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CHIRAL ASYMMETRY IN MAGNETIZED DENSE RELATIVISTIC MATTER AND PULSAR KICKS

The weak interactions of neutrinos with charged fermions in a magnetized dense relativistic matter are shown to generate a non-zero chiral shift parameter for neutrinos that produces their asymmetric distribution in the momentum space in the equilibrium state. It is found that this asymmetry is too small in order to explain the largest pulsar velocities observed. The hot-spot scenario involving the topological current or some other mechanism of the hot spot formation is suggested, and it is argued that this scenario can provide the necessary large pulsar kicks.

Keywords: dense relativistic matter, magnetic field, pulsar kick.

1. Introduction

Since neutrinos extremely weakly interact with matter, they cannot be trapped under normal conditions in a certain region, because they simply will stream out of this region. The only place where neutrinos can be trapped for a sufficiently long time is the central regions of neutron stars, which are characterized by the highest matter densities that occur in the nature. They can exceed the density of the nuclear matter up to ten times. Such densities are large enough for that the neutrinos interacting through the weak interactions with this dense matter are trapped and come to an equilibrium state. It is also very important that the neutron stars are usually characterized by very strong magnetic fields that could reach up to 10^{15} G in magnetars [1, 2] (for a recent review of theoretical developments in the studies of a dense matter in compact stars, see Ref. [3]).

Pulsars are highly magnetized rapidly rotating neutron stars that emit beams of electromagnetic radiation. Some pulsars are observed moving with very high velocities, which can exceed 1000 km/s [4]. It is very difficult to explain how such high velocities can be reached. For example, the asymmetric supernova explosions can provide velocities up to 200 km/s only. Since neutrinos carry away 99% of the energy released during a supernova explosion, a simple estimate shows that the necessary velocities can be attained, if the neutrino emission during a supernova explosion is asymmetric only by 3% in the momentum space. However, as we will discuss below, it is difficult to provide such asymmetry in the equilibrium state.

Many physical properties of neutron stars are understood theoretically and could be tested to some extent through observational data. Still, the dense relativistic matter in a strong magnetic field may hold some new theoretical surprises. In particular, it was revealed in Refs. [5, 6] that the relativistic matter in a magnetic field is characterized by the presence of a non-dissipative axial current in its ground state. More recently, it was shown in Ref. [7] that there exists a chiral shift parameter Δ in the normal ground state of such matter. It enters the effective action as a Lagrange multiplier in front of the operator of the axial current $j_5^3 = \bar{\psi}\gamma^3\gamma^5\psi$ (we assume that the magnetic field points in the $+z$ direction). The meaning of this parameter is clearest in the chiral limit: it determines a relative shift of the longitudinal momenta in the dispersion relations of opposite-chirality fermions, $k^3 \to k^3 \pm \Delta$, where the momentum k^3 is directed along the magnetic field. This suggests a possible connection between the chiral shift parameter Δ and the axial current along the direction of the magnetic field.

Since only the left-handed fermions participate in the weak interactions, the asymmetry with respect to the longitudinal momentum k^3 of the opposite-chirality fermions in the ground state of a dense magnetized matter implies that the neutrinos will scatter asymmetrically off the magnetized relativistic matter. This provides a qualitatively new mechanism for pulsar kicks [8]. This mechanism was proposed in Ref. [7].

The idea behind this mechanism of pulsar kicks is the following. In the presence of a magnetic field, almost any type of relativistic matter inside a protoneutron star should develop axial currents. The main carriers of such currents are electrons in the nuclear matter and quarks together with electrons in the deconfined quark matter. Since the induced currents and the chiral shift parameter have only a weak temperature dependence (assuming $T \ll \mu$) [9], this phenomenon may provide a robust anisotropic medium even at the crucial earliest stages of the protoneutron star. This is of great importance, because only the hot matter with $T \lesssim 50$ MeV may have a large enough amount of the thermal energy to power the strongest pulsar kicks observed [8]. In contrast, the constraints of the energy conservation make it hard, if not impossible, to explain such kicks, if the interior matter is cold ($T \lesssim 1$ MeV). The common difficulty of using a hot matter, however, is the very efficient thermal isotropization that erodes a non-isotropic distribution of neutrinos produced by almost any mechanism [10, 11]. In the new mechanism proposed, however, the asymmetric distribution of neutrinos in the momentum space arises as a result of the weak interactions with the left-handed electrons or quarks flowing in the stellar matter in the direction along the magnetic field. In passing, let us mention that the robustness of the axial currents in a hot magnetized matter may also provide an additional neutrino push to facilitate the successful supernova explosions, as suggested in Ref. [12]. The specific details of such scenario are yet to be worked out.

Another mechanism of generation of pulsar kicks was suggested in [13, 14]. This mechanism is based on the existence of non-dissipative electric and axial currents in a dense magnetized relativistic matter related to the quantum anomalies. Historically, the first example of a non-dissipative current was revealed in [5], where it was shown that the ground state of the system of free relativistic fermions in a magnetic field is characterized by the presence of an axial current. Later, it was argued [6, 15] that the existence of this current is closely related to the chiral anomaly. This effect is known in the literature as the chiral separation effect (for a brief review, see Ref. [16]).

Using the chiral anomaly, it was suggested in Refs. [17, 18] that there exists a non-dissipative electric current in a relativistic matter with the non-zero chiral chemical potential μ_5 in a magnetic field $\mathbf{B}$, which is given by

$$\langle \mathbf{j} \rangle = \frac{\mu_5 e^2 \mathbf{B}}{2\pi^2}. \qquad (1)$$

This phenomenon is known in the literature as the chiral magnetic effect (CME), and current (1) is called topological, because it arises due to the lowest Landau level contribution connected with the chiral anomaly. It was argued in [19] that the QCD topological fluctuations in heavy-ions collisions produce metastable domains with $\mathcal{P}$ and $\mathcal{CP}$ breaking with a chirality induced in the quark-gluon plasma by the chiral anomaly. To mimic the effect of topological charge changing transitions, it was proposed phenomenologically in [20] to introduce a chiral chemical potential (this chemical potential couples to the difference between the number of left- and right-handed fermions). The charge-dependent correlations and flow, experimentally observed in heavy-ions collisions at RHIC [21–24] and LHC [25], appear to be in a qualitative agreement with the predictions of the CME [26, 27].

It was proposed in Refs. [13, 14] (see also Ref. [28]) that the pulsar kicks can be generated by the topological current in a dense relativistic matter. It was argued that a chiral chemical potential appears, because an imbalance in left- and right-handed particles exists in the neutron star matter due to the weak interactions. The idea of this mechanism of pulsar kicks is the following. If the electrons carried by the current can transfer their momentum into space, the current could push the star like a rocket.

In this paper, we study the mechanism of pulsar kicks suggested in Ref. [7] and propose a generalization of the mechanism considered in Refs. [13, 14, 28] in the form of a hot-spot scenario. The paper is orga-

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nized as follows. The kinetic equation and the principle of detailed balance are considered in Sec. 2. In Sec. 3, the neutrino energy dispersion and the neutrino momentum asymmetry in a dense magnetized relativistic matter are studied. The hot-spot scenario of pulsar kicks is proposed in Sec. 4. The results and the conclusions are given in Sec. 5.

2. Kinetic Equation and the Principle of Detailed Balance

Horowitz and Li were first to suggest [29] that a global asymmetry of the neutrino emission of a nascent neutron star could originate from the cumulative effect of multiple neutrino scatterings off polarized nucleons in the magnetized neutron star medium. Indeed, let us assume that the differential neutrino cross section on polarized neutrons equals

$$\frac{d\sigma}{d\Omega} = \sigma_0\,(1 + P\cos\theta), \tag{2}$$

where $\sigma_0 = G_{\rm F}^2 E_\nu^2/4\pi^2$ is the neutrino cross section on unpolarized neutrons, $P = eB/m_n T$ is the polarization of the neutrons by a strong magnetic field of the neutron star, which is given by

$$P \approx 2\times 10^{-5}\left[\frac{B}{10^{13}\ \mathrm{G}}\right]\left[\frac{3\,\mathrm{MeV}}{T}\right].$$

The cross section (2) is clearly asymmetric and, being integrated over the spherical angle, produces the asymmetry $A = P/3$. Using $P \approx 10^{-5}$, this would give very small kick velocities less than 0.1 km/s. However, Horowitz and Li argued that a large asymmetry can arise because the anisotropy of the neutrino flux increases with multiple neutrino scatterings on the neutrons and found that the subsequent neutrino emission may produce velocities up to $v \approx 400$ km/s. Lai and Qian investigated further this idea and obtained similar results [30].

In order to address the problem of multiple neutrino scatterings and neutrino transport, Kusenko, Segre, and Vilenkin [10] considered the kinetic equation for neutrinos. They argued that the principle of detailed balance requires that, even in the presence of parity-violating processes and with anisotropic scattering amplitudes, no asymmetry is generated in the thermal equilibrium. Arras and Lai [31] came also to the same conclusion.

Let us consider these results in more detail. The kinetic equation for the neutrino distribution function has the form [10, 30]

$$\frac{\partial f_\nu(\mathbf{r},\mathbf{k},t)}{\partial t} + \frac{c\,\mathbf{k}}{|\mathbf{k}|}\nabla f_\nu(\mathbf{r},\mathbf{k},t) = I_{\rm coll}, \tag{3}$$

where $\mathbf{k}$ is the neutrino momentum, c is the velocity of light (we neglect small neutrino masses), and the collision integral equals

$$I_{\rm coll} = \int \frac{d^3p}{(2\pi)^3}\frac{d^3k'}{(2\pi)^3}\frac{d^3p'}{(2\pi)^3}\,W_{i\to f}\,\times$$
$$\times\,\{[1 - f_\nu(\mathbf{k})]\,[1 - f_n(\mathbf{p})]\,f_n(\mathbf{p}')f_\nu(\mathbf{k}') -$$
$$-\,f_\nu(\mathbf{k})f_n(\mathbf{p})\,[1 - f_n(\mathbf{p}')]\,[1 - f_\nu(\mathbf{k}')]\}, \tag{4}$$

where $f_n(\mathbf{r},\mathbf{p},t)$ is the neutron distribution function, and $W_{i\to f}$ is the S-matrix element for neutrino-neutron scattering $\mathbf{k}$, $\mathbf{p} \to \mathbf{k}'$, $\mathbf{p}'$. We omitted also the summation over the spins and assumed that $W_{i\to f} = W_{f\to i}$. Factors like $1 - f_\nu$ and $1 - f_n$ in Eq. (4) take into account the Pauli blocking.

The authors of Refs. [10,31] stated that the equilibrium state of neutrinos is given by the conventional Fermi–Dirac distribution function for any asymmetry in the neutrino-neutron cross section. This conclusion is based on the principle of detailed balance in the thermal equilibrium. Mathematically, it is indeed easy to check that the equation

$$[1 - f_\nu(\mathbf{k})][1 - f_n(\mathbf{p})]f_n(\mathbf{p}')f_\nu(\mathbf{k}') =$$
$$= f_\nu(\mathbf{k})f_n(\mathbf{p})\,[1 - f_n(\mathbf{p}')]\,[1 - f_\nu(\mathbf{k}')]$$

for $f_\nu = (e^{(E_\nu-\mu_\nu)/T} + 1)^{-1}$ and $f_n = (e^{(E_n-\mu_n)/T} + 1)^{-1}$ implies the equality

$$e^{\frac{E_\nu(k)+E_n(p)}{T}} = e^{\frac{E_\nu(k')+E_n(p')}{T}},$$

which holds obviously due to the conservation of energy.

Consequently, although it seems that the multiple neutrino scatterings off polarized neutrons (or quarks if the quark matter exists in the cores of neutron stars) in the magnetized neutron star medium due to the asymmetric cross section (2) should produce a large asymmetry of the neutrino distribution in the momentum space, the study utilizing the kinetic equation shows that no such an asymmetry develops in view of the principle of detailed balance. Arras and Lai [31] estimated also the neutrino emission asymmetry due to a deviation of the neutrino distribution from equilibrium in the region close to the neutrino-matter decoupling layer and found that very large magnetic fields of order 10^{16} G are needed in order to get kick velocities of order 100 km/s. Thus, we conclude that the mechanism suggested by Horowitz and Li cannot produce the required large pulsar kick velocities.

3. Neutrino Energy Dispersion and Momentum Asymmetry in Dense Magnetized Relativistic Matter

The kinetic equation describes the evolution of particle distribution functions and is used to describe fluids, when they are not in a local equilibrium state. The interaction between particles is taken into account in the kinetic equation through the collision integral $I_{\rm coll}$. As we saw above, the collision integral is automatically zero if the fermion distribution functions are the Fermi–Dirac ones (the same is true in the case of the Bose–Einstein distribution functions for bosons). However, the kinetic equation approach considered in the previous section implicitly assumes that the energy dispersions of particles coincide with that in the free space. Indeed, the kinetic equation was first used for the description of gases, where the particle energy dispersions are the same as in the free space, and the collisions between particles only redistribute the momenta and energies of individual particles.

However, we know that interactions between particles do, in general, change their energy dispersion in media. For example, the kinetic equation is widely used in solid-state physics, where the quasiparticles energy dispersions are, in general, not like those in the free space. As to neutrinos propagating in a dense matter, we know that their energy dispersion does change compared to that in the free space. In fact, the dense matter effect in the neutrino dispersion relation is one of the crucial ingredients in the well known Mikheyev–Smirnov–Wolfenstein effect [32, 33].

Let us discuss the medium effects on the neutrino propagation [34]. Since the neutrinos in neutron stars have energies much less than the masses of the W and Z bosons, the effective low-energy Hamiltonian describing the relevant neutrino interactions [35] is given by

$$H_{\rm int} = \frac{G_{\rm F}}{2\sqrt{2}}\left(J^{(+)\mu}J^{(-)}_{\mu} + \frac{1}{4}J^{(N)\mu}J^{(N)}_{\mu}\right), \qquad (5)$$

where $J^{(+)}_{\mu}$, $J^{(-)}_{\mu}$, and $J^{(N)}_{\mu}$ are the standard charged and neutral currents of the weak interactions. Then it is not difficult to check that, e.g., the electrons present in the medium lead to the following effective Hamiltonian describing the interaction of neutrinos with electrons:

$$H_{\nu-e} = \frac{G_{\rm F}}{\sqrt{2}}\,\bar{\nu}_e(x)\gamma^{\mu}(1-\gamma^5)\nu_e(x)\,\times \\ \times\,\bar{e}\,\gamma_{\mu}\left(\frac{1-\gamma^5}{2} + 2\sin^2\theta_W\right)e. \qquad (6)$$

Here, $G_{\rm F} = 1.17\times 10^{-11}$ MeV^{-2} is the Fermi coupling constant, and $\sin^2\theta_W = 0.23$ is the weak mixing angle. The interaction Hamiltonian (6) leads for the neutrinos interacting with electrons in a magnetized relativistic plasma to the following terms in the neutrino effective kinetic term:

$$H^{(\rm eff)}_{\nu} = \\ = \frac{G_{\rm F}n_e}{2\sqrt{2}}(1+4\sin^2\theta_W)\,\bar{\nu}_e(x)\gamma^0(1-\gamma^5)\nu_e(x) - \\ - \frac{G_{\rm F}\langle j^5_3\rangle}{\sqrt{2}}\,\bar{\nu}_e(x)\gamma^3\frac{1-\gamma^5}{2}\nu_e(x). \qquad (7)$$

Here, $n_e = \langle\bar{e}\gamma^0 e\rangle$ is the electron density, and $\langle j^5_3\rangle = \langle\bar{e}\gamma_3\gamma^5 e\rangle$ is the component of the electron axial current along the direction of the magnetic field. The first term in Eq. (7) describes a correction to the effective neutrino chemical potential. According to the MSW effect [32, 33], such corrections are relevant for neutrino oscillations, because they are different for the electron, muon, and tau neutrinos propagating in a stellar medium. The second term in Eq. (7) corresponds to a shift of the third component of the neutrino momentum in the neutrino dispersion relation. This means that the thermal equilibrium neutrino distribution will be asymmetric. Note that a similar contribution was discussed after Eq. (2.25) in [36]. Obviously, the neutrino emission with such asymmetric momentum distribution may contribute to the kick contrary to the conclusion of Refs. [10, 31].

Let us estimate the corresponding contribution. According to Refs. [5, 6], the topological axial current due to the lowest Landau level contribution of the electrons equals $\langle j^5_3\rangle = -eB\mu_e/(2\pi^2)$. Using it, we obtain that $\delta_3 = G_{\rm F}eB\mu_e/(2\sqrt{2}\pi^2)$ is of order 10^{-11} MeV/c for $B = 10^{15}$ G and $\mu_e = 100$ MeV and is ten orders less than that needed in order to explain the largest pulsar velocities observed.

4. Hot-Spot Scenario

Although the neutrinos do have an asymmetric distribution in the momentum space due to the weak interactions with charged fermions in neutron stars with a magnetic field, we found in the previous section that this asymmetry is too small to produce the necessary pulsar kicks. Still, there is a large chiral asymmetry stored in charged fermions in a protoneutron star. It was suggested in Refs. [13, 14, 28] that the topological currents of charged fermions could be responsible for the large proper motion (kicks) of neutron stars. The star collapse during the neutron star formation is connected with the capture of electrons by nuclei through the process of inverse beta-decay. Since only the left-handed fermions take part in this interaction, the remaining electrons are mostly right-handed. Consequently, there exists a significant chiral asymmetry for the electrons in the initial state of a protoneutron star, which can be described through the introduction of the chiral chemical potential μ_5.

Although it is not clear what will happen to the electrons when they reach the surface of the star, it was assumed in Refs. [13,14] that somehow the entire current (or at least its sizeable part) carried by the topological current will be transferred into space by some means. The authors of Refs. [13, 14] mentioned that this assumption is likely to be correct only for bare quark stars, where the crust is about 1000 fm wide and is likely wrong for typical neutron stars, where the crust is 1 km in thickness. The energy of the electrons would be absorbed in the latter case by the crust and would not contribute to the kick. Still, we think that it is possible that the absorbed energy will heat the area, where the topological current reaches the crust producing a hot spot on the surface of neutron star.

We would like to emphasize that hot spots may be formed not necessarily only in the scenario with the topological current. For example, the magnetic field of a neutron star funnels charged particles back toward the surface that could result in the formation of hot spots in the polar regions (the Ruderman mechanism [38]). Alternatively, hot spots in the polar regions could be formed due to the anisotropic heat transfer from the neutron star core, which depends strongly on the magnetic field direction and is maximal along the magnetic field lines (see, e.g., Ref. [39]). There might exist also other physical mechanisms of formation of hot spots. Still at present, it is difficult to estimate reliably the efficiency of the formation of hot spots and their characteristics.

In the astrophysical literature, the hot spots were used also phenomenologically in order to explain the results of certain observations. It was argued in Ref. [40] that a sinusoidal light curve of the neutron star candidate 1E 161348-5055 located at the center of the supernova remnant RCW 103 could be explained by a freely precessing neutron star with a hot spot. Moreover, using the data from the XMM-Newton spacecraft, the rotating hot spots on the surfaces of three nearby neutron stars were observed for the first time [41]. The spots vary in size from less than 100 meters to about one kilometer.

Assuming that a hot spot is formed, the emission from this spot will be stronger than from other areas that produces a kick to the neutron star. Let us estimate this kick. For this, we assume that the hot spot has radius R and temperature $T + \Delta T$ that exceeds the temperature T of the rest emitting surface of the neutron star.

The Stefan–Boltzmann law determines the total energy radiated per unit surface area over all wavelengths per unit time. Integrating it over the area of a hot spot and time and subtracting the corresponding value for temperature T, we find the following pulsar kick velocity:

$$v = 4\,\mathrm{km/s}\left(\frac{T}{1\,\mathrm{MeV}}\right)^3\left(\frac{\Delta T}{0.1\,\mathrm{MeV}}\right)\left(\frac{R}{1\,\mathrm{km}}\right)^2\left(\frac{t}{1\,\mathrm{s}}\right). \quad (8)$$

For $T = 3$ MeV, $R = 1$ km, $\Delta T = 0.4$ MeV, and $t = 1$ s, this estimate gives the quite large velocity $v = 430$ km/s.

4.1. Momentum Density

For the mechanism proposed in Refs. [13, 14, 28], as well as for the hot-spot scenario proposed above, to work, it is necessary that the electrons connected with the topological current do carry a non-zero momentum. Let us check this explicitly. The energy-momentum density tensor for fermions equals (see, e.g., Eq. (3.153) in Ref. [35])

$$T^{\mu\nu} = \frac{i}{2}\left(\bar{\psi}\gamma^{\mu}\partial^{\nu}\psi - \partial^{\nu}\bar{\psi}\gamma^{\mu}\psi\right). \quad (9)$$

The fermion propagator in a magnetic field takes the form of a product of the Schwinger phase factor Φ

and a translation invariant part, i.e.,

$$G(r,r') = e^{i\Phi(\mathbf{r},\mathbf{r}')}\bar{G}(r-r'), \tag{10}$$

where the translation invariant part of the propagator in the momentum space expanded over the Landau levels is given by (for simplicity, we consider massless fermions)

$$\bar{G}(\omega,k^3,\mathbf{k}) = ie^{-k^2l^2}\sum_{\chi=\pm}\sum_{n=0}^{\infty}D_n(\omega,k^3,\mathbf{k}) \times \\ \times \frac{1+\chi\gamma^5}{2}\frac{(-1)^n}{(\omega+\mu-\chi\mu_5)^2-k_3^2-2n|eB|}, \tag{11}$$

where μ is the chemical potential, $l = |eB|^{-1/2}$ is the magnetic length, $\mathbf{k} = (k^1,k^2)$ is the "transverse momentum", $k = |\mathbf{k}|$, and the n-th Landau level contribution is determined by

$$D_n(\omega,k^3,\mathbf{k}) = 4(\mathbf{k}\cdot\boldsymbol{\gamma})L^1_{n-1}\left(2k^2l^2\right) + \\ +2[(\omega+\mu+\chi\mu_5)\gamma^0 - k^3\gamma^3] \times \\ \times\left[\mathcal{P}_-L_n\left(2k^2l^2\right) - \mathcal{P}_+L_{n-1}\left(2k^2l^2\right)\right]. \tag{12}$$

Here, L_n^α are the generalized Laguerre polynomials, $L_{-1}^\alpha = 0$ by definition,

$$\mathcal{P}_\pm = \frac{1}{2}\left[1\pm i\gamma^1\gamma^2 s_\perp\right]$$

are spin projectors, and $s_\perp = \mathrm{sgn}(eB)$. Using Eqs. (9), (10), and (11), we find the following momentum density in the direction of the magnetic field:

$$P^3 = \langle T^{03}\rangle = \int\frac{d\omega dk^3 d^2\mathbf{k}}{(2\pi)^4}\,k^3\mathrm{tr}\left[\gamma^0\bar{G}(\omega,k^3,\mathbf{k})\right] = \\ = -i\sum_{\chi=\pm}\sum_{n=1}^{\infty}(-1)^n\int\frac{d\omega dk^3 d^2\mathbf{k}}{(2\pi)^4}\,k_3^2 e^{-k^2l^2}\times \\ \times\mathrm{tr}\left[\frac{\gamma^0\gamma^3\left[\mathcal{P}_-L_n\left(2k^2l^2\right)-\mathcal{P}_+L_{n-1}\left(2k^2l^2\right)\right]}{(\omega+\mu-\chi\mu_5)^2-k_3^2-2n|eB|}\times \\ \times(1+\chi\gamma^5)\right] - 2i\int\frac{d\omega dk^3 d^2\mathbf{k}}{(2\pi)^4}\,k^3\,e^{-k^2l^2}\times \\ \times\mathrm{tr}\left[\frac{\gamma^0[(\omega+\mu)\gamma^0-k^3\gamma^3+\mu_5\gamma^0\gamma^5]\mathcal{P}_-}{(\omega+\mu)^2-k_3^2+\mu_5^2-2(\omega+\mu)\mu_5\gamma^5}\right], \tag{13}$$

where we separated the lowest Landau level contribution in the last term. Integrating over $\mathbf{k}$, one may check that only the lowest Landau level contributes. Using

$$\frac{(\omega+\mu)\gamma^0-k^3\gamma^3+\mu_5\gamma^0\gamma^5}{(\omega+\mu)^2-k_3^2+\mu_5^2-2(\omega+\mu)\mu_5\gamma^5}\mathcal{P}_- = \\ = \frac{1}{(\omega+\mu)\gamma^0-k^3\gamma^3-\mu_5\gamma^0\gamma^5}\mathcal{P}_- = \\ = \frac{1}{(\omega+\mu)\gamma^0-(k^3-s_\perp\mu_5)\gamma^3}\mathcal{P}_-, \tag{14}$$

we find the following momentum density:

$$P^3 = -4i\int\frac{d\omega dk^3}{(2\pi)^3l^2}\frac{(\omega+\mu)k^3}{(\omega+\mu)^2-(k^3-s_\perp\mu_5)^2} = \\ = -4i\mu_5 eB\int\frac{d\omega dk^3}{(2\pi)^3}\frac{\omega+\mu}{(\omega+\mu)^2-k_3^2} = \\ = \frac{\mu_5 eB\mathrm{sgn}(\mu)}{2\pi^2}\int dk^3\,\theta(\mu^2-k_3^2) = \frac{\mu\mu_5 eB}{\pi^2}. \tag{15}$$

Thus, the momentum density for charged fermions in a magnetized relativistic matter is not equal to zero and is proportional to the product of the corresponding electric μ and chiral μ_5 chemical potentials. Consequently, if the dense relativistic matter has a chiral asymmetry generated by a chiral chemical potential, then the corresponding electric current due to the chiral magnetic effect does carry a non-zero momentum. Therefore, if the thermal conductivity of the crust is not too large so that it cannot efficiently remove the heat produced by the energy deposited by the topological current, then the hot-spot scenario may produce large pulsar velocities.

4.2. Neutrino emission and hot spots

It is well known that the neutrino emission is the most efficient mechanism of cooling of neutron stars [42]. The hot-spot scenario relies on the stability of hot spots for a period of several seconds. Therefore, it is crucial to check that the neutrino emission will not eliminate a hot spot during this period. Neutrinos are emitted through many reactions. The strongest of them are the direct Urca processes, but they are threshold reactions open only at sufficiently large densities. Otherwise, the modified Urca processes are the main reactions relevant for the cooling of neutron stars (see, e.g., review [43]).

According to Table 2 in Ref. [43], the neutrino emission from a neutron star matter at temperature T due to the modified Urca processes leads to the following loss of energy per second and cubic centimeter of volume:

$$Q = 2 \times (10^{30}\text{–}10^{33}) \left(\frac{T}{10^5\,\mathrm{eV}}\right)^8 \frac{\mathrm{eV}}{\mathrm{cm}^3 \cdot \mathrm{s}}. \qquad (16)$$

The thermal energy stored in one cubic centimeter of volume of this matter equals

$$E = \frac{1.3 \times 10^{13}\,\mathrm{eV}}{\mathrm{cm}^3} \left(\frac{\mathrm{m}}{\mathrm{eV}}\right)^4 \int_0^\infty \sqrt{x^2+1}\,x^2 dx \times \times \left(\frac{1}{e^{\frac{\sqrt{x^2+1}-\mu_n/m}{T/m}}+1} - \theta\left(\frac{\mu_n}{m} - \sqrt{x^2+1}\right)\right), \qquad (17)$$

where $\theta(x)$ is the step function, m is the neutron mass, and μ_n is the neutron chemical potential.

Using Eq. (16), we easily find that the additional energy released per second through the neutrino emission of one cubic centimeter of the hot spot matter at a temperature $T + \Delta T$ equals

$$\Delta Q = 1.6 \times (10^{31}\text{–}10^{34}) \left(\frac{T}{10^5\,\mathrm{eV}}\right)^8 \frac{\Delta T}{T} \frac{\mathrm{eV}}{\mathrm{cm}^3 \cdot \mathrm{s}}. \qquad (18)$$

On the other hand, the excess of the thermal energy stored in one cubic centimeter of the hot spot matter is given by

$$\Delta E = \frac{1.3 \times 10^{13}\,\mathrm{eV}}{\mathrm{cm}^3} \left(\frac{\mathrm{m}}{\mathrm{eV}}\right)^4 \frac{m\Delta T}{T^2} \int_0^\infty \sqrt{x^2+1}\,x^2 dx \times \times \frac{(\sqrt{x^2+1} - \mu_n/m)\, e^{\frac{\sqrt{x^2+1}-\mu_n/m}{T/m}}}{\left(e^{\frac{\sqrt{x^2+1}-\mu_n/m}{T/m}}+1\right)^2}. \qquad (19)$$

For $T = 3$ MeV, $\Delta T = 0.4$ MeV, $m = 940$ MeV, and $\mu_n = 1$ GeV, we obtain

$$\Delta Q = 1.4 \times (10^{42}\text{–}10^{45}) \frac{\mathrm{eV}}{\mathrm{cm}^3 \cdot \mathrm{s}},$$

$$\Delta E = 2 \times 10^{44} \frac{\mathrm{eV}}{\mathrm{cm}^3}.$$

Thus, the thermal energy excess ΔE stored in one cubic centimeter of excess hot spot matter is two orders of magnitude larger than the lower limit of the additional energy $\Delta Q \cdot 1\,\mathrm{s}$ released through the neutrino emission for one second. Still, ΔE is one order of magnitude smaller than the upper limit of $\Delta Q \cdot 1\,\mathrm{s}$. Therefore, we conclude that the hot spots may or may not survive for the necessary period of several seconds. Clearly, in view of the uncertainty as high as three orders of magnitude in the energy released per second through the neutrino emission, a more careful study of neutrino emission is necessary in order to settle the question of the stability of hot spots.

5. Conclusion

We have shown that the trapped neutrinos in a protoneutron star, which interact with charged fermions in a magnetized relativistic matter, build up an anisotropic distribution in the momentum space due to the interaction-induced chiral shift parameter. However, we have found that this asymmetry is approximately ten orders of magnitude less than that needed to explain the largest pulsar velocities observed.

We have calculated the momentum density of the electric current due to the chiral magnetic effect and have found that it is not zero and is proportional to the product of the electric and chiral chemical potentials and the magnetic field. Assuming that the energy delivered by this current is absorbed by the crust, we have suggested a hot-spot scenario in order to explain large pulsar kicks. We have estimated the pulsar velocities that can be generated through the emission from a hot spot and have found that they are sufficiently large to explain the largest pulsar velocities observed. The problem of the stability of hot spots for a period of several seconds with respect to the cooling due to the neutrino emission is considered, and it is found that the hot spots may or may not survive. In view of the significant uncertainty in the energy released per second through the neutrino emission that exists in the literature, a more careful study of the neutrino emission is necessary. Still, we think that the hot-spot scenario may be considered as one of the realistic mechanisms of the generation of large pulsar kicks.

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1. P.M. Woods and C. Thompson, in *Compact Stellar X-ray Sources*, edited by W.H.G. Lewin and M. van der Klis, (Cambridge Univ. Press, Cambridge, 2006) p. 547 [astro-ph/0406133].
2. S. Mereghetti, Astron. Astrophys. Rev. **15**, 225 (2008).
3. D. Page and S. Reddy, Ann. Rev. Nucl. Part. Sci. **56**, 327 (2006).
4. S. Chatterjee *et al.*, Astrophys. J. **630**, L61 (2005).
5. A. Vilenkin, Phys. Rev. D **22**, 3080 (1980).

6. M.A. Metlitski and A.R. Zhitnitsky, Phys. Rev. D **72**, 045011 (2005).
7. E.V. Gorbar, V.A. Miransky, and I.A. Shovkovy, Phys. Rev. C **80**, 032801(R) (2009).
8. A.G. Lyne and D.R. Lorimer, Nature **369**, 127 (1994); J.M. Cordes and D.F. Chernoff, Astrophys. J. **505**, 315 (1998); B.M.S. Hansen and E.S. Phinney, Mon. Not. Roy. Astron. Soc. **291**, 569 (1997); C. Fryer, A. Burrows, and W. Benz, Astrophys. J. **496**, 333 (1998); Z. Arzoumanian, D.F. Chernoffs, and J.M. Cordes, Astrophys. J. **568**, 289 (2002).
9. E.V. Gorbar, V.A. Miransky, and I.A. Shovkovy, Phys. Rev. D **83**, 085003 (2011).
10. A. Kusenko, G. Segre, and A. Vilenkin, Phys. Lett. B **437**, 359 (2008).
11. I. Sagert and J. Schaffner-Bielich, J. Phys. G **35**, 014062 (2008); Astron. Astrophys. **489**, 281 (2008).
12. C.L. Fryer and A. Kusenko, Astrophys. J. Suppl. **163**, 335 (2006).
13. J. Charbonneau and A. Zhitnitsky, Phys. Rev. C **76**, 015801 (2007).
14. J. Charbonneau and A. Zhitnitsky, JCAP **1008**, 010 (2010).
15. G.M. Newman and D. Son, Phys. Rev. D **73**, 045006 (2006).
16. K. Fukushima, Lect. Notes Phys. **871**, 241 (2013).
17. D. Kharzeev and A. Zhitnitsky, Nucl. Phys. A **797**, 67 (2007).
18. D.E. Kharzeev, L.D. McLerran, and H.J. Warringa, Nucl. Phys. A **803**, 227 (2008).
19. D. Kharzeev, Phys. Lett. B **633**, 260 (2006).
20. K. Fukushima, D.E. Kharzeev, and H.J. Warringa, Phys. Rev. D **78**, 074033 (2008).
21. B.I. Abelev *et al.* [STAR Collaboration], Phys. Rev. Lett. **103**, 251601 (2009); Phys. Rev. C **81**, 054908 (2010).
22. L. Adamczyk *et al.* [STAR Collaboration], arXiv:1303.0901 [nucl-ex].
23. G. Wang [STAR Collaboration], arXiv:1210.5498 [nucl-ex].
24. H. Ke [STAR Collaboration], J. Phys. Conf. Ser. **389**, 012035 (2012).
25. I. Selyuzhenkov [ALICE Collaboration], Prog. Theor. Phys. Suppl. **193**, 153 (2012).
26. S.A. Voloshin, Phys. Rev. C **70**, 057901 (2004).
27. D.E. Kharzeev, Ann. Phys. **325**, 205 (2010); K. Fukushima, D.E. Kharzeev, and H.J. Warringa, Nucl. Phys. A **836**, 311 (2010).
28. J. Charbonneau, K. Hoffman, and J. Heyl, Mon. Not. R. Astron. Soc. **404**, L119 (2010).
29. C.J. Horowitz and G. Li, Phys. Rev. Lett. **80**, 3694 (1998); erratum-ibid. **81**, 1985 (1998).
30. D. Lai and Y.-Z. Qian, Astrophys. J. **495**, L103 (1998).
31. Ph. Arras and D. Lai, arXiv:astro-ph/9806285.
32. L. Wolfenstein, Phys. Rev. D **17**, 2369 (1978); ibid., **20**, 2634 (1979).
33. S.P. Mikheyev and A.Yu. Smirnov, Sov. J. Nucl. Phys. **42**, 913 (1985); Nuovo Cim. C **9**, 17 (1986); Sov. Phys. JETP **64**, 4 (1986).
34. M.C. Gonzalez-Garcia and Y. Nir, Rev. Mod. Phys. **75**, 345 (2003).
35. C. Itzykson and J.-B. Zuber, *Quantum Field Theory* (McGraw-Hill, New York, 1980).
36. T.K. Kuo and J. Pantaleone, Rev. Mod. Phys. **61**, 937 (1989).
37. E.V. Gorbar, V.A. Miransky, I.A. Shovkovy, and Xinyang Wang, Phys. Rev. **88**, 025043 (2013).
38. M. Ruderman and P.G. Sutherland, ApJ **196**, 51 (1975).
39. G. Greenstein and G.J. Hartke, ApJ **271**, 283 (1983).
40. J.S. Hey and L. Hernquist, Astrophys. J. **567**, 510 (2002).
41. A. De Luca, P.A. Caraveo, S. Mereghetti, M. Negroni, and G.F. Bigmani, Astrophys. J. **623**, 1051 (2005).
42. S.L. Shapiro and S.A. Teukolsky, *Black Holes, White Dwarfs, and Neutron Stars* (Wiley, New York, 1983).
43. D.G. Yakovlev, A.D. Kaminker, O.Y. Gnedin, and P. Haensel, Phys. Reports **354**, 1 (2001).

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КІРАЛЬНА АСИМЕТРІЯ
У ГУСТІЙ НАМАГНІЧЕНІЙ РЕЛЯТИВІСТСЬКІЙ
МАТЕРІЇ І ШВИДКОСТІ ПУЛЬСАРІВ

Р е з ю м е

Показано, що внаслідок слабкої взаємодії нейтрино із зарядженими частинками у густій релятивістській плазмі із індукованим за рахунок взаємодії параметром кірального зсуву у зовнішньому магнітному полі нейтрино мають асиметричний розподіл в імпульсному просторі у рівноважному стані. Знайдено, що відповідна асиметрія є дуже слабкою для того, щоб пояснити найвищі швидкості пульсарів, які спостерігаються. Запропоновано сценарій для гарячих точок, який пов'язаний з топологічним струмом або іншим механізмом їх утворення, і аргументовано, що цей сценарій може забезпечити генерацію потрібних великих швидкостей пульсарів.