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THE SHEAR MODULUS AND STRUCTURE OF CARTILAGE TISSUE

Cartilage tissue has been considered as a polymeric gel network formed from chains of fibrillar proteins and proteoglycans. A theoretical model of the network consisting of network units connected by inter-unit chains is proposed, the corresponding deformation mechanism for cartilage tissue is developed, and a formula for the shear modulus is obtained. The shear modulus for elastic cartilage tissue is also determined experimentally. The number of inter-unit chains in the model of the elastic cartilage tissue is evaluated to be equal to 10.

Keywords: cartilage tissue, shear modulus, deformation, network model.

1. Introduction

It is well known (see, e.g., [1, 2]) that cartilage tissue plays an important role in the life of the human organism. It performs a supporting function by creating the resistance to external loads. In this paper, we study a molecular mechanism that provides the supporting function. The chemical structure of cartilage tissue has been studied at length. It was found that this tissue consists of cells (2%) and an intercellular substance (98%). The latter contains water (75%), inorganic salts (8%), fibrillar proteins (collagen and elastin, 10%), proteoglycans, and other organic substances (5%).

The structural unit of proteoglycans is an aggregate that includes various polymer chains. Hyaluronic acid serves as the main chain. About a hundred protein chains are attached to it. Each of the latter is connected to up to 20 chains of sulfated glycosaminoglycans (chondroitin sulfate and others).

The spatial structure of cartilage tissue, especially at the molecular level, has been studied to a much lesser extent in comparison with its chemical structure. Moreover, modern concepts about the spatial structure of cartilage tissue are often contradictory. One of such contradictions consists in that, on the one hand, there is a hypothesis (see, e.g., [1]) that in order to counteract an external load, fibrillar proteins and proteoglycans have to form a framework, although the specific structure of this framework is not discussed; but on the other hand, a model was proposed [3] – below, it will be referred to as the fluid model – where there is no such a framework and cartilage tissue is considered as a fluid where chains of fibrillar proteins and proteoglycan aggregates "float" separately from one another. A model similar to the latter (fluid) model was applied in [4] to determine internal forces arising in cartilage tissue. The orientation of water molecules near the surface of proteoglycan aggregates was supposed to be the main factor responsible for the appearance of those forces.

In this work, we substantiate the necessity of the framework, propose a model of its structure, and consider the mechanism of deformation in this model.

2. Stress States of Cartilage Tissue

The term "stress state" is used in the mechanics of continuous media (see, e.g., [5, 6]), where the examined physical system is considered. as a continuum. Deformations arising in the continuum under the action of external loads are described using the strain tensor ϵ_{ik}. As a result of strains in the continuum, there arise stresses that counteract the external force. The stress state is described by introducing the stress tensor σ_{ik} into consideration.

In this article, we confine the consideration to the case of elastic deformations and assume that cartilage tissue, in terms of the continuous medium theory, is an isotropic elastic continuum. For such a continuum, the relationship between the tensors ϵ_{ik} and σ_{ik} is described by the formula

$$\sigma_{ik} = K\epsilon_{ll}\Delta_{ik} + 2G\left(\epsilon_{ik} - \frac{1}{3}\epsilon_{ll}\right), \quad (1)$$

where K is the bulk elastic modulus, ϵ_{ll} the first invariant of the tensor ϵ_{ik}, and Δ_{ik} the Kronecker delta. The components of the tensor $(\epsilon_{ik} - \frac{1}{3}\epsilon_{ll})$ are called shear strains, and G is called the shear modulus. The first summand in sum (1) corresponds to the stress state, which is a uniform compression. As one can see from formula (1), this state is realized if

$$G = 0. \quad (2)$$

In the mechanics of continuous media, equality (2) is a characteristic feature of fluid. This means that models introduced in [3, 4] can be used only if the case of uniform compression is realized in cartilage tissue.

However, this state does not arise in cartilage tissue, as a rule, under the action of external loads. In particular, it is not realized in the intervertebral disc, although its behavior, as is assumed in [3], is described by the fluid model. In fact, the disc is squeezed by the vertebrae in the vertical direction and is freely deformed in the horizontal one. This means that a state of simple compression takes place in the disc. In this case, if the vertical axis is denoted by the number 1, formula (1) reads

$$\sigma_{11} = E\epsilon_{11}, \quad (3)$$

where E is Young's modulus. This module is known to satisfy the equality

$$E = \frac{9KG}{3K + G}, \quad (4)$$

whence one can see that if equality (2) holds, then $E = 0$ so that

$$\sigma_{11} = 0. \quad (5)$$

From equality (5), it follows that cartilage tissue of the intervertebral disc does not perform its main, supporting, function: no stresses arise in the disc that would counteract the pressure from the vertebrae. Therefore, the fluid model proposed in [3] to describe the behavior of intervertebral disc does not correspond to the real situation.

The same is applicable to any stress state: equality (2) means that there are no stresses in cartilage tissue that would counteract shear deformations and, as a result, cartilage tissue ceases to perform the supporting function.

3. Network Model of Cartilage Tissue

As is seen from the aforesaid, the shear modulus G plays the role of a key parameter that governs the structure of cartilage tissue. It was already mentioned that G must differ from zero. But at the same time, the value of G should not be too high in order to provide cartilage tissue with the required ability to sustain considerable deformation. What should be the structure of cartilage tissue to satisfy both conditions?

Based on the chemical composition of cartilage tissue, it can be argued that, from the physical viewpoint, this tissue, in essence, is an aqueous polymer solution. Such a solution can be in either a sol or a gel phase [7]. In the former case, $G = 0$; in the latter,

$$G > 0, \quad (6)$$

with, the G-values being small in comparison with the corresponding values for solids. This fact makes it possible to say that cartilage tissue has a gel structure in general. It consists of macromolecules that form a network [7], in which the chains are separated by nodes into subchains. As was already mentioned, fibrillar proteins comprise the most fraction of cartilage tissue polymers. Therefore, it is reasonable to assert that the cartilage tissue network is mainly formed by the chains of those polymers. In proteoglycan aggregates, the chains are interconnected, effectively also forming a network. Therefore, these aggregates can be considered as a part of the total cartilage network.

Suppose that in the framework of the network model that we try to construct, the chains have identical physical properties, so we ignore the difference induced by their different chemical compositions. The space between the network subchains is occupied by molecules of water and other substances that were not used when constructing the network. In our opinion, such a network can be a framework mentioned in Introduction, which provides the supporting function of cartilage tissue.

As was already mentioned, the concept of "shear modulus" is introduced in the mechanics of continuous media, where the strain tensor is considered to be a continuous function $\epsilon_{ik}(\mathbf{r})$ of the vector $\mathbf{r} = (x_1, x_2, x_3)$ described by its projections (x_1, x_2, x_3) in the Cartesian coordinate frame. An infinitesimally small region $d\mathbf{r}$ around a point described by the radius vector $\mathbf{r}$ is called the mathematically infinitesimal volume. The tensor $\epsilon_{ik}(\mathbf{r})$ is a quantity that characterizes the deformation of the indicated region. Accordingly, the shear modulus is also the deformation characteristic of this region. Therefore, when if we deal with the shear modulus of cartilage, the latter is considered as a continuum.

According to thermodynamics (see, e.g., [8]), such a consideration is possible if the system can be considered as a set of weakly interacting small regions with a local equilibrium in each of them. Every small region is called the physically infinitesimal volume. We will use the term "block" for it. The size of the block will be denoted by L. Mathematically, the infinitesimal volume is an idealized image of a block. Therefore, the shear modulus characterizes the deformation of the block as a whole.

The existence of weak interaction means that the blocks are separated by interblock gaps, the structure of which is substantially disordered in comparison with the structure of the block itself. Let us denote the thickness of such a gap by h.

Now, the mentioned thermodynamic model for cartilage tissue acquires the form of the network depicted in Fig. 1. Here, the nodes are shown as solid circles, the chains as solid curves, and block boundaries as dashed lines. The space occupied by the blocks is colored. Actually, the arrangement of nodes in space is disordered, but in Fig. 1, for clarity, the nodes are so arranged that they form a lattice.

A characteristic feature of the network shown in Fig. 1 is the availability of subchains of three types: (*1*) surface, one end of which is connected to the block surface node; (*2*) internal, both ends of which are connected to the neighbor nodes of the same block; and (*3*) intermediate, which connect the nearest surface nodes of different blocks.

Since water comprises 75% of cartilage tissue, let us assume that, by the order of magnitude, the value of the parameter l for cartilage tissue is equal to the corresponding value for fluids, $l = 10^{-7}$ m [9]. Let us estimate the distance L_1 between the nodes. Let d denotes the link size. A chain is considered as a cylinder of diameter d. Assume that the chains form a simple cubic lattice with a unit cell of volume

$$V_1 = L_1^3. \tag{7}$$

The unit cell is formed by 12 subchains, and $\frac{1}{4}$ of the volume occupied by those chains is located within the cell. Accordingly, for the volume V_1 of polymer in the cell, we have

$$V_1 = \frac{1}{4} 12 \frac{\pi d^2 L_1}{4}. \tag{8}$$

As was already mentioned, the relative volume occupied by polymer in cartilage tissue is of an order of 0.1, i.e.,

$$\frac{V_1}{V} \sim 0.1. \tag{9}$$

Substituting inequalities (7) and (8) into expression (9), we obtain the estimate

$$L_1 \sim 10d. \tag{10}$$

Taking $d \sim 10^{-9}$ m, we numerically estimate $L_1 \sim 10^{-8}$ m. As one can see from Fig. 1, by order of magnitude, the thickness of the interblock gap equals L_1, i.e., $h \sim 10^{-8}$ m.

4. Mechanism of Deformation in the Network Model

As was indicated above, cartilage tissue can perform its inherent supporting function due to condition (6). Owing to the same condition, transverse waves can propagate in cartilage tissue. There is the well-known formula

$$G = \rho c^2, \tag{11}$$

where ρ is the cartilage tissue density and c the propagation velocity of the transverse wave, as well as the formula

$$c = \frac{\omega}{k}, \tag{12}$$

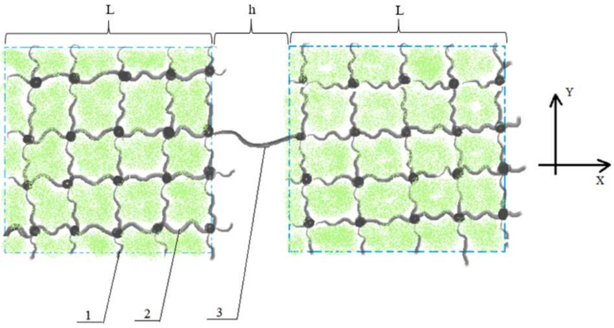

Fig. 1. Network model of cartilage tissue

where ω is the oscillation frequency and k the wave number.

The shear modulus characterizes the deformation of a block as a whole. In other words, the introduction of the concept "shear modulus" is associated with the introduction of a spatial scale equal to L. In such a way, we confine the wavelength λ by the inequality

$$\lambda \geq 2L, \tag{13}$$

which can be rewritten in the form

$$k \leq \frac{\pi}{L}. \tag{14}$$

Let the wave propagate along the X-axis. Accordingly, the blocks will move along the Y-axis (Fig. 1). We consider a plane wave, so the displacements of the blocks whose centers of inertia correspond to the same coordinate x are also identical. This circumstance makes it possible to determine the characteristics of the wave by studying any linear set of blocks for which the coordinates y of their centers of inertia are identical (for example, $y = 0$) in the wave absence.

Denoting the displacements of the i-th and $(i+1)$-th blocks in such a linear set as u_i and u_{i+1}, respectively, and the difference $U_{i+1} - U_i$ as ΔU_i, we can write the following expression for the force F_i that deforms the intermediate chains connecting two blocks:

$$F_i = -f\Delta U_i, \tag{15}$$

where f is the force constant associated with the relative block displacement. The limiting frequency of oscillations, ω_m, in this set of blocks is determined by the equality

$$\omega_m = 2\sqrt{\frac{f}{m}}, \tag{16}$$

where m is the block mass, which will be calculated according to the formula

$$m = \rho L^3. \tag{17}$$

Substituting equalities (16) and (17) into formula (12) and taking into account expression (11), we get

$$G = \frac{4}{\pi^2}\frac{f}{L}. \tag{18}$$

Let n be the number of intermediate chains connecting the adjacent blocks. Accordingly, for the force F_i, we have the expression

$$F_i = nQ_i, \tag{19}$$

where Q is the force per intermediate chain. By comparing equations (15) and (19), we write

$$\Delta U_i = -\frac{n}{f}Q_i. \tag{20}$$

The number of links in the intermediate chain equals $\xi = \frac{L_1}{d}$. If this chain is deformed owing to the displacement of the blocks along the Y-axis, the links of the intermediate chain shift in the same direction. Denoting by the displacements of the j-th and $(j+1)$-th links as $W_{i,j}$ and $W_{i,j+1}$, respectively, and the difference $W_{i,j+1} - W_{i,j}$ as $\Delta W_{i,j}$, we have

$$\Delta U_i = \sum_{j=1}^{\xi} \Delta W_{i,j}. \tag{21}$$

This expression can be replaced by the approximate equality

$$\Delta U_i \approx \xi \Delta W_i, \tag{22}$$

where ΔW_i is the average value of the difference $\Delta W_{i,j}$.

The bonds connecting the links undergo the action of the same force Q_i, which makes it possible to write the expression

$$Q_i = -q\Delta W_i, \tag{23}$$

where q is the force constant associated with the relative displacement of the links. Substituting formula (23) into equality (22), we obtain

$$\Delta U_i = -\frac{\xi}{q}Q_i. \tag{24}$$

By comparing expressions (20) and (24), we get

$$f = \frac{nq}{\xi}. \tag{25}$$

Accordingly, formula (18) acquires the form

$$G = \frac{4}{\pi^2}\frac{nqd}{LL_1}. \tag{26}$$

On the basis of the concept of the virtual connections between protein chains, which was proposed in work [10], we will assume that the deformation of intermediate chains occurs via the rotation of those connections. The value $q = 3.4$ N/m corresponding to deformation of this type was taken from work [11].

5. Experiment

The article deals with the elastic properties of cartilage tissue. Therefore, from the experimental viewpoint, it is important to choose such a tissue type to study for which these properties would be most pronounced. As was already mentioned, the elasticity of cartilage tissue is mainly determined by fibrillar proteins, collagen and elastin. The stiffness of the latter is much lower than that of collagen. Therefore, in view of the research aim, it is most pertinent to choose cartilage tissue with the maximum concentration of elastin. Such elastic cartilage tissue is a component of various organs, in particular, the ear.

The pig's ear was used to prepare specimens. Stripes of the width $a = 6$ mm were cut from the upper side of the auricle (Fig. 2, *a*). After removing skin layers and cleaning the surfaces, plates of cartilage tissue with the thickness $b = 4$ mm were obtained.

The shear modulus G was determined using the torsional pendulum method. Its theory and the construction of the torsional pendulum device are described in works [12, 13]. The specimen was rigidly fixed in the device making use of clamps, as is shown in Fig. 2, *b*. The working length of the specimen l was 2.7 mm.

The primary experimental information in the applied method is the dependence of the pendulum rotation angle ϕ on the time t. An example of the obtained dependences is shown in Fig. 3. Oscillations were registered by means of video recording and further processed using *Tracker* software. The measurements

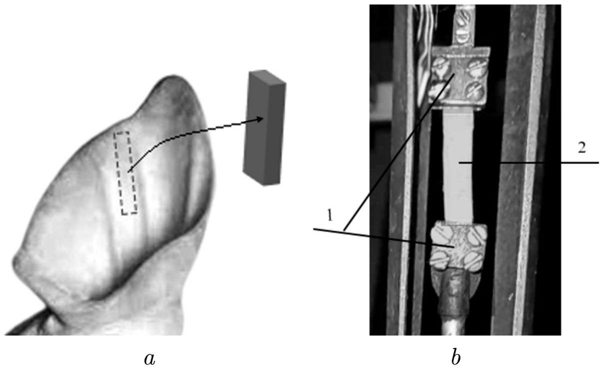

Fig. 2. (*a*) Preparation of specimen and (*b*) its fixation in the device. (*1*) clamps, (*2*) specimen

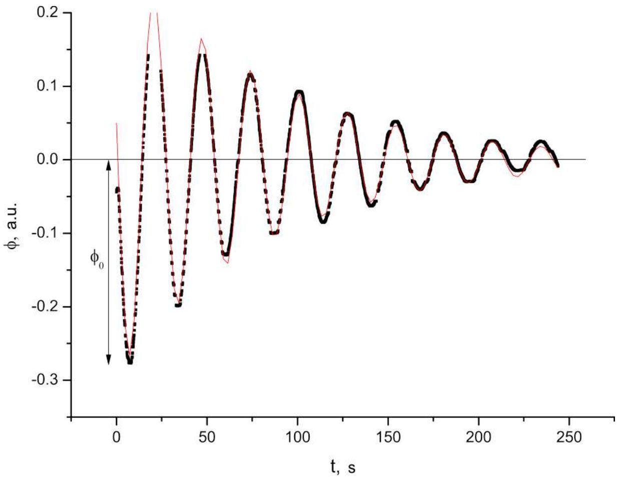

Fig. 3. Dependence of the pendulum rotation angle ϕ on the time t

were carried out at a temperature of 20 °C. The dependences $\phi(t)$ were used to determine the cyclic frequency of oscillations Ω.

The shear modulus G was calculated via the following formula proposed by the theory of the applied method:

$$G = \frac{J_s l}{J}\Omega^2, \tag{27}$$

where J_s is the moment of inertia of the pendulum rotating part, and J is the moment of inertia of the specimen cross-section. The moment of inertia J was calculated using the formula (see, e.g., [14])

$$J = 0.3\, b^3 a. \tag{28}$$

Formula (27) was obtained for the case when the specimen deformations induced by oscillations are

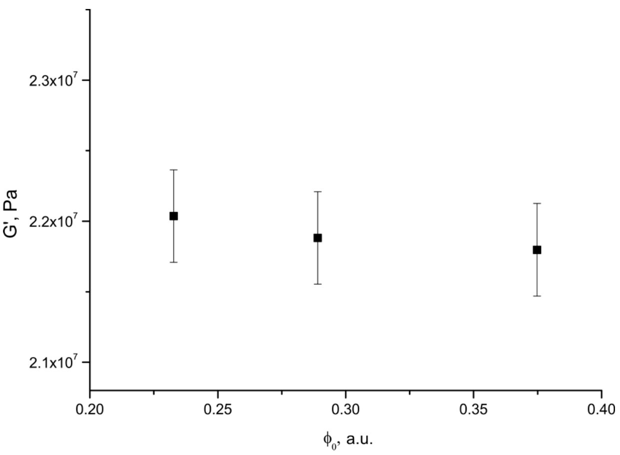

Fig. 4. Dependence of the shear modulus on the initial oscillation amplitude ϕ_0

elastic. Provided this condition, the shear modulus does not depend on the oscillation amplitude. Let us verify whether our experiment satisfied the indicated condition. From the plot shown in Fig. 4, it follows that the value of G' remains constant within the confidence interval, which confirms that formula (27) was applied soundly.

According to formula (26), the number of intermediate chains connecting neighbor blocks is determined by the expression

$$n = G\frac{\pi^2}{4}\frac{LL_1}{qd}. \quad (29)$$

Substituting the average value of $G' = 2.1749\times10^7$ Pa and the numerical values given above for other quantities into this expression, we find that the number of intermediate chains for elastic cartilage tissue equals ten.

6. Conclusions

Neglecting the presence of cells, cartilage tissue is polymer gel by its structure. This structure is based on a network formed by chains of fibrillar proteins and proteoglycans. It consists of network blocks about 10^{-7} m in size connected by intermediate chains. At a deformation induced by external forces, the blocks move as a whole. As a result, the external load becomes actually distributed among the intermediate chains.

The number of intermediate chains is much smaller than the number of chains per cross-section of adjacent blocks. Therefore, the intermediate chains undergo a substantial deformation, which leads to small values of the shear modulus of cartilage tissue in comparison with that of solids. The experimental study of elastic cartilage tissue brought about a value of its shear modulus of an order of 10^7 Pa, and the number of intermediate chains connecting the adjacent blocks was found to equal about ten. In time, cartilage tissue becomes stiffer, which will manifest itself in an increase in the shear modulus. In the framework of the proposed deformation mechanism, the growth of the shear modulus is induced by the growth in the number of intermediate chains. According to the proposed model, the flexibility of cartilage tissue can be restored by introducing such a substance into cartilage tissue that can penetrate into the interblock gaps and break the intermediate chains.

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ЗСУВНИЙ МОДУЛЬ
ТА СТРУКТУРА ХРЯЩОВОЇ ТКАНИНИ

Хрящова тканина розглядається як полімерний гель, сітка якого утворена ланцюгами фібрилярних білків та протеогліканів. Запропоновано модель такої сітки, що складається із сітчастих блоків, з'єднаних прохідними ланцюгами. В рамках запропонованої моделі досліджено механізм деформації хрящової тканини. Розроблено механізм деформації в рамках такої моделі. Отримано формулу для зсувного модуля у згаданій моделі. Експериментально визначено величину зсувного модуля для еластичної хрящової тканини. Встановлено, що число прохідних ланцюгів у запропонованій моделі для еластичного типу хрящової тканини становить 10.

Ключові слова: хрящова тканина, модуль зсуву, деформація, сіткова модель.