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SIMPLIFIED (COARSE-GRAINED) DESCRIPTION OF MAGNON TRANSPORT KINETICS ALONG A FERROMAGNETIC CHAIN

Using a coarse-grained description of the kinetic process – based on the hierarchy of transition rates in a quantum system – the kinetics of the establishment of a steady-state magnon flow from a magnon source to a magnon receiver has been investigated. It is shown that if the characteristic hopping rate of magnons between adjacent chain units significantly exceeds their decay rate, the final stage of the establishment of a steady-state regime of magnon transport along the chain can exhibit a double-exponential or even single-exponential character. Analytical expressions have been obtained for magnon transport rates and the steady-state magnon flow. It is shown that increasing the magnon lifetime significantly improves the transport properties of the chain.

Keywords: magnons, kinetics, microwave cavities, nanomagnets.

1. Introduction

Recent experimental studies have shown that magnonics – an independent branch of spintronics – can realize its potential in nanotechnology applications involving magnetophotonics, thermal nanoconversion, magnonic capacitors, magnonic transistors, and other fields [1, 2]. Magnons are very good candidates for use in quantum communication. This is because the transport of magnons – as uncharged quasiparticles – can occur without the generation of Joule heat, and, just as with charged carriers, this transport can be controlled using magnetic and electric fields.

The implementation of quantum communication based on magnonics depends primarily on the efficiency of magnon generation and the ability to

transmit them over long distances. The traditional method of magnon generation involves thermal activation as well as spin pumping [3–5]. A specific mechanism for magnon generation has recently been implemented in microwave cavities with embedded nanomagnets. In magnonics based on yttrium iron garnet (YIG) [1, 2, 6], the large magnetic moment of a YIG nanomagnet enables the conversion of a microwave cavity electromagnetic field quantum into a quantum of localized spin excitation within the nanomagnet (Kittel magnon) [7–10]. It has recently been shown theoretically that a Kittel magnon can be transferred from one microwave cavity to another via a ferromagnetic chain connecting nanomagnets placed in these cavities [11, 12]. Such long-range magnon transport is based on the superexchange coupling between spin excitations localized on different nanomagnets. Physically, this coupling arises from the participation of virtual delocalized magnonic states of the ferromagnetic chain in the transport of a Kittel magnon, with the transport process itself interpretable as magnon tunneling. Conditions were

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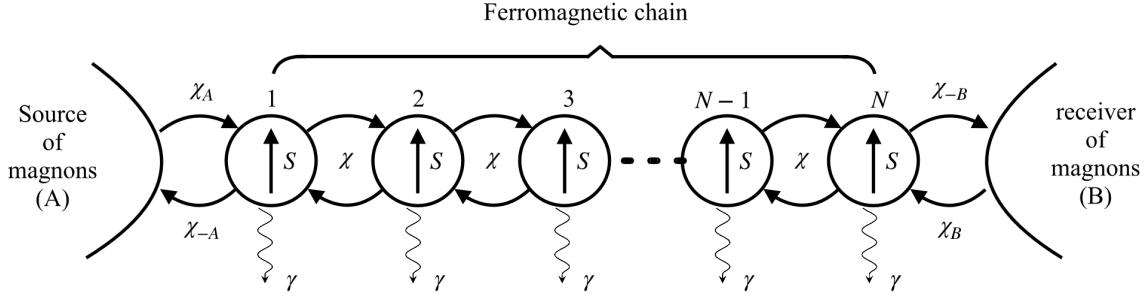


Fig. 1. Schematic of magnon transfer from source to receiver via a ferromagnetic chain. The rates χ_A (χ_B) and χ_{-A} (χ_{-B}) characterize, respectively, the forward and backward transfer of magnons between the terminal unit 1 (N) of the chain and the magnon source (receiver); the parameters χ and γ determine, respectively, magnon hopping between adjacent chain units and magnon decay

found for off-resonant (exponentially decaying) and resonant (peak-like) tunneling mechanisms. A dependence of the magnon flow on the number of units in the ferromagnetic chain was observed both in the case of magnon tunneling between nanomagnets and during the hopping transport of spin excitations between adjacent paramagnetic units of the entire “nanomagnet A – ferromagnetic chain – nanomagnet B” (AFB) system.

The studies [11,12] considered only the steady-state regime of magnonic transport. Therefore, although the proposed theoretical models of magnonic tunneling and long-range sequential transport can serve as a physical basis for quantum information transfer in hybrid AFB systems, the realization of such transfer fundamentally depends on the ratio between the magnon lifetime τ_{lt} and the characteristic time τ_{tr} required for magnon transport between localization centers. It is evident that quantum information transmission using magnons can be efficient only if $\tau_{lt} \gg \tau_{tr}$. In YIG-based structures, the lifetime typically does not exceed a few hundred nanoseconds [13]. It has recently been shown that, under certain conditions, this characteristic time increases by two orders of magnitude, reaching a value of $\tau_{lt} \sim 10$ microseconds [14]. This experimental result confirms the hypothesis regarding the use of magnons as carriers of quantum information.

This paper investigates the behavior of a magnon flow in a hybrid AFB system at various characteristic values of τ_{lt} . It is assumed that the flow $\Pi(t)$ is switched on at time $t = 0$ and reaches its steady-state value $\Pi^{(st)}$ at $t \gg \tau_{tr}$. Estimates of the values of τ_{tr} for sequential long-range magnon transport were obtained, and the conditions under which magnons

can serve as uncharged carriers of quantum information in hybrid structures containing nanomagnets were determined.

2. Model and Basic Equations

This work is devoted to determining the efficiency of sequential magnon transport through a ferromagnetic chain consisting of an arbitrary number N of units. Within the framework of a semi-phenomenological approach, the kinetics of the magnon flow along the chain is determined by the processes of direct and reverse magnon transfer between magnon source A and terminal unit 1 (rates χ_A and χ_{-A} , respectively), magnon hopping between adjacent units of the chain (rate χ), and magnon transfer between terminal unit N and magnon receiver B (rates χ_B and χ_{-B} , respectively) (Fig. 1). Among the observed rates, the dependence of long-range magnonic transport on the number of chain units is determined by the rate χ . It is assumed that in the semi-phenomenological model employed, adjacent paramagnetic chain units are coupled by a Heisenberg spin-spin interaction, regardless of the chain structure. We use the Heisenberg Hamiltonian in its standard form

$$\mathcal{H}_{\text{chain}} = \sum_l \left[\mu_B g \mathbf{H} \hat{\mathbf{S}}_l - D (\hat{S}_l^z)^2 - \frac{1}{2} J \sum_{l'(\neq l)} \hat{\mathbf{S}}_l \hat{\mathbf{S}}_{l'} \right]. \quad (1)$$

Here, μ_B is the Bohr magneton, D is the easy-axis anisotropy parameter, and $\mathbf{H}$ is the static magnetic field that changes the projection M of the spin S of the l -th unit of the paramagnetic chain ($l = 1, 2, \dots, N$). The quantities g and $\hat{\mathbf{S}}_l$ represent the g -factor and the spin operator of the l -th unit, respectively. In this paper, we consider single-magnon

long-range transport under the condition that at a given temperature the average number of magnons in the AFB system is small. Therefore, we can use the Holstein–Primakoff transformation [20], restricting ourselves to the lowest-order terms of the expansion in the creation and annihilation operators a_l^+ and a_l , respectively, of spin excitation on the l -th paramagnetic unit. These operators are related to the spin operators by the expressions $\hat{S}_l^x + i\hat{S}_l^y \simeq \sqrt{2S}a_l^+$, $\hat{S}_l^x - i\hat{S}_l^y \simeq \sqrt{2S}a_l$ and $\hat{S}_l^z = -S + a_l^+a_l$, where x , y , and z are Cartesian coordinates, so that the lowest excited spin state for the l -th unit is $|S, S_l^z = -(S-1)\rangle = |1_l\rangle = a_l^+|0_l\rangle$, and the vacuum state $|0_l\rangle = |S, S_l^z = -S\rangle$ is determined by the condition $a_l|0_l\rangle = 0$. As for the Hamiltonian (1), only the terms relating to the ground state of the chain and the terms corresponding to single-particle spin excitations in the chain are retained. Measuring the energy of spin excitations relative to the ground-state energy, we obtain the Hamiltonian of the lowest spin excitations of the ferromagnetic chain in the form: $\mathcal{H}_{\text{sp-ex}} = \sum_{l,l'=1}^N \Theta_{l,l'} a_l^+ a_{l'}$. The diagonal term $\Theta_{l,l} = E_c \equiv \mu_B g H + JS + D(2S-1)$ represents the spin excitation energy at one of the identical sites of the chain, whereas the off-diagonal term $\Theta_{l,l\pm 1} = t_c \equiv JS$ is the intersite magnon coupling parameter. It is precisely this parameter that characterizes the magnon hopping rate between adjacent sites l and $l' = l \pm 1$ of the chain. In the Born approximation, this rate can be expressed as $\chi = (2\pi/\hbar)t_c^2 \mathcal{F}$. Here, the factor $\mathcal{F}$ characterizes the processes of quantum coherence loss during isoenergetic magnon transitions between adjacent units of the chain. The analytical expression for $\mathcal{F}$ depends on the specific structure of the chain. One of the determining factors leading to the loss of coherence is the effect of stochastic fields on the dynamical system [21]. In our case, these fields influence the spin-excitation energy E_c of the paramagnetic fragment and the coupling parameter t_c . The approach presented in works [22–24] demonstrates that, in the case of dichotomous noise, the inter-center transition rate depends significantly on the value of the Kubo number $K = \Delta/\nu$, where Δ and ν are the mean amplitude and frequency of the dichotomous noise, respectively. Therefore, for instance, at $K \gg 1$ we obtain $\chi \approx 2(t_c/\hbar)^2(\nu/\Delta^2)$.

There are various types of quasi-one-dimensional ferromagnetic chains of organic and inorganic origin (see examples in Refs. [15–19]). To the best of the au-

thor's knowledge, there are currently no experimental data on the transfer of magnons between nanomagnets using a ferromagnetically ordered chain as a mediator of the transmission process. Nevertheless, regardless of the specific chain structure, to determine the dependence of the magnon flow on the chain length (i.e., the number of units N), it suffices to treat the rates of the contact processes $\chi_{A(-A)}$ and $\chi_{B(-B)}$, as well as the hopping rate χ , as phenomenological parameters of the AFB system. When studying magnon transport kinetics, it is necessary to take into account that a magnon is a quasiparticle. Therefore, a spin excitation localized on an arbitrary unit of the chain has a finite lifetime, meaning it can decay at a rate $\gamma \sim \tau_{\text{lt}}^{-1}$.

The magnon flows emerging from source A and entering receiver B are determined, respectively, by the expressions

$$\begin{aligned} \Pi_{\text{input}}(t) &= \chi_A - (\chi_{-A} + \gamma)P_1(t), \\ \Pi_{\text{output}}(t) &= (\chi_{-B} + \gamma)P_N(t) - \chi_B. \end{aligned} \quad (2)$$

It is evident that the temporal dynamics of these flows are determined by the time evolution of the site occupancy probabilities $P_1(t)$ and $P_N(t)$, which characterize the presence of a magnon at the terminal sites of the chain. The quantities $P_1(t)$ and $P_N(t)$, like the analogous quantities $P_l(t)$ (where $l = 2, 3, \dots, N-1$), vary with time in accordance with a system of N coupled kinetic equations:

$$\begin{aligned} \dot{P}_1(t) &= -(\chi_{-A} + \chi + \gamma)P_1(t) + \chi P_2(t) + \chi_A, \\ \dot{P}_l(t) &= -(2\chi + \gamma)P_l(t) + \chi(P_{l-1}(t) + P_{l+1}(t)), \\ \dot{P}_N(t) &= -(\chi_{-B} + \chi + \gamma)P_N(t) + \chi P_{N-1}(t) + \chi_B. \end{aligned} \quad (3)$$

The solution to the system, given the absence of magnons in the chain at time $t = 0$, can be written in the form

$$\begin{aligned} P_l(t) &= P_l^{(\text{st})} + \sum_{\lambda} [F_l(s_{\lambda})/\prod_{\lambda' \neq \lambda} (s_{\lambda} - s_{\lambda'})] e^{s_{\lambda} t}, \\ P_l^{(\text{st})} &= F_l(0)/\prod_{\lambda} s_{\lambda}. \end{aligned} \quad (4)$$

Here, $F_l(s) = \int_0^{\infty} e^{st} P_l(t) dt = D_l(s)/sD(s)$ is the Laplace transform of the function $P_l(t)$. The steady-state site occupancy probabilities are denoted as $P_l^{(\text{st})} \equiv P_l(t \gg \tau_{\text{tr}})$. The quantities s_{λ} are the roots

of the determinant $D(s)$ of the matrix

$$\hat{D}(s) = \begin{pmatrix} a_1(s) & -\chi & 0 \dots & 0 \\ -\chi & a(s) & -\chi \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 \dots & -\chi & a(s) & -\chi \\ 0 \dots & 0 & -\chi & a_N(s) \end{pmatrix}, \quad (5)$$

where $a_{1(N)}(s) = s + \chi + a_{1(N)}$, $a(s) = s + 2\chi + \gamma$ and $a_{1(N)} = \chi_{-A(-B)} + \gamma$. (6)

After some algebra, we get

$$D(s) = a_1(s)a_N(s)D(N-2; s) - \chi^2(a_1(s) + a_N(s))D(N-3; s) + \chi^4 D(N-4; s), \quad (7)$$

and

$$\begin{aligned} D_{1(N)}(s) &= \chi_{A(B)}[a_{N(1)}(s)D(N-2; s) - \chi^2 D(N-3; s)] + \chi_{B(A)}\chi^{N-1}, \\ D_l(s) &= \chi_B\chi^{N-l}[a_1(s)D(l-2; s) - \chi^2 D(l-3; s)] + \chi_A\chi^{l-1}[a_N(s)D(N-1-l; s) - \chi^2 D(N-2-l; s)]. \end{aligned} \quad (8)$$

(In Eq. (8), the expression $D_l(s)$ is valid for chain units $l = 2, 3, \dots, N-1$). As for expression

$$D(l, s) = \frac{\sinh[(l+1)\zeta(s)/2]}{\sinh[\zeta(s)/2]}, \quad (9)$$

it characterizes the process of sequential hopping transport of a magnon, where

$$\zeta(s)/2 = \ln \{[a(s) + \sqrt{a^2(s) - 4\chi^2}]/2\} \quad (10)$$

can be regarded as an attenuation coefficient.

3. Discussion of Results

The analytical expressions (4)–(10), describing the kinetics of establishing a steady-state flow of magnons, can be regarded as the main result of this study. These expressions allow us to understand the physics of the sequential transmission of magnons depending on the number of units N of the ferromagnetic chain connecting the magnon source to the magnon receiver. The roots s_λ of the determinant (7) can be found by noting that the determinant of the tridiagonal matrix (5) is a polynomial $D(s) = \sum_{l=0}^N s^l d_l$ and, consequently, can be written in the form $D(s) = \prod_{\lambda=1}^N (s - s_\lambda)$. Moreover, there is a hierarchy of roots, for instance, $|s_1| < |s_2| < \dots |s_N|$. Since the

rates characterizing the transport process are equal to $k_\lambda = -s_\lambda$, the overall magnon transport kinetics can proceed on various time scales of the order of $\tau_{tr}^{(\lambda)} \sim K_\lambda^{-1}$. Let $K_N > K_{N-1} \dots > K_2 > K_1$, such that the temporal hierarchy is as follows: $\tau_{tr}^{(N)} < \tau_{tr}^{(N-1)} \dots < \tau_{tr}^{(2)} < \tau_{tr}^{(1)}$. Thus, the fast stage of the kinetic process can be associated with the shortest characteristic times $\tau_{tr}^{(N)} < \tau_{tr}^{(N-1)} \dots < \tau_{tr}^{(\lambda_*)}$. It can be said that this kinetic stage is realized on a time scale $\Delta t \sim \tau_{tr}^{(N)}$, establishing specific quasi-equilibrium values $P_l(t \sim \tau_{tr}^{(\lambda_*)})$ for the quantities P_l . The subsequent stages can be observed on time scales $\tau_{tr}^{(\lambda_*)} \ll \Delta t(\sim \tau_{tr}^{(\lambda_{**})})$, $\tau_{tr}^{(\lambda_{**})} \ll \Delta t(\sim \tau_{tr}^{(\lambda_{***})})$, and so on. Stationary values $P_l^{(st)} = P_l(t \gg \tau_{slow})$ are established on a time scale $\Delta t \sim \tau_{slow}$, and therefore the study of the transport process kinetics on this scale is of particular interest. (For the selected hierarchy of transport rates, the characteristic time τ_{slow} is identified with τ_{tr} .) Note that, on the time scale $\Delta t \sim \tau_{slow}$, the fast stages of the kinetic process have already concluded, resulting in the establishment of quasi-stationary values $P_l(t)$. Consequently, the kinetics on this time scale can be described by simpler analytical expressions than those given in expressions (4)–(10). This corresponds to the so-called coarse-grained description of the kinetics.

To demonstrate the approach based on a coarse-grained description, we consider the final (slowest) stage of establishing a steady-state magnon flow. The mechanism of sequential transfer of magnons can be effective if the rate of magnon jumps between adjacent units of the chain χ is significantly higher than the rate of decay of magnons γ . Furthermore, we seek an expression for the slowest rate $K_{slow} = |s| \sim \tau_{slow}^{-1}$, which is significantly lower than the rate χ . Therefore, based on the inequality $\chi \gg \gamma, |s|$, we can set $\zeta(s)/2 \approx \ln \chi + (|s|/2\chi) \approx \ln \chi$. Thus, for rough estimates of the quantity (9) we can use the zeroth-order approximation with respect to $|s|$ and set $D(l, s) \sim (l+1)\chi^l$. This yields

$$D(s) = (N-1)\chi^{N-2}[s^2 + (a_1 + a_N + 2\chi_{seq}s + a_1a_N + (a_1 + a_N)\chi_{seq})], \quad \chi_{seq} = \frac{\chi}{N-1}, \quad (11)$$

and

$$D_l(s) \approx (N-1)\chi^{N-2}[(\chi_A + \chi_B)\chi_{seq} + \frac{N-l}{N-1} \times (s+a_N)\chi_A + \frac{l-1}{N-1}(s+a_1)\chi_B], \quad l = 1, 2, \dots, N. \quad (12)$$

The roots of the determinant (11) are $s_{\pm} = -k_{\pm}$, and, consequently,

$$P_{1(N)}(t) = P_{1(N)}^{(st)} + \frac{1}{K_+ - K_-} \times \\ \times \left\{ \chi_{A(B)} (e^{-K_- t} - e^{-K_+ t}) - \right. \\ \left. - [\chi_{A(B)} a_{N(1)} + (\chi_A + \chi_B) \chi_{seq}] \times \right. \\ \left. \times \left(\frac{1}{K_-} e^{-K_- t} - \frac{1}{K_+} e^{-K_+ t} \right) \right\}, \quad (13)$$

where

$$P_{1(N)}^{(st)} = \frac{1}{K_+ K_-} [\chi_{A(B)} a_{N(1)} + (\chi_A + \chi_B) \chi_{seq}]. \quad (14)$$

The overall magnon transport rates are determined as follows:

$$k_{\pm} = (a_1 + a_N + 2\chi_{seq} \pm \delta)/2, \\ \delta = \sqrt{(\chi_{-A} - \chi_{-B})^2 + 4\chi_{seq}^2}. \quad (15)$$

It is evident that, for $K_+ \sim K_-$, the slow stage of the transport process proceeds on a time scale $\Delta t \sim \tau_{slow}^{(+)} \sim \tau_{slow}^{(-)}$, where $\tau_{slow}^{(\pm)} \sim K_{\pm}^{-1}$; this manifests as a double-exponential time evolution of the quantities $P_{1(N)}(t)$ (as well as the other $P_i(t)$) toward their steady-state values. If $K_+ \gg K_-$, then on the time scale $\Delta t \sim K_-^{-1}$ we have $\exp(-K_+ t) \simeq 0$. As a result, for $t \gg K_+^{-1}$, the double-exponential time dependence of the quantities $P_{1(N)}(t)$ reduces to a single-exponential one:

$$P_{1(N)}(t) = P_{1(N)}^{(st)} (1 - e^{t/\tau_{slow}}), \quad (\tau_{slow} = K_-^{-1}). \quad (16)$$

As noted in the Introduction, the use of magnons as quantum information carriers is possible if the characteristic magnon lifetime $\tau_{lt} \sim \gamma^{-1}$ is much longer than the characteristic time $\tau_{tr} \sim \tau_{slow}$ of magnon transport between the source and the receiver. In the model under consideration, this condition is reflected in the inequality $K_- \gg \gamma$. Since K_- depends on the number of units in the ferromagnetic chain, this inequality defines a condition regarding the length of the chain used for information transmission.

To illustrate the influence of the governing parameter γ on the formation of the magnon flow, let us consider, as an example, direct magnon transport, where $\chi_{-A} = \chi_B = 0$. In such transport, the establishment of steady-state magnon flows (Eq. (2))

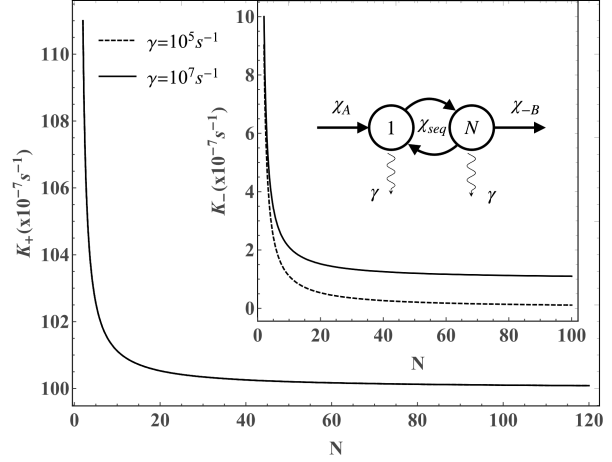


Fig. 2. Dependence of fast (k_+) and slow (k_-) overall magnon transfer rates on the number of units N of a ferromagnetic chain. Calculations with Eq. (15) at $\chi = 10^8 \text{ s}^{-1}$, $\chi_A = 10^7 \text{ s}^{-1}$, $\chi_{-A} = 0$, $\chi_B = 0$, $\chi_{-B} = 10^9 \text{ s}^{-1}$, $\gamma = 10^5 (10^7) \text{ s}^{-1}$

proceeds according to the kinetic scheme shown in Fig. 2 (inset). Thus, it is the elementary rate constants χ_A , χ_{-B} , γ , and τ_{tr} that determine the overall transfer rates (15), which characterize the fast and slow stages of the kinetic process on the corresponding time scales: $\Delta t \sim K_+^{-1} (\sim \tau_{fast})$ and $\Delta t \sim K_-^{-1} (\sim \tau_{slow})$, respectively. When plotting graphs characterizing the features of magnon transport, parameter values were chosen such that the key parameter, the magnon lifetime $\tau_{lt} = \gamma^{-1}$, corresponded to experimental data [14]. As can be seen from Fig. 2, the rate K_+ of the fast stage exceeds the rate K_- of the slow stage by approximately two orders of magnitude. In the region of $N \sim 10$, a sharp change in the nature of the dependence of both rates on chain length is clearly observed. This occurs because, with such a number of chain units, the rate χ_{seq} of magnon transport along the chain becomes lower than the rate γ of their decay. The fast and slow stages of the kinetic process manifest themselves directly in the time dependence of the quantities P_1 and P_N , which determine the input and output magnon flows. It can be seen from Figs 3 and 4 that the fast and slow phases develop on time scales of $\Delta t \sim \tau_{fast} \sim 1 \text{ ns}$ and $\Delta t \sim \tau_{slow} \sim 100 \text{ ns}$, respectively. Such values of Δt follow directly from the fact that, when $\chi_{-B} > \chi_{-A}$, the relations $K_+ \approx a_N + \chi_{seq}$ and $K_- \approx \gamma + \chi_{seq}$ hold for the elementary rate constants. It should be noted that the magnitude of P_1 exceeds the magnitude of

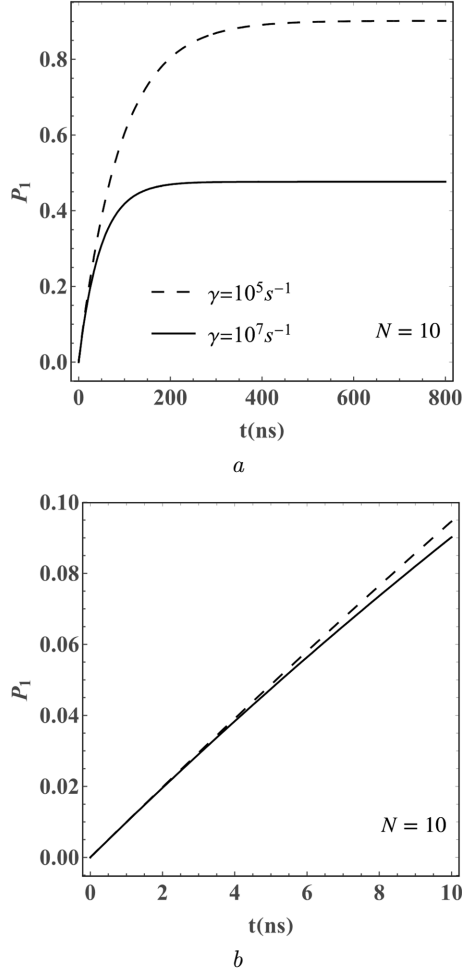


Fig. 3. Time dependence of the probability of finding a magnon at edge unit 1 for various values of the damping parameter γ (panel (a)). Fast temporal evolution occurs on a timescale of the order of a few nanoseconds (panel (b)). Calculations based on Eqs. (13) and (15) using the same parameters as in Fig. 2

P_N by nearly two orders of magnitude. This is due to the rapid transport of magnons (at a rate of χ_{-B}) from edge unit N to the magnon receiver.

The magnon flow reaches its steady-state value at $t \gg \tau_{\text{slow}} \equiv \tau_{\text{tr}}$. In this case, the input and output flows become equal to each other, i.e., $\Pi_{\text{input}}^{(\text{st})} = \Pi_{\text{output}}^{(\text{st})} \equiv \Pi^{(\text{st})}$. Based on the expressions (2) and (13) we obtain

$$\begin{aligned} \Pi^{(\text{st})} &= \chi_A \Phi, \\ \Phi &= \frac{\chi_{\text{seq}} a_N}{a_1 a_N + \chi_{\text{seq}} (a_1 + a_N)}. \end{aligned} \quad (17)$$

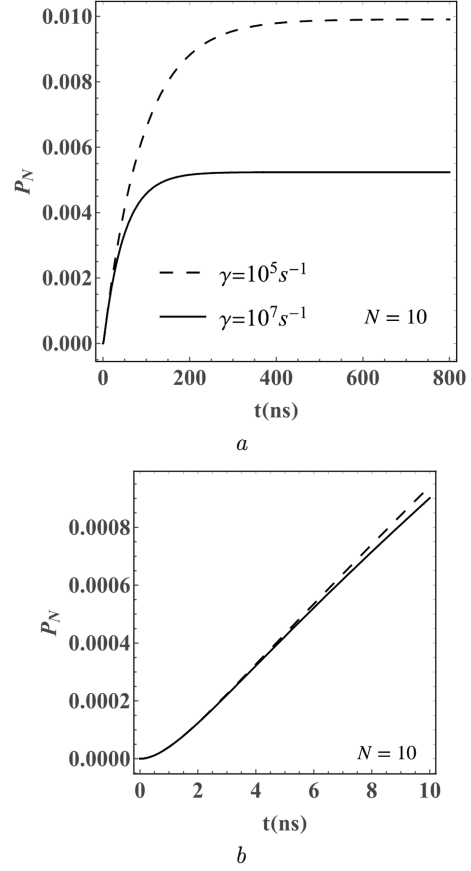


Fig. 4. Time dependence of the probability of finding a magnon at edge unit N for various values of the damping parameter γ (panel (a)). Fast temporal evolution occurs on a timescale of the order of a few nanoseconds (panel (b)). Calculations based on Eqs. (13) and (15) using the same parameters as in Fig. 2

In our case of a direct magnon flow, we have $a_1 \approx \gamma$ and $a_N \approx \chi_{-B} \gg \gamma$. These quantities characterize the long-range transport of magnons along the chain in the absence and presence of decay processes, respectively. The function Φ can be expressed in a different form by using the dependence of χ_{seq} on N (Eq. (11)). Taking $\tau_t = \gamma^{-1}$, we obtain

$$\Phi = \frac{\chi \tau_t}{N - 1 + \chi \tau_t}. \quad (18)$$

Fig. 5 shows the dependence $\Phi = \Phi(N)$ for two experimentally known values of τ_t . It is evident that for a longer magnon lifetime τ_t and the same chain length, the reduction in the flux magnitude becomes noticeably less pronounced. For example, if $N = 10$

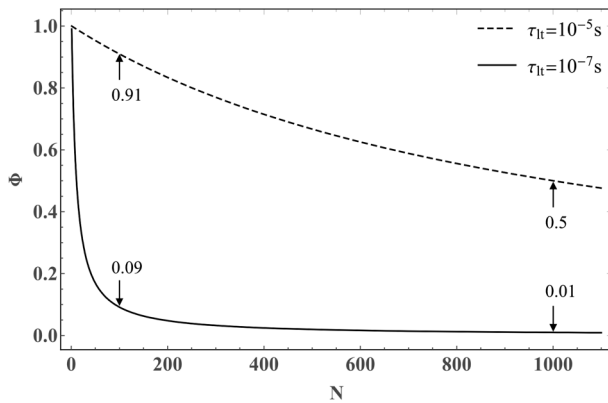


Fig. 5. Dependence of the steady-state magnon flow on the number of chain units N for various values of the magnon lifetime $\tau_{lt} = \gamma^{-1}$. Calculations based on Eq. (17) using the same parameters as in Fig. 2. The approximate expression for Φ (Eq. (18)) yields practically the same result

(100; 1000), the value of Φ is 0.52 (0.09; 0.01) and 0.99 (0.91; 0.51) for $\tau_{lt} \sim 10^{-7}$ s and $\tau_{lt} \sim 10^{-5}$ s, respectively.

4. Conclusion

A model of sequential magnon transport along a ferromagnetic chain was used to investigate the kinetics of the establishment of a steady-state magnon flow under conditions of magnon decay. The result obtained demonstrates that if the kinetic process in the system under consideration is determined by varying rates of transitions between its structural units, then the process of establishing a steady-state kinetic regime can be described in stages. During the transport of magnons along a chain consisting of N units, the kinetics of the establishment of the steady-state transport regime is, in the general case, characterized by N exponentials (Eq. (4)). We have shown that if the magnon hopping rate χ significantly exceeds the decay rate γ , the N -exponential kinetics reduces to bi-exponential kinetics. The corresponding two overall rates, K_+ and $K_- (< K_+)$, are expressed in terms of rates characterizing the influx of magnons into the chain, their efflux from the chain, and the effective magnon rate χ_{seq} between the terminal units 1 and N of the chain. Thus, the sequential transfer of a magnon along a chain of N sites appears as the transfer of a magnon directly between the terminal sites of the chain (see the inset of Fig. 2). Physically, this means that the coarse-grained kinetic process manifests itself on time scales Δt determined by the over-

all rates K_+ and K_- (Figs 2 and 3). Moreover, we show that when there is a large difference between these rates, the primary magnon transport occurs on a timescale associated with the lower rate. Therefore, the value of $\tau_{tr} = K_-^{-1}$ can be used to estimate the characteristic time of magnon transport from the source to the receiver. We have also demonstrated that the coarse-grained kinetic description method makes it possible to derive analytical expressions not only for the rates characterizing the main stage of the transition to the steady-state transport regime but also for the steady-state flow. In particular, for the direct transmission of magnons from source to receiver, an analytical dependence of the magnon flow on the magnon lifetime τ_{lt} was determined; it was shown that as τ_{lt} increases, the flow attenuation caused by an increase in the number of chain units N occurs significantly more slowly (Fig. 5).

The obtained result demonstrates how the magnon decay process can influence the transmission of quantum information using magnons.

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СПРОЦЕНІЙ (ОГРУБЛЕНІЙ) ОПИС КІНЕТИКИ ПЕРЕНЕСЕННЯ МАГНОНІВ ВЗДОВЖ ФЕРОМАГНІТНОГО ЛАНЦЮЖКА

Використовуючи метод крупнозернистого опису кінетичного процесу, заснованого на ієрархії швидкостей переходів у квантовій системі, ми дослідили кінетику встановлення стаціонарного потоку магнонів від джерела до приймача. Показано, що якщо характерна швидкість перескоку магнонів між сусідніми ланками ланцюжка суттєво перевищує швидкість їхнього загасання, то кінцева стадія встановлення стаціонарного режиму магнонного транспорту вздовж ланцюжка може мати двоєкспоненціальний або навіть одноекспоненціальний характер. Отримано аналітичні вирази для швидкостей перенесення магнонів і магнонного потоку, що встановився. Показано, що збільшення часу життя магнонів суттєво поліпшує транспортні властивості ланцюжка.

Ключові слова: магнони, кінетика, мікрохвильові резонатори, наномагніти.