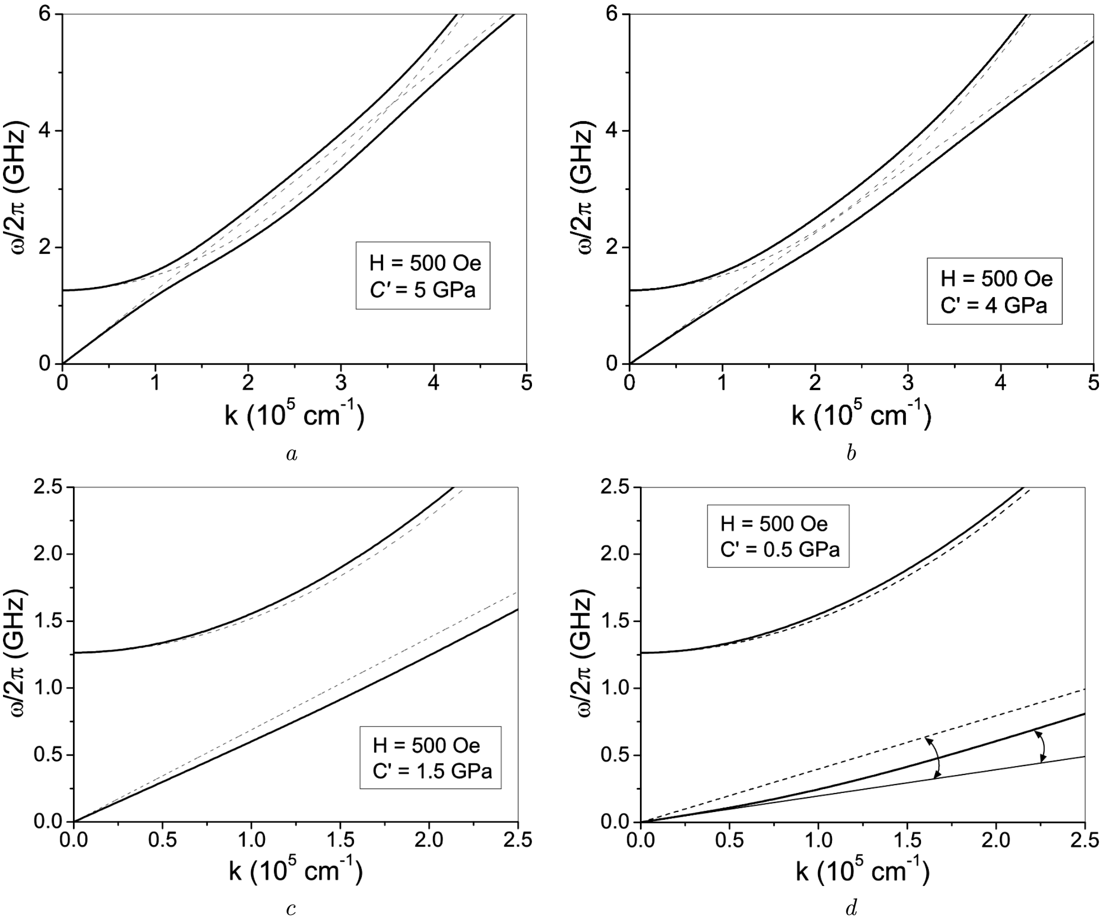

Fig. 2.3. Comparison of the dispersion curves for coupled magnetoelastic waves (solid curves) with the dispersion curves obtained for free spin and acoustic waves (dashed curves). The solid curves correspond to the magnetic field direction **H** $\|$[101], and the dashed curves to **H** $\|$[001]

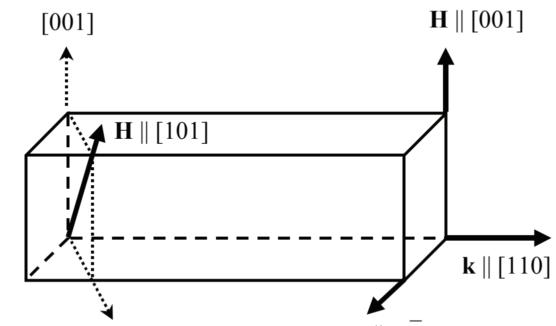

Fig. 2.4. Relative orientation of the prism-like specimen and the wave and polarization vectors of the acoustic wave that is suitable for the experimental observation of the magnetoelastic interaction effects in the external magnetic fields **H** $\|$[101] and **H** $\|$[001]

second transverse sound speed s_{t2} and the shear elastic modulus in the single-crystalline Ni–Mn–Ga. The reduction can be observed experimentally, by changing the direction of the external magnetic field from [001] to [101].

The orientation of a single crystal that is suitable for the observation of this effect is shown in Fig. 2.4. The directions of the wave vector, the magnetic field, and the polarization vector are also shown in the figure for illustration. The specimen has to be a strongly elongated prism, with its long side being oriented along the crystallographic direction [110]. If the magnetic field is directed along the crystallographic direction [101], the quasiacoustic wave frequency is determined by the values of the effective modulus C^{eff} and the specimen length. The value of the effective modulus is close to that of the shear elastic modulus, if the latter equals 5 GPa (see Fig. 2.2, *d*). If the shear modulus values areclose to 1.5 GPa or lower, the effective modulus can reach values of 100 MPa (see Figs. 2.2, *d* and 2.3, *d*). A typical length of experimental specimens is about 1 cm. Therefore, the resonance frequency is about 10^5 Hz, if $C^{\text{eff}} \approx C' = 5$ GPa, and close to 10^4 Hz, if $C^{\text{eff}} \approx 100$ MPa.

It is worth noting that if the magnetic field is applied in the direction [101] and the magnitude of the resonance field does not exceed 500 Oe, the phonon-magnon coupling manifests itself in a wide interval of wave vector magnitudes. In this case, the magnetoelastic interaction can considerably affect the thermodynamic properties of ferromagnetic alloys of the Ni–Mn–Ga family.

3. Coupled Magnetoelastic Waves in a Ferromagnet with Uniaxial Symmetry

3.1. Magnetoelastic gap in a uniaxial ferromagnet as a manifestation of the Higgs effect

In various physical domains, there are examples of systems that demonstrate the spontaneous violation of their symmetry. These are systems, whose energy is characterized by a certain symmetry, whereas the real physical state of the system, which corresponds to a partial solution of the equation of motion, is not characterized by this symmetry. Such a situation takes place, when the energy minimum of the system corresponds to a number of states with a continuous degeneracy parameter. The symmetry can be violated owing to an arbitrarily small perturbation of a special type [31, 32].

Currently, it is well known that there is a magnetoelastic gap in the spin-wave spectra of magnetically ordered materials owing to the interaction between the spin and acoustic waves. In work [10], a hypothesis was put forward that the appearance of the magnetoelastic gap is associated with the violation of the magnetic Hamiltonian symmetry, if the magnetoelastic interaction is made allowance for. However, specific calculations of this phenomenon were not carried out. Earlier, such an effect was discovered by Higgs in the quantum field theory [31].

Using the uniaxial ferromagnet as an example, let us consider comprehensively the appearance of a magnetoelastic gap in the spectrum of spin waves, which occurs owing to the spontaneous symmetry violation in the spin system. We recall how the mass of Higgs bosons arises. In the framework of a simplified model [31], the spontaneous violation of system's symmetry is achieved due to the transition from the potential energy written in the form $m^2\varphi^2/2$ to a potential energy that is an even function with two symmetric minima. Here are speculations concerning a multicomponent field that is described by the field function $\boldsymbol{\phi}(\varphi_1, \varphi_2, ..., \varphi_n)$ and has the following important property: its potential energy contains only terms with even power exponents of $\boldsymbol{\phi}$. The corresponding system has a continuous rotational symmetry, which does not change the value of φ^2.

In order to obtain a spontaneous symmetry violation, the potential energy has to be taken in the form

$$V(\varphi) = -\frac{\mu^2}{2}(\boldsymbol{\phi}\boldsymbol{\phi}) + \frac{h^2}{2}(\boldsymbol{\phi}\boldsymbol{\phi})^2, \tag{3.1}$$

where $\boldsymbol{\phi}\boldsymbol{\phi} = \sum_a \varphi_a^2$. The potential energy minimum is attained at

$$\boldsymbol{\phi}\boldsymbol{\phi} = \varphi_0^2 = \frac{\mu^2}{h^2}. \tag{3.2}$$

This is an equation with n variables. Without losing the generality, let us choose the solution of this equation in the form

$$\boldsymbol{\phi}_0 = (\varphi_0, 0, ..., 0), \tag{3.3}$$

where $\varphi_0 = \mu/h$. Now we should shift the field function $\boldsymbol{\phi}$ by the constant vector $\boldsymbol{\phi}_0$ that satisfies condition (3.3):

$$\boldsymbol{\phi}(x) = \boldsymbol{\phi}_0 + \mathbf{u}(x). \tag{3.4}$$

Then we obtain the following expression for the potential energy:

$$V(\boldsymbol{\phi}_0 + \mathbf{u}(x)) = V_0 + \mu^2 u_1^2 + \mu h u_1(\mathbf{uu}) + \frac{h^2}{4}(\mathbf{uu})^2. \tag{3.5}$$

In this expression, the quadratic term of only the component u_1 is contained. Hence, owing to the symmetry violation, which is expressed by displacement (3.4), the component u_1 acquires the free mass $m_1 = \mu\sqrt{2}$, whereas the other components are massless. It is obvious that the symmetry of this solution cannot be classified as the rotational symmetry in the n-dimensional space of the field function $\boldsymbol{\phi}(\varphi_1, \varphi_2, ..., \varphi_n)$.

If the consideration is carried out in the quantum-mechanical framework, a displacement by a constant value breaks the commutation relation for Fermi particles and does not break it for Bose particles. In other words, the mechanism of mass appearance at the spontaneous symmetry violation takes place only for bosons. The Higgs effect consists in that only one kind of Bose particles acquires a mass [31]. The massless particles are called Goldstone particles.

This situation is realized in magnets with uniaxial symmetry, where the ground states with the magnetic moment directed not along the easy axis are degenerate [3, 32, 33]. Magnetoelastic interactions in a uniaxial ferromagnet were taken into consideration rather long ago and under various conditions [3, 34]. However, only the "easy-axis" ground state was considered, which is not degenerate.

Let us analyze how the magnetoelastic interaction affects the spectrum of spin waves in the case of the "easy-plane" degenerate state. The energy of a uniaxial ferromagnet in the absence of an external magnetic field can be written as follows [3]:

$$F_m = \frac{\alpha}{2}\frac{\partial \boldsymbol{\mu}}{\partial x_i}\frac{\partial \boldsymbol{\mu}}{\partial x_k} - \frac{1}{2}K_1\mu_z^2 - \frac{1}{4}K_2\mu_z^4, \quad (3.6)$$

where α is the inhomogeneous exchange interaction constant, K_1 and K_2 are the uniaxial anisotropy constants, $\boldsymbol{\mu} = \mathbf{M}/M_0$ is the normalized magnetization vector, and M_0 is the saturation magnetization. Here, the constants have the energy dimensionality.

In order to obtain the dispersion law for spin waves in a ferromagnet, the equation of motion for a magnetic moment, the Landau–Lifshitz equation (2.8) [19], has to be used. By applying the standard method [3, 33] to Eq. (2.8), the spin wave frequencies for the ground states of a uniaxial ferromagnet can be obtained. We are interested in the "easy-plane" ground state, when the magnetization lies in the basis plane (e.g., $\mathbf{M} \parallel \langle 100\rangle$), and the stability condition for this state looks like $K_1 < 0$. This state is degenerate with a continuous degeneracy parameter corresponding to rotations in the basis plane. The dispersion law in this case has the form

$$\omega^2 = \gamma^2 M_0^2 \left(\frac{\alpha k^2}{M_0^2} - \frac{K_1}{M_0^2}\right)\frac{\alpha k^2}{M_0^2}, \quad (3.7)$$

where ω and k are the frequency and the wave vector of a spin wave, respectively. This spin wave is a Goldstone boson, because the magnon energy vanishes at $k = 0$. The symmetry of this ground state for a definite value of the magnetization in the basis plane is evidently lower than the symmetry of the initial Hamiltonian.

The degeneration is eliminated, if the external magnetic field oriented in the basis plane (e.g., $\mathbf{H} \parallel \langle 100\rangle$) is taken into consideration [32]:

$$\omega^2 = \gamma^2 M_0^2 \left(\frac{\alpha k^2}{M_0^2} - \frac{K_1}{M_0^2} + \frac{H}{M_0}\right)\left(\frac{\alpha k^2}{M_0^2} + \frac{H}{M_0}\right). \quad (3.8)$$

The inclusion of the magnetic field violates the symmetry of the Hamiltonian. The latter is no more invariant with respect to rotations around the symmetry axis [energy (3.6) should be summed up with the term $-H_x M_x$]. More interesting is the consideration of the case where the perturbation does not change the Hamiltonian symmetry. The magnetoelastic interaction in the crystal is just a perturbation of this kind [32].

When taking the magnetoelastic interaction into account, it is convenient again to write the total energy of a ferromagnet in the form (2.1), where the first term is the magnetic energy of the crystal, which is determined by expression (3.6), and the second term is the elastic energy that looks like [18]

$$F_e = \frac{1}{2}C_{11}(\mathrm{E}_{xx} + \mathrm{E}_{yy})^2 + \frac{1}{2}C_{33}\mathrm{E}_{zz}^2 + C_{13}(\mathrm{E}_{xx} + \mathrm{E}_{yy})\mathrm{E}_{zz} + 2C_{44}(\mathrm{E}_{xz} + \mathrm{E}_{yz})^2 + \frac{1}{2}C_{66}(\mathrm{E}_{xx}^2 + \mathrm{E}_{yy}^2 + 2\mathrm{E}_{xy}^2), \quad (3.9)$$

where E_{ik} are the strain tensor components, and C_{ik} the elastic moduli of the second order for the uniaxial crystal. The third component describes the interaction between the magnetic and elastic subsystems [34, 35]:

$$F_{me} = \frac{1}{2}B_{11}(\mu_x^2 + \mu_y^2)(\mathrm{E}_{xx} + \mathrm{E}_{yy}) + \frac{1}{2}B_{13}\mu_z^2(\mathrm{E}_{xx} + \mathrm{E}_{yy}) + \frac{1}{2}B_{31}(\mu_x^2 + \mu_y^2)\mathrm{E}_{zz} + \frac{1}{2}B_{33}\mu_z^2\mathrm{E}_{zz} + \frac{1}{2}B_{44}(\mu_x\mu_z\mathrm{E}_{xz} + \mu_y\mu_z\mathrm{E}_{yz}) + \frac{1}{2}B_{66}(\mu_x^2\mathrm{E}_{xx} + \mu_y^2\mathrm{E}_{yy} + 2\mu_x\mu_y\mathrm{E}_{xy}), \quad (3.10)$$

where B_{ik} are the magnetoelastic interaction constants in the uniaxial symmetry case.

We emphasize that, in this case, the total energy (2.1) remains invariant with respect to rotations around the symmetry axis. When calculating the spectra of coupled oscillations, two equations of motion have to be taken into consideration. These are the Landau–Lifshitz equation (2.8) and the dynamical equation for the displacement vector (2.9) [3, 18]. In effect, we change from mere spin and mere elastic waves to coupled magnetoelastic oscillations [32]. Let us consider the degenerate "easy-plane" ground state and elucidate how the frequency of spin waves varies, if the magnetoelastic interaction is taken into account. In the indicated ground state, there are nonzero equilibrium values of the strain tensor components. These values can be easily obtained from the condition $\partial F/\partial \mathrm{E}_{ik} = 0$. As a result, we have

$$\begin{aligned}
\mathrm{E}^0_{xx} &= -\frac{B_{66}}{4C_{66}} - \frac{2B_{31}C_{13} - C_{33}(2B_{11}+B_{66})}{4(2C_{13}^2 - C_{33}(2C_{11}+C_{66}))},\\
\mathrm{E}^0_{yy} &= \frac{B_{66}}{4C_{66}} - \frac{2B_{31}C_{13} - C_{33}(2B_{11}+B_{66})}{4(2C_{13}^2 - C_{33}(2C_{11}+C_{66}))},\\
\mathrm{E}^0_{zz} &= \frac{B_{31}(2C_{11}+C_{66}) - C_{13}(2B_{11}+B_{66})}{2(2C_{13}^2 - C_{33}(2C_{11}+C_{66}))}.
\end{aligned} \qquad (3.11)$$

Similarly to what was done when considering the spectrum of mere spin waves (3.7) – i.e. small oscillations of the magnetic moment (2.5) are considered, where $\mathbf{M}(\mathbf{r},t)$ are small deviations from the equilibrium value $\boldsymbol{\mu}_0 = (0,0,1)$ – the strain tensor components can also be written as the sums of equilibrium values and small deviations from them (2.6), where E^0_{ik} are defined by expressions (3.11), and ε_{ik} are small deviations, which can be expressed in terms of the displacement vector $\mathbf{U}$ by formula (2.7) [18].

Let us expand the energy density of a uniaxial ferromagnet in a power series in small deviations m_i and ε_{ik} and take into account that, in this ground state, the component $m_x \approx -(m_y^2 + m_z^2)/2$ is of the second order of smallness. Then, considering the terms up to the second order of smallness with respect to small deviations, we obtain

$$F_{me2} = B_{66}e_{xy}m_y + \frac{1}{2}B_{44}e_{xz}m_z, \qquad (3.12)$$

$$F_{m2} = \frac{B_{66}}{4C_{66}}m_y^2 - \frac{1}{2}(K_{me} + K_1)m_z^2, \qquad (3.13)$$

where the notation $K_{me} = (B_{11} - B_{13} + B_{66})\mathrm{E}^0_{xx} + (B_{11} - B_{13})\mathrm{E}^0_{yy} + (B_{31} - B_{33})\mathrm{E}^0_{zz}$ was introduced. We also omitted the equilibrium energy of the zeroth order of smallness, because it does not contribute to the dynamical equations. Expression (3.9) for the elastic energy remains the same with an accuracy of substituting E_{ik} by ε_{ik}.

From expression (3.13), one can see that a magnetoelastic gap must appear, because the coefficient before m_y is not zero. It is important to mark that this gap arises exclusively owing to the magnetoelastic interaction. The coefficient before m_z demonstrates that the anisotropy constant is also renormalized.

By substituting the expansion of the total energy into the dynamical equations (2.8) and (2.9) and by changing to the Fourier components of the small deviations $\mathbf{M} = \mathbf{M}_0 \exp\{i(\mathbf{kr} - \omega t)\}$ and $\mathbf{U} = \mathbf{U}_0 \times \exp\{i(\mathbf{kr} - \omega t)\}$ with respect to the time t and the coordinates $\mathbf{r}$, we can obtain the dispersion laws for coupled magnetoelastic oscillations [3, 33]. However, we are interested now in modifications of the spectrum of quasispin waves.

Omitting standard intermediate calculations, we present the following results for the frequencies of quasispin waves in the examined ground state (they can be obtained from the dispersion law for coupled magnetoelastic oscillations [32]):

$$\omega^2 = \gamma^2 M_0^2 \left(\frac{\alpha k^2}{M_0^2} - \frac{K_1}{M_0^2} - \frac{K_{me}}{M_0^2}\right)\left(\frac{\alpha k^2}{M_0^2} + \frac{B_{66}^2}{2M_0^2 C_{66}}\right). \qquad (3.14)$$

From expression (3.14), it is easy to see that, at $k = 0$, the spin wave frequency is different from zero [32]:

$$\omega^2 = \gamma^2 \left(-\frac{K_1}{M_0^2} - \frac{K_{me}}{M_0^2}\right)\frac{B_{66}^2}{2C_{66}}. \qquad (3.15)$$

This is the very magnetoelastic gap that appears in the "easy-plane" ground state of a uniaxial ferromagnet, if the magnetoelastic interaction is taken into account.

From the obtained results, it follows that if the magnetoelastic interaction is taken into account, the degeneration of the initially degenerate "easy-plane" ground state of a uniaxial ferromagnet becomes eliminated, and massless magnons (Goldstone bosons) disappear. In other words, the magnetoelastic interaction "transforms" the Goldstone mode into the Higgs boson. It is also important to mark that the appearance of a magnetoelastic gap does not depend on the direction of the wave vector of an elastic wave.

Expression (3.14) testifies that the magnetoelastic interaction eliminates the ground state degeneration even in the isotropic magnet ($K_1 = 0$), which is in full agreement with general principles expounded in work [10]. Thus, the magnetoelastic interaction results in the appearance of a ground state, whose symmetry is lower than the symmetry of Hamiltonian [32].

3.2. Spectra of coupled magnetoelastic waves in a uniaxial ferromagnet

From the minimization condition (3.6) for the magnetic component of the energy, it is easy to show that, in a uniaxial ferromagnet in the absence of an external magnetic field ($H = 0$), there are three ground states for the magnetization vector [33]:

- along the easy magnetization axis: $\mathbf{M} \parallel \langle 001\rangle$; this is the "easy axis" phase; its existence condition is $K_1 + K_2 > 0$;
- in the basis plane: e.g., $\mathbf{M} \parallel \langle 100\rangle$; this is the "easy-plane" phase; its existence condition is $K_1 < 0$;
- at the angle θ with respect to the easy magnetization axis, which is determined by the expression $\cos^2\theta = -\frac{K_1}{K_2}$; this is the "angular" phase; its existence conditions are $K_2 < 0$ and $0 < K_1 < -K_2$.

In real experiments dealing with the elastic and magnetic properties of materials, the external magnetic field $\mathbf{H}$ is usually directed along the direction $\langle 001\rangle$ or $\langle 100\rangle$. Therefore, we will consider the corresponding "easy-axis" and "easy-plane" ground states.

In order to determine the dispersion laws for coupled magnetoelastic waves, let us use the dynamical equations for the magnetization vector $\boldsymbol{\mu}$ [Eq. (2.8)] and the particle displacement vector $\mathbf{U}$ [Eq. (2.9)]. For further calculations, we expand the total energy density (2.1) in a series in small deviations m_i and ε_{ik}, substitute them into the dynamical equations (2.8) and (2.9), and linearize the latter. In the obtained equations, we change to the Fourier components of small deviations $\mathbf{M} = \mathbf{M}_0 \exp\{i(\mathbf{kr} - \omega t)\}$ and $\mathbf{U} = \mathbf{U}_0 \exp\{i(\mathbf{kr} - \omega t)\}$, where ω is the frequency, and $\mathbf{k}$ the wave vector of collective waves, with respect to the time t and the coordinates $\mathbf{r}$. Then Eqs. (2.8) and (2.9), bring us to a system of six equations for the components of the vectors $\mathbf{M}_0$ and $\mathbf{U}_0$. The obtained systems of equations are given in Appendix B for each ground state of a uniaxial ferromagnet.

From the condition that the determinant of the system of dynamical equations equals zero, we obtain the dispersion laws for coupled magnetoelastic waves in the ground states of a cubic ferromagnet. Let us consider a few directions for the wave vector of elastic waves, which are used in experimental studies of acoustic waves in ferromagnets with uniaxial symmetry.

Phase "easy-axis": $\mathbf{H} \parallel \mathbf{M} \parallel \langle 001\rangle$

The equilibrium values of the strain tensor components in this ground state look like

$$\mathrm{E}^0_{xx} = \mathrm{E}^0_{yy} = \frac{B_{13}C_{33} - B_{33}C_{13}}{2(2C_{13}^2 - C_{33}(2C_{11} + C_{66}))},$$
$$\mathrm{E}^0_{zz} = \frac{-B_{13}C_{13} - B_{33}(2C_{11} + C_{66})}{2(2C_{13}^2 - C_{33}(2C_{11} + C_{66}))}. \qquad (3.16)$$

The dispersion laws are as follows. If $\mathbf{k} \parallel \langle 100\rangle$ or $\mathbf{k} \parallel \langle 010\rangle$,

$$\left(\omega^2 - \frac{(C_{11} + C_{66})}{\rho}k^2\right)\left(\omega^2 - \frac{C_{66}}{2\rho}k^2\right)\left[\left(\omega^2 - \frac{C_{44}}{\rho}k^2\right) \times \right.$$
$$\left. \times (\omega^2 - \gamma^2 M_0^2 \omega_{m\parallel}^2) - B_{44}^2 \left\{\frac{\omega_{m\parallel}\gamma^2 k^2}{16\rho}\right\}\right] = 0. \qquad (3.17)$$

If $\mathbf{k} \parallel \langle 001\rangle$,

$$\left(\omega^2 - \frac{C_{33}}{\rho}k^2\right)\left[\omega^2\left(\omega^2 - \frac{2C_{44}}{\rho}k^2\right)(\omega^2 - \gamma^2 M_0^2 \omega_{m\parallel}^2) - \right.$$
$$\left. - B_{44}^2 \left\{\frac{\omega_{m\parallel}\gamma^2 k^2}{8\rho}\left(\omega^2 - \frac{C_{44}}{\rho}k^2\right)\right\}\right] = 0. \qquad (3.18)$$

If $\mathbf{k} \parallel \langle 110\rangle$

$$\left(\omega^2 - \frac{(C_{11} + C_{66})}{\rho}k^2\right)\left(\omega^2 - \frac{C_{66}}{2\rho}k^2\right)\left[\left(\omega^2 - \frac{2C_{44}}{\rho}k^2\right) \times \right.$$
$$\left. \times (\omega^2 - \gamma^2 M_0^2 \omega_{m\parallel}^2) - B_{44}^2 \left\{\frac{\omega_{m\parallel}\gamma^2 k^2}{16\rho}\right\}\right] = 0. \qquad (3.19)$$

If $\mathbf{k} \parallel \langle 1\bar{1}0\rangle$,

$$\left(\omega^2 - \frac{(C_{11} + C_{66})}{\rho}k^2\right)\left(\omega^2 - \frac{C_{66}}{2\rho}k^2\right) \times$$
$$\times \left[\omega^2(\omega^2 - \gamma^2 M_0^2 \omega_{m\parallel}^2) - B_{44}^2 \left\{\frac{\omega_{m\parallel}\gamma^2 k^2}{16\rho}\right\}\right] = 0. \qquad (3.20)$$

In expressions (3.17)–(3.20), the following notation was introduced:

$$\omega_{m\|} = \frac{\alpha k^2}{M_0^2} + \frac{H}{M_0} + \frac{K_{me}}{M_0^2} + \frac{K_1}{M_0^2} + \frac{K_2}{M_0^2}, \qquad (3.21)$$

where

$$K_{me} = (B_{11} - B_{13} + B_{66})\mathrm{E}^0_{xx} + $$
$$+ B_{11} - B_{13})\mathrm{E}^0_{yy} + (B_{31} - B_{33})\mathrm{E}^0_{zz}.$$

***Phase "easy-plane":* H ∥ M ∥ ⟨100⟩**

The equilibrium values of the strain tensor components in this ground state are determined by expressions (3.11).

The dispersion laws are as follows. If $\mathbf{k} \parallel \langle 100\rangle$ or $\mathbf{k} \parallel \langle 010\rangle$,

$$\left(\omega^2 - \frac{(C_{11}+C_{66})}{\rho}k^2\right)\left(\omega^2 - \frac{C_{44}}{\rho}k^2\right) \times$$
$$\times \left[\left(\omega^2 - \frac{C_{66}}{2\rho}k^2\right)(\omega^2 - \gamma^2 M_0^2 \omega_{m1\perp}\omega_{m2\perp}) - \right.$$
$$\left. - B_{66}^2 \left\{\frac{\omega_{m1\perp}\gamma^2 k^2}{4\rho}\right\}\right] = 0. \qquad (3.22)$$

If $\mathbf{k} \parallel \langle 001\rangle$,

$$\left(\omega^2 - \frac{C_{33}}{\rho}k^2\right)\left[\omega^2\left(\omega^2 - \frac{2C_{44}}{\rho}k^2\right) \times \right.$$
$$\times (\omega^2 - \gamma^2 M_0^2 \omega_{m1\perp}\omega_{m2\perp}) - $$
$$\left. - B_{44}^2 \left\{\frac{\omega_{m2\perp}\gamma^2 k^2}{16\rho}\left(\omega^2 - \frac{C_{44}}{\rho}k^2\right)\right\}\right] = 0. \qquad (3.23)$$

If: $\mathbf{k} \parallel \langle 110\rangle$,

$$\left(\omega^2 - \frac{C_{66}}{2\rho}k^2\right)\left(\omega^2 - \frac{C_{44}}{\rho}k^2\right)\left[\left(\omega^2 - \frac{(C_{11}+C_{66})}{\rho}k^2\right) \times \right.$$
$$\left. \times (\omega^2 - \gamma^2 M_0^2 \omega_{m1\perp}\omega_{m2\perp}) - B_{66}^2 \left\{\frac{\omega_{m1\perp}\gamma^2 k^2}{4\rho}\right\}\right] = 0. \qquad (3.24)$$

If $\mathbf{k} \parallel \langle 1\bar{1}0\rangle$,

$$\omega^2\left(\omega^2 - \frac{C_{66}}{2\rho}k^2\right)\left[\left(\omega^2 - \frac{(C_{11}+C_{66})}{\rho}k^2\right) \times \right.$$
$$\left. \times (\omega^2 - \gamma^2 M_0^2 \omega_{m1\perp}\omega_{m2\perp}) - B_{66}^2 \left\{\frac{\omega_{m1\perp}\gamma^2 k^2}{4\rho}\right\}\right] = 0. \qquad (3.25)$$

In expressions (3.22)–(3.25), the following notations were introduced:

$$\omega_{m1\perp} = \frac{\alpha k^2}{M_0^2} + \frac{H}{M_0} - \frac{K_1}{M_0^2} - \frac{K_{me}}{M_0^2},$$
$$\omega_{m2\perp} = \frac{\alpha k^2}{M_0^2} + \frac{H}{M_0} + \frac{B_{66}^2}{2M_0^2 C_{66}}. \qquad (3.26)$$

Expressions (3.17)–(3.25) are the dispersion laws for coupled magnetoelastic waves in a uniaxial ferromagnet written in the general form. According to their structure, these dispersion equations have the standard form [3,4]. If the magnetoelastic interaction is neglected ($B_{ik} \to 0$), they are split into classical dispersion laws for spin waves [3] and elastic waves in cubic crystals [18].

The dispersion laws (3.17)–(3.25) calculated for coupled magnetoelastic waves in a ferromagnet with uniaxial symmetry make it possible to estimate the influence of the magnetic subsystem on the elastic properties of the crystal, namely, on the corresponding elastic moduli. From the dispersion laws (3.17)–(3.25), it follows that the following acoustic modes interact with spin waves in a uniaxial ferromagnet: $s_1^2 = C_{44}/\rho$, $s_2^2 = 2C_{44}/\rho$, $s_3^2 = C_{66}/2\rho$, and $s_4^2 = (C_{11}+C_{66})/\rho$. For illustration, Table 3.1 demonstrates the possibility of the magnetoelastic interac-

Table 3.1. **Interaction of acoustic modes with spin waves in a ferromagnet with uniaxial symmetry**

Acoustic mode and wave vector direction	Phase "easy axis": H ∥ M ∥ ⟨001⟩	Phase "easy plane": H ∥ M ∥ ⟨100⟩
s_1 $\mathbf{k} \parallel \langle 100\rangle$ and $\mathbf{k} \parallel \langle 010\rangle$	B_{44}	No interaction
s_2 $\mathbf{k} \parallel \langle 001\rangle$	B_{44}	B_{44}
s_2 $\mathbf{k} \parallel \langle 110\rangle$	B_{44}	No interaction
s_3 $\mathbf{k} \parallel \langle 100\rangle$ and $\mathbf{k} \parallel \langle 010\rangle$	No interaction	B_{66}
s_4 $\mathbf{k} \parallel \langle 110\rangle$	No interaction	B_{66}
s_4 $\mathbf{k} \parallel \langle 1\bar{1}0\rangle$	No interaction	B_{66}

tion for each acoustic mode depending on the magnetic moment direction in a uniaxial ferromagnet.

From Table 3.1, it becomes evident that the acoustic modes s_3 and s_4 do not interact with spin waves in the "easy-axis" ground state, and the acoustic mode s_1 in the "easy-plane" ground state. The magnetoelastic interaction between acoustic and spin waves is characterized exclusively by the constants B_{44} (the acoustic modes s_1 and s_2) and B_{66} (the acoustic modes s_3 and s_4). The other magnetoelastic constants are responsible only for the formation of a magnetoelastic gap in the spectrum of coupled oscillations [see expressions (3.21) and (3.26)].

4. Damping of Magnetoelastic Waves in Ferromagnets

A general equation describing both the dynamic and static properties of magnetically ordered media was first proposed by Landau and Lifshits in work [19]. This paper became one of the most popular works of those authors. It does not lose its relevance till now, and the equation proposed in this work is deservedly referred to in the literature as the Landau–Lifshits equation.

A fundamental result of work [19] consists in the derivation of the quasiequilibrium thermodynamic potential for a ferromagnet at low temperatures. This procedure is based on the consideration of a crystal symmetry and the classification of interactions in ferromagnets into two classes: weak relativistic and strong exchange interactions. Another, not less fundamental, result consists in the introduction of the effective magnetic field as a variational derivative of the ferromagnet thermodynamic potential with respect to the magnetization. At that time, the models that were widely used for the description of a spin wave dissipation did not correspond to those basic phenomenological principles. A term that is responsible for the magnetization relaxation in the Landau–Lifshits equation was proposed by Landau proceeding from general physical ideas concerning dissipative processes [19]. Later Gilbert constructed a dissipative function for the ferromagnet that corresponds to the Landau–Lifshits relaxation and proposed a formula for the relaxation term written in terms of the magnetization derivative with respect to time [20].

Landau and Lifshits in their work and Gilbert in his one used the model of a ferromagnet with a constant, by absolute value, magnetization. In other words, the longitudinal susceptibility of a ferromagnet was considered to equal zero (i.e. it was not taken into account). Irrespective of the vector equation of motion, the Landau–Lifshits–Gilbert relaxation term is characterized by a single relaxation constant, which corresponds to an isotropic medium. A consideration of the expression for the relaxation term in the framework of models [19, 20] demonstrates that it does not make allowance for the symmetry of a magnetic material, which results in plenty of physical contradictions. It is also important that the relaxation term in the Landau–Lifshits or Gilbert form is associated with the spin-spin and spin-orbit interactions. This circumstance, in turn, does not allow dissipative processes that are connected with the exchange interaction in crystals and are important in many cases [21–23] to be taken into account.

In the next decades, the theory of magnetism by Landau has been widely developed further. However, in many cases [36–38], the application of classical models proposed in the works by Landau and Lifshits turned out insufficient for the description of a number of phenomena. The corresponding researches showed that the Landau theory had to be improved, especially if dissipative processes in magnetically ordered structures are dealt with.

When constructing a dissipative function that describes the damping of collective magnetoacoustic waves, let us proceed from the expression for the total energy of a ferromagnet. In the case concerned, the latter should consist of the magnetic, elastic, and magnetoelastic components:

$$F = F_m + F_e + F_{me}. \quad (4.1)$$

In the case of uniaxial symmetry, the magnetic component of the ferromagnet energy has the form

$$F_m = \frac{\alpha}{2}\frac{\partial \boldsymbol{\mu}}{\partial x_i}\frac{\partial \boldsymbol{\mu}}{\partial x_k} + \frac{\left(\boldsymbol{\mu}^2 - 1\right)^2}{8\chi} - \\ -\frac{1}{2}K_1\mu_z^2 - \frac{1}{4}K_2\mu_z^4 - \mathbf{MH}. \quad (4.2)$$

This expression differs from expression (3.6) by the presence of a term that makes allowance for the homogeneous exchange interaction. The latter can insert a significant contribution, when considering the dissipative processes with exchange interaction.

Accordingly, for a ferromagnet with cubic symmetry, we have

$$F_m = \frac{\alpha}{2}\frac{\partial \boldsymbol{\mu}}{\partial x_i}\frac{\partial \boldsymbol{\mu}}{\partial x_k} + \frac{\left(\boldsymbol{\mu}^2 - 1\right)^2}{8\chi} + $$
$$+ K_1\left(\mu_x^2\mu_y^2 + \mu_x^2\mu_z^2 + \mu_y^2\mu_z^2\right) + K_2\mu_x^2\mu_y^2\mu_z^2 - \mathbf{MH}. \quad (4.3)$$

The elastic energy of a uniaxial crystal can be written in the form [18]

$$F_e = \frac{1}{2}C_{11}(\mathrm{E}_{xx} + \mathrm{E}_{yy})^2 + \frac{1}{2}C_{33}\mathrm{E}_{zz}^2 + C_{13}(\mathrm{E}_{xx} + \mathrm{E}_{yy}) \times$$
$$\times \mathrm{E}_{zz} + 2C_{44}(\mathrm{E}_{xz} + \mathrm{E}_{yz})^2 + \frac{1}{2}C_{66}(\mathrm{E}_{xx}^2 + \mathrm{E}_{yy}^2 + 2\mathrm{E}_{xy}^2), \quad (4.4)$$

where E_{ik} are the strain tensor components, and C_{ik} the elastic moduli of the second order for a uniaxial crystal. In the case of cubic symmetry, the elastic energy reads

$$F_e = \frac{1}{2}C_{11}(\mathrm{E}_{xx}^2 + \mathrm{E}_{yy}^2 + \mathrm{E}_{zz}^2) + C_{12}(\mathrm{E}_{xx}\mathrm{E}_{yy} +$$
$$+ \mathrm{E}_{xx}\mathrm{E}_{zz} + \mathrm{E}_{yy}\mathrm{E}_{zz}) + 2C_{44}(\mathrm{E}_{xy}^2 + \mathrm{E}_{xz}^2 + \mathrm{E}_{yz}^2), \quad (4.5)$$

It can also be written in form (2.3), as was proposed in works [23, 24].

The last term on the right-hand side of expression (4.1) describes the interaction between the magnetic and elastic subsystems. In the case of uniaxial symmetry, it looks like [34, 35]

$$F_{me} = \frac{1}{2}B_{11}(\mu_x^2 + \mu_y^2)(\mathrm{E}_{xx} + \mathrm{E}_{yy}) +$$
$$+ \frac{1}{2}B_{13}\mu_z^2(\mathrm{E}_{xx} + \mathrm{E}_{yy}) + \frac{1}{2}B_{31}(\mu_x^2 + \mu_y^2)\mathrm{E}_{zz} +$$
$$+ \frac{1}{2}B_{33}\mu_z^2\mathrm{E}_{zz} + \frac{1}{2}B_{44}(\mu_x\mu_z\mathrm{E}_{xz} + \mu_y\mu_z\mathrm{E}_{yz}) +$$
$$+ \frac{1}{2}B_{66}(\mu_x^2\mathrm{E}_{xx} + \mu_y^2\mathrm{E}_{yy} + 2\mu_x\mu_y\mathrm{E}_{xy}), \quad (4.6)$$

where B_{ik} are the corresponding constants of the magnetoelastic interaction. For a cubic crystal, the magnetoelastic energy can be written in the form [4]

$$F_{me} = B_1(\mu_x^2\mathrm{E}_{xx} + \mu_y^2\mathrm{E}_{yy} + \mu_z^2\mathrm{E}_{zz}) +$$
$$+ 2B_2(\mu_x\mu_y\mathrm{E}_{xy} + \mu_x\mu_z\mathrm{E}_{xz} + \mu_y\mu_z\mathrm{E}_{yz}), \quad (4.7)$$

or expression (2.4), which is more convenient in some cases, can be used.

In works [21, 22], a method was proposed that allows one to obtain a dissipative function for a ferromagnet proceeding from the crystal symmetry and the conservation laws for the magnetization of a crystal. It should be noted that the construction of a dissipative function for a ferromagnet is based on the phenomenological principles that were expounded in the works by Landau and Lifshits. According to those principles, the dissipative function is constructed, following the same rules as when constructing a quasiequilibrium thermodynamic potential, and it must include terms of the same origin as the total crystal energy does [18, 35]. Therefore, it is quite reasonable to represent the dissipative function density similarly to expression (4.1), i.e. as a sum of three terms,

$$q = q_m + q_e + q_{me}, \quad (4.8)$$

which describe the relaxation processes of the magnetic, elastic, and magnetoelastic origins, respectively.

Following works [21, 22, 33], the magnetic component of the dissipative function can be expressed in the form

$$q_m = \frac{1}{2}\lambda_{11}^r\left((H_x^{\mathrm{eff}})^2 + (H_y^{\mathrm{eff}})^2\right) + \frac{1}{2}\lambda^{\mathrm{ex}}\left(\frac{\partial \mathbf{H}^{\mathrm{eff}}}{\partial x_i}\right)^2, \quad (4.9)$$

for a uniaxial ferromagnet, and

$$q_m = \frac{1}{2}\lambda_{11}^r\left((H_x^{\mathrm{eff}})^2 + (H_y^{\mathrm{eff}})^2 + (H_z^{\mathrm{eff}})^2\right) +$$
$$+ \frac{1}{2}\lambda^{\mathrm{ex}}\left(\frac{\partial \mathbf{H}^{\mathrm{eff}}}{\partial x_i}\right)^2, \quad (4.10)$$

for a cubic crystal.

The elastic component of the dissipative function has to depend on the strain tensor derivatives with respect to the time and to be quadratic [18]. Thus, the most general form of this component looks like

$$q_e = \frac{1}{2}\lambda_{ij,sp}^e\frac{\partial E_{ij}}{\partial t}\frac{\partial E_{sp}}{\partial t}. \quad (4.11)$$

The fourth-rank tensor $\lambda_{ij,sp}^e$ is called the viscosity tensor, and its components are determined by the crystal symmetry, similarly to the elastic constant tensor that enters the elastic energy [18]. For a ferromagnet with uniaxial symmetry, we have

$$q_e = \frac{1}{2}\lambda_{11}^e\left(\frac{\partial E_{xx}}{\partial t} + \frac{\partial E_{yy}}{\partial t}\right)^2 + \frac{1}{2}\lambda_{33}^e\left(\frac{\partial E_{zz}}{\partial t}\right)^2 +$$

$$+\lambda_{13}^{e}\left(\frac{\partial E_{xx}}{\partial t}+\frac{\partial E_{yy}}{\partial t}\right)\frac{\partial E_{zz}}{\partial t}+$$
$$+2\lambda_{44}^{e}\left(\frac{\partial E_{xz}}{\partial t}+\frac{\partial E_{yz}}{\partial t}\right)^2+$$
$$+\frac{1}{2}\lambda_{66}^{e}\left[\left(\frac{\partial E_{xx}}{\partial t}\right)^2+\left(\frac{\partial E_{yy}}{\partial t}\right)^2+2\left(\frac{\partial E_{xy}}{\partial t}\right)^2\right]. \quad (4.12)$$

For a ferromagnet with cubic symmetry,

$$q_e=\frac{1}{2}\lambda_{11}^{e}\left(\left(\frac{\partial E_{xx}}{\partial t}\right)^2+\left(\frac{\partial E_{yy}}{\partial t}\right)^2+\left(\frac{\partial E_{zz}}{\partial t}\right)^2\right)+$$
$$+\lambda_{12}^{e}\left(\frac{\partial E_{xx}}{\partial t}\frac{\partial E_{yy}}{\partial t}+\frac{\partial E_{xx}}{\partial t}\frac{\partial E_{zz}}{\partial t}+\frac{\partial E_{yy}}{\partial t}\frac{\partial E_{zz}}{\partial t}\right)+$$
$$+2\lambda_{44}^{e}\left(\left(\frac{\partial E_{xy}}{\partial t}\right)^2+\left(\frac{\partial E_{yz}}{\partial t}\right)^2+\left(\frac{\partial E_{yz}}{\partial t}\right)^2\right). \quad (4.13)$$

The magnetoelastic component of the dissipative function is constructed by analogy with the corresponding component of the total energy of a ferromagnet. Again, proceeding from expressions (4.9), (4.10), and (4.11), this component has to include the strain tensor derivatives with respect to the time and the effective magnetic field components. It is known that the dissipative function must be invariant with respect to the transformations of the crystal symmetry group. Hence, the magnetoelastic component of the dissipative function should be constructed as a quadratic form composed from the invariants consisting of the strain tensor derivatives with respect to the time and the gradients of the effective magnetic field,

$$q_{me}=\frac{1}{2}\lambda_{ij,sp}^{me}\frac{\partial E_{ij}}{\partial t}\left(\frac{\partial H_s^{\rm eff}}{\partial x_p}+\frac{\partial H_p^{\rm eff}}{\partial x_s}\right). \quad (4.14)$$

For a ferromagnet with uniaxial symmetry, it looks like

$$q_{me}=\frac{1}{2}\lambda_{11}^{me}\left(\frac{\partial E_{xx}}{\partial t}+\frac{\partial E_{yy}}{\partial t}\right)\left(\frac{\partial H_x^{\rm eff}}{\partial x}+\frac{\partial H_y^{\rm eff}}{\partial y}\right)+$$
$$+\frac{1}{2}\lambda_{13}^{me}\left(\frac{\partial E_{xx}}{\partial t}+\frac{\partial E_{yy}}{\partial t}\right)\frac{\partial H_z^{\rm eff}}{\partial z}+$$
$$+\frac{1}{2}\lambda_{31}^{me}\frac{\partial E_{zz}}{\partial t}\left(\frac{\partial H_x^{\rm eff}}{\partial x}+\frac{\partial H_y^{\rm eff}}{\partial y}\right)+\frac{1}{2}\lambda_{33}^{me}\frac{\partial E_{zz}}{\partial t}\frac{\partial H_z^{\rm eff}}{\partial z}+$$
$$+\frac{1}{2}\lambda_{44}^{me}\left(\frac{\partial E_{xz}}{\partial t}\frac{\partial H_x^{\rm eff}}{\partial z}+\frac{\partial E_{zx}}{\partial t}\frac{\partial H_z^{\rm eff}}{\partial z}+\right.$$
$$\left.+\frac{\partial E_{yz}}{\partial t}\frac{\partial H_y^{\rm eff}}{\partial z}+\frac{\partial E_{zy}}{\partial t}\frac{\partial H_z^{\rm eff}}{\partial z}\right)+$$
$$+\frac{1}{2}\lambda_{66}^{me}\left(\frac{\partial E_{xx}}{\partial t}\frac{\partial H_x^{\rm eff}}{\partial x}+\frac{\partial E_{yy}}{\partial t}\frac{\partial H_y^{\rm eff}}{\partial y}+\right.$$
$$\left.+\frac{\partial E_{xy}}{\partial t}\frac{\partial H_x^{\rm eff}}{\partial y}+\frac{\partial E_{yx}}{\partial t}\frac{\partial H_y^{\rm eff}}{\partial x}\right). \quad (4.15)$$

Accordingly, for a cubic crystal,

$$q_{me}=\lambda_1^{me}\left(\frac{\partial E_{xx}}{\partial t}\frac{\partial H_x^{\rm eff}}{\partial x}+\frac{\partial E_{yy}}{\partial t}\frac{\partial H_y^{\rm eff}}{\partial y}+\frac{\partial E_{zz}}{\partial t}\frac{\partial H_z^{\rm eff}}{\partial z}\right)+$$
$$+2\lambda_2^{me}\left(\frac{\partial E_{xy}}{\partial t}\frac{\partial H_x^{\rm eff}}{\partial x}+\frac{\partial E_{yx}}{\partial t}\frac{\partial H_y^{\rm eff}}{\partial x}+\frac{\partial E_{xz}}{\partial t}\frac{\partial H_x^{\rm eff}}{\partial z}+\right.$$
$$\left.+\frac{\partial E_{zx}}{\partial t}\frac{\partial H_z^{\rm eff}}{\partial z}+\frac{\partial E_{yz}}{\partial t}\frac{\partial H_y^{\rm eff}}{\partial z}+\frac{\partial E_{zy}}{\partial t}\frac{\partial H_z^{\rm eff}}{\partial x}\right). \quad (4.16)$$

While calculating the damping of coupled magnetoacoustic waves, the dynamical equations (2.8) and (2.9) appended with the corresponding relaxation terms have to be used:

$$\frac{\partial \mathbf{M}}{\partial t}=-\gamma\boldsymbol{\mu}\times\mathbf{H}_{\rm eff}+\mathbf{R}_m, \quad (4.17)$$

$$\rho\ddot{\mathbf{U}}=-\frac{\delta F}{\delta \mathbf{U}}+\mathbf{R}_e, \quad (4.18)$$

where the relaxation terms read [18, 21, 22]

$$\mathbf{R}_m=\frac{\delta q}{\delta \mathbf{H}^{\rm eff}}, \quad (4.19)$$

$$\mathbf{R}_e=\frac{\delta q}{\delta\left(\frac{\partial U}{\partial t}\right)}. \quad (4.20)$$

Changing in Eqs. (4.17) and (4.18) to the Fourier components of the small deviations $\mathbf{M}=\mathbf{M}_0\times\exp\{i(\mathbf{kr}-\omega t)\}$ and $\mathbf{U}=\mathbf{U}_0\exp\{i(\mathbf{kr}-\omega t)\}$, where ω and $\mathbf{k}$ are the frequency and wave vector, respectively, of collective waves, with respect to the time t and the coordinates $\mathbf{r}$, we arrive at a system of six equations for the components of the vectors $\mathbf{M}_0$ and $\mathbf{U}_0$. From the condition for this system of equations to have a non-trivial solution (the determinant of the system is equal to zero), we obtain a dispersion law for coupled magnetoacoustic oscillations, in which their damping is taken into account. It should be noted that if the relaxation processes in the magnet are neglected, the obtained result must coincide with the dispersion laws (2.16)–(2.20) for a cubic ferromagnet and the dispersion laws (3.17)–(3.25) for a uniaxial one. However, if the magnetoelastic interaction is neglected, we separately obtain the spectra of spin and acoustic waves.

It should be noted that a more detailed consideration of the problem concerning the relaxation of coupled magnetoelastic oscillations and the calculation of corresponding spectra making allowance for their damping deserves a separate publication, which is planned by the authors.

5. Conclusions

In this work, the thorough analysis of the main types of models used to describe the interaction between the spin and acoustic waves in magnetically ordered materials has been carried out. The dispersion laws for coupled magnetoacoustic waves in ferromagnets with cubic symmetry are calculated. The behavior of the spectra of coupled magnetoacoustic waves in the vicinity of lattice phase transitions, namely, in the vicinity of the martensitic phase transformations in materials with the shape memory effect, is analyzed. The elastic moduli decrease in the vicinity of those phase transitions, which results in a growth of the magnetoelastic interaction.

It is shown that the interaction (the magnetoelastic interaction coefficient) between acoustic and spin waves depends on the directions of the oscillation wave vector and the magnetic moment in a ferromagnet. The magnetoelastic interaction coefficient can be strongly changed (it even can vanish), by depending on the specific parameters. This fact explains the dependence of the elastic moduli of a ferromagnet on the direction of the external magnetic field, which was revealed in many experiments.

The results are used to interpret experimental data obtained for the Ni–Mn–Ga alloy. They allow the phenomenon of a drastic reduction of the elastic moduli of this alloy when approaching its martensitic phase transition point to be explained theoretically. It is demonstrated that the main influence on the elastic characteristics of this material is exerted by the inhomogeneous magnetostriction.

Similar calculations of the dispersion laws for coupled magnetoacoustic waves are also performed in the case of ferromagnets with uniaxial symmetry. The influence of the magnetoacoustic interaction on the dispersion laws for quasispin waves in the degenerate ground state of an "easy-plane" uniaxial ferromagnet is analyzed. The results of calculations show that the magnetoelastic interaction eliminates the degeneration and gives rise to the appearance of a magnetoacoustic gap in the ferromagnet spectrum. In other words, the magnetoelastic interaction "transforms" the Goldstone mode into a Higgs boson. In this case, the appearance of a magnetoelastic gap does not depend on the direction of the oscillation wave vector.

In the case of uniaxial ferromagnet, it is shown that the magnetoelastic interaction can strongly increase in the vicinity of lattice phase transitions, which is accompanied by a drastic modification of the ferromagnet elastic moduli.

The dependence of the magnetoelastic interaction on the wave-vector and magnetic-moment directions in a uniaxial ferromagnet also takes place and, in some cases, is even more pronounced. The fact that some acoustic modes interact with spin waves in a ground state of a ferromagnet and do not interact in the other one enables an experimenter to accurately determine some magnetoelastic constants for a uniaxial ferromagnet.

We have also constructed a model dissipative function that describes the relaxation processes associated with the damping of coupled magnetoacoustic waves in ferromagnets with various symmetry properties. The obtained model dissipative function makes allowance for the ferromagnet symmetry and describes both the exchange and relativistic interactions in the crystal.

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APPENDIX A:
Systems of equations for the ground states of a cubic ferromagnet

***Phase 1*: $\mathbf{H} \parallel \mathbf{M} \parallel \langle 001 \rangle$**

The system of dynamical equations looks like

$$(\rho\omega^2 - C_{11}k_x^2 - C_{44}(k_y^2 + k_z^2))U_{0x} - $$
$$- (C_{12} + C_{44})k_x k_y U_{0y} - (C_{12} + C_{44})k_x k_z U_{0z} -$$
$$- i\frac{1}{2}\delta_2 k_z m_{0x} - i\frac{2}{3}(\delta_0 - 6\delta_1)k_x m_{0z} = 0; \quad \text{(A1)}$$

$$- (C_{12} + C_{44}) k_x k_y U_{0x} + (\rho\omega^2 -$$
$$- C_{11} k_y^2 - C_{44}(k_x^2 + k_z^2)) U_{0y} - (C_{12} + C_{44}) k_y k_z U_{0z} -$$
$$- i\frac{1}{2}\delta_2 k_z m_{0y} - i\frac{2}{3}(\delta_0 - 6\delta_1) k_y m_{0z} = 0; \qquad \text{(A2)}$$
$$- (C_{12} + C_{44}) k_x k_z U_{0x} - (C_{12} +$$
$$+ C_{44}) k_y k_z U_{0y} + (\rho\omega^2 - C_{11} k_z^2 - C_{44}(k_x^2 + k_y^2)) U_{0z} -$$
$$- i\frac{1}{2}\delta_2 k_x m_{0x} - i\frac{1}{2}\delta_2 k_y m_{0y} - i\frac{2}{3}(\delta_0 + 12\delta_1) k_z m_{0z} = 0; \qquad \text{(A3)}$$
$$i\frac{1}{2M_0}\gamma\delta_2 k_z U_{0y} + i\frac{1}{2M_0}\gamma\delta_2 k_y U_{0z} +$$
$$+ i\omega m_{0x} - \gamma M_0 \omega_{m1} m_{0y} = 0; \qquad \text{(A4)}$$
$$- i\frac{1}{2M_0}\gamma\delta_2 k_z U_{0x} - i\frac{1}{2M_0}\gamma\delta_2 k_x U_{0z} +$$
$$+ \gamma M_0 \omega_{m1} m_{0x} + i\omega m_{0y} = 0; \qquad \text{(A5)}$$
$$i\omega m_{0z} = 0. \qquad \text{(A6)}$$

In expressions (A4) and (A5), the notation

$$\omega_{m1} = \frac{\alpha k^2}{M_0^2} + \frac{H}{M_0} + \frac{2K_1}{M_0^2} + \frac{72\delta_1^2}{M_0^2(C_{11} - C_{12})}$$

was introduced.

***Phase 2:* H ‖ M ‖ ⟨101⟩**

The system of dynamical equations looks like

$$(\rho\omega^2 - C_{11} k_x^2 - C_{44}(k_y^2 + k_z^2)) U_{0x} -$$
$$- (C_{12} + C_{44}) k_x k_y U_{0y} - (C_{12} + C_{44}) k_x k_z U_{0z} -$$
$$- i\sqrt{2}\left(\left(\frac{1}{3}\delta_0 + 4\delta_1\right) k_x + \frac{1}{4}\delta_2 k_z\right) m_{0x} - i\frac{\sqrt{2}}{4}\delta_2 k_y m_{0y} -$$
$$- i\sqrt{2}\left(\left(\frac{1}{3}\delta_0 - 2\delta_1\right) k_x + \frac{1}{4}\delta_2 k_z\right) m_{0z} = 0; \qquad \text{(A7)}$$
$$- (C_{12} + C_{44}) k_x k_y U_{0x} + (\rho\omega^2 - C_{11} k_y^2 -$$
$$- C_{44}(k_x^2 + k_z^2)) U_{0y} - (C_{12} + C_{44}) k_y k_z U_{0z} -$$
$$- i\frac{\sqrt{2}}{3}(\delta_0 - 6\delta_1) k_y m_{0x} -$$
$$- i\frac{\sqrt{2}}{4}\delta_2 (k_x + k_z) m_{0y} - i\frac{\sqrt{2}}{3}(\delta_0 - 6\delta_1) k_y m_{0z} = 0; \qquad \text{(A8)}$$
$$- (C_{12} + C_{44}) k_x k_z U_{0x} - (C_{12} + C_{44}) k_y k_z U_{0y} +$$
$$+ (\rho\omega^2 - C_{11} k_z^2 - C_{44}(k_x^2 + k_y^2)) U_{0z} -$$
$$- i\sqrt{2}\left(\left(\frac{1}{3}\delta_0 - 2\delta_1\right) k_x + \frac{1}{4}\delta_2 k_z\right) m_{0x} - i\frac{\sqrt{2}}{4}\delta_2 k_y m_{0y} -$$
$$- i\sqrt{2}\left(\left(\frac{1}{3}\delta_0 + 4\delta_1\right) k_x + \frac{1}{4}\delta_2 k_z\right) m_{0z} = 0; \qquad \text{(A9)}$$
$$i\frac{1}{4M_0}\gamma\delta_2 k_y U_{0x} + i\frac{1}{4M_0}\gamma\delta_2 (k_x + k_z) U_{0y} +$$
$$+ i\frac{1}{4M_0}\gamma\delta_2 k_y U_{0z} + i\omega m_{0x} - \frac{\sqrt{2}}{2}\gamma M_0 \omega_{m2} m_{0y} = 0; \qquad \text{(A10)}$$
$$- i\frac{6}{M_0}\gamma\delta_1 k_x U_{0x} + i\frac{6}{M_0}\gamma\delta_1 k_z U_{0z} + \frac{\sqrt{2}}{2}\gamma M_0 \omega_{m3} m_{0x} +$$
$$+ i\omega m_{0y} - \frac{\sqrt{2}}{2}\gamma M_0 \omega_{m3} m_{0z} = 0; \qquad \text{(A11)}$$
$$- i\frac{1}{4M_0}\gamma\delta_2 k_y U_{0x} - i\frac{1}{4M_0}\gamma\delta_2 (k_x + k_z) U_{0y} -$$
$$- i\frac{1}{4M_0}\gamma\delta_2 k_y U_{0z} + \frac{\sqrt{2}}{2}\gamma M_0 \omega_{m2} m_{0y} + i\omega m_{0z} = 0. \qquad \text{(A12)}$$

In expressions (A10)–(A12), the notations

$$\omega_{m2} = \frac{\alpha k^2}{M_0^2} + \frac{H}{M_0} + \frac{K_1}{M_0^2} + \frac{K_2}{2M_0^2} +$$
$$+ \frac{36\delta_1^2}{M_0^2(C_{11} - C_{12})} + \frac{\delta_2^2}{8M_0^2 C_{44}},$$
$$\omega_{m3} = \frac{\alpha k^2}{M_0^2} + \frac{H}{M_0} - \frac{2K_1}{M_0^2}{}_0 + \frac{\delta_2^2}{4M_0^2 C_{44}}$$

were introduced.

***Phase 3:* H ‖ M ‖ ⟨111⟩**

The system of dynamical equations looks like

$$(\rho\omega^2 - C_{11} k_x^2 - C_{44}(k_y^2 + k_z^2)) U_{0x} -$$
$$- (C_{12} + C_{44}) k_x k_y U_{0y} - (C_{12} + C_{44}) k_x k_z U_{0z} -$$
$$- i\frac{\sqrt{3}}{3}\left(\left(\frac{2}{3}\delta_0 + 8\delta_1\right) k_x + \frac{1}{2}\delta_2 (k_y + k_z)\right) m_{0x} -$$
$$- i\frac{\sqrt{3}}{3}\left(\left(\frac{2}{3}\delta_0 - 4\delta_1\right) k_x + \frac{1}{2}\delta_2 k_y\right) m_{0y} -$$
$$- i\frac{\sqrt{3}}{3}\left(\left(\frac{2}{3}\delta_0 - 4\delta_1\right) k_x + \frac{1}{2}\delta_2 k_z\right) m_{0z} = 0; \qquad \text{(A13)}$$
$$- (C_{12} + C_{44}) k_x k_y U_{0x} + (\rho\omega^2 - C_{11} k_y^2 -$$
$$- C_{44}(k_x^2 + k_z^2)) U_{0y} - (C_{12} + C_{44}) k_y k_z U_{0z} -$$
$$- i\frac{\sqrt{3}}{3}\left(\left(\frac{2}{3}\delta_0 - 4\delta_1\right) k_y + \frac{1}{2}\delta_2 k_x\right) m_{0x} -$$
$$- i\frac{\sqrt{3}}{3}\left(\left(\frac{2}{3}\delta_0 + 8\delta_1\right) k_y + \frac{1}{2}\delta_2 (k_x + k_z)\right) m_{0y} -$$
$$- i\frac{\sqrt{3}}{3}\left(\left(\frac{2}{3}\delta_0 - 4\delta_1\right) k_y + \frac{1}{2}\delta_2 k_z\right) m_{0z} = 0; \qquad \text{(A14)}$$
$$- (C_{12} + C_{44}) k_x k_z U_{0x} - (C_{12} + C_{44}) k_y k_z U_{0y} +$$
$$+ (\rho\omega^2 - C_{11} k_z^2 - C_{44}(k_x^2 + k_y^2)) U_{0z} -$$
$$- i\frac{\sqrt{3}}{3}\left(\left(\frac{2}{3}\delta_0 - 4\delta_1\right) k_z + \frac{1}{2}\delta_2 k_x\right) m_{0x} -$$
$$- i\frac{\sqrt{3}}{3}\left(\left(\frac{2}{3}\delta_0 - 4\delta_1\right) k_z + \frac{1}{2}\delta_2 k_y\right) m_{0y} -$$
$$- i\frac{\sqrt{3}}{3}\left(\left(\frac{2}{3}\delta_0 + 8\delta_1\right) k_z + \frac{1}{2}\delta_2 (k_x + k_y)\right) m_{0z} = 0; \qquad \text{(A15)}$$
$$i\frac{1}{6M_0}\gamma\delta_2 (k_y - k_z) U_{0x} + i\frac{1}{6M_0}\gamma(24\delta_1 k_y + \delta_2 k_x) U_{0y} -$$
$$- i\frac{1}{6M_0}\gamma(24\delta_1 k_z + \delta_2 k_x) U_{0z} +$$
$$+ i\omega m_{0x} - \frac{\sqrt{3}}{3}\gamma M_0 \omega_{m4} m_{0y} + \frac{\sqrt{3}}{3}\gamma M_0 \omega_{m4} m_{0z} = 0; \qquad \text{(A16)}$$
$$- i\frac{1}{6M_0}\gamma(24\delta_1 k_x + \delta_2 k_y) U_{0x} - i\frac{1}{6M_0}\gamma\delta_2 (k_x -$$

$$-k_z)U_{0y}+i\frac{1}{6M_0}\gamma(24\delta_1k_z+\delta_2k_y)U_{0z}+$$

$$+\frac{\sqrt{3}}{3}\gamma M_0\omega_{m4}m_{0x}+i\omega m_{0y}-\frac{\sqrt{3}}{3}\gamma M_0\omega_{m4}m_{0z}=0; \quad \text{(A17)}$$

$$i\frac{1}{6M_0}\gamma(24\delta_1k_x+\delta_2k_z)U_{0x}-i\frac{1}{6M_0}\times$$

$$\times\gamma(24\delta_1k_y+\delta_2k_z)U_{0y}+i\frac{1}{6M_0}\gamma\delta_2(k_x-k_y)U_{0z}-$$

$$-\frac{\sqrt{3}}{3}\gamma M_0\omega_{m4}m_{0x}+\frac{\sqrt{3}}{3}\gamma M_0\omega_{m4}m_{0y}+i\omega m_{0z}=0. \quad \text{(A18)}$$

In expressions (A16)–(A18), the notation

$$\omega_{m4}=\frac{\alpha k^2}{M_0^2}+\frac{H}{M_0}-\frac{4K_1}{3M_0^2}-\frac{4K_2}{9M_0^2}+\frac{\delta_2^2}{4M_0^2C_{44}}$$

was introduced.

APPENDIX B: Systems of equations for the ground states of a uniaxial ferromagnet

Phase "easy axis": $\mathbf{H}\parallel\mathbf{M}\parallel\langle 001\rangle$

The system of dynamical equations looks like

$$(\rho\omega^2-(C_{11}+C_{66})k_x^2-\frac{1}{2}C_{66}k_y^2-C_{44}k_z^2)U_{0x}-$$

$$-\left(\left(C_{11}+\frac{1}{2}C_{66}\right)k_xk_y+C_{44}k_z^2\right)U_{0y}-$$

$$-((C_{13}+C_{44})k_xk_z+C_{44}k_yk_z)U_{0z}+$$

$$+i\frac{1}{4}B_{44}k_zm_{0x}+iB_{13}k_xm_{0z}=0; \quad \text{(B1)}$$

$$-\left(\left(C_{11}+\frac{1}{2}C_{66}\right)k_xk_y+C_{44}k_z^2\right)U_{0x}+$$

$$+\left(\rho\omega^2-\frac{1}{2}C_{66}k_x^2-(C_{11}+C_{66})k_y^2-C_{44}k_z^2\right)U_{0y}-$$

$$-((C_{13}+C_{44})k_yk_z+C_{44}k_xk_z)U_{0z}+$$

$$+i\frac{1}{4}B_{44}k_zm_{0y}+iB_{13}k_ym_{0z}=0; \quad \text{(B2)}$$

$$-((C_{13}+C_{44})k_xk_z+C_{44}k_yk_z)U_{0x}-$$

$$-((C_{13}+C_{44})k_yk_z+C_{44}k_xk_z)U_{0y}+$$

$$+(\rho\omega^2-C_{44}(k_x+k_y)^2-C_{33}k_z^2)U_{0z}+$$

$$+i\frac{1}{4}B_{44}k_xm_{0x}+i\frac{1}{4}B_{44}k_ym_{0y}+iB_{33}k_ym_{0z}=0; \quad \text{(B3)}$$

$$-i\frac{1}{4M_0}\gamma B_{44}k_xU_{0y}-i\frac{1}{4M_0}\gamma B_{44}k_yU_{0z}+i\omega m_{0x}-$$

$$-\gamma M_0\omega_{m||}m_{0y}=0; \quad \text{(B4)}$$

$$i\frac{1}{4M_0}\gamma B_{44}k_zU_{0x}+i\frac{1}{4M_0}\gamma B_{44}k_xU_{0z}+$$

$$+\gamma M_0\omega_{m||}m_{0x}+i\omega m_{0y}=0; \quad \text{(B5)}$$

$$i\omega m_{0z}=0. \quad \text{(B6)}$$

In expressions (B4) and (B5), the notation

$$\omega_{m\parallel}=\frac{\alpha k^2}{M_0^2}+\frac{H}{M_0}+\frac{K_{me}}{M_0^2}+\frac{K_1}{M_0^2}+\frac{K_2}{M_0^2},$$

where

$$K_{me}=(B_{11}-B_{13}+B_{66})\mathrm{E}^0_{xx}+$$

$$+(B_{11}-B_{13})\mathrm{E}^0_{yy}+(B_{31}-B_{33})\mathrm{E}^0_{zz},$$

was introduced.

Phase "easy plane": $\mathbf{H}\parallel\mathbf{M}\parallel\langle 100\rangle$

The system of dynamical equations looks like

$$(\rho\omega^2-(C_{11}+C_{66})k_x^2-\frac{1}{2}C_{66}k_y^2-C_{44}k_z^2)U_{0x}-$$

$$-\left(\left(C_{11}+\frac{1}{2}C_{66}\right)k_xk_y+C_{44}k_z^2\right)U_{0y}-$$

$$-((C_{13}+C_{44})k_xk_z+C_{44}k_yk_z)U_{0z}+i(B_{11}+B_{66})\times$$

$$\times k_xm_{0x}+i\frac{1}{2}B_{66}k_ym_{0y}+i\frac{1}{4}B_{44}k_zm_{0z}=0, \quad \text{(B7)}$$

$$-\left(\left(C_{11}+\frac{1}{2}C_{66}\right)k_xk_y+C_{44}k_z^2\right)U_{0x}+$$

$$+\left(\rho\omega^2-\frac{1}{2}C_{66}k_x^2-(C_{11}+C_{66})k_y^2-C_{44}k_z^2\right)U_{0y}-$$

$$-((C_{13}+C_{44})k_yk_z+C_{44}k_xk_z)U_{0z}+$$

$$+iB_{11}k_ym_{0x}+i\frac{1}{2}B_{66}k_xm_{0y}=0, \quad \text{(B8)}$$

$$-((C_{13}+C_{44})k_xk_z+C_{44}k_yk_z)U_{0x}-$$

$$-((C_{13}+C_{44})k_yk_z+C_{44}k_xk_z)U_{0y}+$$

$$+(\rho\omega^2-C_{44}(k_x+k_y)^2-C_{33}k_z^2)U_{0z}+$$

$$+iB_{31}k_zm_{0x}+i\frac{1}{4}B_{44}k_xm_{0z}=0, \quad \text{(B9)}$$

$$i\omega m_{0x}=0, \quad \text{(B10)}$$

$$-i\frac{1}{4M_0}\gamma B_{44}k_zU_{0x}-i\frac{1}{4M_0}\gamma B_{44}k_xU_{0z}+i\omega m_{0y}-$$

$$-\gamma M_0\omega_{m1\perp}m_{0z}=0, \quad \text{(B11)}$$

$$i\frac{1}{4M_0}\gamma B_{66}k_yU_{0x}+i\frac{1}{4M_0}\gamma B_{66}k_xU_{0y}+$$

$$+\gamma M_0\omega_{m2\perp}m_{0y}+i\omega m_{0z}=0. \quad \text{(B12)}$$

In expressions (B11) and (B12), the notations

$$\omega_{m1\perp}=\frac{\alpha k^2}{M_0^2}+\frac{H}{M_0}-\frac{K_1}{M_0^2}-\frac{K_{me}}{M_0^2},$$

$$\omega_{m2\perp}=\frac{\alpha k^2}{M_0^2}+\frac{H}{M_0}+\frac{B_{66}^2}{2M_0^2C_{66}}$$

were introduced.

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В.Г. Бар'яхтар, О.Г. Данилевич

МАГНІТОПРУЖНІ ХВИЛІ
В ФЕРОМАГНЕТИКАХ В ОКОЛІ
СТРУКТУРНИХ ФАЗОВИХ ПЕРЕХОДІВ У ҐРАТЦІ

Р е з ю м е

Розраховані закони дисперсії зв'язаних магнітопружних хвиль для феромагнетиків кубічної та одновісної симетрії. Проведено аналіз особливостей отриманих законів дисперсії в околі спін-переорієнтаційних фазових переходів. Показано, що взаємодія між звуковими та спіновими хвилями залежить від напрямку магнітного моменту феромагнетика. Досліджено вплив магнітопружної взаємодії на закон дисперсії квазіспінових хвиль у виродженому основному стані одновісного феромагнетика "легка площина". Розрахунки показують, що магнітопружна взаємодія знімає виродження та приводить до появи магнітоакустичної щілини у спектрі. Проаналізовано поведінку спектрів зв'язаних магнітопружних хвиль в околі фазових переходів в гратці, а саме в околі мартенситних фазових перетворень в матеріалах з ефектом пам'яті форми. Отримані результати використані для інтерпретації експериментальних даних для сплаву Ni–Mn–Ga. Теоретично пояснене явище різкого зменшення пружних модулів даного сплаву при наближенні до мартенситних фазових переходів. Показано, що при цьому основний вплив на пружні характеристики матеріалу відіграє неоднорідна магнітострикція. Побудована модель дисипативної функції, що описує релаксаційні процеси обумовлені затуханням зв'язаних магнітопружних хвиль у феромагнетиках кубічної та одновісної симетрії. Отримана модель дисипативної функції базується на врахуванні симетрії магнетика та описує як обмінну, так і релятивістську взаємодію в кристалі.