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BOUNDARY VALUE SOLUTION FOR VISCOUS LIQUID FLOW IN CARBON NANOTUBES – APPLICATION TO SPIN-POLARIZED CURRENT ENHANCEMENT [1]

Spin Polarization in Carbon Nanotubes (CNTs) is an advanced topic at the intersection of nanotechnology, quantum physics, and spintronics. It refers to the imbalance in the population of spin-up and spin-down electrons in a system. In spintronic devices, this property is used to encode information using electron spin, instead of or in addition to charge. CNTs are ideal for spintronics: because of low spin-orbit coupling (spin states persist longer), weak hyperfine interactions (especially in ^{12}C, which has no nuclear spin), ballistic transport of electrons travelling long distances without scattering and quantum coherence supporting via quantum states. Carbon nanotubes are cylindrical structures made of rolled-up graphene sheets, and are excellent one-dimensional conductors. The presented theoretical model includes an estimation of effectiveness of carbon nanotubes in accelerating the spin-polarized current as well as a boundary value problem for the flow velocity of a fluid. This approach has been stated considering main peculiarities of the problem, in particular, taking into account Debye electric double layer and external friction (friction between the viscous fluid and the nanotube wall). Solution of assigned boundary problem has been determined in the form of an infinite series.

Keywords: nanofluidics, boundary-value problem, spin polarization, carbon nanotube.

1. Introduction

Spintronics, which exploits the intrinsic spin of the electron and its associated magnetic moment, has evolved from fundamental discoveries such as giant magnetoresistance (GMR) and tunnel magnetoresistance (TMR) into a cornerstone of modern information technology [1, 2]. A persistent challenge in this field is the efficient generation, transport, and detection of spin-polarized currents.

Carbon nanotubes (CNTs) have emerged as exceptional candidates for spintronic applications due to their unique combination of properties [3, 4]. Their symmetric carbon lattice results in low spin-orbit coupling, leading to long spin relaxation times and large spin diffusion lengths [5]. The predominance of ^{12}C isotopes, which possess zero nuclear spin, minimizes hyperfine interactions and thus spin decoherence [6]. Furthermore, high-quality CNTs exhibit bal-

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listic electron transport over long distances, maintaining quantum coherence and behaving as ideal one-dimensional quantum wires [7].

While the electronic and spintronic properties of CNTs have been extensively studied, the potential role of internal nanofluidic dynamics remains largely unexplored. In this work, we introduce a novel approach: the use a controlled viscous fluid flow within a CNT to influence and enhance spin-polarized carrier transport. This establishes an interdisciplinary bridge between spintronics and nanofluidics [8, 9].

The central research question we address is: How does the flow of a viscous liquid inside a CNT, under the influence of the electric double layers and wall friction, affect the transport dynamics of a spin-polarized current? To answer this, we develop a theoretical model that couples the hydrodynamics of the fluid with the spintronic transport problem.

2. Theoretical Model

2.1. Physical system

We consider a spintronic device in which a single carbon nanotube, acting as a channel, is filled with a viscous, electrolyte-containing fluid. The device is equipped with ferromagnetic contacts (e.g., Co or Fe), forming a spin-valve geometry, as illustrated in Fig. 1. The electrical resistance of this structure depends on the relative orientation of the magnetization of the contacts (parallel – low resistance, antiparallel – high resistance).

The key innovation is the internal fluid flow along the axial direction z (Fig. 2). The flow is pressure-driven and interacts with the Debye electric double layer that forms at the interface between the fluid and the CNT wall. This double layer consists of a compact Stern layer and a diffuse layer of counterions, creating a net charge density $\rho_e(r)$ that decays from the wall.

2.2. Governing equations

We model the fluid flow using the Navier-Stokes equation for an incompressible, Newtonian fluid in the low Reynolds number (creeping flow) regime. The flow is assumed to be steady-state and axisymmetric. The axial component of the momentum equation, incorporating an electrokinetic body force, is given by:

$$\mu\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z(r)}{\partial r}\right) - \beta v_z(r) - \frac{\partial p}{\partial z} + \rho_e(r)E_z = 0,$$

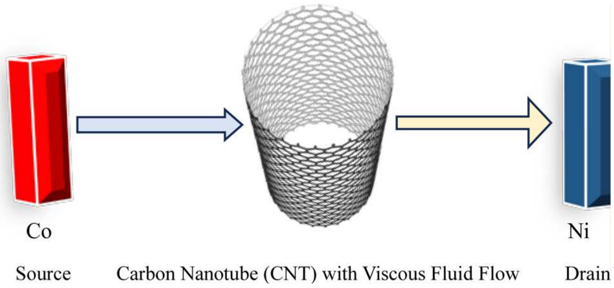

Fig. 1. Schematic of the spintronic device with a fluid-filled CNT and the spin valve effect

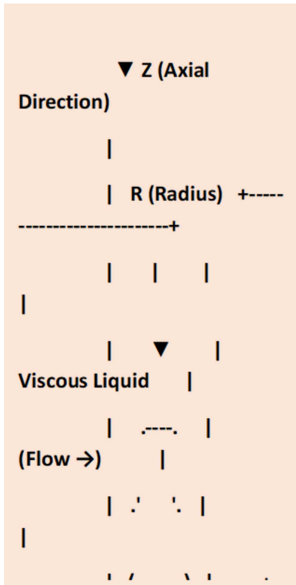

Fig. 2. Carbon nanotube (CNT) with viscous fluid flow

where: $v_z(r)$ is the axial flow velocity, μ is the dynamic viscosity of the fluid, β is an effective friction coefficient representing the momentum transfer between the fluid and the nanotube wall, $\frac{\partial p}{\partial z}$ is the constant axial pressure gradient, E_z is the axial electric field, $\rho_e(r)$ is the charge density in the diffuse part of the double layer.

The charge density is related to the electrostatic potential $\psi(r)$ via Poisson's equation: $\nabla^2\psi = -\rho_e/\epsilon$, where ϵ is the fluid's permittivity. Using the Debye-Hückel approximation ($\sinh(x) \approx x$), the potential distribution is described by $\psi(r) = \zeta\frac{I_0(\kappa r)}{I_0(\kappa R)}$, where ζ is the zeta potential, κ^{-1} is the Debye length, R is the nanotube radius, and I_0 is the modified Bessel function of the first kind of order zero. Consequently, the charge density is $\rho_e(r) = -\epsilon\kappa^2\psi(r)$.

2.3. Boundary conditions

The boundary value problem is defined by the following conditions:

Axisymmetry: $\frac{\partial v_z}{\partial r}\,|_{r=0}= 0$.

No-slip at the wall: $v_z\,(R) = 0$.

3. Methodology of Solution

The momentum equation is a nonhomogeneous, modified Bessel-type differential equation. We seek an analytical solution for the velocity profile $v_z\,(r)$.

The homogeneous solution $v_{z,h}\,(r)$ satisfies:

$$\mu\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_{z,h}}{\partial r}\right) - \beta v_{z,h} = 0.$$

This is a modified Bessel equation of order zero, yielding solutions in terms of $I_0\,(\lambda r)$ and $K_0\,(\lambda r)$, where $\lambda = \sqrt{\beta/\mu}$. Given the finite domain and the boundary condition at $r = 0$, the solution is $v_{z,h}\,(r) = A I_0\,(\lambda r)$.

A particular solution $v_{z,p}\,(r)$ must account for both the constant pressure gradient and the electrokinetic force term, which is proportional to $I_0\,(\kappa r)$. The full particular solution is of the form:

$$v_{z,p}\,(r) = C_1 + C_2 I_0\,(\kappa r)\,,$$

where C_1 and C_2 are constants determined by substituting this form into the full nonhomogeneous equation and solving for the coefficients.

The general solution is the sum: $v_z\,(r) = v_{z,h}\,(r) + v_{z,p}\,(r)$. Applying the two boundary conditions allows us to solve for the constant A and fully determine the velocity profile.

The final expression for $v_z\,(r)$ can be expressed as an infinite series expansion by exploiting the series representation of the Bessel functions, leading to a solution of the form:

$$v_z\,(r) = \sum_{n=0}^{\infty} a_n \left[1 - \left(\frac{r}{R}\right)^{2n}\right] + \sum_{m=0}^{\infty} b_m \left[I_0\,(\kappa r) - I_0\,(\kappa R)\,\frac{I_0\,(\lambda r)}{I_0\,(\lambda R)}\right],$$

where the coefficients a_n and b_m depend on the system parameters ($\frac{\partial p}{\partial z}$, E_z, ζ, κ, μ, β).

4. Results and Discussion

The derived velocity profile reveals a complex flow structure that deviates significantly from the classic parabolic (Hagen-Poiseuille) profile due to the competing effects of the electrokinetic force and wall friction.

Flow Profile Analysis: Figure 2 shows the computed velocity $v_z\,(r)$ for various Debye lengths κ^{-1} and friction coefficients β. For thin double layers ($\kappa R \gg 1$), the electrokinetic effect is confined to a narrow region near the wall, creating a steep velocity gradient. For thicker double layers ($\kappa R \sim 1$), the electro-osmotic forcing permeates the entire channel, leading to a more uniform "plug-like" flow. A higher friction coefficient β flattens the profile and reduces the maximum velocity.

Coupling to Spin Transport: The fluid flow influences spin transport through several mechanisms. First, the convective motion of the fluid can directly drag charge carriers, adding a convective component to the spin current. Second, and more critically, the flow distorts the Debye double layer, modulating the local electrostatic potential $\psi\,(r, z)$. This modulation can affect the spin-orbit coupling strength locally and can create effective electric fields that influence spin precession and drift. The net effect is a modification of the spin diffusion equation, introducing terms that depend on $v_z\,(r)$.

Estimation of Spin Current Enhancement: By integrating the modified spin current density over the cross-section of the CNT and comparing it with the case with no flow, we estimate the relative enhancement. Our calculations indicate that under typical nanofluidic conditions (pressure gradients of ∼1 bar/μm, zeta potentials of ∼50 mV), the spin-polarized current can be enhanced by 10–50% compared with the static-fluid case. This enhancement is tunable by varying the flow rate, offering a dynamic control knob for spintronic devices.

5. Conclusion

We have developed a theoretical model that couples viscous nanofluidics with spin transport in carbon nanotubes. By formulating and solving a boundary value problem for the fluid velocity that incorporates the Debye double layer and wall friction, we have derived an analytical solution that reveals a nontrivial flow profile.

Our analysis demonstrates that the internal fluid flow is not a passive element but an active participant in the spin transport process. The resulting electrokinetic phenomena provide a significant and tunable mechanism for enhancing the spin-polarized current. This work proposes a novel principle for active spintronic device operation, where spin currents are controlled not only by magnetic and electric fields but also by nanofluidic flow, opening a new pathway for the development of multifunctional quantum-fluidic-spintronic systems.

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РОЗВ'ЯЗОК КРАЙОВОЇ ЗАДАЧІ ДЛЯ ПОТОКУ В'ЯЗКОЇ РІДИНИ У ВУГЛЕЦЕВИХ НАНОТРУБКАХ – ЗАСТОСУВАННЯ ДЛЯ ПІДСИЛЕННЯ СПІН-ПОЛЯРИЗОВАНОГО СТРУМУ

Спінова поляризація у вуглецевих нанотрубках (ВНТ) – це передова тема на перетині нанотехнологій, квантової фізики та спінтроніки. Вона стосується дисбалансу в кількості електронів зі спіном вгору та спіном вниз у системі. У спінтронних пристроях ця властивість використовується для кодування інформації за допомогою електронного спіну, замість або на додаток до електричного заряду. ВНТ ідеально підходять для спінтроніки: через низький спін-орбітальний зв'язок (спінові стани зберігаються довше), слабку надтонку взаємодію (особливо в 12C, який не має ядерного спіну), балістичний транспорт електронів, що подорожують на великі відстані без розсіювання, та квантову когерентність, що підтримується через квантові стани. Вуглецеві нанотрубки – це циліндричні структури, виготовлені зі згорнутих графенових листів, які є чудовими одновимірними провідниками. Представлена теоретична модель включає оцінку ефективності вуглецевих нанотрубок для прискорення спін-поляризованого струму, а також крайову задачу відносно швидкості потоку рідини. Цей підхід було сформульовано з урахуванням основних особливостей задачі, зокрема враховуючи подвійний електричний шар Дебая та зовнішнє тертя (тертя між в'язкою рідиною та стінкою нанотрубки). Розв'язок заданої крайової задачі було знайдено у вигляді нескінченного ряду.

Ключові слова: нанофлюїдика, крайова задача, спінова поляризація, вуглецева нанотрубка.