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SPIN WAVES IN A FERROMAGNETIC NANOTUBE. ACCOUNT OF DISSIPATION AND SPIN-POLARIZED CURRENT

Dipole-exchange spin waves in a ferromagnetic nanotube with a circular cross-section have been studied in the presence of a spin-polarized electric current. The exchange and dipole-dipole magnetic interactions, anisotropy, dissipation effects, and the influence of a spin-polarized current are taken into consideration. An equation for the magnetic potential of spin excitations in the system concerned is derived, and the dispersion relation for spin waves is obtained. Depending on its direction, the spin-polarized current is demonstrated to either strengthen or weaken the effective dissipation. A condition, under which the presence of the spin-polarized current can lead to a generation of a spin wave, is determined.

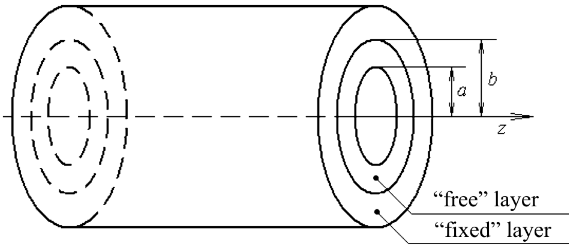
Keywords: spin wave, ferromagnetic nanotube, dipole-exchange theory, nanomagnetism, spin-polarized current.

1. Introduction

Spin waves, i.e. waves of magnetization in magnetically ordered materials [1, 2], have constituted a challenging popular topic of researches in the last decades. Spin waves in thin ferromagnetic films [3–5] and nanostructures, such as nanowires [6–9], micron-sized magnetic quantum dots [10–12], and other nanosystems, in which magnetically ordered materials are used, are of particular interest in the recent years. Spin waves in nanostructures are a promising object for numerous practical applications, such as the creation of new devices to store [13, 14] and to transfer [13, 14] information, new information processing devices [15], and so forth.

The magnetic properties of nanostructures are known to depend substantially on their shape and size. That is why the spin waves are studied separately in nanoparticles of different shapes. Magnetic nanotubes, which have been synthesized recently [16–22], found a wide spectrum of practical applications (in particular, in magnetobiology [23, 24]). However, the spin waves in magnetic nanotubes remain little-studied till now. The known works on this topic are mainly devoted to the research of spin solitons [25] and waves at the boundaries of magnetic domains [26, 27]. Our recent paper [28] on this subject dealt with the research of dipole-exchange spin waves in a single-layer ferromagnetic nanotube. However, no attention was paid to the analysis of a possible influence of dissipative effects and spin-polarized current, provided that the latter is present in the system.

Depending on the spin wave frequency, the size, shape, and material of a nanosystem, as well as other factors, the effects associated with the energy dissipation can either substantially affect the spin wave pattern in the system or be negligibly small (see, e.g., [29]). Therefore, while studying the spin waves in nanosystems and, in particular, in nanotubes, the account of dissipative effects is necessary in the general case.



Geometry of the examined nanotube

It is known that magnetic nanostructures and, in particular, magnetic nanotubes can be used as wave guides for spin waves. In this case, there emerges a problem of spin wave generation in such structures. It is known that, when a spin-polarized current passes through a thin ferromagnetic layer, the orientation of the magnetization vector in this layer can change [30, 31]. As a result, the spin wave pattern in the system also changes, which can give rise to the generation of spin excitations [32, 33]. However, the known researches of spin waves in the presence of a spin-polarized current in nanosystems dealt with thin magnetic films, whereas nanosystems with other geometries, in particular, nanotubes, have not been studied till now. Hence, the properties of spin waves in magnetic nanotubes in the presence of a spin-polarized current, in particular, the generation of spin waves in such systems, are a challenging issue for researches.

In this work, the theoretical research of spin waves in ferromagnetic nanotubes, which was started in [28], is continued. Here, the dipole-exchange spin waves in a two-layer ferromagnetic nanotube in the presence of the dissipation and a spin-polarized current are studied. The corresponding equation for the magnetic potential and the dispersion relation for the spin waves is obtained with regard for the magnetic dipole-dipole interaction, exchange interaction, anisotropy, dissipation effects, and the influence of a spin-polarized current. It is shown that the spin-polarized current changes the "effective dissipation" in the system. It is also shown that spin waves can be excited in the described system by passing a spin-polarized current through it. Moreover, a condition of this generation is found.

2. Formulation of the Problem

Let us consider a two-layer ferromagnetic nanotube, the length of which strongly exceeds its external radius. One layer of the nanotube is "fixed" in the sense of magnetization direction, whereas the other is "free", so that a spin-polarized current can run through the "free" layer in the radial direction. The internal radius of the "free" layer will be designated as a, and the external one as b (see Figure). The "free" layer is assumed to consist of a ferromagnet of the "easy-axis" type with the following parameters: the exchange interaction constant α, the parameter of uniaxial anisotropy β, and the gyromagnetic ratio γ. The anisotropy axis of the ferromagnet is directed along the symmetry axis.

Let the equilibrium magnetization of the "free" layer, $\mathbf{M}_0$, and the direction of "fixed" layer magnetization be directed along the symmetry axis, which is convenient to be selected as the axis Oz. We suppose that the nanotube is embedded into an external magnetic field $\mathbf{H}_0^{(e)}$, which is also directed along the axis Oz.

Let us consider a spin wave propagating in the "free" layer of the nanotube along the axis Oz. Since the system under consideration is nano-sized, the exchange interaction can substantially affect the spin wave pattern. Hence, the Landau–Lifshits equation should take into account not only the magnetic dipole-dipole interaction, but also the exchange one. (The dipole-exchange approximation used in this work is reasonable to be applied in the wave number interval of $10^5 \div 10^7$ cm^{-1}, because exchange effects can be neglected at smaller wave numbers. For nanotubes with standard sizes and materials, the wave numbers lie in the interval from $10^3 \div 10^4$ cm^{-1} to 10^7 cm^{-1}, which makes it necessary to take both magnetic dipole-dipole and exchange effects into account.) Dissipation effects are taken into consideration by introducing a relaxation term into the Landau–Lifshits equation.

We will use the linear approximation assuming the magnetization $\mathbf{m}$ and the magnetic field $\mathbf{h}$ of a wave to be small perturbations of the equilibrium magnetization and the total magnetic field, respectively. Hence, the total magnetization is equal to $\mathbf{M} = \mathbf{M_0} + \mathbf{m}$, where $|\mathbf{m}| \ll |\mathbf{M}_0|$, and the total magnetic field in the "free" layer to $\mathbf{H}^{(i)} = \mathbf{H}_0^{(i)} + \mathbf{h}$, where $|\mathbf{h}| \ll \left|\mathbf{H}_0^{(i)}\right|$, and $\mathbf{H}_0^{(i)}$ is the internal ground state magnetic field. Our task consists in obtaining a dispersion equation for the spin wave with regard for the dissipation and effects induced by the spin-polarized current and in finding a condition of a spin wave generation.

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3. System of Equations for a Spin Wave in the "Free" Layer of the Nanotube

Let us apply the linearized theory of spin waves, by writing down the linearized Landau–Lifshits equation in the magnetostatic approximation. First, by analogy with the previous work [28], let us write down the linearized Landau–Lifshits equation in the case where the dissipation and the spin-polarized current are absent. The equation for spin waves in the "free" layer of the nanotube described in the previous section, looks like [1]:

$$\frac{\partial \mathbf{m}}{\partial t} = \gamma(\mathbf{M}_0 \times (\mathbf{h} + \alpha\Delta \vec{m} + \beta\mathbf{n}(\mathbf{mn}) - \\ - \frac{1}{M_0^2}(\mathbf{M}_0\mathbf{H}_0^{(i)} + \beta(\mathbf{M}_0\mathbf{n})^2)\mathbf{m})), \tag{1}$$

where $\mathbf{n}$ is a unit vector directed along the anisotropy axis (in our case, it coincides with the unit vector $\mathbf{e}_z$). To take the energy dissipation into account, let us introduce a relaxation term in the Gilbert form, $\mathbf{T}_\mathrm{G} = \frac{\alpha_\mathrm{G}}{M}\left[\mathbf{M} \times \frac{\partial \mathbf{M}}{\partial t}\right]$, where, α_G is the dissipation parameter. When changing to the linearized Landau–Lifshits equation, this term reads

$$\mathbf{t}_\mathrm{G} = \alpha_\mathrm{G}\left[\mathbf{M}_0 \times \frac{\partial \mathbf{m}}{\partial t}\right]. \tag{2}$$

The thickness of nanotube walls, $b - a$, is assumed to be small in comparison with the internal nanotube radius a. Therefore, in the Landau–Lifshits equation for the spin wave, we can use the Slonczewski–Berger term derived for a planar layer (see, e.g., work [32]),

$$\mathbf{T}_s = \frac{\varepsilon\gamma\hbar J}{2eM_0^2\,(b-a)}[\mathbf{M} \times [\mathbf{M} \times \mathbf{e}_p]], \tag{3}$$

where ε is the dimensionless spin-polarization efficiency, J the current density (it is assumed to be constant), $\hbar$ the reduced Planck constant, e the elementary charge, and $\mathbf{e}_p$ a unit vector in the direction of a "fixed" layer magnetization (in our case, $\mathbf{e}_p = \mathbf{e}_z$). In the linearized form of the Landau–Lifshits equation, this term looks like

$$\mathbf{t}_s = \frac{\varepsilon\gamma\hbar J}{2eM_0^2\,(b-a)}[\mathbf{M}_0 \times [\mathbf{m} \times \mathbf{e}_z]], \tag{4}$$

here the relations $\mathbf{M}_0 \parallel \mathbf{e}_z$ and $\mathbf{m} \perp \mathbf{e}_z$ are taken into account. Therefore, the linearized Landau–Lifshits equation, in which the energy dissipation and the influence of a spin-polarized current are considered, takes the form

$$\frac{\partial \mathbf{m}}{\partial t} = \gamma\Bigg(\mathbf{m}_0 \times \Bigg(\mathbf{h} + \alpha\Delta \vec{m} + \beta\mathbf{n}\,(\mathbf{mn}) - \\ - \frac{1}{M_0^2}\left(\mathbf{m}_0\mathbf{H}_0^{(i)} + \beta\,(\mathbf{m}_0\mathbf{n})^2\right)\mathbf{m} + \frac{\alpha_\mathrm{G}}{\gamma M_0}\frac{\partial \mathbf{m}}{\partial t} + \\ + \frac{\varepsilon\hbar J}{2eM_0^2\,(b-a)}\,[\mathbf{m} \times \mathbf{e}_z]\Bigg)\Bigg). \tag{5}$$

In particular, if the perturbation is periodic in time,

$$\vec{m}\,(\mathbf{r}, t) = \mathbf{m}_0\,(\mathbf{r})\;(i\omega t)\,, \mathbf{h}\,(\mathbf{r}, t) = \mathbf{h}_0\,(bfr)\;(i\omega t). \tag{6}$$

Taking the symmetry into account, the Landau–Lifshits equation acquires the following form:

$$i\omega\mathbf{m}_0 = \gamma\Bigg(M_0\mathbf{e}_z \times \Bigg(\mathbf{h}_0 + \alpha\Delta \vec{m}_0 - \\ - \left(\beta + \frac{H_0^{(e)}}{M_0} - i\frac{\alpha_\mathrm{G}}{\gamma M_0}\omega\right)\vec{m}_0 + \\ + \frac{\varepsilon\hbar J}{2eM_0^2\,(b-a)}\,[\mathbf{m}_0 \times \mathbf{e}_z]\Bigg)\Bigg). \tag{7}$$

Here, we used the fact that, in the case of long nanotube, $4\pi\hat{N}\mathbf{M}_0 = 0$, where $\hat{N}$ is the tensor of demagnetizing coefficients. Consequently, the internal magnetic field $\mathbf{H}_0^{(i)}$ is equal to the external one, $\mathbf{H}_0^{(e)}$, being therefore uniform.

By analogy with the previous work [28], in order to obtain another necessary relation between the magnetization and the magnetic field, let us apply the magnetostatic approximations (see, e.g., work [1]) and introduce the magnetic potential. In our case, this procedure has to be additionally substantiated, because of the presence of an electric current in the system. However, a possibility of introducing the magnetic potential in the "free" layer of a nanotube follows from the fact that the current configuration described in the formulation of the problem does not create an additional magnetic field.

Hence, by introducing the magnetic potential Φ in such a way that $\mathbf{h} = -\nabla\Phi$, $\mathbf{h}_0 = -\nabla\Phi_0$, and $\Phi = \Phi_0\,(\mathbf{r})\exp\,(i\omega t)$, and using Maxwell's equation $\operatorname{div}\,\mathbf{h} = -4\pi\operatorname{div}\mathbf{m}$, we will obtain the required relation $\Delta\Phi - 4\pi\ \operatorname{div}\,\mathbf{m} = 0$ or, if the perturbation is periodic in time,

$$\Delta\Phi_0 - 4\pi\operatorname{div}\mathbf{m}_0 = 0. \tag{8}$$

The system of equations (7) and (8) provides a required relationship between the magnetization perturbation and the magnetic potential. By excluding $\vec{m}_0$ from this system, we obtain an equation for the magnetic potential.

4. Equation for the Magnetic Potential

Analogously to the previous work [28], in order to derive a dispersion relation, let us exclude the magnetization perturbation from the system of equations (7) and (8). Using the formula $\mathbf{h}_0 = -\nabla\Phi_0$, let us multiply Eq. (7) by the unit vector $\mathbf{e}_z$ (the cross product) and divide the result by γM_0. Taking into account that $m_{0z} = 0$, we obtain

$$-\frac{i}{\gamma M_0}\left(\omega + \frac{i\gamma\varepsilon\hbar J}{2eM_0\,(b-a)}\right)[\mathbf{e}_z \times \mathbf{m}_0] = -\nabla\Phi_0 + \\ + \alpha\Delta\,\vec{m}_0 - \left(\beta + \frac{H_0^{(e)}}{M_0} - i\frac{\alpha_\mathrm{G}}{\gamma M_0}\omega\right)\vec{m}_0 + \frac{\partial\Phi_0}{\partial z}\mathbf{e}_z \quad (9)$$

or, taking the divergence of both sides of this equation,

$$-\frac{i}{\gamma M_0}\,(\omega \pm i\kappa)\,\mathrm{div}\,[\mathbf{e}_z \times \mathbf{m}_0] = -\Delta\Phi_0 + \frac{\partial^2\Phi_0}{\partial z^2} + \\ + \frac{1}{4\pi}(\alpha\Delta - \tilde{\beta})\Delta\Phi_0. \quad (10)$$

Here, $\kappa = \gamma\varepsilon\hbar\,|J|\,/\,(2eM_0\,(b-a))$, the sign before κ on the left-hand side is positive for the current running from the "fixed" magnetic layer into the "free" one ($J > 0$) and negative in the opposite case, and

$$\tilde{\beta} = \beta + \frac{H_0^{(e)}}{M_0} - i\frac{\alpha_\mathrm{G}}{\gamma M_0}\omega.$$

Let us apply the operator $\alpha\Delta - \tilde{\beta}$ to both sides of obtained equation. Substituting the quantity $\left(\alpha\Delta - \tilde{\beta}\right)\mathbf{m}_0$ from Eq. (9), we obtain

$$-\frac{i\,(\omega \pm i\kappa)}{\gamma M_0}\mathrm{div}\left[\mathbf{e}_z \times \left(-\frac{i(\omega \pm i\kappa)}{\gamma M_0}\,[\mathbf{e}_z \times \mathbf{m}_0] + \\ + \nabla\Phi_0 - \frac{\partial\Phi_0}{\partial z}\mathbf{e}_z\right)\right] = (\alpha\Delta - \tilde{\beta}) \times \\ \times \left(\frac{1}{4\pi}(\alpha\Delta - \tilde{\beta}) - 1\right)\Delta\Phi_0 + (\alpha\Delta - \tilde{\beta})\frac{\partial^2\Phi_0}{\partial z^2}. \quad (11)$$

Since $\mathrm{div}\,\left(\mathbf{e}_z \times \left(\nabla\Phi_0 - \frac{\partial\Phi_0}{\partial z}\mathbf{e}_z\right)\right) = 0$, $\mathrm{div}(\mathbf{e}_z \times (\mathbf{e}_z \times \mathbf{m}_z)) = -\Delta\Phi_0/4\pi$, we obtain the final equation for the magnetic potential:

$$\left(\frac{(\omega \pm i\kappa)^2}{\gamma^2 M_0^2} - (\tilde{\beta} - \alpha\Delta)(4\pi + \tilde{\beta} - \alpha\Delta)\right)\Delta\Phi_0 + \\ + 4\pi(\tilde{\beta} - \alpha\Delta)\frac{\partial^2\Phi_0}{\partial z^2} = 0. \quad (12)$$

Note that, at $\kappa = 0$, this equation coincides with the analogous equation for a continuous cylindrical wave guide.

One can see that Eq. (12) for the magnetic potential differs from the analogous equation obtained in the previous work [28]. In particular, it contains terms describing the influence of the dissipation and the spin-polarized current. Hence, the indicated effects change the pattern of spin waves in the system. Let us solve this equation and analyze the influence of those terms on the spin wave pattern in the system.

5. Dispersion Relation and Spin Wave Generation Condition

Let us find a dispersion relation for spin waves in the "free" layer of a nanotube. First, we note that Eq. (12) leads to the following solution in the cylindrical coordinates (ρ, θ, z):

$$\Phi = (A_1 J_n\,(k_\perp\rho) + A_2 N_n\,(k_\perp\rho)) \times \\ \times\,\exp\left(i\left(n\theta + k_\parallel z - \omega t\right)\right), \quad (13)$$

where A_1 and A_2 are constants, J_n and N_n are the Bessel and Neumann, respectively, functions of the n-th order, and $k_\perp$ and $k_\parallel$ are the transverse and longitudinal, respectively, wave numbers. Substituting solution (13) into Eq. (12), we obtain the dispersion relation in the form

$$\left(\frac{(\omega \pm i\kappa)^2}{\gamma^2 M_0^2} - (\tilde{\beta} + \alpha k^2)(4\pi + \tilde{\beta} + \alpha k^2)\right)k^2 + \\ + 4\pi(\tilde{\beta} + \alpha k^2)k_\parallel^2 = 0, \quad (14)$$

where the total wave number was introduced, $k^2 = k_\perp^2 + k_\parallel^2$.

One can see that the dispersion equation (14) includes two components of the wave number. In the general case, in order to obtain a dispersion relation for spin waves, we should solve Eq. (12) with the corresponding boundary conditions for the magnetization. However, one of the wave number components can be excluded from Eq. (14), if we pay attention to that the thickness of typical nanotubes is of the same order of magnitude as the characteristic length of the exchange interaction $l_{\mathrm{ex}} = \sqrt{\alpha/4\pi}$. Therefore, we may consider the case where the tube thickness is smaller than the exchange interaction length. This assumption affect the initial equations; in particular, the small thickness of the shell can be taken into account, e.g., by omitting the radial derivatives in Eqs. (5) and (7). However, solution (13) will satisfy the modified equation as well, if we put $k_\perp = 0$ in it. Hence, in the case of a nanotube, whose thickness is narrower than the exchange interaction length, we may neglect the radial dependence of the potential, by putting $k_\perp = 0$ (and, consequently, $n = 0$) and transforming Eq. (14) for $k \neq 0$ as follows:

$$k^2 = \frac{1}{\alpha}\left(\frac{\omega\,(1 - i\alpha_{\mathrm{G}}) \pm i\kappa}{\gamma M_0} - \beta - \frac{H_0^{(e)}}{M_0}\right). \tag{15}$$

From whence, we obtain the sought dispersion relation for the spin wave:

$$\omega = \frac{1}{1+\alpha_{\mathrm{G}}^2}\left(\gamma M_0\left(\alpha k^2 + \beta + \frac{H_0^{(e)}}{M_0}\right) \pm \kappa + \right.$$
$$\left. + i\left(\alpha_{\mathrm{G}}\gamma M_0\left(\alpha k^2 + \beta + \frac{H_0^{(e)}}{M_0}\right) \mp \kappa\right)\right). \tag{16}$$

The damping or growing of the spin wave amplitude is governed by the imaginary part of the frequency. Therefore, let us analyze the imaginary part of the frequency given by the dispersion relation (16) at various current values. Provided that the current is positive ($J > 0$) at

$$\kappa > \alpha_{\mathrm{G}}\gamma M_0\left(\alpha k^2 + \beta + \frac{H_0^{(e)}}{M_0}\right),$$

i.e. under the condition

$$|J| > \frac{2e\alpha_{\mathrm{G}} M_0^2}{\varepsilon\hbar}\,(b-a)\left(\alpha k^2 + \beta + \frac{H_0^{(e)}}{M_0}\right), \tag{17}$$

the spin wave amplitude grows in time, so that we have instability, and a wave generation takes place. If

$$J = \frac{2e\alpha_{\mathrm{G}} M_0^2}{\varepsilon\hbar}\,(b-a)\left(\alpha k^2 + \beta + \frac{H_0^{(e)}}{M_0}\right), \tag{18}$$

the spin wave is self-supporting. Finally, if $J > 0$, i.e.

$$|J| < \frac{2e\alpha_{\mathrm{G}} M_0^2}{\varepsilon\hbar}\,(b-a)\left(\alpha k^2 + \beta + \frac{H_0^{(e)}}{M_0}\right),$$

the dissipation processes dominate over the generation ones, but the effective dissipation will be lower than that in the absence of a spin-polarized current. At $J < 0$, on the contrary, the presence of a spin-polarized current strengthens the damping of the spin wave amplitude.

Note that, while studying the spin waves in a two-layer ferromagnetic film, in which, similarly to our system, one layer is “fixed”, the other is “free”, and a spin-polarized current flows from the former into the latter, similar regularities were obtained, which testifies in favor of the effective dissipation caused by the spin-polarized current [32] and confirms the results obtained above. The negative damping and the generation of spin oscillations, that arises when a spin-polarized current passes through a magnetic film, were also studied in work [33], in which an oscillator rather than a running wave was considered.

Provided that the wave number is real, the characteristic damping time for the spin wave amplitude equals

$$\tau = \frac{2\pi}{\mathrm{Im}\omega} = \frac{2\pi\left(1+\alpha_{\mathrm{G}}^2\right)}{\alpha_{\mathrm{G}}\,|\gamma|\,M_0\left(\alpha k^2 + \beta + \frac{H_0^{(e)}}{M_0}\right) \mp \kappa}. \tag{19}$$

In a similar way, we can determine the characteristic time of spin wave amplitude growth. However, the corresponding obtained value will be negative.

6. Analysis of the Results Obtained

Let us compare the above-obtained expression for the critical current with the formula derived in work by Slavin and Tiberkevich [33]. The cited authors expounded the theory of an oscillator with negative damping and applied it to a “free” layer considered in the form of a planar nanodisk, which was thin enough for the magnetization of spin oscillations in it to be assumed spatially uniform (therefore, it was an oscillator rather than a running wave that was analyzed in work [33]). The formula for the critical current, at which the wave damping transforms into the wave generation, looks like

$$J = \frac{2e\alpha_{\mathrm{G}} M_0 (H_0^{(e)} - 4\pi M_0)\,(b-a)}{\varepsilon\hbar}. \tag{20}$$

When obtaining this expression, the spatial inhomogeneity of a magnetization was ignored, as well as the exchange effects and the effects associated with anisotropy. The above-obtained expression (18) for the critical current in the nanotube after neglecting the corresponding terms acquires the form

$$J = \frac{2e\alpha_{\mathrm{G}} M_0 H_0^{(e)} (b-a)}{\varepsilon\hbar}. \tag{21}$$

One can see that the expressions coincide (the internal field for a disk, $H_0^{(e)} - 4\pi M_0$, corresponds to $H_0^{(e)}$ for a long nanotube), which confirms the validity of the results obtained.

Let us evaluate numerically the wave frequency and the characteristic time of the wave damping, as well as the influence of a spin-polarized current, with the use of the obtained expressions. The typical values of constants for a ferromagnet constituting the "free" layer are as follows: $\beta = 1$, $\alpha = 10^{-12}$ cm^{-2}, $\gamma = 10^7$ Hz/G, and $M_0 = 10^3$ G. The dissipation constant for typical ferromagnetic materials that are used in experiments with the spin-polarized current varies by an order of magnitude in the interval of $10^{-1} \div 10^{-2}$. We assume that the wave number k is restricted, on the one hand, by the nanotube length and, on the other hand, by the exchange interaction length. Therefore, for typical nanotubes, the order of magnitude of this parameter changes from 10^2 to 10^6 cm^{-1}. Hence, in the absence of a spin-polarized current, the spin wave frequency is of an order of 10^{10} Hz within the whole interval of wave numbers, and the characteristic time τ changes from 10^{-8} to 10^{-9} s.

Now, let us consider the terms that are associated with the presence of aa spin-polarized current. The critical value

$$\kappa_{cr} = \alpha_{\mathrm{G}} \gamma M_0 (\alpha k^2 + \beta + H_0^{(i)}/M_0),$$

at which the effective dissipation changes its sign, has an order of 10^8–10^9 Hz depending on the value of α_{G}. This corresponds to a current density ranging from 3×10^6 to 3×10^7 A. The obtained value corresponds to a typical critical current in experiments with thin ferromagnetic films (see, e.g., [33, 34]). Typical values of current density in corresponding experiments amount to $10^7 \div 10^8$ A/cm^2 (see, e.g., [33]), so that the generation condition (17) can be realized experimentally.

7. Conclusions

To summarize, in this work, the linear dipole-exchange spin waves in a ferromagnetic nanotube in the presence of a spin-polarized current and the dissipation have been analyzed. A two-layer ferromagnetic nanotube is considered. The spin-polarized current passes in the radial direction through the "free" (in the sense of the magnetization direction) layer consisting of a ferromagnet of the "easy-axis" type. The equation for the magnetic potential of a spin wave in the "free" layer of the nanotube is derived in the magnetostatic approximation with regard for the dissipation and the influence of a spin-polarized current. The dispersion relation and the characteristic time of spin excitation damping for this wave are obtained provided that the corresponding wave number is real. It is shown that the spin-polarized current can both strengthen and weaken the effective dispersion of a spin wave, depending on the spin-polarized current direction. It is also found that, if the value of current is high enough, it can lead to the spin wave generation. The condition of spin wave generation in the examined system is obtained.

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СПIНОВI ХВИЛI У ФЕРОМАГНIТНIЙ НАНОТРУБЦI. УРАХУВАННЯ ДИСИПАЦIЇ ТА СПIН-ПОЛЯРИЗОВАНОГО СТРУМУ

Р е з ю м е

У роботi дослiджено дипольно-обмiннi спiновi хвилi у феромагнiтнiй нанотрубцi кругового перерiзу за наявностi спiн-поляризованого електричного струму. Враховано обмiнну взаємодiю, диполь-дипольну взаємодiю, ефекти анiзотропiї, дисипативнi ефекти та вплив спiн-поляризованого струму. Отримано рiвняння для магнiтного потенцiалу спiнових хвиль у такiй системi, знайдено дисперсiйне вiдношення. Показано, що залежно вiд напрямку спiн-поляризованого струму наявнiсть останнього може пiдсилювати або послаблювати ефективну дисипацiю спiнової хвилi. Показано, що за певних умов наявнiсть спiн-поляризованого струму може приводити до генерацiї спiнової хвилi. Записано умову такої генерацiї.