https://doi.org/10.15407/ujpe64.11.983

SHI-DONG LIANG,[1] T. HARKO[1,2]

[1] School of Physics, Sun Yat-sen University
(Guangzhou, 510275, China; e-mail: stslsd@mail.sysu.edu.cn)
[2] Department of Physics, Babes-Bolyai University
(Kogalniceanu Str., 400084 Cluj-Napoca, Romania; e-mail: t.harko@ucl.ac.uk)

TOWARDS AN OBSERVABLE TEST OF NONCOMMUTATIVE QUANTUM MECHANICS[1]

The conceptual incompatibility of spacetime in gravity and quantum physics implies the existence of noncommutative spacetime and geometry on the Planck scale. We present the formulation of a noncommutative quantum mechanics based on the Seiberg–Witten map, and we study the Aharonov–Bohm effect induced by the noncommutative phase space. We investigate the existence of the persistent current in a nanoscale ring with an external magnetic field along the ring axis, and we introduce two observables to probe the signal coming from the noncommutative phase space. Based on this formulation, we give a value-independent criterion to demonstrate the existence of the noncommutative phase space.

Keywords: noncommutative quantum mechanics, Seiberg–Witten map, Aharonov–Bohm effect, persistent current.

1. Introduction

The concepts of noncommutative spacetime and geometry in theoretical physics are proposed for settling two fundamental problems in science. The first is the incompatibility of spacetime in gravity and quantum physics, while the second is represented by some puzzles in particle physics and cosmology, like the singularity, nonlocality, dark energy, and dark matter problems. A solution to these problems can be obtained by assuming the finite nature of length and time (Planck's scales), which allows us to understand consistently all phenomena in the physical world [1–3]. The finite Planck's length and time units hint towards the noncommutative spacetime, and to noncommutative algebra and geometry [1–3]. These noncommutative concepts are generalized further to the noncommutative phase space as a generalization of the Heisenberg algebra in quantum mechanics [5, 7]. In Fig. 1, we show the set of noncommutative spacetimes and their algebras. The use of noncommutative concepts and techniques can be expected to lead to a solution to the singularity and nonlocality problems in particle physics, quantum gravity, and quantum cosmology [1–3].

In fact, noncommutative phase-space phenomena also arise in condensed matter physics [1–3], like, for example, the analogy between the two-dimensional electron gas in the presence of a magnetic field and the free-particle system in the noncommutative phase space [7]. This analogy reveals that the noncommutative phase space effect can occur beyond the Planck scale, even though the physics in a noncommutative space has not been understood completely.

Several techniques have been proposed to implement the idea of noncommutative phase space [5], such as the canonical formulation, Bopp shift, Moyal product, path integration, Weyl–Wigner phase space [8, 9], and Seiberg–Witten map [6, 10–12]. The noncommutative phase space generalizes the uncertainty principle and smears the phase space, leading to some new physical effects arising from the noncommutative nature of the phase space, such as the Aharonov–Bohm effect [13–16], quantum Hall effect [17], magnetic monopoles [18], and Berry phase [6].

However, up to now, no direct experimental evidence to detect the effect of a noncommutative space is known. The main difficulty in directly observing the phenomena in noncommutative spaces is caused by that the effects of noncommutative space or phase space are too weak and are relevant on the Planck scale only.

[1] This work is based on the results presented at the XI Bolyai–Gauss–Lobachevskii (BGL-2019) Conference: Non–Euclidean, Noncommutative Geometry and Quantum Physics.

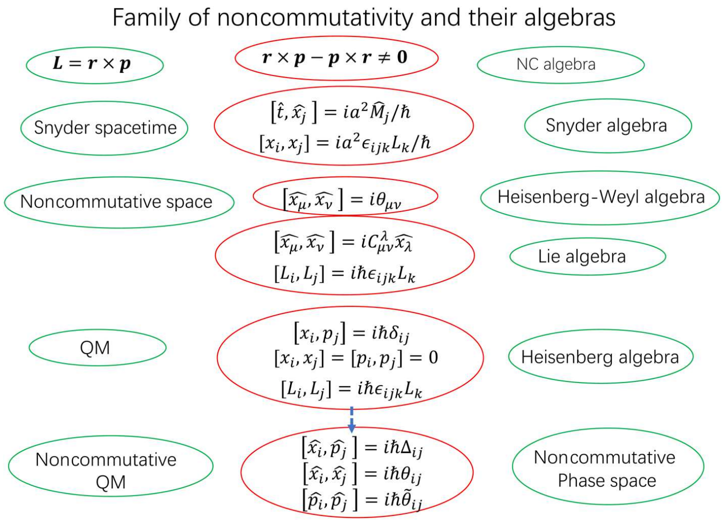

Fig. 1. Family of noncommutative geometries and their algebras

The present paper is organized as follows. In Section 2, we will present the mathematical formulation of the noncommutative phase space and give the Heisenberg representation of a noncommutative phase space based on the Seiberg–Witten map [6,10–12]. The noncommutative algebra yields an effective magnetic vector potential. In Section 3, we will present the Aharonov–Bohm effect induced by the effective magnetic vector potential. In Section 4, we will study the persistent current in a nanoscale ring with an external magnetic field along the ring axis in the noncommutative phase space. The effective magnetic vector potential leads equivalently to an effective magnetic flux in the ring, which drives the extra persistent current. In Section 5, we introduce two variables associated to the persistent current in the ring to probe the signal coming from the noncommutative phase space. We will give a value-independent criterion to detect the existence of a noncommutative phase space in Section 6. Since the persistent current and magnetic flux in mesoscopic rings have been already used in nanotechnology, it can be expected to probe the noncommutative phase-space effect by this scheme. We conclude our results in Section 7.

2. Noncommutative Quantum Mechanics

2.1. General formulation

Even though there are many schemes of noncommutative spacetime or algebra in physics, we focus here on the so-called noncommutative phase space, which is a generalization of the Heisenberg noncommutative phase space,

$$[x^\mu, x^\nu] = 0, \tag{1a}$$
$$[p^\mu, p^\nu] = 0, \tag{1b}$$
$$[x^\mu, p^\nu] = i\hbar\delta^{\mu\nu}, \tag{1c}$$

to the noncommutative phase space written as [6,10–12]

$$[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}, \tag{2a}$$
$$[\hat{p}^\mu, \hat{p}^\nu] = i\eta^{\mu\nu}, \tag{2b}$$
$$[\hat{x}^\mu, \hat{p}^\nu] = i\hbar\Delta^{\mu\nu}, \tag{2c}$$

where $\theta^{\mu\nu}$ and $\eta^{\mu\nu}$ are the noncommutative strength parameters. The hatted operators in Eq. (2) act in the noncommutative phase space, while the nonhatted operators in Eq. (1) act in the Heisenberg phase space.

The generalized Schrödinger equation in a noncommutative phase space can be obtained as

$$i\hbar\frac{\partial}{\partial t}\Psi(\hat{x}, \hat{y}, \hat{z}) = \hat{H}\Psi(\hat{x}, \hat{y}, \hat{z}), \tag{3}$$

where the Hamiltonian in the noncommutative phase space is written in general as

$$\hat{H} = \frac{1}{2m}\left(\hat{p}_x^2 + \hat{p}_y^2 + \hat{p}_z^2\right) + V(\hat{x}, \hat{y}, \hat{z}). \tag{4}$$

Based on the quantization principle, the physical observables are described by operators in terms of the

 ISSN 2071-0194. Ukr. J. Phys. 2019. Vol. 64, No. 11

position and momentum operators, $\widehat{O}\left(\widehat{x}_i, \widehat{p}_i\right)$. For example, the angular momentum operator in a noncommutative phase space is generalized to $\widehat{\mathbf{L}} = \widehat{\mathbf{r}} \times \widehat{\mathbf{p}}$. The expectation value of observables can be obtained as

$$\langle \widehat{O} \rangle = \langle \Psi | \widehat{O} | \Psi \rangle = Tr(\widehat{\rho}\widehat{O}), \tag{5}$$

where $\widehat{\rho} = |\Psi\rangle\langle\Psi|$ is the density matrix in the noncommutative phase space. In principle, in the noncommutative QM, we have a formulation similar to Heisenberg's QM, but it is given in the noncommutative algebra of Eq. (2). However, to implement the noncommutative algebra of Eq. (2), we need the operator representations of the position and momentum operators in the noncommutative algebra of Eq. (2). There are many methods to implement this noncommutative algebra [5]. Seiberg and Witten proposed a map between the noncommutative algebra and Heisenberg's noncommutative algebra, [6, 10–12], which provides an efficient way to implement the noncommutative QM.

2.2. Heisenbereg's representation of the noncommutative quantum mechanics

The Seiberg–Witten (SW) map provides an efficient way to map the operators in the noncommutative phase space to the Heisenberg's algebra. The basic idea of the Seiberg–Witten (SW) map is that the quantum objects can be represented in the noncommutative phase space with the help of a map that maps the noncommutative algebra into the Heisenberg algebra, so that we can still work with the Heisenberg algebra even in the noncommutative phase space. However, the noncommutative effects emerge as a result of the use of the SW map.

The SW map can be expressed as [6, 10–12]

$$\begin{pmatrix} \widehat{X} \\ \widehat{P} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} X \\ P \end{pmatrix}, \tag{6}$$

where $\widehat{X}^\top = (\widehat{x}, \widehat{y}, \widehat{z})^\top$, $\widehat{P}^\top = (\widehat{p}_x, \widehat{p}_y, \widehat{p}_z)^\top$, $X^\top = (x, y, z)^\top$ and $P^\top = (p_x, p_y, p_z)^\top$. The coefficients A, B, C, and D satisfy the conditions

$$AD^\top - BC^\top = \Delta, \tag{7a}$$

$$AB^\top - BA^\top = \frac{\Theta}{\hbar}, \tag{7b}$$

$$CD^\top - DC^\top = \frac{\widetilde{\Theta}}{\hbar}, \tag{7c}$$

where $\Delta = (\delta_{ij})$ is the unit matrix in the approximation $\theta\eta/\hbar^2 \to 0$, which sets the diagonal elements to be 1; $\Theta = (\theta_{ij})$ and $\widetilde{\Theta} = (\eta_{ij})$, where $\theta_{ji} = -\theta_{ij}$ and $\eta_{ji} = -\eta_{ij}$ are antisymmetric. The transformation matrices can be obtained as

$$A = D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}; \quad B = \begin{pmatrix} 0 & -\theta/2\hbar & -\theta/2\hbar \\ \theta/2\hbar & 0 & -\theta/2\hbar \\ \theta/2\hbar & \theta/2\hbar & 0 \end{pmatrix}; \tag{8}$$

$$C = \begin{pmatrix} 0 & \eta/2\hbar & \eta/2\hbar \\ -\eta/2\hbar & 0 & \eta/2\hbar \\ -\eta/2\hbar & -\eta/2\hbar & 0 \end{pmatrix}.$$

By substituting the matrix representations of the coefficients in (8), the SW map can be expressed as

$$\begin{pmatrix} \widehat{x} \\ \widehat{y} \\ \widehat{z} \end{pmatrix} = \begin{pmatrix} x - \frac{\theta}{2\hbar}(p_y + p_z) \\ y + \frac{\theta}{2\hbar}(p_x - p_z) \\ z + \frac{\theta}{2\hbar}(p_x + p_y) \end{pmatrix}, \tag{9a}$$

$$\begin{pmatrix} \widehat{p}_x \\ \widehat{p}_y \\ \widehat{p}_z \end{pmatrix} = \begin{pmatrix} p_x + \frac{\eta}{2\hbar}(y + z) \\ p_y - \frac{\eta}{2\hbar}(x - z) \\ p_z - \frac{\eta}{2\hbar}(x + y) \end{pmatrix}. \tag{9b}$$

This representation of the coordinate and momentum operators can be regarded as a Heisenberg-type representation of the noncommutative QM. Any operator in the noncommutative phase space can be mapped to Heisenbereg's representation, $\widehat{O}(\widehat{r}, \widehat{p}) \to O(r, p)$ by the SW map.

The Heisenberg representation of the momentum operators in the noncommutative phase space can be rewritten as

$$\widehat{\mathbf{p}} = \mathbf{p} + q\widetilde{\mathbf{A}}, \tag{10}$$

where

$$\widetilde{\mathbf{A}} = \begin{pmatrix} \frac{\eta}{2q\hbar}(y + z) \\ -\frac{\eta}{2q\hbar}(x - z) \\ -\frac{\eta}{2q\hbar}(x + y) \end{pmatrix} \tag{11}$$

is the effective magnetic vector potential.

Based on the SW map given by Eqs. (6) and (9), the external potential $V(\widehat{\mathbf{r}})$ can be expanded in a Taylor series in $\mathbf{r}$. By noticing that $p_y p_x - p_x p_y = 0$ in the Heisenberg algebra, in the first-order approximation, the potential in the noncommutative phase space is given by [19]

$$V(\widehat{\mathbf{r}}, t) \approx V(\mathbf{r}, t) + \mathcal{O}\left[\left(\frac{\theta}{\hbar}\right)^2\right] \approx V(\mathbf{r}, t). \tag{12}$$

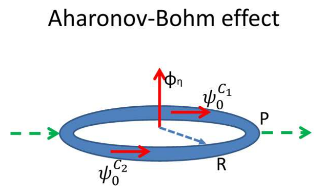

Fig. 2. Aharonov–Bohm effect

Similarly, the wave function in the noncommutative phase space can be also expanded in Taylor series in $\mathbf{r}$. In the first-order approximation, the wave function can be expressed approximately as

$$\Psi(\widehat{\mathbf{r}}, t) \approx \psi(\mathbf{r}, t) + \mathcal{O}\left[\left(\frac{\theta(\varepsilon)}{\hbar}\right)^2\right] \approx \psi(\mathbf{r}, t). \qquad (13)$$

The Hamiltonian in the noncommutative phase space can be mapped to the Heisenberg algebra by

$$\widehat{H} = \frac{1}{2m}\left(\mathbf{p} + q\widetilde{\mathbf{A}}\right)^2 + V(\mathbf{r}, t), \qquad (14)$$

where $\widehat{H}$ is the Hamiltonian in the Heisenberg QM (or, equivalently, as the representation).

Consequently, we can use the standard quantum mechanical formulation to study the noncommutative effects in the Heisenberg representation of the noncommutative QM.

It should be remarked that the SW map provides an efficient way to reveal the basic physics of the noncommutative QM even though it is not unitary and not canonical.

3. Aharonov–Bohm Effect in a Noncommutative Phase Space

Let us consider a two-dimensional free electronic system in a noncommutative phase space with the Hamiltonian given by

$$\widehat{H} = \frac{1}{2m}(\mathbf{p} + e\widetilde{\mathbf{A}})^2, \qquad (15)$$

where

$$\widetilde{\mathbf{A}} = \frac{\eta}{2e\hbar}\begin{pmatrix} y \\ -x \end{pmatrix} \qquad (16)$$

is the effective magnetic vector potential. The solution of the stationary Schrödinger equation can be expressed as

$$\psi(\mathbf{r}) = \psi_0(\mathbf{r}) \exp\left(-\frac{ie}{\hbar}\int_C \widetilde{\mathbf{A}}(\mathbf{r}')\, d\ell\right). \qquad (17)$$

Suppose that an electron moves along two semicircles that meet together at a point P on the circle. We denote the radius of the circle by R. By using the polar coordinate system, the effective magnetic vector potential is expressed as

$$\widetilde{\mathbf{A}} = \frac{\eta R}{2e\hbar}\begin{pmatrix} \sin\varphi \\ -\cos\varphi \end{pmatrix}. \qquad (18)$$

The effective magnetic flux is given by

$$\Phi_\eta = \oint_C \widetilde{\mathbf{A}}(\mathbf{r}')\, d\ell = -\frac{S\eta}{e\hbar}, \qquad (19)$$

where $S = \pi R^2$ is the area of the ring. Supposing that $\eta = \hbar^2\kappa_\eta^2$, we have $\Phi_\eta = -S\kappa_\eta^2\phi_0$, where $\phi_0 = h/e$ is the magnetic flux quantum.

Thus, the wave function at the point P can be expressed as

$$\psi(\mathbf{r}_P) = e^{-iS\kappa_\eta^2/2}\left(\psi_0^{C_1}(\mathbf{r}_P) + \psi_0^{C_2}(\mathbf{r}_P)e^{iS\kappa_\eta^2}\right). \qquad (20)$$

The probability density at the point P can be expressed as

$$\rho(\mathbf{r}_P) = \varrho(\mathbf{r}_P)(1 + 2\lambda\cos(S\kappa_\eta^2 + \widetilde{\varphi})), \qquad (21)$$

where

$$\varrho(\mathbf{r}_P) = \rho^{C_1}(\mathbf{r}_P) + \rho^{C_2}(\mathbf{r}_P), \qquad (22)$$

$$\lambda = \frac{\sqrt{\rho^{C_1}(\mathbf{r}_P)\rho^{C_2}(\mathbf{r}_P)}}{\varrho(\mathbf{r}_P)}, \qquad (23)$$

$$\tan\widetilde{\varphi} = \frac{\psi_0^{C_1}(\mathbf{r}_P)^*\psi_0^{C_2}(\mathbf{r}_P)}{\psi_0^{C_1}(\mathbf{r}_P)\psi_0^{C_2}(\mathbf{r}_P)^*}. \qquad (24)$$

It can be seen that the presence of the noncommutative parameter κ_η yields an effective magnetic flux on the two-dimensional closed path, which induces the Aharonov–Bohm effect (see Fig. 2).

4. Persistent Currents in Nanorings in the Noncommutative Phase Space

Let us consider a nanoring system in the noncommutative phase space with an external magnetic field along the ring axis. The Hamiltonian is written as

$$\widehat{H} = \frac{1}{2m}\left[\mathbf{p} + e(\widetilde{\mathbf{A}} + \mathbf{A})\right]^2, \qquad (25)$$

where $\widetilde{\mathbf{A}}$ is the effective magnetic vector potential induced by the noncommutative phase space and $\mathbf{A}$ is

ISSN 2071-0194. Ukr. J. Phys. 2019. Vol. 64, No. 11

the external magnetic vector potential. We assume that the magnetic field is homogeneous inside the ring. Hence, the electron states depend only on the total magnetic flux in the ring. In the polar coordinate system, $x = R\cos\varphi$, $y = R\sin\varphi$, the Hamiltonian in the noncommutative phase space is written as [14]

$$\widehat{H} = -\frac{\hbar^2}{2mR^2}\left[\frac{\partial}{\partial\varphi} + i\left(\frac{\phi}{\phi_0} - \frac{\phi_\eta}{\phi_0}\right)\right]^2 - \frac{3\hbar^2}{8mR^2}\frac{\phi_\eta^2}{\phi_0^2}, \quad (26)$$

where $\phi_\eta = \frac{2\pi R^2\eta}{e\hbar}$ is the effective magnetic flux coming from the noncommutative phase space, $\phi_0 = \frac{h}{e}$ is the flux quanta ($e < 0$), and ϕ is the external magnetic flux in the ring (see Fig. 2). For the sake of convenience, we introduce the dimensionless magnetic flux, $f_\eta \equiv \frac{\phi_\eta}{\phi_0}$ and $f = \frac{\phi}{\phi_0}$. Then the Hamiltonian of the quantum ring can be rewritten as

$$\widehat{H} = -\varepsilon_0\left[\frac{\partial}{\partial\varphi} + i\left(f - f_\eta\right)\right]^2 - \frac{3\varepsilon_0}{4}f_\eta^2, \quad (27)$$

where $\varepsilon_0 \equiv \frac{\hbar^2}{2mR^2}$. The energy can be obtained as [14]

$$E_n = \varepsilon_0\left(n + f - f_\eta\right)^2 - \frac{3\varepsilon_0}{4}f_\eta^2, \quad (28)$$

where $n = 0, \pm1, \pm2, \dots$.

It can be seen that the effective magnetic flux induced by the noncommutative space modifies the energy levels. We note that $n = 0, \pm1, \pm2, ..$, and the energies in Eq. (28) are invariant under the transformation $f - f_\eta \to f - f_\eta + 1$. Hence, we can consider only the domain of $f - f_\eta$ within $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (the first Brillouin flux zone) [20,21]. Suppose there are N electrons in the ring, and they occupy the energy levels at zero temperature. By observing that $N = 2k + 1$ for the odd-electron ring, the ground-state energy is $E_g = \sum_{n=0,\pm1,\pm2,\dots}^{\pm k} E_n$. For the even-electron ring, $N = 2k$, $E_g = \left(\sum_{n=0,1,2,\dots}^{k-1} + \sum_{n=-1,-2,\dots}^{-k}\right)E_n$, the ground-state energy of the ring is obtained as [14]

$$E_g(f) = \varepsilon_0\begin{cases}\frac{N^3 - N}{12} + N\left[(f - f_\eta)^2 - \frac{3}{4}f_\eta^2\right], \\ \text{for } N = 2k+1, \\ \frac{N^3 + 2N}{12} - N(f - f_\eta) + \\ + N\left[(f - f_\eta)^2 - \frac{3}{4}f_\eta^2\right], \quad \text{for } N = 2k.\end{cases} \quad (29)$$

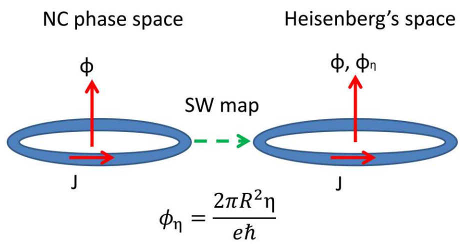

Fig. 3. Persistent current in the nanoring

Moreover, the ground-state energy is symmetric for $f - f_\eta = 0$. Hence, we can restrict our attention to a half of the first Brillouin flux zone, $\left[0, \frac{1}{2}\right]$. The persistent current in the ground state is defined by $J(f) = -\frac{\partial E_g}{\partial\phi}$, and it can be obtained as [14]

$$J(f) = J_0\begin{cases}-2Nf\left(1 - \frac{f_\eta}{f}\right), \text{ for } N = 2k+1, \\ N - 2Nf\left(1 - \frac{f_\eta}{f}\right), \text{ for } N = 2k,\end{cases} \quad (30)$$

where $J_0 = \frac{e}{\hbar}\varepsilon_0$. The persistent current depends on both the external magnetic flux and the effective magnetic flux induced by the noncommutative phase space (Fig. 3). This relationship between the persistent current and the magnetic flux in Eq. (30) provides a way to detect the noncommutative phase space effect.

5. Signals from Noncommutative Phase Space

In order to detect the effects of the noncommutative phase space experimentally, we introduce two variables defined by [14]

$$\lambda(f) := \frac{\partial}{\partial f}\left(\frac{J}{f}\right), \quad \text{for } N = 2k+1, \quad (31)$$

$$\sigma(f) := \frac{\partial}{\partial f}\left(\frac{J - NJ_0}{f}\right), \quad \text{for } N = 2k. \quad (32)$$

These variables can provide two signatures to detect the effects of the noncommutative phase space experimentally. Thus, we get

$$\lambda(f) = -2NJ_0\frac{f_\eta}{f^2}, \quad \text{for } N = 2k+1, \quad (33)$$

$$\sigma(f) = -2NJ_0\frac{f_\eta}{f^2}, \quad \text{for } N = 2k. \quad (34)$$

It can be seen that if the noncommutative phase space does exist, both λ and σ are proportional to $\frac{-1}{f^2}$, which diverges in the limit of small external magnetic fluxes.

For a given parameter $\eta \leq 1.76 \times 10^{-61}$ kg^{2}m^{2}s^{-2} [6, 12], and a mesoscopic ring with the radius $R = 1$ μm, $f_\eta \leq \frac{2\pi R^2\eta}{e\hbar h/e} = \frac{R^2\eta}{\hbar^2} = 1.5828 \times 10^{-5}$. Suppose that the effective electron number in the ring is about $N \sim 10^4 \div 10^5$. Then both $\lambda(f)$ and $\sigma(f)$ are proportional to $-2J_0 s_\eta/f^2$, where $s_\eta \equiv N f_\eta \approx 1$ is the signal from the noncommutative phase space.

Based on this result, we propose a criterion to directly detect the existence of the noncommutative phase space.

Criterion: if, by varying the external magnetic flux f λ, σ becomes divergent with decreasing f, then this effect yields the existence of the noncommutative phase space.

6. Experimental Scheme for Detecting a Noncommutative-Phase-Space Effect

The persistent current in the nano-sized ring inspires us to propose an experimental scheme to probe explicitly the existence of the noncommutative phase space via a specific signal.

Experimental Scheme: The experimental scheme includes the following steps:

1) Setting up an experimental system containing a nano-sized ring with an external magnetic field.

2) Measuring the persistent current J versus the external magnetic flux f.

3) Calculating λ and σ based on the experimental data of the persistent current and the external magnetic flux.

4) Plotting λ and σ versus f.

When we observe that λ or σ diverges as f approaches zero, we capture the signal coming from the existence of the noncommutative phase space.

It should be remarked that the criterion is based on the divergent behaviors of λ and σ as the external magnetic flux decreases, a behavior which does not depend on the numerical values of λ and σ. This result should be meaningful and valid for all qualitative levels, because we do not know actually the exact values of these parameters. In other words, the divergent behavior $1/f^2$ of λ or σ from the experimental data provides a way to predict the values of the noncommutative parameter η. In practice, we can plot $\log \lambda \sim \log f$ such that we can verify the divergent behavior $1/f^2$ as a linear relation.

The persistent current in mesoscopic rings has been studied both theoretically and experimentally in the past two decades [22–24]. Buttiker first predicted that a persistent current occurs in mesoscopic rings and oscillates with an AB flux [22]. The amplitude of the persistent current reaches the value $(10^{-2} \div 2)ev_{\rm F}/2\pi R$, where $v_{\rm F}$ is the electronic velocity at the Fermi level. The experimental results of an isolated Au ring and GaAs/Al$_x$Ga$_{1-x}$As in the diffusive region at low temperatures agrees with the theoretical predictions [23–25]. However, they did not study the effect of the noncommutative phase space. Actually, nanotechnology provides an efficient way to probe the effects coming from the noncommutative phase space. We strongly suggest to rerun the persistent-current experiment to turn out more precise data on the persistent current versus the external magnetic flux, by which we can plot the relation between (λ, σ) and f and prove the existence of the noncommutative phase space.

On the other hand, Carroll *et al.* studied the noncommutative field theory and Lorentz violation. They gave an upper bound of the noncommutative parameter, $\eta \leq (10\ \text{TeV})^{-2}$ [26]. Falomir *et al.* also proposed a scheme to explore the spatial noncommutativity of the scattering differential cross-section by the AB effect [27]. It relies on the particle physics experiment involving energies between 200 and 300 GeV for $\eta \leq (10\ \text{TeV})^{-2}$ and estimating the typical order of the cross-section for neutrino events 10^{-3} [27]. Obviously, those experimental schemes are much more difficult to be implemented than this scheme, because the experimental scheme proposed in the present paper involves eV energy scales and nanoscale physics.

In fact, up to now, there has not been any direct experimental evidence to prove the existence of noncommutative space and phase space. Hence it should be worth studying and exploring different aspects and different energy scales, even though the concept of noncommutativity originates from the Planck scale physics. Actually, many phenomena in condensed matter physics contain the characteristics of a noncommutative space in nonrelativistic quantum mechanics, such as the analogy between the Landau levels of a two-dimensional electron gas in the presence of a magnetic field and the free electron in the two-dimensional noncommutative phase space [8,9,28], as well as the quantum Hall effect [29]. Hence it should be expected that some possibilities to capture the signal of noncommutativity may exist especially in the nanoscale condensed matter physics.

7. Conclusions

The conceptual incompatibility of spacetime in gravity and quantum physics inspires to propose the existence of a noncommutative spacetime and a noncommutative geometry for the understanding of the fundamental physics behind gravitational and quantum behaviors. The physical spacetime territory extends from 10^{-35} m and 10^{-44} s (Planck scale) to $10^{61}\ell_p$ (Universe scale). Even if the noncommutative spacetime and geometry appear on the Planck and/or Universe scales, this influences the macroscopic physical behavior. However, up to now, there have not been any direct observational evidences to indicate the existence of noncommutative spacetime effects.

In the present paper, we have presented the formulation of the noncommutative quantum mechanics based on the Seiberg–Witten map, and we have studied the effects of the noncommutative phase space on the Aharonov–Bohm effect and on the persistent current in a nano-sized ring. We propose an experimental scheme to prove the existence of a noncommutative phase space by using a nano-sized quantum ring. We have introduced two variables λ and σ as indicators for the detection of the effects coming from the noncommutative phase space, based on the relation between the persistent current and the external magnetic flux in the ring. The divergent behavior of (λ, σ) versus f provides a value-independent method for the experimental measurement of the effects of the noncommutative phase space. This method can be expected to give answer to the question whether the noncommutative phase space does exist in nature.

S.-D. Liang thanks the Natural Science Fundation of Guangdong Province (Grant No. 2016A030313313) for the financial support.

1. M.R. Douglas, N.A. Nekrasov. Noncommutative field theory. *Rev. Mod. Phys.* **73**, 977 (2001).
2. R.J. Szabo. Quantum field theory on noncommutative spaces. *Phys. Rep.* **378**, 207 (2003)
3. H. Snyder. Quantized space-time. *Phys. Rev.* **71**, 38 (1947).
4. A. Connes. *Noncommutative Geometry* (Academic, 1994).
5. L. Gouba. A comparative review of four formulations of noncommutative quantum mechanics. *Intern. J. Modern Phys. A* **31** (19), 1630025 (2016).
6. O. Bertolami, J.G. Rosa, C.M.L. de Aragão, P. Castorina, D. Zappalà. Noncommutative gravitational quantum well. *Phys. Rev. D* **72**, 025010 (2005).
7. F. Delduc, Q. Duret, F. Gieres, M. Lefrancois. Magnetic fields in noncommutative quantum mechanics. *J. Phys.: Conf. Series* **103**, 012020 (2008).
8. J. Gamboa, M. Loewe, J.C. Rojas. Noncommutative quantum mechanics. *Phys. Rev. D* **64**, 067901 (2001).
9. M. Chaichian, M.M. Sheikh-Jabbari, A. Tureanua. Hydrogen atom spectrum and the Lamb-shift in noncommutative QED. *Phys. Rev. Lett.* **86**, 2716 (2001).
10. N. Seiberg, E. Witten. Electric-magnetic duality, monopole condensation, and confinement in $N = 2$ supersymmetric Yang-Mills theory. *Nucl. Phys. B* **426**, 19 (1994)
11. N. Seiberg, E. Witten. String theory and noncommutative geometry. *J. High Energy Phys.* **09**, 032 (1999).
12. C. Bastos, O. Bertolami. Berry phase in the gravitational quantum well and the Seiberg–Witten map. *Phys. Lett. A* **372** 5556 (2008).
13. A. Das, H. Falomir, M. Nieto, J. Gamboa, F. Mendez. Aharonov–Bohm effect in a class of noncommutative theories. *Phys. Rev. D* **84**, 045002 (2011).
14. Shi-Dong Liang, Haoqi Li, Guang-Yao Huang. Detecting noncommutative phase space by the Aharonov–Bohm effect. *Phys. Rev. A* **90**, 010102 (2014); Shi-Dong Liang. *Poster presentation on Conference of 90 Years of Quantum Mechanics* (NTU, 2017).
15. M. Chaichian, P. Presnajder, M.M. Sheikh-Jabbari, A. Tureanu. Aharonov–Bohm effect in noncommutative spaces. *Phys. Lett. B* **527**, 149 (2002).
16. M. Chaichian, A. Demichev, P. Presnajder, M.M. Sheikh-Jabbari, A. Tureanu. Quantum theories on noncommutative spaces with nontrivial topology: Aharonov–Bohm and Casimir effects. *Nucl. Phys. B* **611**, 383 (2001).
17. A. Kokado, T. Okamura, T. Saito. Noncommutative quantum mechanics and the Seiberg–Witten map. *Phys. Rev. D* **69**, 125007 (2004).
18. S. Kovacik, P. Presnajder. Magnetic monopoles in noncommutative quantum mechanics 2. *J. Math. Phys.* **59**, 082107 (2018).
19. T. Harko, S.-D. Liang. Energy-dependent noncommutative quantum mechanics. *E. Phys. J. C* **79**, (2019) 300.
20. D. Loss, P. Goldbart. Period and amplitude halving in mesoscopic rings with spin. *Phys. Rev. B* **43**, 13762 (1991).
21. G.-Y. Huang, S.-D. Liang. Quantum phase transitions in mesoscopic Rashba rings. *Phys. Lett. A* **375**, 738 (2011).
22. M. Buttiker, Y. Imry, R. Landauer. Josephson behavior in small normal one-dimensional rings. *Phys. Lett. A* **96**, 365 (1983).
23. L.P. Levy, G. Dolan, J. Dunsmuir, H. Bouchiat. Magnetization of mesoscopic copper rings: Evidence for persistent currents. *Phys. Rev. Lett.* **64**, 2074 (1990).
24. D. Mailly, C. Chapelier, A. Benoit. Experimental observation of persistent currents in GaAs–AlGaAs single loop. *Phys. Rev. Lett.* **70**, 2020 (1993).

25. V. Chandrasekhar, R.A. Webb, M.J. Brady, M.B. Ketchen, W.J. Gallagher, A. Kleinsasser. Magnetic response of a single, isolated gold loop. *Phys. Rev. Lett.* **67**, 3578 (1991).
26. S. Carroll, J. Harvey, V.A. Kostelecky, C.D. Lane, T. Okamoto. Noncommutative field theory and Lorentz violation. *Phys. Rev. Lett.* **87**, 141601(2001).
27. H. Falomir, J. Gamboa, M. Loewe, F. Mendez, J.C. Rojas. Testing spatial noncommutativity via the Aharonov–Bohm effect. *Phys. Rev. D* **66**, 045018 (2002).
28. P.-M. Ho, H.-C. Kao. Noncommutative quantum mechanics from noncommutative quantum field theory. *Phys. Rev. Lett.* **88**, 151602 (2002).
29. B. Basu, D. Chowdhury, S. Ghosh. Inertial spin Hall effect in noncommutative space. *Phys. Lett. A* **377**, 1661 (2013).

Received 22.01.09

Ші-Донг Ліянг, Т. Харко

НА ШЛЯХУ ДО ЕКСПЕРИМЕНТАЛЬНОЇ ПЕРЕВІРКИ НЕКОМУТАТИВНОЇ КВАНТОВОЇ МЕХАНІКИ

Р е з ю м е

Концептуальна несумісність простору-часу в теорії гравітації та квантовій фізиці означає існування некомутативних простору-часу та геометрії на планківських відстанях. Ми представляємо формулювання некомутативної квантової механіки на основі відображення Зайберга–Віттена і вивчаємо ефект Ааронова–Бома, породжений некомутативним фазовим простором. Досліджено існування постійного струму в нанорозмірному кільці із зовнішнім магнітним полем вздовж осі кільця і введено дві спостережувані для зондування сигналу, що надходить з некомутативного фазового простору. На основі цього формулювання дано незалежний від величин критерій для підтвердження існування некомутативного фазового простору.