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O.I DAVYDOVS'KA, V.O. NESTEROV
Institute for Nuclear Research, Nat. Acad. of Sci. of Ukraine
(47, Nauky Ave., Kyiv 03028, Ukraine)

ELASTIC SCATTERING CROSS-SECTIONS OF THE ^{18}O + ^{58}Ni AND ^{18}O + ^{60}Ni REACTIONS OBTAINED ON THE BASIS OF THE POTENTIAL OF THE MODIFIED THOMAS–FERMI METHOD, TAKING THE CORE INTO ACCOUNT

The nucleon density distributions and the nuclear interaction potentials for the ^{18}O + ^{58}Ni and ^{18}O + ^{60}Ni reactions have been calculated using the modified Thomas–Fermi method and taking into account all second-order terms in $\hbar$ in the quasi-classical expansion of the kinetic energy. Skyrme forces, depending on the nucleon density, were used as the nucleon-nucleon interaction. An analytical parameterization has been proposed that reproduces well the behavior of the nucleus-nucleus interaction potentials calculated using the modified Thomas–Fermi method with density-dependent Skyrme forces. Using the obtained potentials, the elastic scattering cross-sections have been calculated, yielding good agreement with experimental results.

Keywords: nucleus-nucleus potential, modified Thomas–Fermi method, Skyrme forces, elastic scattering, cross-sections.

1. Introduction

In order to determine the key characteristics of nuclear reactions, such as the scattering cross-sections in various channels, it is necessary to have a good understanding of the peculiarities in the behavior of the potential energy that governs the interaction between the nuclei [1–4]. That is why one of the main tasks of theoretical nuclear physics from its beginnings has been the study of the nucleus-nucleus interaction potential. Information about the value and the radial dependence of such a potential at small distances between the nuclei is especially important.

In general, the nucleus-nucleus interaction potential can be represented as the sum of three components: nuclear, Coulomb, and centrifugal. The properties of the latter two have been determined quite thoroughly to date, whereas the behavior of the nuclear component has been studied to a much lesser extent. As a result, a wide range of different models have been proposed for its approximation [1–25]. However, the barrier heights that directly affect the course of nuclear reactions in those models can demonstrate significant differences. In this regard, information on the nucleus-nucleus interaction potential and the corresponding barrier heights is of fundamental importance for the correct description of nuclear reactions.

The modified Thomas–Fermi method [25–37] is one of many methods that are currently used to calculate the nucleus-nucleus interaction potential. A semi-microscopic approach is applied in it. It is this method that we will use in this work to calculate the

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nucleon density distribution and the nucleus-nucleus interaction potential. In this approach, the density-dependent Skyrme forces [4, 7–9, 11, 12, 14–25] were used as the nucleon-nucleon interaction. Today, there are a considerable number of successful parameterizations of the Skyrme interaction. In our studies, we used the SkP parameterization [32]. In this case, in the semiclassical expansion of the kinetic energy in the powers of $\hbar$, we take into account all terms up to $\hbar^2$ inclusive. As was shown in previous calculations carried out by both other authors and by us, this is a rather accurate approximation. In such an approximation, the modified Thomas–Fermi method using Skyrme forces demonstrates high efficiency in reproducing the distribution of nucleon densities, binding energies, root-mean-square radii, and other important characteristics of the ground and excited states of nuclei [26–32, 34].

The nucleus-nucleus interaction potential obtained in the modified Thomas–Fermi approximation using Skyrme forces demonstrates quite a characteristic behavior. At large distances, it approaches the Coulomb potential, whereas at small distances, the formation of a potential barrier is observed owing to the balance between the Coulomb repulsion and the nuclear attraction. As the distance between the nuclei decreases further, the potential energy gradually decreases. At very small distances between the nuclei, i.e., when their nucleon densities substantially overlap, the potential obtained in the modified Thomas–Fermi approximation with Skyrme forces exhibits the presence of a repulsive core [8, 11, 14, 15, 18–22], which is explained by the substantial incompressibility of nuclear matter [14, 15, 22]. It should be noted that this behavior is not unique feature to this potential; similar repulsion at short distances can also be observed for the Proximity potential [5].

When studying scattering reactions, nucleus-nucleus interaction potentials with a repulsive core are not used very often. In this sense, one can refer to works [14, 15, 22, 38, 39], in which the inclusion of the repulsive component in the potential simultaneously made it possible to describe both elastic scattering and subbarrier fusion of atomic nuclei. In this regard, the consideration of elastic scattering of atomic nuclei in the framework of the modified Thomas–Fermi method with the application of Skyrme forces, which ensures the presence of a repulsive core, seems to be a rather challenging and interesting problem.

The mathematical methods used to implement the chosen approach are considered in sections 2 and 3. Section 4 is devoted to the calculation of elastic scattering cross-sections and a discussion of the obtained results. General conclusions of our work are presented in section 5.

2. Nucleus-Nucleus Interaction Potential in the Framework of the Modified Thomas–Fermi Method

The nucleus-nucleus interaction potential $V(R)$ can be qualitatively divided into nuclear, $V_N(R)$, Coulomb, $V_{\mathrm{Coul}}(R)$, and centrifugal, $V_l(R)$, parts:

$$V(R) = V_N(R) + V_{\mathrm{Coul}}(R) + V_l(R), \tag{1}$$

where R is the distance between the centers of mass of the nuclei. To date, the Coulomb and centrifugal parts of the potential have been studied quite thoroughly, and standard expressions, which can be found, e.g., in Refs. [22, 23], can be used for them.

Our goal is to calculate the nuclear component of the interaction potential, $V_N(R)$, using the modified Thomas–Fermi method and taking into account all second-order terms in $\hbar$ in the semiclassical expansion of the kinetic energy. Density-dependent Skyrme forces–in particular, the SkP parameterization [32]–are chosen to describe the nucleon-nucleon interaction. In this case, we will use the "frozen-density" approach, which is well justified in the energy range close to the potential barrier.

The nucleus-nucleus interaction potential $V_N(R)$ for two nuclei is taken in the form [9, 11]

$$V_N(R) = E_{12}(R) - (E_1 + E_2). \tag{2}$$

It is the difference between the energy $E_{12}(R)$ of a system of two nuclei located at a distance R from each other,

$$E_{12}(R) = \int \varepsilon[\rho_{1p}(\bar{r}) + \rho_{2p}(\bar{r}, R), \rho_{1n}(\bar{r}) + + \rho_{2n}(\bar{r}, R)]\, d\bar{r}, \tag{3}$$

and their energy $E_1 + E_2$ when they are at an infinite distance (it should be noted that the energy of a system of two nuclei at an infinite distance is simply the sum of the binding energies of two separate nuclei, denoted as E_1 and E_2),

$$E_{1(2)}(R) = \int \varepsilon[\rho_{1(2)p}(\bar{r}), \rho_{1(2)n}(\bar{r})]\, d\bar{r}. \tag{4}$$

In Eqs (2) and (3), $\rho_{1(2)p}$ and $\rho_{1(2)n}$ are the proton (p) and neutron (n) densities, respectively, in nucleus 1(2), $\varepsilon[\rho_{1(2)p}(\bar{r}), \rho_{1(2)n}(\bar{r})]$ is the energy density, and R is the distance between the centers of mass of the nuclei.

The energy density can be considered as the sum of kinetic, potential, and Coulomb terms. In the case of Skyrme forces, its form is well known from the literature [24–28, 30, 32, 37]:

$$\begin{aligned}
\varepsilon = \varepsilon_{\text{kin}} + \varepsilon_{\text{pot}} + \varepsilon_{\text{Coul}} = \frac{\hbar^2}{2m}\tau + \\
+ \frac{1}{2}t_0\left[\left(1 + \frac{1}{2}x_0\right)\rho^2 - \left(x_0 + \frac{1}{2}\right) \times \left(\rho_n^2 + \rho_p^2\right)\right] + \\
+ \frac{1}{12}t_3\rho^\alpha\left[\left(1 + \frac{1}{2}x_3\right)\rho^2 - \left(x_3 + \frac{1}{2}\right) \times \left(\rho_n^2 + \rho_p^2\right)\right] + \\
+ \frac{1}{4}\left[t_1\left(1 + \frac{1}{2}x_1\right) + t_2\left(1 + \frac{1}{2}x_2\right)\right]\tau\rho + \\
+ \frac{1}{4}\left[t_2\left(x_2 + \frac{1}{2}\right) - t_1\left(x_1 + \frac{1}{2}\right)\right] \times \\
\times (\tau_n\rho_n + \tau_p\rho_p) + \frac{1}{16}\left[3t_1\left(1 + \frac{1}{2}x_1\right) - \right. \\
\left. - t_2\left(1 + \frac{1}{2}x_2\right)\right](\nabla\rho)^2 - \\
- \frac{1}{16}\left[3t_1\left(x_1 + \frac{1}{2}\right) + t_2\left(x_2 + \frac{1}{2}\right)\right] \times \\
\times \left((\nabla\rho_n)^2 + (\nabla\rho_p)^2\right) + \\
+ \frac{1}{2}W_0\left[J\nabla\rho + J_n\nabla\rho_n + J_p\nabla\rho_p\right] + \varepsilon_{\text{Coul}}.
\end{aligned} \tag{5}$$

In this expression, ε_{kin} denotes the kinetic energy density, ε_{pot} the potential energy density, and $\varepsilon_{\text{Coul}}$ the Coulomb energy density. The quantities t_0, t_1, t_2, t_3, x_0, x_1, x_2, x_3, α, and W_0 are the parameters of the Skyrme interaction. The summands proportional to the parameters t_0 and t_3 are associated with zero-range forces. In particular, the component depending on t_0 characterizes attraction, while the term containing t_3 describes repulsion, which increases as the density of nuclear matter grows. This circumstance ensures the absence of the collapse of nuclear systems. The summands proportional to the parameters t_1 and t_2 provide a correction accounting for the finite range of the nuclear force. As the nucleon density grows, the contribution of those components to the total system energy increases. The parameters x_0, x_1, x_2, and x_3 are responsible for the description of exchange effects and take into account the spin and isospin asymmetries. The quantity W_0 is a constant that determines the character of the spin-orbit interaction.

In the quasi-classical expansion of kinetic energy, we take into account all terms up to the second order in $\hbar$, i.e., the kinetic energy density can be represented as the sum of the density in the standard Thomas–Fermi method and the second-order gradient correction, $\tau = \tau_{\text{TF}} + \tau_2$ [8, 9, 11, 12, 24, 27, 28, 37, 40, 41], whereas the total density $\tau = \tau_p + \tau_n$ is the sum of the proton and neutron kinetic energy densities. The first summand [27, 28],

$$\tau_{\text{TF},n(p)} = k\rho_{n(p)}^{5/3}, \tag{6}$$

is the kinetic energy density of neutrons (protons) in the standard Thomas–Fermi approximation. Here $k = \frac{5}{3}(3\pi^2)^{2/3}$, and τ_2 is the following expression for the second-order gradient correction in powers of $\hbar$ [27, 28]:

$$\begin{aligned}
\tau_{2q} = b_1\frac{(\nabla\rho_q)^2}{\rho_q} + b_2\nabla^2\rho_q + b_3\frac{(\nabla f_q\nabla\rho_q)}{f_q} + \\
+ b_4\rho_q\frac{\nabla^2 f_q}{f_q} + b_5 + \rho_q\left(\frac{\nabla f_q}{f_q}\right)^2 + b_6 h_m^2\rho_q\left(\frac{\mathbf{W}_q}{f_q}\right)^2,
\end{aligned} \tag{7}$$

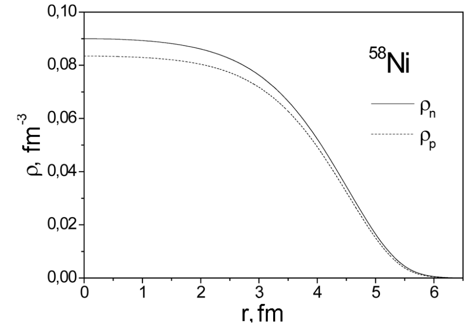
where the numerical coefficients are $b_1 = 1/36$, $b_2 =$ $= 1/3$, $b_3 = 1/6$, $b_4 = 1/6$, $b_5 = -1/12$, and $b_6 =$ $= 1/2$, $h_m = \hbar^2/2m$, the last summand in Eq. (7) is responsible for taking the spin-orbit interaction into account, the subscript q denotes the neutron ($q = n$) or proton ($q = p$) part,

$$\vec{W}_q = \frac{\delta\varepsilon(r)}{\delta\mathbf{J}_q(r)} = \frac{W_0}{2}\nabla(\rho + \rho_q), \tag{8}$$

f_q depends on the parameters of the Skyrme interaction as follows:

$$\begin{aligned}
f_q = 1 + \frac{2m}{\hbar^2}\left[\frac{1}{4}\left[t_1\left(1 + \frac{1}{2}x_1\right) + t_2\left(1 + \frac{1}{2}x_2\right)\right]\rho + \right. \\
\left. + \frac{1}{4}\left[t_2\left(x_2 + \frac{1}{2}\right) - t_1\left(x_1 + \frac{1}{2}\right)\right]\rho_q\right],
\end{aligned} \tag{9}$$

and the constant W_0 is related to the spin-orbit interaction and is determined by the choice of a specific Skyrme force parameterization. Note that the contribution of the term associated with the standard



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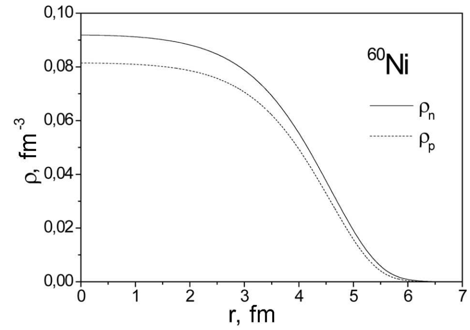

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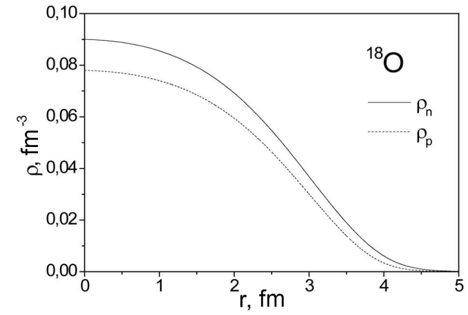

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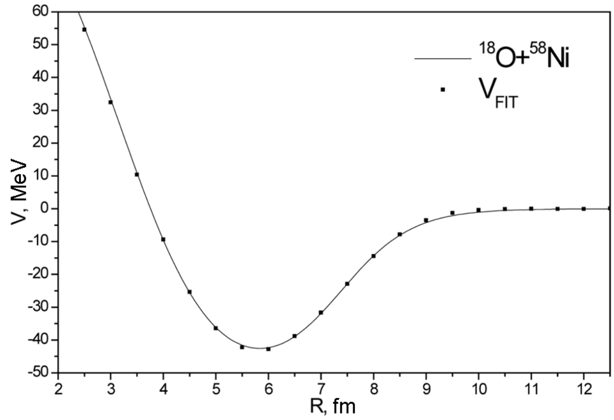
Fig. 1. Nucleon density distributions calculated for the ^{58}Ni (*a*), ^{60}Ni (*b*), and ^{18}O (*c*) nuclei in the framework of the modified Thomas–Fermi method



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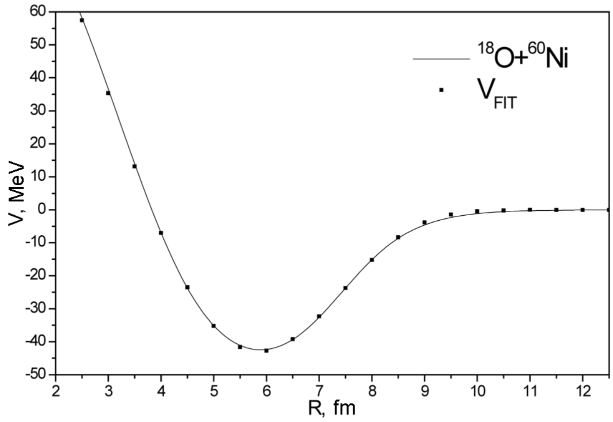

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Fig. 2. Interaction potentials for the reactions ^{18}O + ^{58}Ni (*a*) and ^{18}O + ^{60}Ni (*b*) calculated in the framework of the modified Thomas–Fermi method (solid curves) and the corresponding potentials in the analytical form (13) (symbols)

Thomas–Fermi method prevails, especially in the interior of the nucleus. However, at the nuclear surface, the gradient corrections also become substantial.

The aim of our work is to study the elastic scattering reactions for the ^{18}O + ^{58}Ni and ^{18}O + ^{60}Ni systems. For this purpose, we should calculate the nucleus-nucleus interaction potential in the framework of the modified Thomas–Fermi method. To carry out such calculations, the first step is to determine the nucleon density distributions of the nuclei. In our study, we applied the nucleon densities that were obtained using the modified Thomas–Fermi method itself with the Skyrme force approximation (SkP parameterization [32]). The nucleon density profiles for the nuclei ^{18}O, ^{58}Ni, and ^{60}Ni obtained using this approach are shown in Fig. 1.

In the second step, after calculating the nucleon densities, it is necessary to construct an expression for the energy density in order to use it for calculating the nucleus-nucleus interaction potential within the modified Thomas–Fermi method with Skyrme forces (1)–(9). Figure 2 shows the nuclear component of the interaction potentials for the reactions ^{18}O + ^{58}Ni and ^{18}O + ^{60}Ni. As one can see, the potentials are characterized by a realistic shape, with a pronounced repulsion at small distances.

3. Analytical Representation of the Interaction Potential

To simplify further calculations, it would be very useful to represent the obtained nucleus-nucleus interaction potentials in an analytical form. At the same time, we note that such an analytical representation must take into account the repulsion effect at short distances, which makes the traditional parameterization of potentials in the Woods–Saxon form unacceptable. Let us try to take into account the repulsion effect by adding another component to the analytical formula. This term should reproduce the form of the expression for the kinetic energy in the Thomas–Fermi method and create the repulsion effect at short distances. This approach is similar to what we used in Ref. [20] for double-convolution potentials, thus significantly improving the results obtained. Hence, the final formula for the analytical potential takes the following form:

$$V_{\text{FIT}}(R) = V_{\text{WS}}(R) + V_{\text{kin}}(R). \tag{10}$$

Here, $V_{\text{WS}}(R)$ is the widely used Woods–Saxon potential

$$V_{\text{WS}}(R) = \frac{-V_0}{1+\exp\frac{R-R_0}{d_0}}, \tag{11}$$

and $V_{\text{kin}}(R)$ is similar in form to the first term in the kinetic energy density of the Thomas–Fermi method, which is proportional to $\rho^{5/3}$; see Eq. (6). When writing down this kinetic term in terms of the density, we use the well-known Fermi distribution:

$$V_{\text{kin}}(R) = \left(\frac{V_c}{1+e^{(R-C)/a}}\right)^{5/3}. \tag{12}$$

Then the final expression for the analytical potential takes the form

$$V_{\text{FIT}}(R) = \frac{-V_0}{1+e^{\frac{R-R_0}{d_0}}} + \left(\frac{V_c}{1+e^{(R-C)/a}}\right)^{5/3}. \tag{13}$$

Formula (13) includes six fitting parameters: V_0, R_0, d_0, V_c, C, and a. Their values are determined using a minimization procedure to ensure the most accurate description of the potentials calculated by us earlier using the modified Thomas–Fermi method with density-dependent Skyrme forces. The parameter values of the analytical potential for the reactions ^{18}O + ^{58}Ni and ^{18}O + ^{60}Ni are quoted in Table 1.

The approximation results for the nuclear part of the interaction potentials in the modified Thomas–Fermi approach with Skyrme forces using expression (13) obtained for the reactions ^{18}O + ^{58}Ni and ^{18}O + ^{60}Ni are presented in Fig. 2. As one can see, the approximation accuracy turned out to be very high so that the discrepancies are practically not distinguishable in the plot. This fact indicates that the proposed analytical formula well reproduces the behavior of nucleus-nucleus interaction potentials with a repulsive core.

4. Calculations of Elastic Scattering Cross-Sections

The elastic scattering cross-sections will be calculated in the framework of the optical model. In this case, the previously defined nucleus-nucleus interaction potentials (13) with the corresponding parameters (see Table 1) will play the role of the real part of the potential, and the imaginary part of the potential will be considered in the following form [2, 4]:

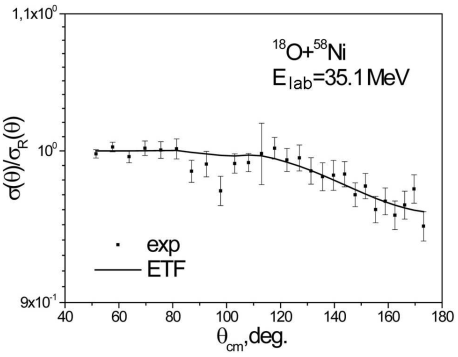
$$W(R) = -\frac{W_W}{1+\exp[R - r_W(A_1^{1/3}+A_2^{1/3})/d_W]} - \\ -\frac{W_S\exp[R - r_S(A_1^{1/3}+A_2^{1/3})/d_S]}{d_S\{1+\exp[R - r_S(A_1^{1/3}+A_2^{1/3})/d_S]\}^2}. \tag{14}$$

Here, the parameters W_W, r_W, d_W, W_S, r_S, and d_S represent the depth W, radius r, and diffuseness d for the bulk (W) and surface (S) components of the imaginary potential. This form of the imaginary potential component is widely used for the description of nuclear reactions.

The aim of our work is to study elastic scattering for the reactions ^{18}O + ^{58}Ni at the beam energies $E_{\text{lab}} = 35.1$, 46, and 63 MeV, and for the ^{18}O + ^{60}Ni reaction at the beam energies $E_{\text{lab}} = 34.5$ and 63 MeV. The elastic scattering cross-sections were calculated based on the potential (13), whose parame-

Table 1. **Parameter values for the analytical representation of the potential for the examined reactions**

Reaction	V_0, MeV	R_0, fm	d_0, fm	V_c, MeV$^{3/5}$	C, fm	a, fm
^{18}O + ^{58}Ni	53.4379	7.3264	0.6859	22.0837	3.7176	1.0858
^{18}O + ^{60}Ni	53.8914	7.3609	0.6957	22.1571	3.7884	1.0898



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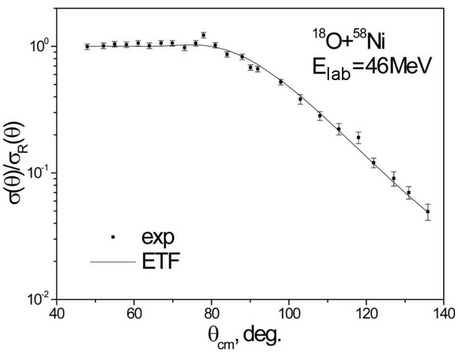

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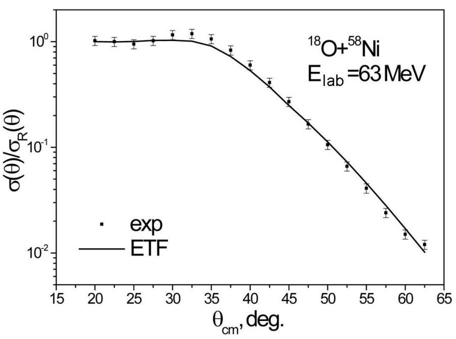

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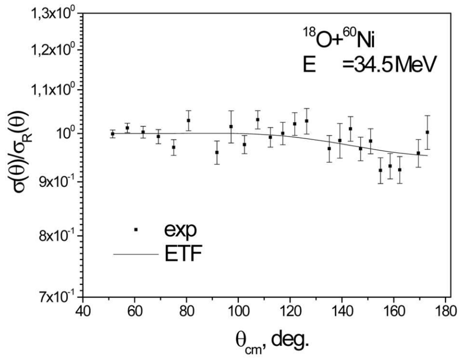
Fig. 3. Elastic differential scattering cross-sections for the ^{18}O + ^{58}Ni system at the beam energies $E_{\rm lab}$ = 35.1 (*a*), 46 (*b*), and 63 MeV (*c*) calculated in the modified Thomas–Fermi approximation with density-dependent Skyrme forces (solid curves). Experimental data (symbols) were taken from works [42–44]

ters are given in Table 1. This potential approximates the nucleus-nucleus interaction potential calculated using the modified Thomas–Fermi method. The parameters W_W, r_W, d_W, W_S, r_S, and d_S of the imaginary part were determined by fitting the calculated elastic scattering cross-sections to the corre-



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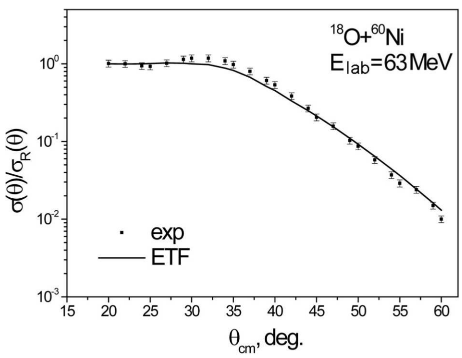

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Fig. 4. Elastic differential scattering cross-sections for the ^{18}O + ^{60}Ni system at the beam energies $E_{\rm lab}$ = 34.5 (*a*) and 63 MeV (*b*) calculated in the modified Thomas–Fermi approximation with density-dependent Skyrme forces (solid curves). Experimental data (symbols) were taken from works [42, 44]

Table 2. **Parameter values for the imaginary part of the potential (14) for the reaction $^{18}\mathbf{O} + {}^{58}\mathbf{Ni}$**

Parameter	W_w, MeV	r_w, fm	d_w, fm	W_s, MeV	r_s, fm	d_s, fm
$E_{\rm lab}$ = 35.1 MeV	20.129	1.183	0.300	4.000	1.150	0.756
$E_{\rm lab}$ = 46 MeV	20.342	1.199	0.495	8.374	1.150	0.754
$E_{\rm lab}$ = 63 MeV	24.446	1.198	0.337	19.913	1.449	0.899

Table 3. **Parameter values for the imaginary part of the potential (14) for the reaction $^{18}\mathbf{O} + {}^{60}\mathbf{Ni}$**

Parameter	W_w, MeV	r_w, fm	d_w, fm	W_s, MeV	r_s, fm	d_s, fm
$E_{\rm lab}$ = 34.5 MeV	20.665	1.199	0.499	4.011	1.194	0.700
$E_{\rm lab}$ = 63 MeV	24.133	1.141	0.494	20.499	1.449	0.899

sponding experimental data. The specific values obtained for those parameters are quoted in Tables 2 and 3.

In Figs 3 and 4, the calculation results are presented for the elastic scattering cross-sections in the $^{18}\mathrm{O} + ^{58}\mathrm{Ni}$ reaction at the beam energies $E_{\mathrm{lab}} = 35.1$, 46, and 63 MeV, and in the $^{18}\mathrm{O} + ^{60}\mathrm{Ni}$ reaction at the energies $E_{\mathrm{lab}} = 34.5$ and 63 MeV. The calculated cross-sections are normalized by the Rutherford elastic differential cross-section. For comparison, experimental data from references [42–44] are also depicted. As one can see, the calculated cross-sections describe the experimental data well.

5. Conclusions

In our study, the nucleus-nucleus interaction potentials have been calculated using the modified Thomas–Fermi method with the application of density-dependent Skyrme forces (the SkP parameterization) for the systems $^{18}\mathrm{O} + ^{58}\mathrm{Ni}$ and $^{18}\mathrm{O} + ^{60}\mathrm{Ni}$. Note that the nucleon densities were also calculated in the framework of this approach. The obtained potentials demonstrate quite a realistic behavior, revealing a substantial repulsion at short distances; this fact plays an important role in the study of elastic scattering processes.

A successful analytical parameterization of the nuclear part of the nucleus-nucleus interaction potential has been proposed, which accurately reproduces the behavior of the potentials calculated in the framework of the modified Thomas–Fermi method with Skyrme forces.

Using the obtained nucleus-nucleus interaction potentials, elastic scattering for the reactions $^{18}\mathrm{O} + ^{58}\mathrm{Ni}$ and $^{18}\mathrm{O} + ^{60}\mathrm{Ni}$ at various energies has been analyzed. Elastic scattering cross-sections were calculated. For each reaction, the same expression for the real part of the potential was used, and only the imaginary part was varied. The obtained results demonstrate a good agreement with experimental data.

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О.І. Давидовська, В.О. Нестеров

ПЕРЕРІЗИ ПРУЖНОГО РОЗСІЯННЯ ДЛЯ РЕАКЦІЙ $^{18}\mathrm{O}+{}^{58}\mathrm{Ni}$ ТА $^{18}\mathrm{O}+{}^{60}\mathrm{Ni}$, ОДЕРЖАНІ НА ОСНОВІ ПОТЕНЦІАЛУ МОДИФІКОВАНОГО МЕТОДУ ТОМАСА–ФЕРМІ З УРАХУВАННЯМ КОРУ

Розподіли густини нуклонів і потенціали взаємодії ядер для реакцій $^{18}\mathrm{O}+{}^{58}\mathrm{Ni}$ і $^{18}\mathrm{O}+{}^{60}\mathrm{Ni}$ обчислено за допомогою модифікованого методу Томаса–Фермі, з урахуванням усіх членів другого порядку за степенями $\hbar$ у квазікласичному розкладанні кінетичної енергії. Для опису нуклон-нуклонної взаємодії використовувалися сили Скірма, залежні від густини нуклонів. Запропоновано аналітичну параметризацію, що добре відтворює поведінку потенціалів ядерно-ядерної взаємодії, розрахованих за допомогою модифікованого методу Томаса–Фермі із залежними від густини силами Скірма. Використовуючи отримані потенціали, ми обчислили перерізи пружного розсіяння, які добре узгоджуються з експериментальними результатами.

Ключові слова: ядерно-ядерний потенціал, модифікований метод Томаса–Фермі, сили Скірма, пружне розсіяння, поперечні перерізи.