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L.L. JENKOVSZKY,[1] C. MERINO[2]

[1] Bogolyubov Institute for Theoretical Physics, Nat. Acad. of Sci. of Ukraine
(14-b, Metrolohichna Str., Kyiv 03143, Ukraine)
[2] Instituto Galego de Física de Altas Enerxías (IGFAE)
María de Maeztu Unit of Excellence, Center of Excellence CIGUS
Universidade de Santiago de Compostela
(Campus Universitario s/n, 15782 Compostela, Galiza, Spain; e-mail: carlos.merino@usc.gal)

COLLECTIVE PHENOMENA IN LARGE AND SMALL PARTONIC SYSTEMS[1]

We compare the collective phenomena in proton-proton, proton-nucleus scattering (large systems) and in lepton-proton deep inelastic scattering (small systems). The most characteristic features in both cases are exposed and compared.

Keywords: strong interactions, deep-inelastic scattering, van der Waals equation of state, collective effects, phase transition, deconfinement.

1. Introduction

We analyze the critical phase transition and crossover phenomena in large (proton-proton, proton-nucleus scattering) systems, and in small (lepton-proton deep inelastic scattering) systems, and we underline the most significant similarities and differences between both scenarios, see Ref. [1].

Collective phenomena in proton-proton scattering and in deep-inelastic (DIS) lepton-proton scattering have been extensively studied, and both similarities and also differences have been noticed between the two cases.

In principle, the transition from individual quark-quark, quark-hadron, hadron-hadron, hadron-nuclei, to lepton-quark, or lepton-hadron collisions, to collective phenomena is quite the same, but also significant differences can be identified related to the fact that while in hadronic collisions two similar extended particles (hadrons or nuclei) interact, in DIS the probe is a point-like particle (a lepton radiating a photon). Furthermore, one substantial difference regards the two formalisms used to describe each case: hadronic collisions are studied in the center-of-mass (c.m.) or laboratory (lab.) frame, whereas the DIS formalism is intrinsically connected to the infinite momentum frame.

However, the common crucial point is the creation in both cases of a new state of matter formed by a soup of quarks and gluons, with saturation and a transition from uncorrelated (leaving apart energy-momentum conservation) collisions of individual particles to collective phenomena. We analyze the physics of these collective phenomenona in pp, pA, AA as well as in ep and eA scattering at high energies and high virtualities of the probe, within the framework of the well known and established van der Waals (VdW) approach, which may serve as a clear and well-defined bridge between the two scenarios.

2. Collective Effects in Hadronic Scattering

The treatment of the van der Waals forces in hadronic systems has been consistently established in the litterature (see Refs. [2–4]).

The van der Waals equation of state is a model used to describe the pressure function in equilibrium systems of particles with both repulsive and attractive interactions, which predicts a first-order liquid-gas phase transition and the corresponding critical point Refs. [4–8].

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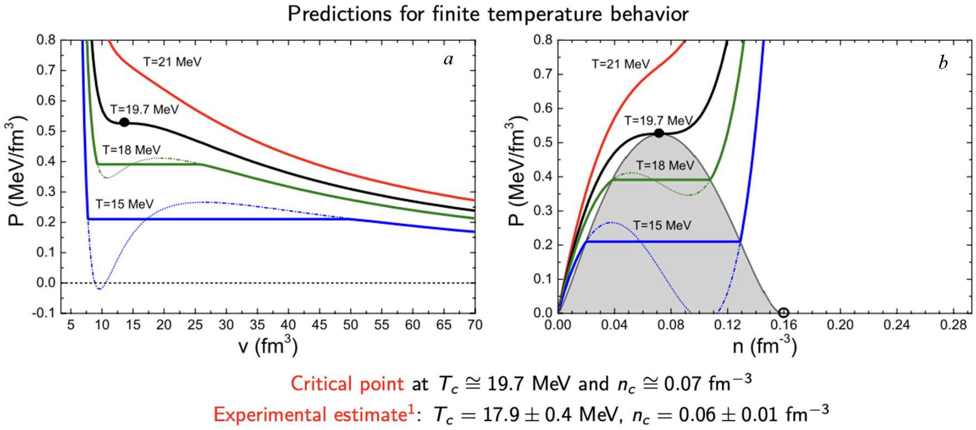

Fig. 1. Pressure isotherms in (a) (p, v) and (b) (p, n), calculated in the quantum van der Waals equation of state (see details in Ref. [4]). The full circle on the $T = T_c$ isotherm corresponds to the critical point. Shaded grey area in (b) represents the mixed-phase region. This figure has been borrowed from Ref. [4]

In the canonical ensemble (CE), the VdW equation of state has the following form:

$$p(T,n) = \frac{NT}{V - bN} - a\frac{N^2}{V^2} \equiv \frac{nT}{1 - bn} - an^2, \qquad (1)$$

where $a > 0$ and $b > 0$ are the VdW parameters describing attractive and repulsive interactions, respectively, and $n \equiv N/V$ is the particle number density. In order to apply the VdW equation of state to systems with a variable number of particles, one has to consider the grand canonical ensemble (CGE), where the quantum statistics is easier to introduce and the VdW equation of state with Fermi statistics is used to describe nuclear matter, see Refs. [9–13].

The thermodynamics of nuclear matter and its application to heavy-ion collisions have been studied for more than forty years. In particular, models employing a self-consistent mean-field approach Refs. [14–19] are used to describe the properties of nuclear matter. The presence of the liquid-gas phase transition in nuclear matter has been experimentally detected in Ref. [20], and direct measurements of the nuclear caloric curve have been published in Refs. [21, 22].

The VdW pressure isotherms in (T, v) and (T, n) coordinates ($v \equiv 1/n$) obtained in Ref. [4] are shown in Fig. 1 (extracted from Ref. [4]). The critical temperature is found to be $T_c \simeq 19.7$ MeV, close to the experimental estimates in references [23, 24]. At $T < T_0$ two phases appear: the gas and liquid phases, separated by a first-order phase transition, the mixed phase region being shown by horizontal lines in Fig. 1, a and by shaded gray area in Fig. 1, b.

Thus, the VdW equation with Fermi statistics applied to a system of interacting nucleons predicts a first-order liquid-gas phase transition with a critical endpoint.

Then, to deal with the problem of entropy production during the hadronization process, and based on the observation of Regge trajectories, possible entropy production mechanisms far from equilibrium can be developed in terms of stochastic dynamics Ref. [25]

3. Collective Effects in Deep Inelastic Scattering

We will assume that in inclusive deep inelastic scattering (DIS), or exclusive deeply virtual Compton scattering (DVCS), the interior of a nucleon (or a nucleus) is seen as a thermodynamic system that, as in the case of nuclear and heavy-ion collisions bears collective (thermodynamic) properties governed by a relevant equation of state (EoS). The idea that DIS structure functions (SF) can be treated thermodynamically by means of statistical mechanics, though not new, continues to attract attention; but several subtle points still remain unclear, in particular with regard to the choice of the appropiate coordinate system and the corresponding variables.

3.1. Statistical models of DIS structure functions

For simplicity, we will focus on the small-x singlet (gluon) component of the SF, with the extension to low-x and/or the non-singlet (valence quark) contributions being straightforward

$$x \cdot G(x, Q^2) \sim \frac{X_0 x^b}{\exp\left[(x - X_0)/\bar{x}\right] + 1}, \qquad (2)$$

where x is the Bjorken (light-cone) variable, X_0 is the chemical potential, which, for the gluon component can be set to zero, and $\bar{x}$ is interpreted as the temperature inside the proton.

In the literature, see Refs. [26–29], the dimensional energy E (or momentum k) variable has been used in the statistical model of the SF, instead of x, as in Eq. (2). This is not a trivial kinematical problem, since thermodynamics implies the presence of the dimensional temperature in the statistical distribution such as k/T (be it of the Fermi–Dirac, Bose–Einstein, or Boltzmann type), while the appearance of x as in Eq. (2) requires some extra modification. This can be circumvented (see Ref. [30]) by using a dimensionless temperature $\bar{x} = 2T/m$, where m is the proton mass, which is a consequence of the transition from the rest frame to the infinite-momentum frame (IMF). Accordingly,

$$G(x) \sim \exp\left(-\frac{mx}{2T}\right). \quad (3)$$

The Boltzmann factor in the denominator of Eq. (2) can mimic the large-x factor $(1-x)^n$ in the SF, although be reconciled also with the quark counting rules appearing with the power n.

One final point is also connected with the EoS expected from the statistical distribution of the type given in Eq. (2). Let us recall that for an ideal gas of particles

$$P(T) = \int k d^3k \, \exp\left(-k/T\right),$$

that due to radial symmetry can be rewritten as

$$4\pi \int\limits_0^\infty k^3 dk \, \exp\left(-k/T\right),$$

and by making the change of variable $y = k/T$ one trivially arrives at the Stefan–Boltzmann (S-B) EoS:

$$P \sim T^4.$$

This fact has a physical interpretation: the large-x component of the SF corresponds to a dilute perfect gas of partons. The low-x factor in Eq. (2) will affect the ideal Stefan–Boltzmann EoS only when it will be written as

$$(x/\bar{x})^b = \left(\frac{mx}{2T}\right)^b,$$

instead of x^b. As a consequence, the ideal Stefan–Boltzmann EoS will be modified to

$$P(T) \sim T^{4+b}. \quad (4)$$

The relative contribution of this correction is negligible at small x, but it increases with Q^2 and decreasing x, resulting in a gas-liquid phase transition.

Usually, the Q^2 dependence is neglected, either for simplicity or, conceptually, by assuming that the statistical approach applies to the SF for some fixed input value of Q^2 from which it evolves according to the DGLAP equation. We do not exclude high Q^2 evolution of the SF, however, this is subject to the following caveats:

a) the structure functions show strong Q^2 dependence, already at low x, below the perturbative DGLAP domain;

b) at large Q^2, instead of the monotonic DGLAP evolution, and due to the proliferation of partons, the inverse process of their recombination is manifest, this process being essential in our interpretation of the saturation as a gas-liquid phase transition (see the next section).

Thus we prefer to keep explicit Q^2 dependence for all x and Q^2. This dependence is mild in the gaseous region of point-like partons (at large x), but it becomes significant towards the saturation region (depending on both x and Q^2), where the point-like partons are replaced by finite-size droplets of the partonic fluid. In the next section we will treat this transition by using the classical van de Waals equation.

3.2. Gas-fluid phase transition in the van der Waals equation of state

Once the statistical properties of the SF have been defined, we now proceed to write an equation of state (EoS) describing the transition from a parton gas to the partonic liquid, via a mixed foggy phase. To do so we use the van der Waals equation (see Refs. [2,3]):

$$(P + N^2 a/V^2)(V - Nb) = NT, \quad (5)$$

where a and b are parameters depending on the properties of the system, N is the number of particles, and V is the volume of the container:

$$V(s) = \pi R^3(s), \quad R(s) \sim \ln s$$

is the nucleon radius. For point-like particles (perfect gas), $a = b = 0$, and Eq. (5) reduces to

$$pV = NT,$$

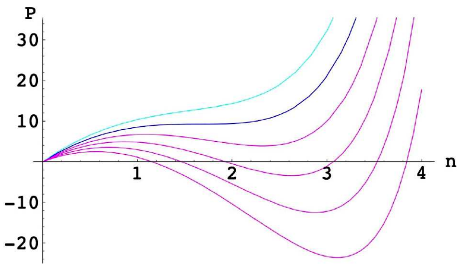

Fig. 2. The pressure-density dependence, calculated in arbitrary units, from Eq. (8)

and, since $N/V \sim T^3$, we get in this approximation

$$p \sim T^4,$$

to be compared with $p \sim T^{(4+b)}$ in Eq. (4).

On the other way, Eq. (5) can be also written as in Ref. [2]

$$(P + a/V^2)(V - b) = RT,$$

or, equivalently

$$P = \frac{RT}{V - b} - \frac{a}{V^2}.$$

The parameter b is responsible for the finite size of the constituents, related to $1/Q$ in our case, and the term a/V^2 is connected to the (long-range) forces between the constituents. From this cubic equation one finds, see Ref. [2], that the critical values for the main quantities $V = V_c$, $P = P_c$, and $T = T_c$, can be written in terms of the parameters a and b as:

$$V_c = 3b, \quad p_c = a/(27b^2), \quad T_c = 8a/(27Rb).$$

The number of particles $N(s)$ can be calculated, see Refs. [9, 31] as

$$N(s) = \int_0^1 dx F_2(x, Q^2),$$

where $F_2(x, Q^2)$ is the nucleon structure function, measured in DIS.

Now, we recall the kinematics:

$$s = Q^2(1 - x)/x + m^2,$$

which at small x reduces to $s \approx Q^2/x$. The radius of the constituent as seen in DIS is $r_0 \sim 1/Q$, hence its two-dimensional volume is $\sim Q^{-2}$.

By introducing the reduced volume, pressure and temperature:

$$\mathcal{P} = P/P_c, \quad \mathcal{V} = V/V_c = \rho_c/\rho, \quad \mathcal{T} = T/T_c,$$

the van der Waals equation (5) can be rewritten as

$$(\mathcal{P} + 3/\mathcal{V}^3)(\mathcal{V} - 1/3) = 8\mathcal{T}/3. \tag{6}$$

Note that Eq. (6) contains only numerical constants, and therefore it is universal. States of various substances with the same values of $\mathcal{P}$, $\mathcal{V}$ and $\mathcal{T}$ are called corresponding states and Eq. (6) is known as the van der Waals equation for corresponding states. The universality of the liquid-gas phase transition and the corresponding principle are typical for any system with short-range repulsive and long-range attractive forces. This property is shared by both ordinary liquids and by nuclear matter.

Let us present two examples of EoS, one based on the Skyrme effective interaction and finite-temperature Hartree–Fock theory, and the other one is the van der Waals EoS. In Refs. [32, 33] one uses the EoS

$$P = \rho kT - a_0\rho^2 + a_3(1 + \sigma)\rho^{(2+\sigma)}, \tag{7}$$

where $\rho = N/V$ is the density and a_0, a_3 and σ are parameters, $\sigma = 1$ accounting for the usual Skyrme interaction.

According to the law of the corresponding states Eq. (7) is universal for scaled (reduced) variables, for which it takes the form

$$P = 3\mathcal{T}/\mathcal{V} - 3/\mathcal{V}^2 + 1/\mathcal{V}^3,$$

to be compared with the van der Waals EoS

$$P = 8\mathcal{T}/(3\mathcal{V} - 1) - 3/\mathcal{V}^2.$$

If we now write the van der Waals EoS in the form

$$P(T; N, V) = -\left(\frac{\partial F}{\partial V}\right)_{TN} =$$

$$= \frac{NT}{V - bN} - a\left(\frac{N}{V}\right)^2 = \frac{nT}{1 - bn} - an^2, \tag{8}$$

with $n = N/V$ denoting the particle number density, a the strength of the mean-field attraction, and b governing the short-range repulsion, we identify the particle number density with the SF $F_2(x, Q^2)$. Fig. 2 shows the pressure-density dependence calculated from Eq. (8), with $a = 5$ GeV^{-2} and $b = 0.2$ GeV^{-3}.

Notice that while Eq. (8) is sensitive to b (short-range repulsion), it is less so to a (long-range attraction). Representative isotherms are shown in this figure, the second from the top line (the dark blue line in the online version of this contribution) being the critical one, $T_c = 8a/(27b)$. Above this temperature (top line, pale blue in the online version of this paper), the pressure rises uniformly with density, corresponding to a single thermodynamic state for each P and T. For subcritical temperatures, $0 < T < T_c$, (the lower lines, red in the online version of this paper), by contrast the function $P(n)$ presents a maximum followed by a minimum. The coexistence phase can be determined by a Maxwell construction.

To map the saturation region in DIS onto the spinodal region in the VdW EoS, that is, to make the correspondence between the EoS with its variables P, T, μ, *etc.* to the experimental observables, depending on the reaction kinematics, is the most delicate and complicated point in the thermodynamic description of any high-energy collisions, in particular of DIS. It requires further careful studies and accurate numerical tests. Attempts to link two different approaches, one based on the S-matrix (scattering amplitudes cross sections) and the other one based on their collective properties (statistical mechanics, thermodynamics, equations of state), to hadron dynamics, have already been proposed (see Ref. [9] and references therein).

4. Conclusions

The description of critical phenomena (phase transitions) in hadronic processes and deep inelastic scattering are presented, and the formalism and most characteristic features in both cases are presented and compared emphasizing the main similarities and differences. To establish the mapping of the saturation region in DIS onto the spinodal region in the van der Waals equation of state further detailed studies and advanced numerical calculations are needed. In addition, to further develop possible entropy production mechanisms far from equilibrium in terms of stochastic dynamics can be of significant interest in describing critical phenomena, both in proton-proton and DIS processes.

C.M. wants to pay tribute to the figure of Prof. László L. Jenkovszky. Those who had the privilege to know him and to collaborate with him are witnesses of his generosity and integrity. His work and legacy will have a longstanding impact in the field of Particle Physics for the new generations of physicists to come.

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Л.Л. Єнковський, К. Меріно

КОЛЕКТИВНІ ЯВИЩА
У ВЕЛИКИХ І МАЛИХ АДРОННИХ СИСТЕМАХ

Порівнюються колективні явища протон-протонного та протон-ядерного розсіювання (великі системи) і глибоко-непружного лептон-протонного розсіювання (малі системи). Виявлені та порівнюються найбільш характерні особливості в обох випадках.

Ключові слова: сильні взаємодії, глибоко-непружне розсіювання, рівняння стану Ван-дер-Ваальса, колективні ефекти, фазовий перехід, деконфайнмент.