SCALAR FIELD POTENTIAL DISTRIBUTION FOR A "THICK" NULL STRING OF CONSTANT RADIUS

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PACS 04.60.CF

The general form of the scalar field potential distribution for a "thick" null string of constant radius moving along the axis z and completely lying in a plane orthogonal to this axis at every time moment is proposed. The conditions, under which a contraction of the field to a one-dimensional object (circle of radius R) results in the asymptotic coincidence of components of the energy-momentum tensor of a scalar field with those of a closed null string of the same radius, are found.

1. Introduction

String theories show the steady progress during several recent decades. In spite of problems inevitable for any developing theory, they arouse admiration both due to the results already obtained and their great possibilities in the future. On the one hand, the interest in cosmic strings and other topological solutions is initiated by the role possibly played by topological defects in the process of evolution of the Universe (e.g., string mechanisms of generation of primary inhomogeneities of the matter density in the early Universe or ideas of the topological inflation). On the other hand, it is due to the physical properties of these objects significantly differing from those of common matter [1]–[7].

Null strings realize the zero tension limit in the string theory [5], [8]. The components of the energy-momentum tensor for a null string have the following form [8]:

$$T^{mn}\sqrt{-g} = \gamma \int d\tau d\sigma \; x^m_{,\tau} x^n_{,\tau} \delta^4 \left(x^l - x^l\left(\tau,\sigma\right)\right), \qquad (1)$$

where the indices m, n, and l take on the values 0, 1, 2, and 3, the functions $x^m = x^m(\tau,\sigma)$ determine the trajectory of a null string, τ and σ are the parameters on the light surface of the null string, $x^m_{,\tau} = \partial x^m/\partial\tau$, $g = |g_{mn}|$, g_{mn} is the metric tensor of the environment, and $\gamma = \text{const}$.

In the cylindrical system of coordinates,

$$x^0 = t, \quad x^1 = \rho, \quad x^2 = \theta, \quad x^3 = z,$$

the functions $x^m(\tau,\sigma)$ that determine the trajectory of a closed string with constant (time-invariant) radius R moving along the axis z and completely lying in a plane orthogonal to this axis at every time moment have the following form:

$$t = \tau, \quad \rho = R = \text{const}, \quad \theta = \sigma, \quad z = \pm\tau, \qquad (2)$$

where the signs $\pm$ correspond to the choice of the direction of motion. For definiteness, the negative sign will be supposed in (2) hereinafter. It is worth noting that trajectory (2) is rather often realized for a closed null string moving in background gravitational fields, for example in the space-time of a plane gravitational wave [9] and in Lorentz spaces with a nontrivial conform group which describe the propagation of shock gravitational waves [10].

Under conditions (2), the nonzero components of the energy-momentum tensor (1) are as follows:

$$T^{00} = T^{33} = -T^{03} = \frac{\gamma}{\sqrt{-g}}\delta(q)\delta(\rho - R), \qquad (3)$$

where $q = t + z$.

For trajectory (2), all directions on the hypersurfaces $z = \text{const}$ are equivalent; therefore, the metric functions $g_{mn} = g_{mn}(t,\rho,z)$. Using the invariance of the quadratic form with respect to the inversion of θ to $-\theta$, we obtain

$$g_{02} = g_{12} = g_{32} = 0. \qquad (4)$$

One can also see that the space-time quadratic form must be invariant with respect to the simultaneous inversion $t \to -t$, $z \to -z$. Hence,

$$g_{mn}(t,\rho,z) = g_{mn}(-t,\rho,-z), \tag{5}$$

which yields

$$g_{01} = g_{31} = 0. \tag{6}$$

Finally, using the free choice of the systems of coordinates in the general relativity theory, we partially fix it by the requirement

$$g_{03} = 0. \tag{7}$$

Thus, the quadratic form for the problem to be solved can be presented as

$$dS^2 = e^{2\nu}(dt)^2 - A(d\rho)^2 - B(d\theta)^2 - e^{2\mu}(dz)^2, \tag{8}$$

where ν, μ, A, and B depend on the variables t, ρ, and z.

Since trajectory (2) must be one of the solutions of the motion equations of a null string, additional restrictions imposed on the metric functions can be obtained, whose fulfillment provides the constancy of a trajectory of the null string specified by (2).

The motion of a null string in the pseudo-Riemannian space is determined by the system of equations [5]

$$x^m_{,\tau\tau} + \Gamma^m_{pq} x^p_{,\tau} x^q_{,\tau} = 0, \tag{9}$$

$$g_{mn} x^m_{,\tau} x^n_{,\tau} = 0, \quad g_{mn} x^m_{,\tau} x^n_{,\sigma} = 0, \tag{10}$$

where Γ^m_{pq} are the Christoffel symbols. Putting down the first of Eqs. (10) for (8), one can make sure that it has the form $e^{2\nu} - e^{2\mu} = 0$ for trajectory (2). Consequently,

$$\nu \equiv \mu, \tag{11}$$

whereas the rest of equations of system (9), (10) for (2), (8) under condition (11) are reduced to the single equation $\nu_{,t} - \nu_{,z} = 0$, which yields

$$\nu = \nu(q,\rho). \tag{12}$$

Then, according to (4),

$$\nu(q,\rho) = \nu(-q,\rho), \tag{13}$$

i.e., the function $\nu(q,\rho)$ is even in q.

Analyzing the system of Einstein equations constructed for (3), (8) and using conditions (11)–(13), the dependence of functions of the quadratic form (8) can be redefined as

$$A = A(q,\rho), \quad B = B(q,\rho). \tag{14}$$

In this case, the Einstein system itself is reduced to the equations

$$\frac{A_{,qq}}{A} + \frac{B_{,qq}}{B} - 2\nu_{,q}\left(\frac{A_{,q}}{A} + \frac{B_{,q}}{B}\right) - $$

$$-\frac{1}{2}\left(\left(\frac{A_{,q}}{A}\right)^2 + \left(\frac{B_{,q}}{B}\right)^2\right) = -2\chi T_{00}, \tag{15}$$

$$2\nu_{,\rho\rho} + 2(\nu_{,\rho})^2 + \frac{B_{,\rho\rho}}{B} - \frac{1}{2}\left(\frac{B_{,\rho}}{B}\right)^2 +$$

$$+\nu_{,\rho}\left(\frac{B_{,\rho}}{B} - \frac{A_{,\rho}}{A}\right) - \frac{1}{2}\frac{A_{,\rho}}{A}\frac{B_{,\rho}}{B} = 0, \tag{16}$$

$$\frac{B_{,q\rho}}{B} + 2\nu_{q,\rho} - \nu_{,\rho}\left(\frac{A_{,q}}{A} + \frac{B_{,q}}{B}\right) -$$

$$-\frac{1}{2}\frac{B_{,\rho}}{B}\left(\frac{A_{,q}}{A} + \frac{B_{,q}}{B}\right) = 0, \tag{17}$$

$$2\nu_{,\rho\rho} + 3(\nu_{,\rho})^2 - \nu_{,\rho}\frac{A_{,\rho}}{A} = 0, \tag{18}$$

$$(\nu_{,\rho})^2 + \nu_{,\rho}\frac{B_{,\rho}}{B} = 0, \tag{19}$$

where $T_{00} = \gamma \frac{e^{2\nu}}{\sqrt{AB}}\delta(q)\delta(\rho - R)$.

With the use of the obtained conditions (11), (12), and (14), expression (8) can be presented in the form

$$dS^2 = e^{2\nu}\left((dt)^2 - (dz)^2\right) - A(d\rho)^2 - B(d\theta)^2, \tag{20}$$

where $\nu = \nu(q,\rho), B = B(q,\rho)$, and $A = A(q,\rho)$. It is also worth noting that, according to (5), the functions $A(q,\rho)$ and $B(q,\rho)$ in (20) are even in q, i.e.,

$$A(q,\rho) = A(-q,\rho), \quad B(q,\rho) = B(-q,\rho). \tag{21}$$

Equation (3) implies that, beyond the string, i.e., at $q \neq 0, \rho \neq R$, all components of its energy-momentum tensor are equal to zero, while the non-zero ones (tending to infinity) appear directly at the string. This allows one to investigate the system of Einstein equations in two directions:

1. By restricting oneself to the analysis of the "external" problem in the region, where the components of the energy-momentum tensor (right-hand sides of the Einstein equations) are equal to zero.

2. By considering the components of the energy-momentum tensor of a string as a limit of some "thick" distribution and analyzing the Einstein equations for this "thick" distribution.

As was shown in [11], the analysis of the "external" problem results in a large number of vacuum solutions of the Einstein equations that satisfy the problem symmetry. However, the criteria allowing one to choose those describing the gravitational field of a null string from this totality of solutions remain unclear. When trying to consider the components of the energy-momentum tensor of the string as a limit of some "thick" distribution (e.g., simply replacing the delta functions in the energy-momentum tensor by the corresponding delta-function sequences), some errors can arise due to the indeterminacy of considering the possible appearance of terms (multipliers) tending to zero (constant) under the contraction of this "thick" distribution into a one-dimensional object. That is why it is more suitable to start from some "well-determined" "thick" distribution such as, for example, a real massless scalar field (as we consider a scalar null object) and then to contract it to a string of the required configuration provided that the components of the energy-momentum tensor of the scalar field asymptotically coincide with those of the null-string energy-momentum tensor.

2. System of Einstein Equations for the "Thick" Problem

The components of the energy-momentum tensor for a real massless scalar field have the form [2]

$$T_{\alpha\beta} = \varphi_{,\alpha}\varphi_{,\beta} - \frac{1}{2}g_{\alpha\beta}L, \tag{22}$$

where $L = g^{\omega\lambda}\varphi_{,\omega}\varphi_{,\lambda}$, $\varphi_{,\alpha} = \partial\varphi/\partial x^{\alpha}$, φ is the scalar field potential, and the indices α, β, ω, and λ take on the values $0, 1, 2$, and 3. To provide the self-consistency of the Einstein equations and tensor (22), we demand that

$$T_{\alpha\beta} = T_{\alpha\beta}\left(q, \rho\right) \rightarrow \varphi = \varphi\left(q, \rho\right). \tag{23}$$

Putting down Eq.(22) for quadratic form (20), we obtain

$$T_{00} = (\varphi_{,q})^2 + \frac{e^{2\nu}}{2A}(\varphi_{,\rho}^2), \quad T_{01} = T_{13} = \varphi_{,q}\varphi_{,\rho},$$

$$T_{33} = (\varphi_{,q})^2 - \frac{e^{2\nu}}{2A}(\varphi_{,\rho}^2), \quad T_{03} = (\varphi_{,q})^2,$$

$$T_{11} = \frac{1}{2}(\varphi_{,\rho})^2, \quad T_{22} = -\frac{B}{2A}(\varphi_{,\rho})^2. \tag{24}$$

The system of Einstein equations for (20), (24) can be presented as follows:

$$2\nu_{,\rho\rho} + 3(\nu_{,\rho})^2 - \nu_{,\rho}\frac{A_{,\rho}}{A} = -\frac{\chi}{2}(\varphi_{,\rho})^2, \tag{25}$$

$$(\nu_{,\rho})^2 + \nu_{,\rho}\frac{B_{,\rho}}{B} = \frac{\chi}{2}(\varphi_{,\rho})^2, \tag{26}$$

$$\frac{A_{,qq}}{A} + \frac{B_{,qq}}{B} - 2\nu_{,q}\left(\frac{A_{,q}}{A} + \frac{B_{,q}}{B}\right) -$$

$$-\frac{1}{2}\left(\left(\frac{A_{,q}}{A}\right)^2 + \left(\frac{B_{,q}}{B}\right)^2\right) = -2\chi\left(\varphi_{,q}\right)^2, \tag{27}$$

$$2\nu_{,\rho\rho} + 2(\nu_{,\rho})^2 + \frac{B_{,\rho\rho}}{B} - \frac{1}{2}\left(\frac{B_{,\rho}}{B}\right)^2 +$$

$$+\nu_{,\rho}\left(\frac{B_{,\rho}}{B} - \frac{A_{,\rho}}{A}\right) - \frac{1}{2}\frac{A_{,\rho}}{A}\frac{B_{,\rho}}{B} = -\frac{\chi}{2}(\varphi_{,\rho})^2, \tag{28}$$

$$\frac{B_{,q\rho}}{B} + 2\nu_{q,\rho} - \nu_{,\rho}\left(\frac{A_{,q}}{A} + \frac{B_{,q}}{B}\right) -$$

$$-\frac{1}{2}\frac{B_{,\rho}}{B}\left(\frac{A_{,q}}{A} + \frac{B_{,q}}{B}\right) = -2\chi\varphi_{,q}\varphi_{,\rho}. \tag{29}$$

Let us consider system (25)–(29) for the distribution of the scalar field already concentrated inside a "thin" ring with the variables q and ρ taking values in the interval

$$q \in \left[-\Delta q, +\Delta q\right], \rho \in \left[R - \Delta\rho, R + \Delta\rho\right]. \tag{30}$$

Here, R stands for the radius of a closed null string, while Δq and $\Delta\rho$ are small positive constants that determine the "thickness" of the ring, i.e.,

$$\Delta q \ll 1, \quad \Delta\rho \ll 1. \tag{31}$$

With a further contraction of this "thin" ring into a one-dimensional object (null string),

$$\Delta q = 0, \quad \Delta\rho = 0, \tag{32}$$

the space, where such a "thick" null string moves and for which the variables q and ρ take on values in the interval $q \in (-\infty, +\infty)$, $\rho \in [0, +\infty)$, can be conditionally divided into three regions:
– region I, for which

$$q \in (-\infty, -\Delta q) \cup (+\Delta q, +\infty)\,, \rho \in [0, +\infty)\,, \qquad (33)$$

– region II, for which

$$q \in (-\Delta q, +\Delta q)\,, \rho \in [0, R - \Delta\rho) \cup (R + \Delta\rho, \infty)\,, \qquad (34)$$

– region III, for which

$$q \in [-\Delta q, +\Delta q]\,, \rho \in [R - \Delta\rho, R + \Delta\rho]\,. \qquad (35)$$

Moreover, since the scalar field is concentrated inside such a "thin" ring specified by (30)–(32), the scalar field potential is equal to zero in regions I and II, contrary to region III (inside the "thin" ring), where $\varphi \neq 0$.

Since the contraction of the scalar field into a string must result in the asymptotic coincidence of system (25)–(29) with the system for a closed null string (15)–(19), we obtain for the regions specified by (33), (34) (regions I and II):

$$\varphi = 0, \quad \varphi_{,\rho} = 0, \quad \varphi_{,q} = 0. \qquad (36)$$

For the region specified by (35) (inside the "thin" ring), we have, in the general case,

$$\varphi \neq 0, \quad \varphi_{,\rho} \neq 0, \quad \varphi_{,q} \neq 0. \qquad (37)$$

Comparing the system of Einstein equations for a closed null string (15)–(19) with system (25)-(29), we may conclude that, under the contraction of the scalar field into a string of the required configuration, i.e., at $\Delta q = 0$, $\Delta\rho = 0$,

$$(\varphi_{,\rho})^2\big|_{q=0,\rho=R} = 0, \quad (\varphi_{,q})^2\big|_{q=0,\rho=R} \to \infty,$$

$$(\varphi_{,\rho}\varphi_{,q})\big|_{q=0,\rho=R} = 0. \qquad (38)$$

According to (36), the scalar field potential in region I at any fixed value of $q = q_0 \in (-\infty, -\Delta q) \cup (+\Delta q, +\infty)$, and all values of $\rho \in [0, +\infty)$

$$\varphi(q_0, \rho) = 0. \qquad (39)$$

Considering the scalar field potential distribution at any fixed value of $q = q_0 \in (-\Delta q, +\Delta q)$ (regions II and III), we obtain that the condition

$$\varphi(q_0, \rho) = 0 \qquad (40)$$

must be realized if $\rho \in [0, R - \Delta\rho) \cup (R + \Delta\rho, +\infty)$ (region II), whereas, for $\rho \in (R - \Delta\rho, R + \Delta\rho)$ (region III),

$$\varphi(q_0, \rho) \neq 0. \qquad (41)$$

3. Scalar Field Potential Distribution for a "Thick" Null String

For the obtained conditions (39)–(41), it is suitable to present the scalar field potential distribution in the form

$$\varphi(q, \rho) = \ln\left(\frac{1}{\alpha(q) + \lambda(q) f(\rho)}\right), \qquad (42)$$

where the functions $\alpha(q)$ and $\lambda(q)$ are symmetric with respect to the inversion of q to $-q$, i.e.,

$$\alpha(q) = \alpha(-q), \quad \lambda(q) = \lambda(-q), \qquad (43)$$

the function $\alpha(q) + \lambda(q) f(\rho)$ is bounded

$$0 < \alpha(q) + \lambda(q) f(\rho) \leq 1, \qquad (44)$$

and the scalar field potential specified by (42), in accordance with (44), can assume values from

$$\varphi = 0, \quad \text{at} \quad \alpha(q) + \lambda(q) f(\rho) = 1, \qquad (45)$$

to

$$\varphi \to \infty, \quad \text{at} \quad \alpha(q) + \lambda(q) f(\rho) \to 0. \qquad (46)$$

Moreover, according to (39), (45),

$$\alpha(q) = 1, \quad \lambda(q) = 0 \qquad (47)$$

in region I.

Since the scalar field potential equals zero in region II, the following condition must be met at $q \in (-\Delta q, +\Delta q)$ and any fixed value of $\rho \in [0, R - \Delta\rho) \cup (R + \Delta\rho, +\infty)$:

$$\alpha(q) + \lambda(q) f(\rho_0) = 1. \qquad (48)$$

In region III, $\varphi \neq 0$. Therefore, for the same values of $q \in (-\Delta q, +\Delta q)$ and at $\rho \in (R - \Delta\rho, R + \Delta\rho)$,

$$0 < \alpha(q) + \lambda(q) f(\rho_0) < 1. \qquad (49)$$

Equation (48) implies that, for all $\rho \in [0, R - \Delta\rho) \cup (R + \Delta\rho, +\infty)$, the values of the function $f(\rho)$ are constant

$$f(\rho)\big|_{\rho \in [0, R-\Delta\rho) \cup (R+\Delta\rho, +\infty)} = f_0 = \text{const.} \qquad (50)$$

Moreover, $f_0 \neq 0$, while the functions $\alpha(q)$ and $\lambda(q)$ are interconnected:

$$\lambda(q) = \frac{1}{f_0}\left(1 - \alpha(q)\right). \qquad (51)$$

Substituting (51) into (49), we obtain

$$0 < \alpha(q) + (1 - \alpha(q))\frac{f(\rho_0)}{f_0} < 1 \tag{52}$$

for region III ($\varphi \neq 0$). This together with (46) and (52) mean that, as $\varphi \to \infty$,

$$\alpha(q) \to 0, \quad f(\rho) \to 0. \tag{53}$$

Thus, the functions $\alpha(q)$ and $f(\rho)$ in the expression for the scalar field potential (42) are bounded and, for any $q \in (-\infty, +\infty)$ and $\rho \in [0, +\infty)$, take on values in the intervals

$$0 \leq \alpha(q) \leq 1, \quad 0 \leq \ f(\rho) \leq f_0. \tag{54}$$

Moreover, according to (47),

$$\alpha(q)|_{q\in(-\infty,-\Delta q)\cup(+\Delta q,+\infty)} = 1 \tag{55}$$

in region I, whereas conditions (53) with regard for the symmetry of the function $\alpha(q)$ (Eq.(43)) yield

$$\lim_{q\to 0} \alpha(q) \to 0. \tag{56}$$

The distribution for the function $f(\rho)$ at $\rho \in [0, R - \Delta\rho) \cup (R + \Delta\rho, +\infty)$ is determined by Eq. (50), and, according to (53),

$$f(\rho)|_{\rho\to R} \to 0. \tag{57}$$

Differentiating (43) with regard for (51), we obtain

$$\varphi_{,q} = -\frac{\alpha_{,q}(1 - f(\rho)/f_0)}{\alpha(q) + (1 - \alpha(q))f(\rho)/f_0},$$

$$\varphi_{,\rho} = -\frac{(1 - \alpha(q))f_{,\rho}/f_0}{\alpha(q) + (1 - \alpha(q))f(\rho)/f_0}. \tag{58}$$

Using (47), (48), and (50) for (58), we obtain that $\varphi_{,\rho} = 0$ and $\varphi_{,q} = 0$ in regions I and II, which coincides with (36). In region III as $\rho \to R$, the first of Eqs. (58) with regard for (57) can be presented in the form

$$\varphi_{,q} = -\alpha_{,q}/\alpha(q). \tag{59}$$

This, according to (38), yields

$$|\alpha_{,q}/\alpha(q)|_{q=0} \to \infty \tag{60}$$

at $\Delta q = 0, \Delta\rho = 0$.

With regard for (56), the second of Eqs.(58) as $q \to 0$ can be presented as

$$\varphi_{,\rho} = -f_{,\rho}/f(\rho). \tag{61}$$

According to (38), at $\Delta q = 0, \Delta\rho = 0$,

$$f_{,\rho}/f(\rho)|_{\rho=R} = 0. \tag{62}$$

On the other hand, considering Eqs.(58) in some small neighborhood of the circle $q = 0$, $\rho = R$, i.e., inside the region, where the scalar field is concentrated with $f(\rho)/f_0 \ll 1$, $\alpha(q) \ll 1$ (according to (56), (57)), we can put down

$$\varphi_{,q}\varphi_{,\rho} = \frac{(\alpha_{,q}/\alpha(q))}{\left(1 + \frac{1}{f_0}\frac{f(\rho)}{\alpha(q)}\right)} \times \frac{(f_{,\rho}/f(\rho))}{\left(1 + f_0\frac{\alpha(q)}{f(\rho)}\right)}. \tag{63}$$

Then, according to (38), the following condition must be satisfied at $\Delta q = 0, \Delta\rho = 0$:

$$\left(\frac{\alpha_{,q}}{\alpha(q)}\right) \times \left(\frac{f_{,\rho}}{f(\rho)}\right)\Bigg|_{q=0,\rho=R} = 0. \tag{64}$$

As an example, the functions $\alpha(q)$ and $f(\rho)$ satisfying the found conditions can be chosen as follows:

$$\alpha(q) = \exp\left(\frac{-1}{\eta + (\xi q)^2}\right), \tag{65}$$

$$f(\rho) = f_0 \exp\left(-\gamma\left(1 - \exp\left(\frac{-1}{(\zeta(\rho - R))^2}\right)\right)\right), \tag{66}$$

where the constants ξ and ζ determine the size ("thickness") of the ring with the scalar field concentrated inside with respect to the variables q and ρ, respectively. Namely, as follows from (65), (66) as $\Delta q \to 0$, $\Delta\rho \to 0$,

$$\xi \to \infty, \quad \zeta \to \infty, \tag{67}$$

while the positive constants η and γ provide the fulfillment of conditions (56), (57), (60), and (62) at $\Delta q = 0, \Delta\rho = 0$, $q = 0$, and $\rho = R$. Namely, at $\Delta q \ll 1$ $\Delta\rho \ll 1$, we have

$$\eta \ll 1, \quad \gamma \gg 1. \tag{68}$$

With a further contraction into a one-dimensional object (null string), i.e., at $\Delta q = 0$, $\Delta\rho = 0$,

$$\eta = 0, \quad \gamma \to \infty. \tag{69}$$

Using (51), (65), and (66) for (42), we obtain the expression for one of the possible distributions of the potential of the massless scalar field, whose components of the energy-momentum tensor asymptotically coincide with

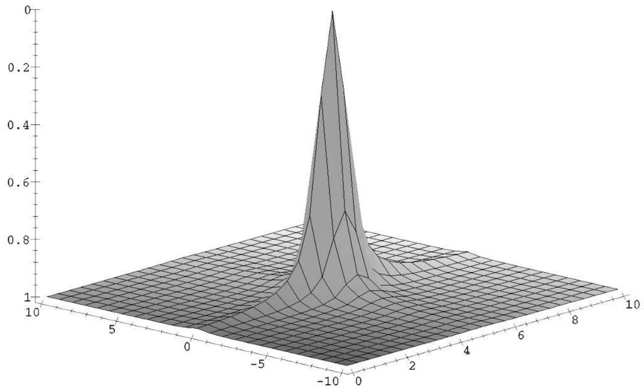

Fig. 1. Distribution of the function $\alpha(q)+(1-\alpha(q))f(\rho)/f_0$ for (65) and (66) at $R=5$, $\eta=0.01$, $\xi=2$, and $\zeta=\gamma=4$; $q\in[-10,10]$, $\rho\in[0,10]$

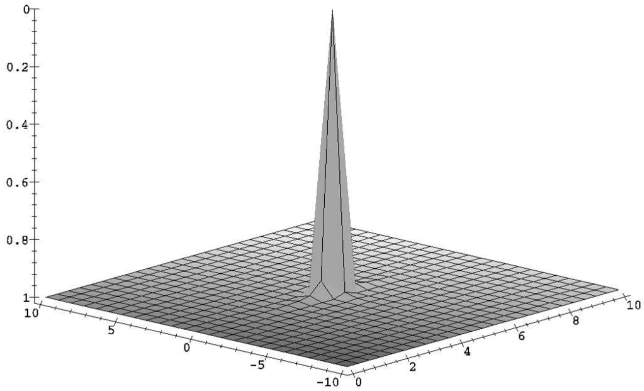

Fig. 2. Distribution of the function $\alpha(q)+(1-\alpha(q))f(\rho)/f_0$ for (65) and (66) at $R=5$, $\eta=0.01$, $\gamma=4$, $\xi=10$, and $\zeta=20$; $q\in[-10,10]$, $\rho\in[0,10]$

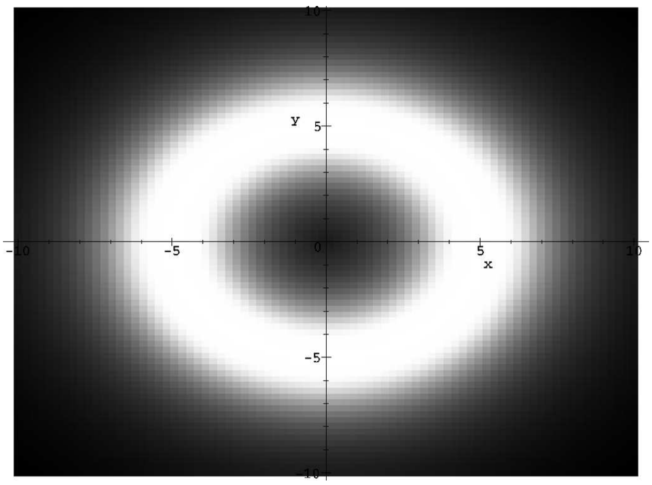

Fig. 3. Scalar field potential distribution specified by (42), (51), (65), and (66) with respect to ρ ($\rho\in[0,10]$) at $q=0.01$; $R=5$, $\eta=0.01$, $\gamma=4$, $\xi=0.5$, and $\zeta=0.5$

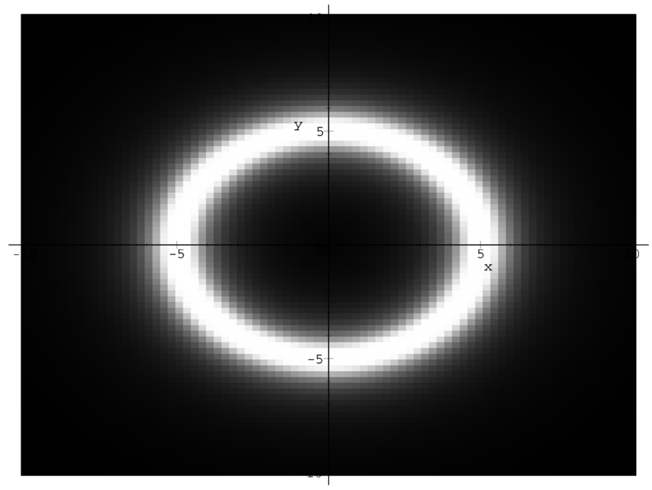

Fig. 4. Scalar field potential distribution specified by (42), (51), (65), and (66) with respect to ρ ($\rho\in[0,10]$) at $q=0.01$; $R=5$, $\eta=0.01$, $\gamma=4$, $\xi=1.3$, and $\zeta=1.3$

those of a closed null string of radius R under contraction into a one-dimensional object (circle of the same radius).

Figures 1 and 2 present the distributions of the function $\alpha(q)+(1-\alpha(q))f(\rho)/f_0$ in the region $q\in[-10,10]$, $\rho\in[0,10]$ for the functions $\alpha(q)$ and $f(\rho)$ specified by Eqs. (65) and (66) at $R=5$, $\eta=0.01$, $\gamma=4$ corresponding to the following choice of the constants: $\xi=2$, $\zeta=4$ (Fig. 1) and $\xi=10$, $\zeta=20$ (Fig. 2). One can see from these figures that, with increasing values of the constants ξ and ζ, the region of the non-unity function $\alpha(q)+(1-\alpha(q))f(\rho)/f_0$ (i.e., the region, where the scalar field is concentrated, and the scalar field potential differs from zero) contracts, which corresponds to a decrease of the "thickness" of the ring with the scalar field concentrated inside.

Figures 3 and 4 show the distributions of the scalar field potential specified by Eqs. (42), (51), (65), and (66) with respect to the variable ρ ($\rho\in[0,10]$) at $R=5$, $\eta=0.01$, $\gamma=4$, and $q=0.01$ with the following constants: $\xi=0.5$, $\zeta=0.5$ (Fig. 3) and $\xi=1.3$, $\zeta=1.3$ (Fig. 4). The black region corresponds to $\varphi=0$. One can see that, with increasing constants ξ and ζ, the region of the nonzero scalar field potential contracts, which corresponds to a decrease of the "thickness" of the ring with the scalar field concentrated inside with respect to ρ.

In Figs. 5 and 6, one can see the distributions of the scalar field potential specified by Eqs. (42), (51), (65), and (66) on the surface $\theta=\text{const}$ at $R=5$, $\eta=0.01$, and $\gamma=4$ with $\xi=0.5$, $\zeta=0.5$ (Fig. 5) and $\xi=1.3$,

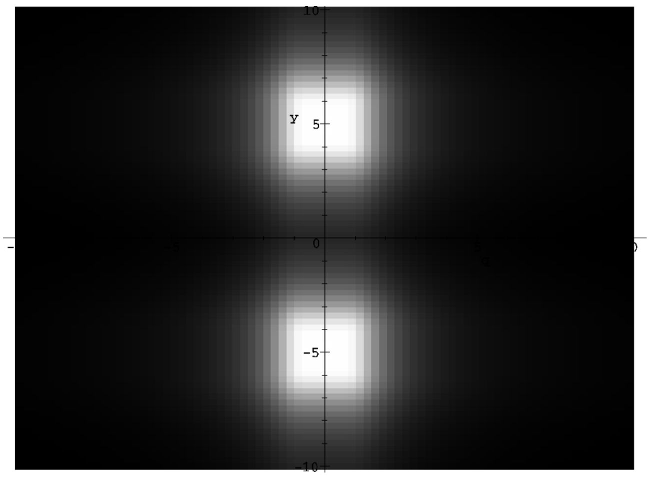

Fig. 5. Scalar field potential distribution specified by (42), (51), (65), and (66) on the surface $\theta = const$; $R = 5$, $\eta = 0.01$, $\gamma = 4$, $\xi = 0.5$, $\zeta = 0.5$, $q \in [-10, 10]$, and $\rho \in [0, 10]$

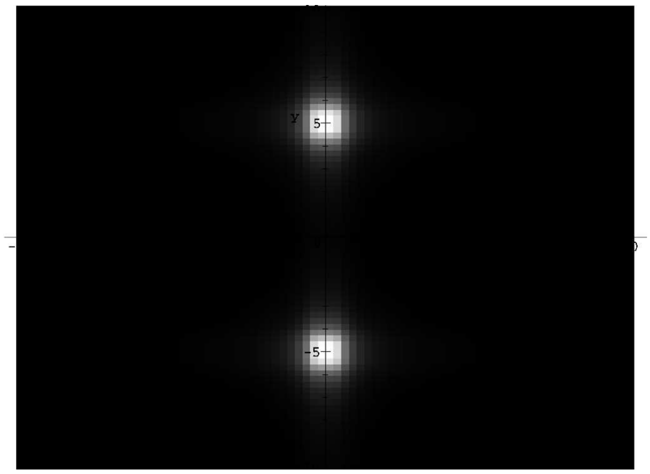

Fig. 6. Scalar field potential distribution specified by (42), (51), (65), and (66) on the surface $\theta = const$; $R = 5$, $\eta = 0.01$, $\gamma = 4$, $\xi = 1.3$, $\zeta = 1.3$, $q \in [-10, 10]$, and $\rho \in [0, 10]$

$\zeta = 1.3$ (Fig. 6). Here, $q \in [-10, 10]$, $\rho \in [0, 10]$. It is obvious that an increase of the constants ξ and ζ results in the contraction of the region with the non-zero scalar field potential. In other words, the "thickness" of the ring, where the scalar field is concentrated, decreases.

4. Conclusions

Comparing the systems of Einstein equations for the distribution of the real massless scalar field concentrated inside a thin ring and for a closed null string with radius R moving along the axis z and completely lying in a plane orthogonal to this axis at every time moment, we have obtained the conditions for the scalar field potential, under which a contraction of the scalar field into a one-dimensional object (circle of radius R) results in the asymptotic coincidence of the components of the energy-momentum tensor of the scalar field with those of the closed null string of the same radius. The general form of the potential distribution describing the motion of the scalar field concentrated inside a thin ring of constant radius along the z axis is proposed. An example of the scalar field potential distribution satisfying the obtained conditions is given.

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Received 26.10.10
Translated from Ukrainian by H.G. Kalyuzhna

РОЗПОДІЛ ПОТЕНЦІАЛУ СКАЛЯРНОГО ПОЛЯ ДЛЯ "РОЗМАЗАНОЇ" НУЛЬ-СТРУНИ СТАЛОГО РАДІУСА

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Р е з ю м е

У роботі запропоновано загальний вигляд розподілу потенціалу скалярного поля для "розмазаної" нуль-струни сталого радіуса, яка прямує уздовж осі z і в кожен момент часу цілком знаходиться у площині, ортогональній цій осі. Знайдено умови, за яких компоненти тензора енергії-імпульсу скалярного поля, при стисканні поля в одновимірний об'єкт (коло радіуса R), асимптотично збігаються з компонентами тензора енергії-імпульсу замкненої нуль-струни того ж радіуса.