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NEW UNDERSTANDING OF THE PHASE DIAGRAM OF QCD[1]

The phase diagram of QCD at finite temperature is understood using large N_c methods. For zero baryon density and for temperatures below $T \leq 160$ MeV, the three-dimensional string model is shown to describe the thermodynamics of QCD, as well as the spectrum of mesons and glueballs in vacuum. This description involves no undetermined parameters. It is argued that there are at least three phases at zero baryon number density characterized by the N_c dependence of extensive thermodynamic quantities. At high baryon number density and low temperature, there are again three phases. One of these phases, the quarkyonic phase, with energy density of order N_c, is distinguished from its counterpart at low baryon density and temperature by its chiral properties.

Keywords: phase diagram of QCD, N_c methods, baryon density, spectrum of mesons and glueballs.

1. Introduction

This talk presents the results of work done in Refs. [1, 2]. The scientific questions addressed in this presentation are:

1. Is there a phase of matter between the quark gluon plasma and the hadron gas?

2. Can one quantitatively describe the thermodynamics in the hadron phase, which at zero baryon density is for temperatures $T \leq 160$ MeV?

3. What is the physics beyond the hadron phase?

This work is partly motivated by some observations of Cohen and Glozman [3], and earlier observations by myself and Rob Pisarski [4].

In Ref. [1], we argued that there were three phases of QCD at finite temperature and zero baryon number density. This was first argued using large N_c, where N_c is the number of quark colors, and then was followed by a computation of the thermodynamic quantities using a three-dimensional string theory. The three-dimensional string theory has only one parameter which is the string tension, and this is well constrained phenomenologically. The string model provides an excellent description of lattice data concerning thermodynamic quantities [5, 6]. In Ref. [2], we refined the quantitative evaluation of thermodynamic quantities from 3 dimensional string theory and verified that our results provided an excellent determination of thermodynamic quantities as measured on the lattice, and also provided an excellent description of the integrated mass spectrum of mesons and glueballs.

In the Hagedorn resonance gas description of hadrons, there is a limiting temperature T_H. For temperatures beyond the Hagedorn temperature, the integration over possible states that contribute to thermal quantities diverges. However, before such a temperature is reached, interactions of mesons and of glueballs become strong, and the Hagedorn description loses its applicability [7]. In the large N_c limit, the amplitude for the interaction of mesons with mesons has strength of oder $1/N_c$, and glueballs with glueballs is of order $1/N_c^2$. A measure of interaction strength for thermodynamic quantities is the particle density squared times the interaction amplitude and when this quantity is of order the density, the interactions are strong. This occurs for mesons when the density of mesons is of order N_c and for gluons when the density is of order N_c^2. The den-

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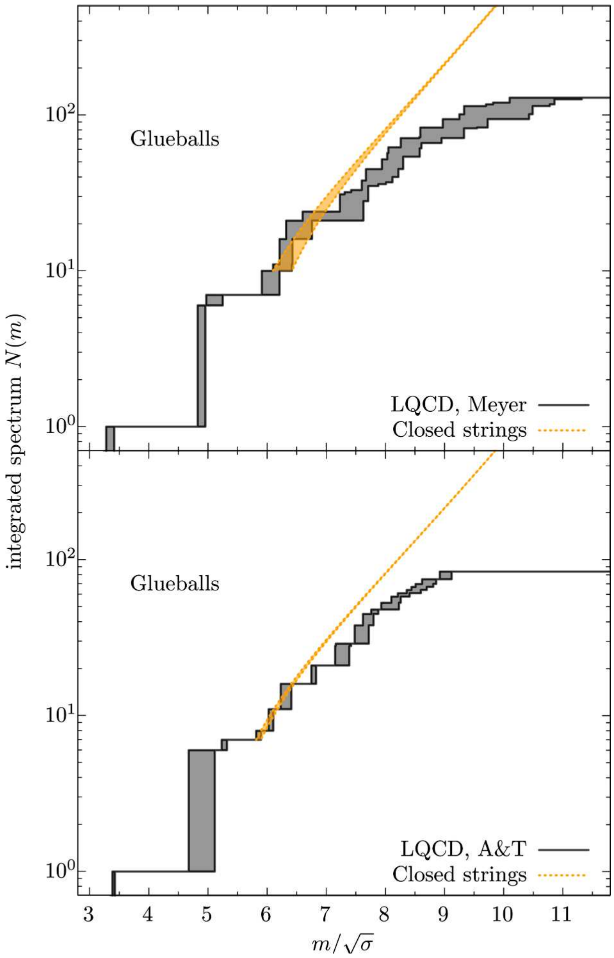

Fig. 1. Continuum-extrapolated cumulative mass spectra of glueballs from LQCD simulations (black, solid bands). The spectra are taken from Ref. [8] (top panel) and [9] (bottom panel) and are depicted in units of the string tension $\sqrt{\sigma}$. The bands represent the uncertainties of the continuum-limit extrapolation. The spectra for closed strings are shown as orange, dashed bands. Their uncertainties come from the uncertainties of the continuum-limit extrapolation of the mass of the lightest resonance in the LQCD spectra (see text). Figure taken from Ref. [2]

sity of a gas of quarks is of order N_c and that of gluons is of order N_c^2. This very simple argument suggests that at finite temperature, there are three phases, a hadron gas with extensive thermal quantities parametrically of order unity, a quark phase with confined gluons where thermal quantities are of order N_c, and a third phase where the energy of quark and gluons where the energy density is of order N_c^2.

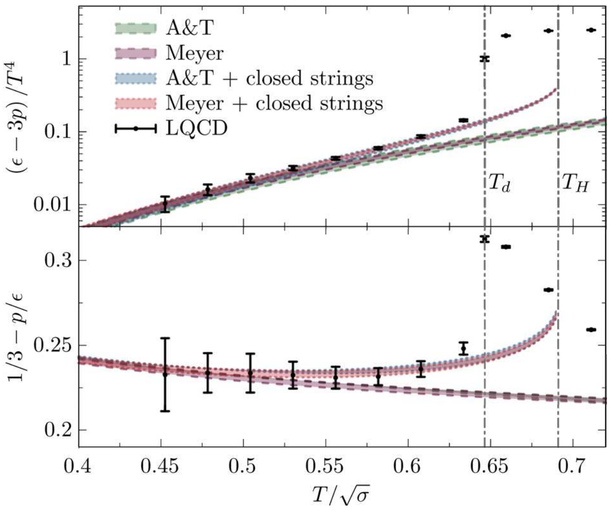

Fig. 2. Trace anomaly $(\epsilon - 3p)/T^4$ (top panel) and energy-density-normalized trace anomaly $1/3 - p/\epsilon$ (bottom panel). The SU(3) pure gauge LQCD data on pressure and energy density are taken from Ref. [10]. The uncertainty bands in both panels are obtained by propagating the reported errors on pressure and energy density. The vertical lines $T_d/\sqrt{\sigma} = 0.646$ and $T_H/\sqrt{\sigma} = 0.691$ represent the critical deconfinement and Hagedorn temperatures, respectively. Note that the closed strings are shown only up to T_H (see text for details). Figure taken from Ref. [2]

The Hagedorn temperature T_H for the three spatial dimension string is determined in terms of the string tension σ by $T_H = \sqrt{3\sigma/2\pi}$. The string tension is $\sigma \sim 440$ MeV, corresponding to a Hagedorn temperature of $T_H \sim 300$ MeV. We repeated the analysis of Meyer in Ref. [11] for the spectrum of glueball states and thermodynamics of pure QCD determined by lattice gauge theory computations. These results are shown in Figs. 1 and 2. The spectrum computation is slightly above the lattice data for glueball masses $M_{\text{glueball}} \geq 7\sqrt{\sigma}$, but this is where glueballs begin to become unstable with respect to decay into two glueballs, and a lattice extraction of glueball states is expected to miss such excited states. The lattice data are shown for the trace anomaly, from which one can construct the pressure, energy density, and entropy. The data is for pure glue theory for $N_c = 3$. For finite N_c, the deconfinement temperature, T_d, is slightly below the Hagedorn temperature, but approaches it in the large N_c limit. The glueball states are taken to be closed strings. The agreement between the string theory and the lattice thermodynamic quantities is remarkably good.

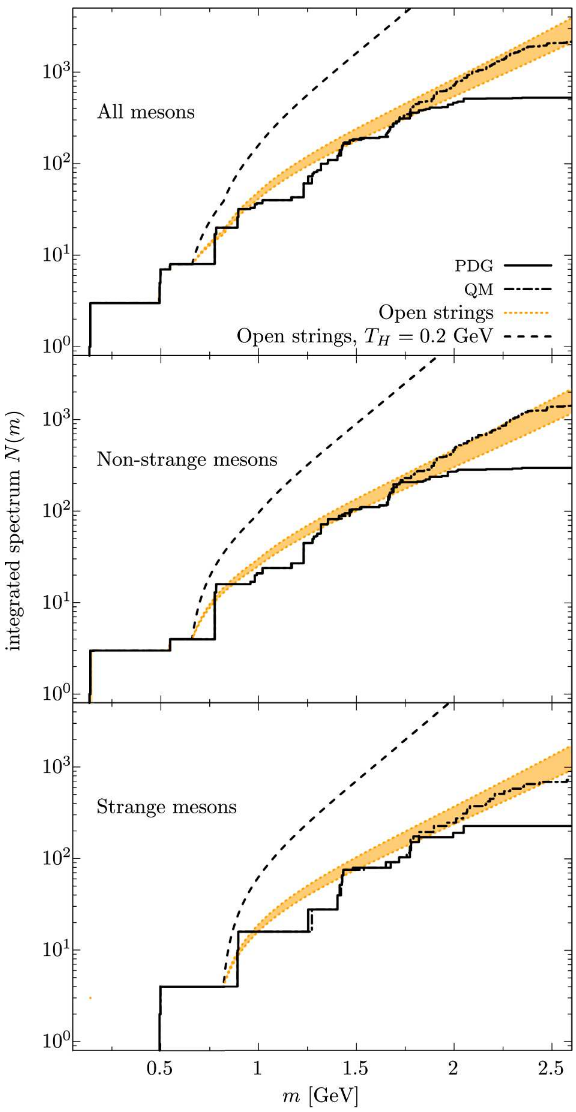

Fig. 3. Cumulative mass spectra of all (top), non-strange (middle), and strange (bottom) mesons in the PDG (black, solid lines). Also shown are spectra that include the predictions from the quark model [12, 13] (QM) (black, dash-dotted lines). The yellow bands represent the uncertainty in the Hagedorn limiting temperature in the exponential spectrum (see text for details). We note that $f(500)$ and $\kappa_0^\star(700)$ mesons are not included in the discrete spectra due to their ambiguous nature [14, 15]. The black, dashed lines show spectra obtained for $T_H = 0.2$ GeV. Figure taken from Ref. [2]

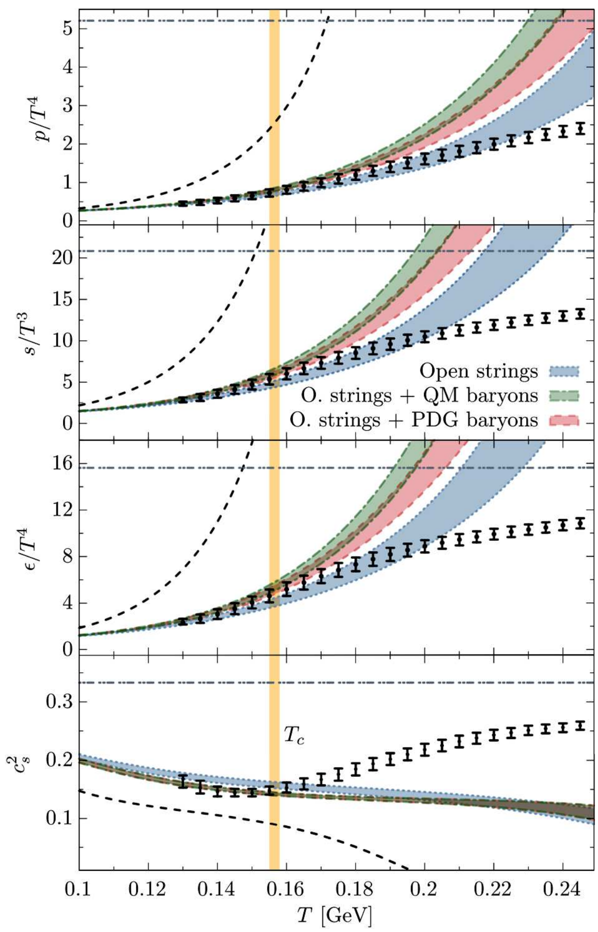

Fig. 4. The equation of state at finite temperature and vanishing chemical potential. The LQCD results are taken from Ref. [16]. Mesons are modeled via the continuous spectrum of open strings, and baryons are taken as discrete states from the Particle Data Group (PDG) or quark model (QM). The blue, red, and green bands indicate the uncertainty in the string tension (see text). The yellow vertical band marks the estimation of the pseudo-critical temperature for the chiral crossover transition $T_c = 156.5 \pm 1.5$ MeV [17]. The gray, horizontal, doubly-dotted–dashed lines mark the Stefan–Boltzmann limit considering quarks of 3 flavors and gluons. The black, dashed lines show results for open strings with $T_H = 0.2$ GeV. Figure taken from Ref. [2]

We also took the open string formula to describe mesons with $T_H \sim 300$ MeV. The results for the meson spectrum and for the thermodynamic quantities are shown (see Figs. 3 and 4). The comparison of the open string formula with the measured meson spectrum is excellent. This is somewhat surprising, since the conventional wisdom about the Hagedorn temperature is that it is near 150 MeV. The three-dimensional string model works well because of mass-dependent prefactors to the expnonential mass dependence of the density of states. The comparison of the string computation with the pressure, entropy density, energy density and sound speed also shows excellent agreement with lattice data for temperatures below $T \leq 156$ MeV. At this temperature, the lattice gauge theory computations indicate that there is a chiral transition. It is here that the measure of the number of degrees of freedom σ/T^3, where here σ is the entropy density, becomes of order N_c and that mesons begin to strongly interact with one another. This is where the string theory computation should break down and this is the transition to this region.

We also computed the thermal expectation value of the chiral order parameter, $\langle\overline{\psi}\psi\rangle$, as a function of temperature [1]. This computation was done by differentiating the pressure with respect to the quark mass, and one needs to know the meson mass dependence upon quark mass. We took this to be a constant to fit the data. With this one parameter fit, we describe the lattice data very accurately, as shown in Fig. 5.

These computations show the remarkable agreement between the three-dimension string computations and the lattice data on thermodynamics and glueball spectra, and the measured meson spectrum for a Hagedorn temperature $T_H \sim 300$ MeV. Below a temperature of $T \sim 160$ MeV, one has a chirally broken hadron gas. For $T \geq 300$ MeV, the system is a deconfined gas of gluons and quarks, that is a quark-gluon plasma. In the intermediate region, the number of degrees of freedom is that of quarks, but the system is still confining since the glueballs remain bound. In the following paragraphs we will argue that this intermediate region may have confining-like properties as well as properties of a deconfined system. We might think of the quarks as confined strings, but with contribution to the thermodynamic quantities typical of a deconfined system. The gluons are all combined into glueballs. Because of the large glueball mass, the glueballs make only very small contributions. This is seen by computing the σ/T^3 for gluons and quarks. As seen in Fig. 8, the gluon contribution relative to that of quarks is very small. We call this new phase the spaghetti of quarks with glueballs or SQGB.

The expectation value of the Wilson line gives the free energy of an isolated test quark,

$$\langle L\rangle = e^{-F_q/T}. \quad (1)$$

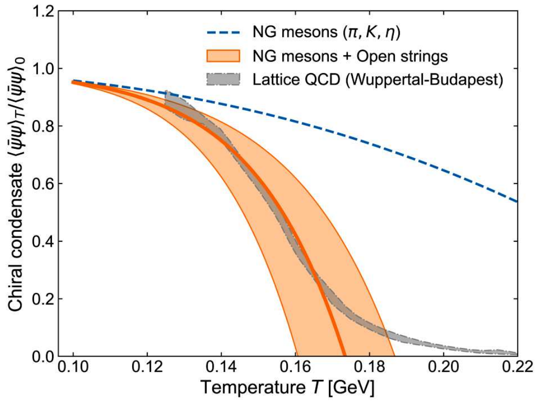

Fig. 5. Ratio of the chiral condensate at finite temperature to its value at zero temperature. The lattice data are taken from Ref. [18]. Figure taken from Ref. [1]

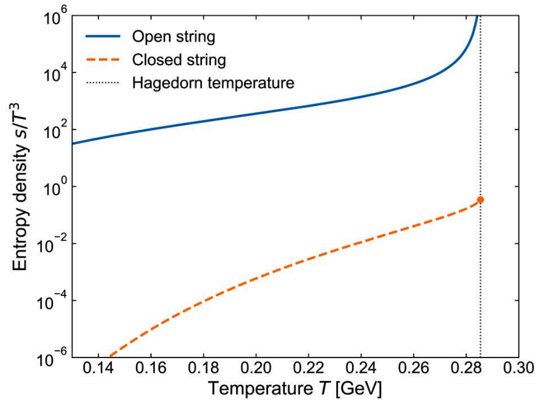

Fig. 6. Comparison between the entropy densities from an open-string gas (drawn as the solid curve) and a closed-string gas (drawn by the dashed curve) up to the Hagedorn temperature of $T = 285$ MeV. Figure taken from Ref. [1]

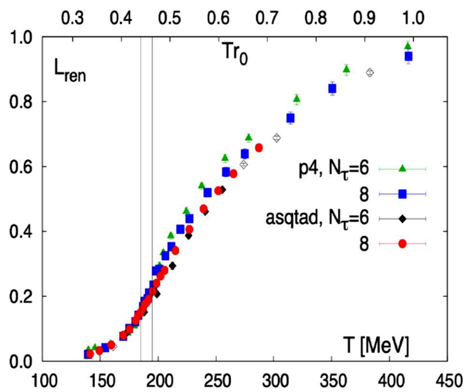

Fig. 7. The expectation value of the Wilson line which measures the exponential of the free energy of an isolated quark, $e^{-\beta F_Q}$ as a function of temperature, taken from Karsch *et al.*, see Ref. [16]

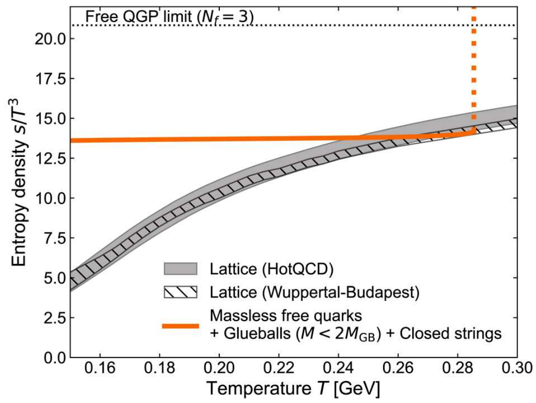

Fig. 8. Comparison of the entropy density between the lattice-QCD data and the gas of massless, free quarks and light glueballs, as well as heavy glueballs that obey the closed-string spectrum. Figure taken from Ref. [1]

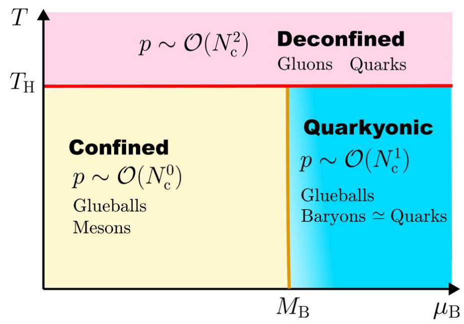

Fig. 9. Quarkyonic phase diagram in the strict large-N_c limit. Figure is taken from Ref. [1]

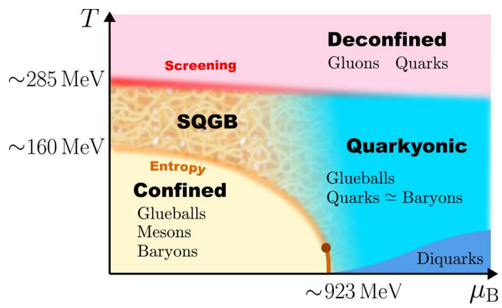

Fig. 10. New and realistic phase diagram for $N_c = 3$, including a window of the SQGB regime. Figure is taken from Ref. [1]

In the large N_c limit or in theories with no dynamical quarks, this quantity is zero in the confined phase and non-zero in the deconfined phase. It has been measured on the lattice [16] for an SU(3) gauge theory with dynamical quarks. It is very small at the chiral temperature and of order unity at the Hagedorn temperature. Therefore we should most likely think of the system as being approximately confined just above the chiral transition, making a transition to a more deconfining-like state near the Hagedorn temperature.

This interpretation is reinforced by measurements of the entropy σ/T^3, as shown in Fig. 8. In this figure, σ/T^3 is compared to that of a massless ideal gas of quarks. Glue is not important below the Hagedorn temperature, because until the Hagedorn temperature gluons are confined into glueballs and glueballs are much heavier than the temperature. It is seen that above $T = 160$ MeV, this quantity is of the correct magnitude to be described by quark degrees of freedom, but above 240 MeV the computed σ/T^3 from the lattice agrees well with that of an ideal massless quark gas..

The value of the Debye screening mass is taken as a measure of confinement. When the Debye mass is less than the QCD scale $M_D < \Lambda_{\rm QCD}$, screening occurs at too large a distance scale to cutoff the effect of confinement. In the large N_c limit, the Debye mass for a quark–gluon plasma is (N_f is the number of quark flavors)

$$M_D^2 \sim \alpha_{'\rm tHooft} \left(\frac{1}{3} + +\frac{N_f}{6N_c}\right). \qquad (2)$$

The 't Hooft coupling is $g^2_{'\rm tHooft} = g^2 N_c$, which is held finite in the large N_c limit. Because the intermediate phase is made of quarks, in the strict large N_c the Debye mass vanishes and the system is confined until gluons appear. This occurs at the Hagedorn temperature. For realistic N_f and N_c where Debye screening sets in is not so well determined, and as N_f increases, it will appear at lower temperature, until presumably at some large N_f, the chiral and deconfinement temperature will become the same. Of course, the entire concept of deconfinement at finite temperature is even qualitatively ill-defined at large N_f, so these concepts become somewhat fuzzy. For the case of QCD, this is presumably at some temperature between the chiral and Hagedorn temperature. Since the distinction between confinement and deconfinement cannot be rigorously given, we imagine a gradual change in the confining properties of matter at $160 \le T \le 300$ MeV, where the system changes from being better described as an interacting meson gas to a gas of quarks.

At zero temperature and finite baryon density, it was argued that a similar type of phase to the SQGB occurs [4]. At zero temperature there are no gluons. The Debye mass therefore is of order $M_D^2 \sim \sim \alpha_{\text{'tHooft}}\, \mu_Q^2$ where μ_Q is the quark baryon chemical potential. Therefore, the Debye screening mass cannot exceed Λ_{QCD} until the chemical potential $\mu_Q \geq \sqrt{N_c}\Lambda_{\text{QCD}}$. The system is confined until a momentum scale that is parametrically large compared to the QCD scale. At this scale, we would expect interactions to be weakly coupled. But this is not true at the Fermi surface, where interactions are sensitive to small momentum scales or long distances. At high densities, the system is a Fermi sphere of quarks surrounded by a thin shell of confined baryons with mesonic and glueball excitations.

At the chemical potential $\mu_B = M_N$, $(\mu_B = N_c\mu_Q)$ baryons first appear at zero temperature. At finite temperature they might also appear at $\mu_B \leq M_N$. However, since $e^{-M_N/T} \sim e^{-N_c\,\Lambda_{\text{QCD}}/T}$, at large N_c, the matter with chemical potential $\mu_B < M_n$ is baryon free and has an energy density which is parametrically of order unity in powers of N_c. In the range $\sqrt{N_c}\,\Lambda_{\text{QCD}} \geq \mu_Q \geq \Lambda_{\text{QCD}}$, the energy density is parametrically of order N_c in powers of N_c, and is confined. For $\mu_Q \geq \Lambda_{\text{QCD}}$, the system becomes deconfined. We call this intermediate phase quarkyonic. Due to effects at the Fermi surface, quarkyonic matter may break chiral symmetry [19].

The phase diagram we expect in the strict large N_c limit is shown in Fig. 9. The chiral temperature and the deconfinement temperature are both at the Hagedorn temperature in this limit. A figure with realistic parameters of QCD is shown in Fig. 10. Note that the quarkyonic phase and the SQGB are more or less continuously connected. There is probably a chiral symmetry breaking line of transitions that separates the SQGB from quarkyonic matter [19]. Also, at very high densities there is color superconductivity [20, 21].

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НОВЕ РОЗУМІННЯ ФАЗОВОЇ ДІАГРАМИ КХД

Фазова діаграма КХД при скінченних температурах стає зрозумілою за допомогою методів великих N_c. Для нульової баріонної густини та для температур нижче $T \leq 160$ МеВ показано, що тривимірна струнна модель описує термодинаміку КХД, а також спектр мезонів та глюболів у вакуумі. Запропонований опис не містить невизначених параметрів. Стверджується, що при нульовій баріонній густині існують щонайменше три фази, що характеризуються залежністю від N_c від екстенсивних термодинамічних величин. За високої баріонної густини та низької температури знову ж таки існують три фази. Одна з цих фаз, кваркіонна фаза, з густиною енергії порядку N_c, відрізняється від свого аналога за низької баріонної густини та температури своїми хіральними властивостями.

Ключові слова: фазова діаграма КХД, N_c методи, баріонна густина, спектр мезонів та глюболів.