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# EFFECTS OF SLOW MODE PHASE VELOCITY ON NON-LINEAR WAVE STRUCTURES IN TWO-TEMPERATURE NON-ISOTHERMAL PLASMA

*In a collisionless, unmagnetized and non-relativistic plasma consisting of warm adiabatic positive and negative ions, warm isothermal positrons and two-temperature non-isothermal electrons, slow mode non-linear wave structures are investigated by the Sagdeev pseudopotential method. The "dispersion relation" is derived from the basic set of normalized fluid equations under the condition of solitary wave solutions and the slow mode phase velocity ($V_S$) is then obtained analytically. The effect of the normalized phase velocity ($V_S$) on large amplitude slow mode solitons and small amplitude slow mode double layers is comprehensively analyzed using the profiles of the Sagdeev potential function $\psi(\phi)$, first- ($\phi_1$) and second-order ($\phi_2$) solitary-wave solutions and the double layer solutions $\phi_{\rm DL}$ in a two-temperature non-isothermal electron plasma.*

*Keywords*: slow mode and fast mode phase velocity, non-isothermal plasma, Sagdeev pseudopotential, slow mode compressive solitons and double layers.

## 1. Introduction

Ion-acoustic slow and fast mode solitons and double layers in nonequilibrium plasmas have been studied in a magnetized or unmagnetized multicomponent plasmas comprising positive ions, negative ions and electrons by some researchers [1–18]. The temperatures and concentrations of the two kinds of ions are fully responsible for the development of two modes of solitons and double layers. For equal ion temperatures, Tagare [19–20] studied slow and fast mode solitons along with their widths and amplitudes in an unmagnetized isothermal and non-isothermal electron plasma containing positive and negative ions. Using the reductive perturbation method, Mishra *et al.* [21] investigated the slow and fast mode phase velocities in a magnetized low $\beta$ – plasma consisting of warm negative and positive ions for a single temperature isothermal electrons. Maharaj *et al.* [22] studied large amplitude slow and fast mode solitons and double layers for cool and hot ions with cool and hot electrons by the Sagdeev pseudopotential method. In a five-component plasma, Sijo Sebastian *et al.* [23] also investigated slow and fast mode solitons. Ion-acoustic slow and fast mode solitons and double layers in magnetized or unmagnetized plasmas with or without dust particles are studied by many researchers [24–25]. Dubinov *et al.* [26] investigated the slow and fast mode solitons in their studies. Using the Sagdeev pseudopotential method, Mushinzimana *et al.* [27]

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studied the ion thermal and electron superthermal effects on large amplitude slow ion-acoustic solitons, supersolitons and double layers in a warm positive and negative ion plasma for Kappa-distributed electrons.

Mishra *et al.* [28] investigated the ion-acoustic compressive and rarefactive double layers in a warm multicomponent plasma with negative ions and obtained some interesting results. Kim and Schamel [29–30] studied the slow ion-acoustic double layers which move with a velocity near the ion thermal speed. Under appropriate double layer conditions, Sutradhar and Bujarbarua [31] also obtained slow ion-acoustic double layer solutions with non-isothermal electron distributions.

In this paper, the author investigates the large amplitude slow mode solitary waves and small amplitude slow mode double layers corresponding to slow mode phase velocities in a two-temperature non-isothermal electron plasma consisting of warm positrons and warm adiabatic positive and negative ions using the Sagdeev pseudopotential method. The effects of the normalized phase velocity ($V_S$) on nonlinear wave structures such as large amplitude slow mode compressive solitons and small amplitude slow mode compressive double layer profiles of the Sagdeev pseudopotential function $\psi(\phi)$ as a function of the electrostatic potential $\phi$, the first- ($\phi_1$) and second-order ($\phi_2$) solitary wave solutions as functions of $\eta$ and the double-layer solutions ($\phi_{\mathrm{DL}}$) as functions of $\eta$ are comprehensively analyzed, where $\eta$ is defined by the Galilean transformation.

## 2. Formulation

Considering a collisionless, unmagnetized and nonrelativistic plasmas consisting of warm positive ($i$) and negative ($j$) ions, warm positrons and two-temperature non-isothermal electrons ($n_e$) and treating positive ($i$) and negative ion ($j$) species to be the colder and hotter adiabatic fluids, the normalized basic equations [19] along x-axis for positive ($i$) and negative ions ($j$) are written in the following way:

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i u_i) = 0, \tag{1}$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} + \frac{\sigma_i}{n_i}\frac{\partial p_i}{\partial x} + \frac{\partial \phi}{\partial x} = 0, \tag{2}$$

$$\frac{\partial p_i}{\partial t} + u_i \frac{\partial p_i}{\partial x} + 3p_i \frac{\partial u_i}{\partial x} = 0, \tag{3}$$

$$\frac{\partial n_j}{\partial t} + \frac{\partial}{\partial x}(n_j u_j) = 0, \tag{4}$$

$$\frac{\partial u_j}{\partial t} + u_j \frac{\partial u_j}{\partial x} + \frac{\sigma_j}{Q n_j}\frac{\partial p_j}{\partial x} - \frac{Z}{Q}\frac{\partial \phi}{\partial x} = 0, \tag{5}$$

$$\frac{\partial p_j}{\partial t} + u_j \frac{\partial p_j}{\partial x} + 3 \quad p_j \frac{\partial u_j}{\partial x} = 0. \tag{6}$$

The Poisson equation is formulated for warm positive ($i$) and negative ($j$) ions, warm positrons and two-temperature non-isothermal electrons [32–33] in the following normalized form:

$$\frac{\partial^2 \phi}{\partial x^2} = n_e - n_i + Z n_j - n_p. \tag{7}$$

The normalized positron density is

$$n_p = \chi e^{-\sigma_p \phi}. \tag{8}$$

The normalised electron density for two-temperature non-isothermal electrons is

$$n_e = 1 + \phi - \frac{4}{3}\frac{\left(\mu b_l + \nu b_h \beta_1^{\frac{3}{2}}\right)}{(\mu+\nu\beta_1)^{\frac{3}{2}}}\phi^{\frac{3}{2}} + \frac{1}{2}\frac{\left(\mu+\nu\beta_1^2\right)}{(\mu+\nu\beta_1)^2}\phi^2 - $$
$$- \frac{8}{15}\frac{\left(\mu b_l^{(1)} + \nu b_h^{(1)} \beta_1^{\frac{5}{2}}\right)}{(\mu+\nu\beta_1)^{\frac{5}{2}}}\phi^{\frac{5}{2}} + \frac{1}{6}\frac{\left(\mu+\nu\beta_1^3\right)}{(\mu+\nu\beta_1)^3}\phi^3 - ..., \tag{9}$$

where

$$\sigma_i = \frac{T_i}{T_{\mathrm{eff}}}, \quad \sigma_j = \frac{T_j}{T_{\mathrm{eff}}}, \quad \sigma_p = \frac{T_{\mathrm{eff}}}{T_p}, \quad Q = \frac{m_j}{m_i},$$
$$b_l = \frac{1-\beta_l}{\sqrt{\pi}}, \quad b_h = \frac{1-\beta_h}{\sqrt{\pi}}, \quad b_l^{(1)} = \frac{1-\beta_l^2}{\sqrt{\pi}},$$
$$b_h^{(1)} = \frac{1-\beta_h^2}{\sqrt{\pi}}, \quad \beta_1 = \frac{T_{el,f}}{T_{eh,f}}, \quad \beta_l = \frac{T_{el,f}}{T_{el,t}},$$
$$\beta_h = \frac{T_{eh,f}}{T_{eh,t}}, \quad T_{\mathrm{eff}} = \frac{T_{el}\,T_{eh}}{\mu T_{eh} + \nu T_{el}}, \quad \mu+\nu = 1.$$

Here $n_i$, $n_j$, $n_e$, $n_p$; $u_i$, $u_j$; $p_i$, $p_j$; $\sigma_i$, $\sigma_j$; $Z$, $Q$, $x$, $t$, $\phi$, $\chi$, $\sigma_p$, $\mu$, $\nu$, $\beta_1$, $\beta_l$, $\beta_h$; $b_l$, $b_h$, $b_l^{(1)}$, $b_h^{(1)}$; $T_i$, $T_j$, $T_p$ and $T_{\mathrm{eff}}$ denote the concentrations of positive ions, negative ions, electrons and positrons; velocities of positive and negative ions; pressures of positive and negative ions; temperature ratios of positive (negative) ions to effective electrons; charge of ions, mass ratio of negative ($m_j$) to positive ($m_i$) ions; space coordinate, time, electrostatic potential, concentration of positrons at $\phi = 0$, temperature ratio of effective electrons to positrons, unperturbed number density


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of low- and high-temperature electrons, temperature ratio of free electrons at low and high temperatures, temperature ratio of free and trapped electrons at low temperatures, temperature ratio of free and trapped electrons at high temperatures; non-isothermal parameters connected with $\beta_l$ and $\beta_h$; temperatures of positive ions, negative ions, positrons and effective temperatures of electrons; $T_{el,f}$ and $T_{eh,f}$ are the temperatures of free electrons at low and high temperatures; $T_{el,t}$ and $T_{eh,t}$ are the temperatures of trapped electrons at low and high temperatures.

For two-temperature non-isothermal electron plasmas

$0 < b_l$ or $b_h < \frac{1}{\sqrt{\pi}}$ and $0 < b_l^{(1)}$ or $b_h^{(1)} < \frac{1}{\sqrt{\pi}}$.

In the above equations, all physical quantities are normalized in the following way:

The densities $n_i$, $n_j$, $n_e$ and $n_p$ are normalized by the unperturbed electron (or negative ions) density $n_0$ $[n_0 = n_{io} + n_{jo} = n_{ceo} + n_{heo} = \mu + \nu = 1]$, ion pressures $p_i$ and $p_j$ are normalized by the characteristic ion pressure $p_0(= n_0 K_B T_j)$, the length $x$ is normalized by the characteristic Debye length $\lambda_{De} = \left(K_B T_j / 4\pi e^2 n_{jo}\right)^{\frac{1}{2}}$ and the time t is normalized by the plasma period $\omega_{pe}^{-1} = \left(m_j / 4\pi e^2 n_{jo}\right)^{\frac{1}{2}}$ where $K_B$ is Boltzmann constant. The ion velocities $u_i$ and $u_j$ are normalized by ion-sound velocity $C_S = (K_B T_{eff}/m_\alpha)^{\frac{1}{2}}$ where $m_\alpha$ is the mass of ions [$\alpha = i$ for positive ions and $\alpha = j$ for negative ions] and the electrostatic potential $\phi$ by $(K_B T_{eff}/e)$.

The boundary conditions are

$$n_i \to n_{i0}, \quad n_j \to n_{j0}, \quad u_i \to 0, \quad u_j \to 0,$$
$$p_i \to p_{io}, \quad p_j \to p_{jo}, \quad n_e \to 1, n_p \to \chi, \qquad (10)$$
$$\phi \to 0 \text{ at } |x| \to \infty.$$

The charge neutrality condition is

$$n_{i0} + \chi = 1 + Z n_{j0}. \qquad (11)$$

Using the above boundary conditions (10) and the Galilean transformation $\eta = x - Vt$, where $V = \left(\frac{v}{C_S}\right)$ is the phase velocity of the wave i.e., the phase velocity V is obtained from the velocity $(v)$ of the non-linear structure in the inertial frame normalized with respect to ion-sound velocity $C_S$, it is observed that the equations (3) and (6) are consistent with the equation of state $p_i = p_{i0}\left(\frac{n_i}{n_{i0}}\right)^3$ and $p_j = p_{j0}\left(\frac{n_j}{n_{j0}}\right)^3$ in one-dimensional motion and henceforth the author takes $p_{i0} = 1$ and $p_{j0} = 1$.

By using the Galilean transformation and boundary conditions (10), the Eqs. (1)–(7) are transformed as follows

$$-V\frac{dn_i}{d\eta} + \frac{d}{d\eta}(n_i u_i) = 0,$$
$$-V\frac{du_i}{d\eta} + u_i\frac{du_i}{d\eta} + \frac{\sigma_i}{n_i}\frac{dp_i}{d\eta} + \frac{d\phi}{d\eta} = 0,$$
$$-V\frac{dp_i}{d\eta} + u_i\frac{dp_i}{d\eta} + 3p_i\frac{du_i}{d\eta} = 0,$$
$$-V\frac{dn_j}{d\eta} + \frac{d}{d\eta}(n_j u_j) = 0, \qquad (12)$$
$$-V\frac{du_j}{d\eta} + u_j\frac{du_j}{d\eta} + \frac{\sigma_j}{Qn_j}\frac{dp_j}{d\eta} - \frac{Z}{Q}\frac{d\phi}{d\eta} = 0,$$
$$-V\frac{dp_j}{d\eta} + u_j\frac{dp_j}{d\eta} + 3p_j\frac{du_j}{d\eta} = 0,$$
$$\frac{d^2\phi}{d\eta^2} = n_e - n_i + Zn_j - n_p.$$

From the above equations after solving and using the boundary conditions, one can find $n_i^2$ and $n_j^2$ in terms of the electrostatic potential $\phi$.

$$n_i^2 = \frac{n_{io}^3}{6\sigma_i}\left[\left\{V^2 + \frac{3\sigma_i}{n_{io}} - 2\phi\right\} \pm \pm\sqrt{\left\{V^2 + \frac{3\sigma_i}{n_{io}} - 2\phi\right\}^2 - \frac{12\sigma_i V^2}{n_{io}}}\right], \qquad (13)$$

$$n_j^2 = \frac{Qn_{jo}^3}{6\sigma_j}\left[\left\{V^2 + \frac{3\sigma_j}{Qn_{jo}} + \frac{2Z\phi}{Q}\right\} \pm \pm\sqrt{\left\{V^2 + \frac{3\sigma_j}{Qn_{jo}} + \frac{2Z\phi^2}{Q}\right\}^2 - \frac{12\sigma_j V^2}{Qn_{jo}}}\right]. \qquad (14)$$

In this paper the author considers only the slow mode solitary waves and double layers. The slow and fast wave modes are distinguished based on the range of their phase speeds with respect to ion thermal speeds in the following way:

$$v_{tc} < v_{slow} < v_{th} < v_{fast} < v_{tce} < < v_{el.acoust} < v_{the}, \qquad (15)$$

where $v_{tc}$, $v_{slow}$, $v_{th}$, $v_{fast}$, $v_{tce}$, $v_{el.acoust}$ and $v_{the}$ are thermal speeds of cold positive ions, slow mode ion velocity, thermal speeds of hot negative ions, fast mode ion velocity, cold electron thermal speed, electron acoustic speed and hot electron thermal speed in non-normalized form. For further study, the author adopts the following inequality for the fast ($v_F$) and slow ($v_S$) mode phase speeds in which the fast mode phase speed lies between the thermal speeds of hotter ion species and the colder electrons in non-normalized form:

$$v_{\text{tc}} < v_S < v_{\text{th}} < v_F < v_{\text{tce}}.$$

The above inequality can be normalized by the ion-sound velocity $C_S$ in the following form:

$$\sqrt{\frac{3\sigma_i}{n_{io}}} < V_S < \sqrt{\frac{3\sigma_j}{Qn_{jo}}} < V_F < \frac{v_{\text{tce}}}{C_S}. \tag{16}$$

Now by basic assumptions and from normalized inequality (16), the present author obtains after simplifying from equations (13) and (14)

$$n_i = \sqrt{\frac{n_{i0}^3}{12\,\sigma_i}}\left[\left\{\left(V+\sqrt{\frac{3\sigma_i}{n_{i0}}}\right)^2 - 2\phi\right\}^{\frac{1}{2}} - \left\{\left(V-\sqrt{\frac{3\sigma_i}{n_{i0}}}\right)^2 - 2\phi\right\}^{\frac{1}{2}}\right], \tag{17}$$

$$n_j\sqrt{\frac{Qn_{j0}^3}{12\,\sigma_j}}\left[\left\{\left(\sqrt{\frac{3\sigma_j}{Qn_{j0}}}+V\right)^2 + \frac{2Z}{Q}\phi\right\}^{\frac{1}{2}} + \left\{\left(\sqrt{\frac{3\sigma_j}{Qn_{j0}}}-V\right)^2 + \frac{2Z}{Q}\phi\right\}^{\frac{1}{2}}\right]. \tag{18}$$

Substituting the values of $n_e$, $n_i$, $n_j$ and $n_p$ into Eq. (12), a differential equation involving the electrostatic potential $\phi$ is obtained which can be written as Newton's equation in the following form:

$$\frac{d^2\phi}{d\eta^2} = -\frac{\partial\psi}{\partial\phi},$$

where $\psi(\phi)$ is the Sagdeev potential function. Multiplying this equation by $\frac{d\phi}{d\eta}$ and integrating with boundary conditions (10), the energy-like equation is obtained as

$$\Rightarrow \frac{1}{2}\left(\frac{d\phi}{d\eta}\right)^2 + \psi\left(\phi\right) = 0. \tag{19}$$

Here the Sagdeev potential function $\psi\left(\phi\right)$ is given in the following form:

$$\psi(\phi) = \Bigg[-\phi - \frac{1}{2}\phi^2 + \frac{8}{15}\frac{\mu b_l + \nu b_h\beta_1^{\frac{3}{2}}}{(\mu+\nu\beta_1)^{\frac{3}{2}}}\,\phi^{\frac{5}{2}} - \frac{1}{6}\frac{\mu+\nu\beta_1^2}{(\mu+\nu\beta_1)^2}\phi^3 + \frac{16}{105}\frac{\mu b_l^{(1)} + \nu b_h^{(1)}\beta_1^{\frac{5}{2}}}{(\mu+\nu\beta_1)^{\frac{5}{2}}}\,\phi^{\frac{7}{2}} - \frac{1}{24}\frac{\mu+\nu\beta_1^3}{(\mu+\nu\beta_1)^3}\,\phi^4 + \ldots\Bigg] +$$
$$+ \frac{1}{6}\sqrt{\frac{n_{io}^3}{3\sigma_i}}\Bigg[\left\{\left(V-\sqrt{\frac{3\sigma_i}{n_{io}}}\right)^2 - 2\phi\right\}^{\frac{3}{2}} - \left\{\left(V+\sqrt{\frac{3\sigma_i}{n_{io}}}\right)^2 - 2\phi\right\}^{\frac{3}{2}} + \left(V+\sqrt{\frac{3\sigma_i}{n_{io}}}\right)^3 - \left(V-\sqrt{\frac{3\sigma_i}{n_{io}}}\right)^3\Bigg] -$$
$$- \frac{1}{6}\sqrt{\frac{Q^3n_{io}^3}{3\sigma_j}}\Bigg[\left\{\left(\sqrt{\frac{3\sigma_j}{Qn_{jo}}}-V\right)^2 + \frac{2Z\phi}{Q}\right\}^{\frac{3}{2}} + \left\{\left(\sqrt{\frac{3\sigma_j}{Qn_{jo}}}+V\right)^2 + \frac{2Z\phi}{Q}\right\}^{\frac{3}{2}} - \left(\sqrt{\frac{3\sigma_j}{Qn_{jo}}}+V\right)^3 - \left(\sqrt{\frac{3\sigma_j}{Qn_{jo}}}-V\right)^3\Bigg] +$$
$$+ \frac{\chi}{\sigma_p}(1-e^{-\sigma_p\phi}). \tag{20}$$

The restriction on $\phi$ is

$$-\frac{Q}{2Z}\left(\sqrt{\frac{3\sigma_j}{Qn_{jo}}}-V\right)^2 < \Phi < \frac{1}{2}\left(V-\sqrt{\frac{3\sigma_i}{n_{io}}}\right)^2. \tag{21}$$

The requirements for soliton formation from Sagdeev potential function $\psi\left(\phi\right)$ are

$\psi(\phi) = 0$ at $\phi = 0$ and$\phi = \phi_m$,
$\frac{\partial\psi}{\partial\phi} = 0$ at$\phi = 0$,
$\frac{\partial^2\psi}{\partial\phi^2} \le 0$ at $\phi = 0$,
$\psi\left(\phi\right) < 0$ for $0 < \phi < \phi_m$ and $\phi > \phi_m$,
$\frac{\partial^3\psi}{\partial\phi^3} > 0$ at $\phi = 0$


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for positive (compressive) potentials solitons.

$$\frac{\partial\psi}{\partial\phi} > 0 \text{ at } \phi = \phi_m$$

for positive (compressive) potentials solitons.

The condition for the existence of solitary wave solution is represented by the following non-linear dispersion relation

$$\frac{\partial^2\psi}{\partial\phi^2}\Big|_{\phi=0} \le 0$$

$$\Rightarrow \frac{n_{i0}}{V^2 - \frac{3\sigma_i}{n_{i0}}} + \frac{\frac{Z^2 n_{j0}}{Q}}{V^2 - \frac{3\sigma_j}{Q\, n_{j0}}} \le 1 + \chi\sigma_p. \quad (22)$$

From (22), the linear dispersion relation is

$$\frac{n_{i0}}{V^2 - \frac{3\sigma_i}{n_{i0}}} + \frac{\frac{Z^2 n_{j0}}{Q}}{V^2 - \frac{3\sigma_j}{Q\, n_{j0}}} = 1 + \chi\sigma_p. \quad (23)$$

Equation (23) after simplification gives,

$$V^4 - 3\left[\left\{\left(\frac{\sigma_i}{n_{i0}} + \frac{\sigma_j}{Q\, n_{j0}}\right)\right\} + \frac{Qn_{i0} + Z^2 n_{j0}}{3Q\,(1+\chi\sigma_p)}\right] V^2 + \\ + \frac{9}{Q}\left[\left\{\frac{\sigma_i\sigma_j}{n_{i0}n_{j0}}\right\} + \\ + \frac{1}{3}(1+\chi\sigma_p)\left\{\left(\frac{n_{i0}\sigma_j}{n_{j0}} + \frac{Z^2 n_{j0}\sigma_i}{n_{i0}}\right)\right\}\right] = 0. \quad (24)$$

This is a quadratic equation in $V^2$. The roots of this equation are the critical values of phase speeds ($V_{\rm crit}$) of the linear wave modes. It is found that three distinct positive roots exist of which the smallest value of $V_{\rm crit}$ is identified as a slow ion – acoustic mode ($V_S$), the intermediate is a fast ion – acoustic mode ($V_F$) and the largest root is an electron- acoustic mode ($V_{\rm el.acoust}$). In this paper, the author is discussing the large amplitude slow mode compressive solitons and small amplitude slow mode compressive double layers corresponding to the slow mode phase velocity ($V_S$).

From Eq. (24) after simplification it is found

$$V^2 = \frac{3}{2}\left\{\left[\left(\frac{\sigma_i}{n_{i0}} + \frac{\sigma_j}{Qn_{j0}}\right) + \frac{(Qn_{i0} + Z^2 n_{j0})}{3Q\,(1+\chi\sigma_p)}\right] \pm \\ \pm\left[\left\{\left(\frac{\sigma_i}{n_{i0}} + \frac{\sigma_j}{Qn_{j0}}\right) + \frac{(Qn_{i0} + Z^2 n_{j0})}{3Q\,(1+\chi\sigma_p)}\right\}^2 - \\ - \frac{4}{Q}\left\{\frac{\sigma_i\sigma_j}{n_{i0}n_{j0}} + \frac{1}{3\,(1+\chi\sigma_p)}\left(\frac{n_{i0}\sigma_j}{n_{j0}} + \frac{Z^2 n_{j0}\sigma_i}{n_{i0}}\right)\right\}\right]^{\frac{1}{2}}\right\}. \quad (25)$$

The minimum values of $V$ $(= V_S)$ are

$$V^2_{\rm crit} = V_S^2 = \frac{3}{2}\left\{\left[\left(\frac{\sigma_i}{n_{i0}} + \frac{\sigma_j}{Qn_{j0}}\right) + \frac{(Q\,n_{i0} + Z^2 n_{j0})}{3Q\,(1+\chi\,\sigma_p)}\right] - \\ - \left[\left\{\left(\frac{\sigma_i}{n_{i0}} + \frac{\sigma_j}{Qn_{j0}}\right) + \frac{(Q\,n_{i0} + Z^2 n_{j0})}{3Q\,(1+\chi\,\sigma_p)}\right\}^2 - \\ - \frac{4}{Q}\left\{\frac{\sigma_i\sigma_j}{n_{i0}n_{j0}} + \frac{1}{3}(1+\chi\,\sigma_p)\left(\frac{n_{i0}\sigma_j}{n_{j0}} + \frac{Z^2 n_{j0}\sigma_i}{n_{i0}}\right)\right\}\right]^{\frac{1}{2}}\right\} \quad (26)$$

and

$$V^2_{\rm crit} = V_F^2 = \\ = \frac{3}{2}\left\{\left[\left(\frac{\sigma_i}{n_{i0}} + \frac{\sigma_j}{Qn_{j0}}\right) + \frac{(Q\,n_{i0} + Z^2 n_{j0})}{3Q\,(1+\chi\,\sigma_p)}\right] + \\ + \left[\left\{\left(\frac{\sigma_i}{n_{i0}} + \frac{\sigma_j}{Qn_{j0}}\right) + \frac{(Q\,n_{i0} + Z^2 n_{j0})}{3Q\,(1+\chi\,\sigma_p)}\right\}^2 - \\ - \frac{4}{Q}\left\{\frac{\sigma_i\sigma_j}{n_{i0}n_{j0}} + \frac{1}{3}(1+\chi\,\sigma_p)\left(\frac{n_{i0}\sigma_j}{n_{j0}} + \frac{Z^2 n_{j0}\sigma_i}{n_{i0}}\right)\right\}\right]^{\frac{1}{2}}\right\}. \quad (27)$$

Equation (26) gives the slow mode phase velocity ($V_S$) and it is the minimum velocity. Now from Eq. (26) by Taylor series expansion to the zeroth order, it is found approximately as

$$V_S^2 \approx \frac{3\sigma_i}{n_{io}} + \frac{Z^2 n_{jo}}{Q\,(1+\chi\sigma_p)} + ...,$$

which shows $V_S > \sqrt{\frac{3\sigma_i}{n_{io}}}$.

Equation (27) gives the fast mode phase velocity ($V_F$) which is known as the intermediate root and after Taylor series expansion from that equation to the zeroth order, it is also found approximately from (27) as

$$V_F^2 \approx \frac{3\sigma_j}{Q\,n_{jo}} + \frac{n_{io}}{(1+\chi\sigma_p)} + \frac{3\sigma_i}{n_{io}} + \frac{Z^2 n_{jo}}{Q\,(1+\chi\sigma_p)} + ...,$$

which shows $V_F > \sqrt{\frac{3\sigma_j}{Qn_{jo}}}$.

Again from analytical expressions (26) and (27), it is found that $V_F > V_S$.

Thus, the relation between $V_S$ and $V_F$ is finally obtained as $\sqrt{\frac{3\sigma_i}{n_{io}}} < V_S < \sqrt{\frac{3\sigma_j}{Qn_{jo}}} < V_F$.

## 3. Solitary Wave Solutions

In order to study the solitary wave solutions, the author discusses the nature of the energy equation for different orders of the electrostatic potential $\phi$. By Taylor series expansion of $\psi(\phi)$ in powers of $\phi$ from Eq. (19), it is found that

$$\frac{d^2\phi}{d\eta^2} = A_1\phi - A_2\phi^{\frac{3}{2}} + A_3\phi^2 - A_4\phi^{\frac{5}{2}} + + A_5\phi^3 - ... = -\frac{\partial\psi}{\partial\phi}, \tag{28}$$

where

$$A_1 = \left[1 - n_{i0}\left\{V^2 - \frac{3\sigma_i}{n_{i0}}\right\}^{-1} - -Z^2 n_{j0}\left\{QV^2 - \frac{3\sigma_j}{n_{j0}}\right\}^{-1} + \chi\sigma_p\right],$$

$$A_2 = \frac{4}{3}\frac{\left(\mu b_l + \nu b_h \beta_2^{\frac{3}{2}}\right)}{(\mu + \nu\beta_2)^{\frac{3}{2}}},$$

$$A_3 = \frac{1}{2}\left[\frac{\mu + \nu\beta_2^2}{(\mu + \nu\beta_2)^2} - \frac{n_{i0}^{\frac{3}{2}}}{2\sqrt{3\sigma_i}}\left\{\left(V - \sqrt{\frac{3\sigma_i}{n_{i0}}}\right)^{-3} - -\left(V + \sqrt{\frac{3\sigma_i}{n_{i0}}}\right)^{-3}\right\} + \frac{Z^3 n_{j0}^{\frac{3}{2}}}{2Q\sqrt{3Q\sigma_j}}\left\{\left(V - \sqrt{\frac{3\sigma_j}{Qn_{j0}}}\right)^{-3} - -\left(V + \sqrt{\frac{3\sigma_j}{Qn_{j0}}}\right)^{-3}\right\} - \chi\sigma_p^2\right],$$

$$A_4 = \frac{8}{15}\frac{\left(\mu b_l^{(1)} + \nu b_h^{(1)}\beta_2^{\frac{5}{2}}\right)}{(\mu + \nu\beta_2)^{\frac{5}{2}}},$$

$$A_5 = \frac{1}{2}\left[\frac{1}{3}\frac{\mu + \nu\beta_2^3}{(\mu + \nu\beta_2)^3} - \frac{n_{i0}^{\frac{3}{2}}}{2\sqrt{3\sigma_i}}\left\{\left(V - \sqrt{\frac{3\sigma_i}{n_{i0}}}\right)^{-5} - -\left(V + \sqrt{\frac{3\sigma_i}{n_{i0}}}\right)^{-5}\right\} + \frac{Z^4 n_{j0}^{\frac{3}{2}}}{2Q^2\sqrt{3Q\sigma_j}}\left\{\left(V + \sqrt{\frac{3\sigma_j}{Qn_{j0}}}\right)^{-5} - -\left(V - \sqrt{\frac{3\sigma_j}{Qn_{j0}}}\right)^{-5}\right\} + \frac{\chi\sigma_p^3}{3}\right]. \tag{29}$$

Taking terms up to $\phi^{\frac{3}{2}}$, the Eq. (28) can be written as

$$\frac{d^2\phi}{d\eta^2} = A_1\phi - A_2\phi^{\frac{3}{2}}. \tag{30}$$

The first-order solitary wave solution ($\phi_1$) [34–35] is

$$\phi_1 = \left(\frac{5A_1}{4A_2}\right)^2 \mathrm{Sech}^4\left(\sqrt{\frac{A_1}{16}}\eta\right) \tag{31}$$

and the first-order width ($\phi_1$) is

$$\phi_1 = \frac{4}{\sqrt{A_1}}, \tag{32}$$

where $A_1 > 0$.

Similarly, taking terms up to $\phi^2$, the Eq. (28) can be written as

$$\frac{d^2\phi}{d\eta^2} = A_1\phi - A_2\phi^{\frac{3}{2}} + A_3\phi^2. \tag{33}$$

The second-order solitary wave solution ($\phi_2$) [34–35] is

$$\phi_2 = \left[\frac{3A_2}{5A_3} \pm \sqrt{\left(\frac{3A_2}{5A_3}\right)^2 - \frac{3A_1}{2A_3}} \times \times \mathrm{Sech}\left(\frac{1}{2}\sqrt{A_1 - \frac{9A_2^2}{25A_3}}\,\eta\right)\right]^2. \tag{34}$$

And the second-order width ($\phi_2$) is

$$\phi_2 = \frac{2}{\sqrt{A_1 - \frac{9A_2^2}{25A_3}}}, \tag{35}$$

where $A_1 - \frac{9A_2^2}{25A_3} > 0$.

Finally, after substitution of the appropriate value of $V_S$ for slow mode phase velocity in place of V in Eqs (31) and (34), the large amplitude slow mode compressive solitary wave solutions of first ($\phi_1$) and second ($\phi_2$) order in two-temperature non-isothermal electron plasmas are obtained.

## 4. Double Layers

A double layer is a non-linear wave structure formed sometimes for a short duration of time in plasma. In this study, the author investigates the small amplitude slow mode compressive double layers. The slow mode compressive double layers are found from such values of the slow mode phase velocities ($V_S$) which exceed the upper limiting values of the slow mode solitons.


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The boundary conditions for double layers are $\psi(\phi) = 0$ for $\phi = 0$ & $\phi_{dl}$; $\frac{\partial\psi(\phi)}{\partial\phi} = 0$ for $\phi = 0$ & $\phi_{dl}$ and $\frac{\partial^2\psi}{\partial\phi^2}\big|_{\phi=0,\,\phi_{dl}} < 0$, $\phi_{dl} > 0$.

For small amplitude double layer solutions, after expanding $\psi(\phi)$ in power series in $\phi$ up to $\phi^2$, the energy Eq. (19) reduces to the form

$$\frac{d^2\phi}{d\eta^2} = A_1\phi - A_2\phi^{\frac{3}{2}} + A_3\phi^2 = -\frac{\partial\psi}{\partial\phi}, \tag{36}$$

where

$$\psi(\phi) = -\frac{1}{2}A_1\phi^2 + \frac{2}{5}A_2\phi^{\frac{5}{2}} - \frac{1}{3}A_3\phi^3. \tag{37}$$

After applying the boundary conditions, Eq. (37) is obtained by integrating Eq. (36). Here $A_1$, $A_2$, $A_3$ are defined previously in Eq. (29).

Now by applying the boundary conditions of double layers on Eqs (36) and (37), the values of $A_1$ and $A_2$ in terms of $A_3$ are obtained as $A_1 = \frac{2}{3}\phi_{dl}A_3$ and $A_2 = \frac{5}{3}\sqrt{\phi}_{dl}A_3$.

Substituting these values of $A_1$ and $A_2$ in Eqs (37) and (36), the following equations are obtained finally as:

$$\psi(\phi) = -\frac{1}{3}A_3\phi^2\left(\sqrt{\phi} - \sqrt{\phi}_{dl}\right)^2, \tag{38}$$

$$\frac{\partial\psi(\phi)}{\partial\phi} = -\frac{1}{3}A_3\phi\left(\sqrt{\phi} - \sqrt{\phi}_{dl}\right)\left(3\sqrt{\phi} - 2\sqrt{\phi}_{dl}\right), \tag{39}$$

$$\phi_{\rm DL} = \frac{1}{4}\phi_{dl}\left[1 - \tanh\left(\sqrt{\frac{A_3\phi_{dl}}{24}}\,\eta\right)\right]^2, \tag{40}$$

"where $A_3 > 0$."

The stable structure of double layer solution is found for $A_3 > 0$ and this gives the physically acceptable values of the double layers which also satisfy the necessary boundary conditions.

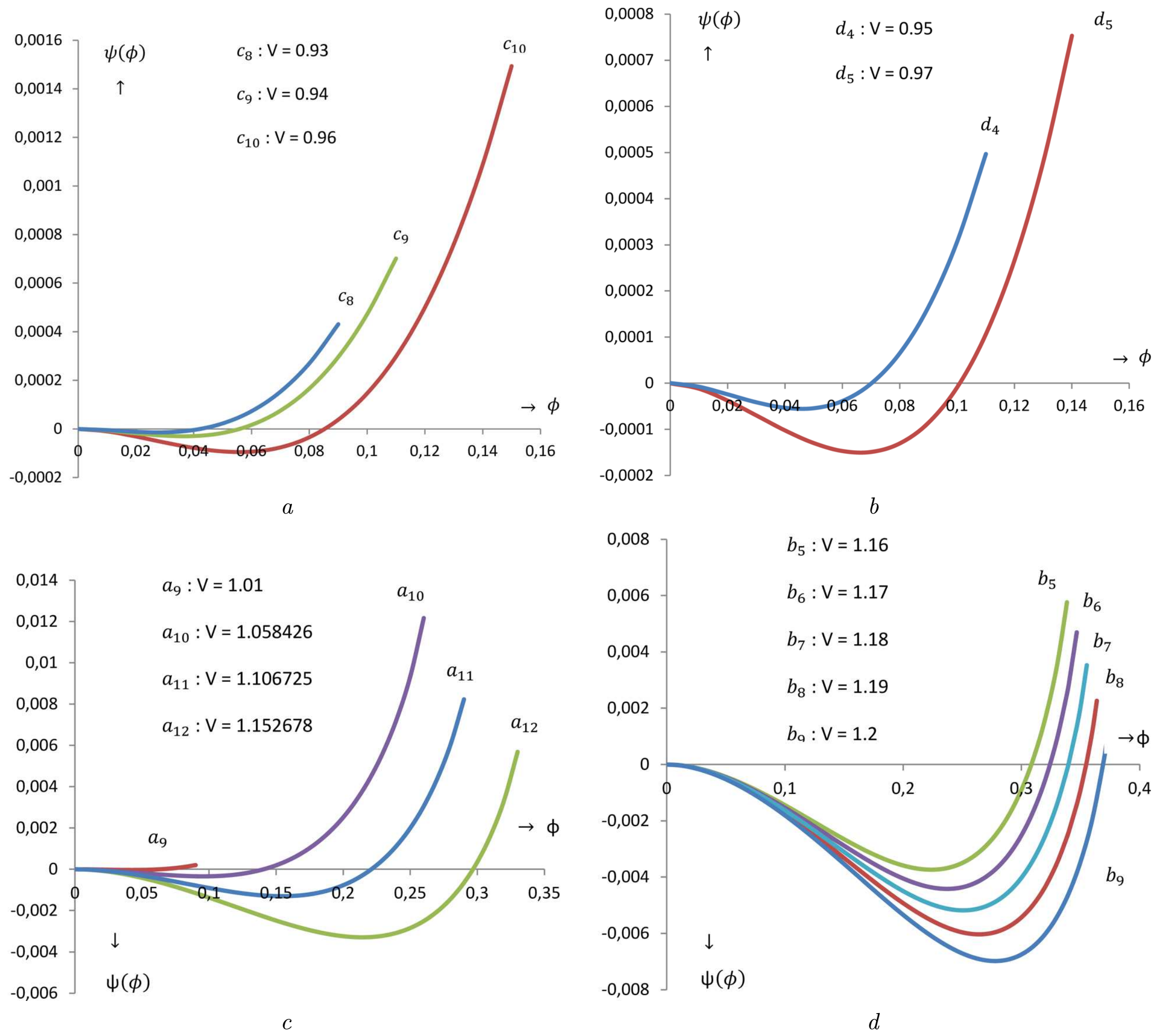
The profiles of Sagdeev potential function $\psi(\phi)$ as a function of $\phi$ and the double layer solutions $\phi_{\rm DL}$ as functions of $\eta$ for small amplitude slow mode compressive double layers are obtained after substituting the appropriate values of slow mode phase velocity $V_S$ in place of $V$.

## 5. Results and Discussions

In this paper, the author discusses the large amplitude slow mode compressive solitons and small amplitude compressive slow mode double layers in two-temperature non-isothermal electron plasmas containing warm adiabatic negative ions, warm adiabatic positive ions and warm isothermal positrons. Under different variations of the slow mode phase velocities ($V_S$), the author has tried to show the formation of fully non-linear slow mode compressive solitons from Sagdeev potential function $\psi(\phi)$ as a function of $\phi$, slow mode compressive first ($\phi_1$) and second ($\phi_2$) order solitary wave solutions as functions of $\eta$, small amplitude slow mode compressive double layer profiles from Sagdeev potential function $\psi(\phi)$ as a function of $\phi$ and their corresponding double layer solutions $\phi_{\rm DL}$ as functions of $\eta$. All these are obtained with respect to the slow mode phase speed and the range of this slow mode phase speed follows the inequality $v_{\rm tc} < v_{\rm slow} < v_{\rm th}$ where $v_{\rm tc}$, $v_{\rm th}$ and $v_{\rm slow}$ are, respectively the non-normalized thermal speed of the colder positive ions, hotter negative ions and slow mode phase speed. Actually the phase speed of slow mode solitons depends on the choice of normalization and plasma parameters. The arbitrary parameters used in this problem satisfy the boundary conditions, charge neutrality conditions and the conditions of solitary wave solutions. In this problem, the different slow mode phase speeds ($V_S$) are taken for showing the effects of the slow mode solitons such that they always satisfy the normalized phase speed inequality $\sqrt{\frac{3\sigma_i}{n_{io}}} < V_S < \sqrt{\frac{3\sigma_j}{Qn_{jo}}} < V_F$. Again the slow mode compressive double layers are found from such values of the slow mode phase velocities ($V_S$) which exceed the upper limiting values of the slow mode solitons.

It is found in this plasma model that slow ion-acoustic solitons appear from the value of the slow mode phase velocity $V_S = 0.93$ to $1.20$ under some restrictions. It means that at some fixed values of $n_{jo}$, $\sigma_i$, $\sigma_j$ and $Q$, slow ion-acoustic solitons occur for some values of $V\,(= V_S)$ which exceed the critical values $V_{\rm crit}$. Slow ion-acoustic solitons cease to exist for such a value of $V\,(= V_S)$ which either lies on $V(= V_S) = 1.21$ or above it such that for $V_S \geq 1.21$, the Sagdeev potential function $\psi(\phi)$ no longer has a positive root. Moreover, for fully non-linear standard slow mode solitons, it is observed in this plasma model that the temperature of negative ions ($\sigma_j$) is greater than that of positive ions ($\sigma_i$).

Fig. 1, *a* shows the Sagdeev potential function $\psi(\phi)$ as a function of $\phi$ for $V\,(= V_S) = 0.93$, $0.94$ and $0.96$ denoted respectively by the curves $c_8$, $c_9$ and $c_{10}$ when $n_{jo} = 0.05$, $n_{i0} = 0.88$, $u_{i0} = 0$, $u_{j0} = 0$, $\sigma_i = \frac{1}{100}$,



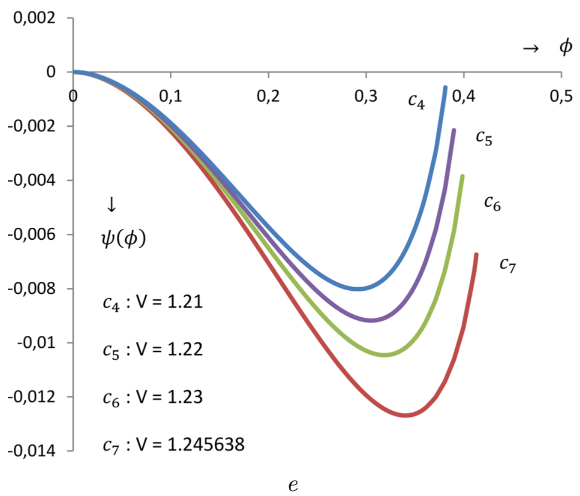




***Fig. 1.*** Sagdeev potential function $\psi(\phi)$ as a function of $\phi$ for slow mode compressive soliton in a two-temperature non-isothermal electron plasma under variation of $V\,(=V_S)$ for $n_{j0}=0.05$, $n_{i0}=0.88$, $u_{i0}=0$, $u_{j0}=0$, $Q=1.9$, $\chi=0.17$, $\sigma_p=0.41$, $\beta_1=0.25$, $Z=1$, $\mu=0.15$, $\nu=0.85$, $b_l=0.15$, $b_h=0.4$, $b_l^{(1)}=0.24$, $b_h^{(1)}=0.51$: $\sigma_i=\frac{1}{100}$, $\sigma_j=\frac{1}{30}$ when $V\,(=V_S)=0.93$, 0.94, 0.96 (*a*), $V\,(=V_S)=0.95$, 0.97 (*b*); $\sigma_i=\frac{1}{30}$, $\sigma_j=\frac{1}{20}$ when $V\,(=V_S)=1.01$, 1.058426, 1.106725, 1.152678 (*c*), when $V\,(=V_S)=1.16$, 1.17, 1.18, 1.19, 1.20 (*d*), when $V\,(=V_S)=1.21$, 1.22, 1.23, 1.245638 (*e*)

$\sigma_j = \frac{1}{30}$, $\chi = 0.17$, $\sigma_p = 0.41$, $Q = 1.9$, $\beta_1 = 0.25$, $\mu = 0.15$, $\nu = 0.85$, $b_l = 0.15$, $b_h = 0.4$, $b_l^{(1)} = 0.24$, $b_h^{(1)} = 0.51$, $Z = 1$. The amplitudes of slow mode solitons increase for higher values of $V\,(= V_S)$.

Fig. 1, *b* shows the Sagdeev potential function $\psi\,(\phi)$ as a function of $\phi$ for $V\,(= V_S) = 0.95$, $0.97$ denoted respectively by the curves $d_4$ and $d_5$ when $n_{j0} = 0.05$, $n_{i0} = 0.88$, $u_{i0} = 0$, $u_{j0} = 0$, $\sigma_i = \frac{1}{100}$, $\sigma_j = \frac{1}{30}$, $Q = 1.9$, $\chi = 0.17$, $\sigma_p = 0.41$, $\beta_1 = 0.25$, $Z = 1$, $\mu = 0.15$, $\nu = 0.85$, $b_l = 0.15$, $b_h = 0.4$, $b_l^{(1)} = 0.24$, $b_h^{(1)} = 0.51$. In this case, the amplitude of the curve $d_5$ is larger than the amplitude of the curve $d_4$ for these plasma parameters.

In Fig. 1, *c*, the Sagdeev potential function $\psi\,(\phi)$ as a function of $\phi$ for large amplitude slow mode solitons is obtained for $V\,(= V_S) = 1.01$, $1.058426$, $1.106725$ and $1.152678$ denoted respectively by the curves $a_9$, $a_{10}$, $a_{11}$ and $a_{12}$ when $n_{jo} = 0.05$, $n_{i0} = 0.88$, $u_{i0} = 0$, $u_{j0} = 0$, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{20}$, $\chi = 0.17$, $\sigma_p = 0.41$, $Q = 1.9$, $\beta_1 = 0.25$, $\mu = 0.15$, $\nu = 0.85$, $b_l = 0.15$, $b_h = 0.4$, $b_l^{(1)} = 0.24$, $b_h^{(1)} = 0.51$, $Z = 1$. It is also observed from this figure that the amplitudes are larger for higher values of $V\,(= V_S)$.

Fig. 1, *d* shows the fully nonlinear large amplitude slow mode solitons from Sagdeev potential function $\psi\,(\phi)$ as a function of $\phi$ for $V\,(= V_S) = 1.16$, $1.17$, $1.18$, $1.19$ and $1.20$ denoted respectively by the curves $b_5$, $b_6$, $b_7$, $b_8$ and $b_9$ when $n_{jo} = 0.05$, $n_{i0} = 0.88$, $u_{i0} = 0$, $u_{j0} = 0$, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{20}$, $\chi = 0.17$, $\sigma_p = 0.41$, $Q = 1.9$, $\beta_1 = 0.25$, $\mu = 0.15$, $\nu = 0.85$, $b_l = 0.15$, $b_h = 0.4$, $b_l^{(1)} = 0.24$, $b_h^{(1)} = 0.51$, $Z = 1$. Once the upper limit V$(= V_S) = 1.20$ corresponding to $n_{j0} = 0.05$ has been exceeded, slow ion-acoustic solitons are no longer possible such as for $V\,(= V_S) = 1.21$ shown in Fig. 1, *e*, the Sagdeev potential function $\psi\,(\phi)$ becomes complex valued long before a positive root of $\psi\,(\phi)$ can be attained.

In Fig. 1, *e*, the Sagdeev potential function $\psi\,(\phi)$ as a function of $\phi$ for $V\,(= V_S) = 1.21$, $1.22$, $1.23$ and $1.245638$ denoted respectively by the curves $c_4$, $c_5$, $c_6$ and $c_7$ is shown for the non-streaming ions when $n_{jo} = 0.05$, $n_{i0} = 0.88$, $u_{i0} = 0$, $u_{j0} = 0$, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{20}$, $\chi = 0.17$, $\sigma_p = 0.41$, $Q = 1.9$, $\beta_1 = 0.25$, $\mu = 0.15$, $\nu = 0.85$, $b_l = 0.15$, $b_h = 0.4$, $b_l^{(1)} = 0.24$, $b_h^{(1)} = 0.51$, $Z = 1$. It is found from this figure that the slow mode soliton formation disappears for these values of $V\,(= V_S)$, since the concentrations are complex after crossing the said values of $V\,(= V_S) = 1.20$ for this plasma model.

In connection with this plasma model, the other important topic is the slow mode compressive solitary wave solutions of first ($\phi_1$) and second ($\phi_2$) orders as a function of $\eta$ which are mentioned in Figs. 2 to 3.

Fig. 2, *a* represents the first-order ($\phi_1$) slow mode compressive solitary wave solutions as a function of $\eta$ under the variation of $V\,(= V_S)$ for $n_{j0} = 0.05$, $n_{i0} = = 0.88$, $u_{i0} = 0$, $u_{j0} = 0$, $Q = 1.9$, $\chi = 0.17$, $\sigma_p = 0.41$, $\beta_1 = 0.25$, $Z = 1$, $\mu = 0.15$, $\nu = 0.85$, $b_l = 0.15$, $b_h = 0.4$, $b_l^{(1)} = 0.24$, $b_h^{(1)} = 0.51$, $\sigma_i = = \frac{1}{30}$, $\sigma_j = \frac{1}{20}$ when $V\,(= V_S) = 1.058426$, $1.106725$, $1.152678$ and $1.16$ denoted respectively by the curves $g_1$, $g_2$, $g_3$ and $g_4$. The first-order slow mode soliton attains larger values as V gradually increases. In this figure, the author is only showing the effect of $V\,(= V_S)$ on the first-order slow mode compressive solitary wave solutions.

Similarly Fig. 2, *b* shows the second-order ($\phi_2$) slow mode compressive solitary wave solutions as a function of $\eta$ under the variation of $V\,(= V_S)$ for $n_{j0} = = 0.05$, $n_{i0} = 0.88$, $u_{i0} = 0$, $u_{j0} = 0$, $Q = 1.9$, $\chi = 0.17$, $\sigma_p = 0.41$, $\beta_1 = 0.25$, $Z = 1$, $\mu = 0.15$, $\nu = 0.85$, $b_l = 0.15$, $b_h = 0.4$, $b_l^{(1)} = 0.24$, $b_h^{(1)} = 0.51$, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{20}$ when $V(= V_S) = 1.058426$, $1.106725$, $1.152678$ and $1.16$ denoted respectively by the curves $g_5$, $g_6$, $g_7$ and $g_8$. In this plasma model, it is seen in this figure that the highest and lowest values of the second-order solitary wave solution profiles are observed when $V\,(= V_S) = 1.16$ and $V\,(= V_S) = 1.058426$ at which the slow mode solitons exist perfectly.

In Fig. 3, *a*, the first-order ($\phi_1$) slow mode compressive solitary wave solutions as a function of $\eta$ are shown under variation of $V\,(= V_S)$ for $n_{j0} = 0.05$, $n_{i0} = 0.88$, $u_{i0} = 0$, $u_{j0} = 0$, $Q = 1.9$, $\chi = 0.17$, $\sigma_p = 0.41$, $\beta_1 = 0.25$, $Z = 1$, $\mu = 0.15$, $\nu = 0.85$, $b_l = 0.15$, $b_h = 0.4$, $b_l^{(1)} = 0.24$, $b_h^{(1)} = 0.51$, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{20}$ when $V\,(= V_S) = 1.17$, $1.18$, $1.19$ and $1.20$ denoted respectively by the curves $h_1$, $h_2$, $h_3$ and $h_4$. As the values of $V\,(= V_S)$ increase, the peak of the first-order slow mode solitary wave solution ($\phi_1$) profile increases gradually, reaching its maximum when $V\,(= V_S)$ attains the value 1.20 at which slow mode soliton exists and it is also mentioned in this plasma model that the slow mode solitons will no longer be observed when $V\,(= V_S) > 1.20$.

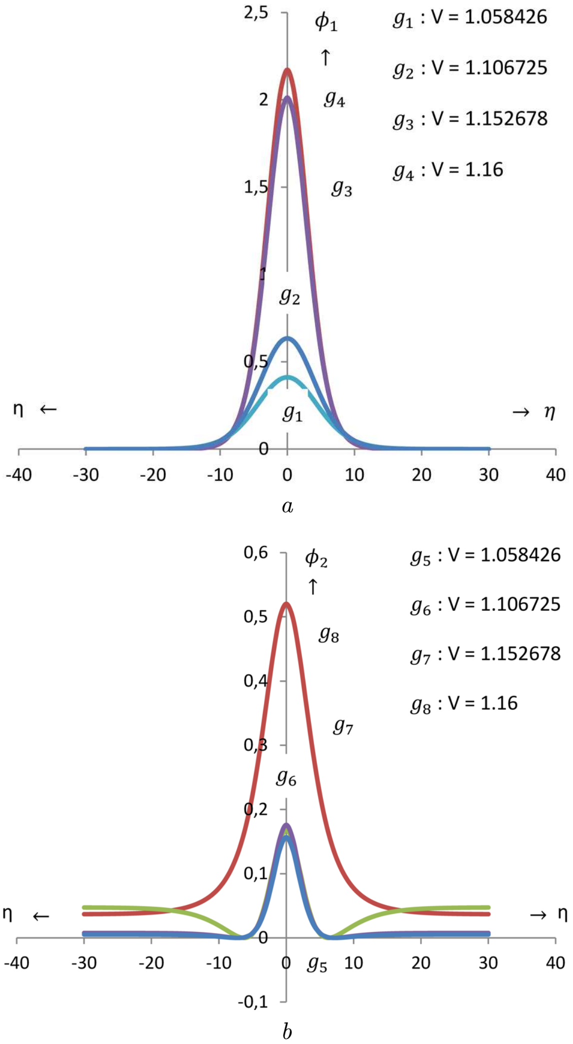



***Fig. 2.*** First-order slow mode solitary wave profiles $\phi_1$ as functions of $\eta$ for two-temperature non-isothermal electron plasma with varying $V\,(= V_S)$ for $n_{j0} = 0.05$, $n_{i0} = 0.88$, $u_{i0} = 0$, $u_{j0} = 0$, $Q = 1.9$, $\chi = 0.17$, $\sigma_p = 0.41$, $\beta_1 = 0.25$, $Z = 1$, $\mu = 0.15$, $\nu = 0.85$, $b_l = 0.15$, $b_h = 0.4$, $b_l^{(1)} = 0.24$, $b_h^{(1)} = 0.51$, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{20}$ when $V\,(= V_S) = 1.058426$, 1.106725, 1.152678, 1.16 (*a*). Second-order slow mode solitary wave solutions $\phi_2$ as functions of $\eta$ for two-temperature non-isothermal electron plasma under variation of $V(= V_S)$ for $n_{j0} = 0.05$, $n_{i0} = 0.88$, $u_{i0} = 0$, $u_{j0} = 0$, $Q = 1.9$, $\chi = 0.17$, $\sigma_p = 0.41$, $\beta_1 = 0.25$, $Z = 1$, $\mu = 0.15$, $\nu = 0.85$, $b_l = 0.15$, $b_h = 0.4$, $b_l^{(1)} = 0.24$, $b_h^{(1)} = 0.51$, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{20}$ when $V(= V_S) = 1.058426$, 1.106725, 1.152678, 1.16 (*b*)

Fig. 3, *b* shows the second-order ($\phi_2$) slow mode compressive solitary wave solutions as functions of $\eta$ under variation of $V\,(= V_S)$ for $n_{j0} = 0.05$,

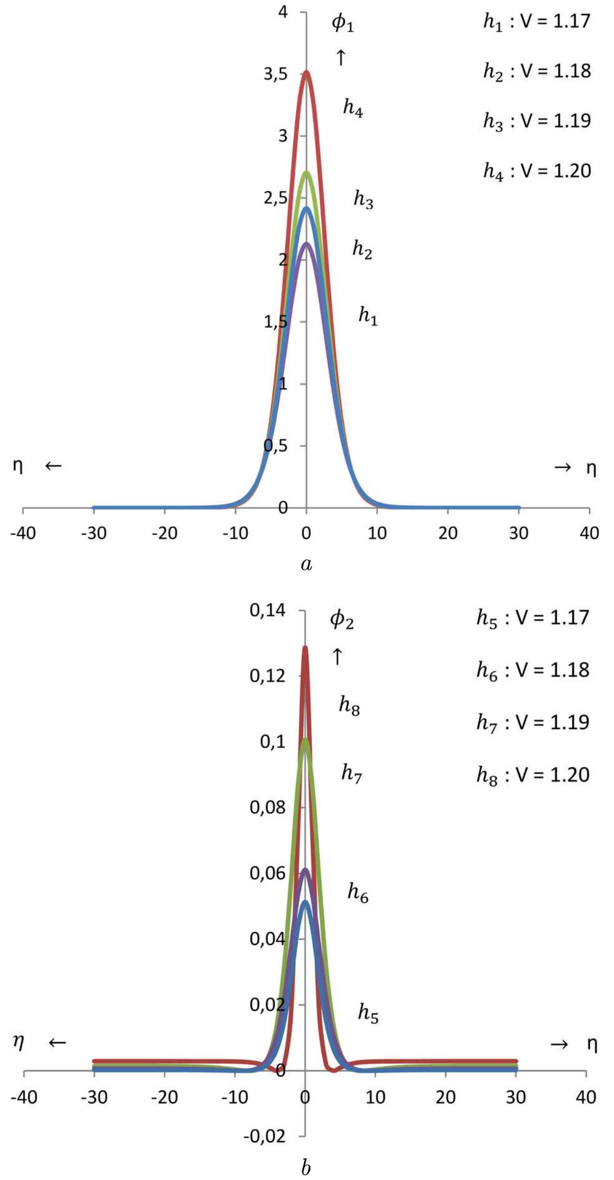



***Fig. 3.*** First order slow mode solitary wave solutions ($\phi_1$) as functions of $\eta$ with varying $V\,(= V_S)$ in two-temperature non-isothermal electron plasma for $n_{j0} = 0.05$, $n_{i0} = 0.88$, $u_{i0} = 0$, $u_{j0} = 0$, $Q = 1.9$, $\chi = 0.17$, $\sigma_p = 0.41$, $\beta_1 = 0.25$, $Z = 1$, $\mu = 0.15$, $\nu = 0.85$, $b_l = 0.15$, $b_h = 0.4$, $b_l^{(1)} = 0.24$, $b_h^{(1)} = 0.51$, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{20}$ when $V\,(= V_S) = 1.17$, 1.18, 1.19, 1.20 (*a*). Second order slow mode ion-acoustic solitary wave solutions ($\phi_2$) as functions of $\eta$ under variation of $V\,(= V_S)$ in a two-temperature non-isothermal electron plasma for $n_{j0} = 0.05$, $n_{i0} = 0.88$, $u_{i0} = 0$, $u_{j0} = 0$, $Q = 1.9$, $\chi = 0.17$, $\sigma_p = 0.41$, $\beta_1 = 0.25$, $Z = 1$, $\mu = 0.15$, $\nu = 0.85$, $b_l = 0.15$, $b_h = 0.4$, $b_l^{(1)} = 0.24$, $b_h^{(1)} = 0.51$, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{20}$ when $V\,(= V_S) = 1.17$, 1.18, 1.19, 1.20 (*b*)

$n_{i0} = 0.88$, $u_{i0} = 0$, $u_{j0} = 0$, $Q = 1.9$, $\chi = 0.17$, $\sigma_p = 0.41$, $\beta_1 = 0.25$, $Z = 1$, $\mu = 0.15$, $\nu = 0.85$, $b_l = 0.15$, $b_h = 0.4$, $b_l^{(1)} = 0.24$, $b_h^{(1)} = 0.51$, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{20}$ when $V\,(= V_S) = 1.17$, 1.18, 1.19 and 1.20 denoted respectively by the curves $h_5$, $h_6$, $h_7$ and $h_8$. The second-order slow mode compressive solitary wave solutions $(\phi_2)$ increase gradually for larger values of $V\,(= V_S)$, provided that slow mode solitary wave solutions must exist for those larger values. The highest value of the slow mode soliton solution profile is achieved for highest value of $V\,(= V_S) = 1.20$ and for $V\,(= V_S) > 1.20$, the second-order slow mode soliton profile is not observed in this plasma model.

The author now considers the small amplitude slow mode compressive double layers in two-temperature non-isothermal electron plasmas. The compressive double layers occur when the slow mode phase velocity $V\,(= V_S)$ of the nonlinear wave structure exceeds the upper limiting values of soliton velocity beyond which the number density of the cool ions $(n_i)$ becomes complex-valued. A theoretical examination for different values of soliton velocity $(V)$ at some values of the concerned plasma parameters, shows that the double layer occurs at some values of $V = V_{dl}\,(> V_S)$ with amplitude $\phi = \phi_{dl}$. This indicates the end of the soliton velocity range before the sonic point with limiting electrostatic potential $\phi = \phi_{lp}$ is reached where $\phi_{dl} < \phi_{lp}$ and concentrations of ions must be real-valued at $\phi = \phi_{lp}$ which defines the existence region of double layers. After crossing $\phi = \phi_{lp}$, the concentrations of ions are no longer real-valued and the double layers do not exist in those regions for this reason.

The profiles of small amplitude slow mode compressive double layers obtained from the Sagdeev potential function $\psi\,(\phi)$ as a function of $\phi$ and its corresponding double layer solutions $\phi_{\mathrm{DL}}$ as functions of $\eta$ under variation of the phase velocity $V = V_{dl}\,(> V_S)$ in two-temperature non-isothermal electron plasmas containing warm positrons, warm negative and positive ions for this model plasmas are shown respectively in Fig. 4 to Fig. 8.

In Fig. 4, *a*, the small amplitude slow mode compressive double layers obtained from the Sagdeev potential function $\psi\,(\phi)$ as a function of $\phi$ are shown under variation of $V = V_{dl}\,(> V_S)$ for $n_{j0} = 0.05$, $n_{i0} = 0.88$, $u_{i0} = 0$, $u_{j0} = 0$, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{20}$ and $Q = 1.9$ when $V = V_{dl}\,(> V_S) = 1.65$, 1.7, 1.75 and 1.8 denoted respectively by the curves $s_1$, $s_2$, $s_3$ and $s_4$. In this figure, the double layers are considered in an existence domain where the concentration of positive ions $(n_i)$ is real-valued so that the slow mode compressive double layers are observed. It is also observed that the amplitudes of the slow mode double layers corresponding to the curves $s_1$, $s_2$, $s_3$ and $s_4$ decrease for higher values of $V = V_{dl}\,(> V_S)$. In this case, the nonlinearity weakens and the potential difference across the double layer decreases, leading to reduced amplitude of the electrostatic potential.

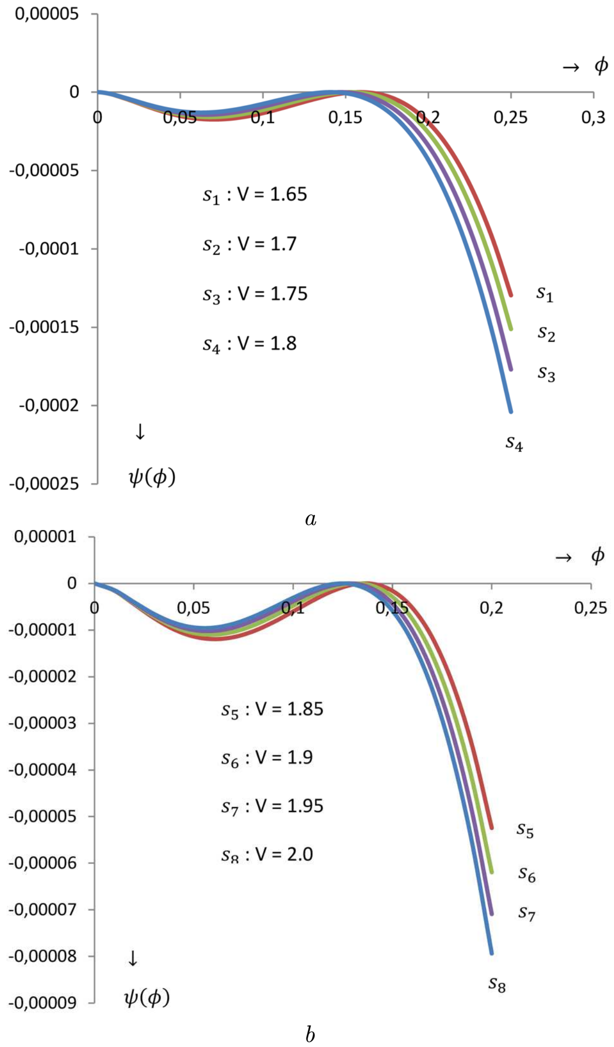



***Fig. 4.*** Sagdeev potential function $\psi\,(\phi)$ as a function of $\phi$ for small amplitude slow mode double layers with varying $V = V_{dl}\,(> V_S)$ for $n_{jo} = 0.05$, $n_{io} = 0.88$, $Q = 1.9$, $\chi = 0.17$, $\sigma_p = 0.41$, $\beta_1 = 0.25$, $Z = 1$, $\mu = 0.15$, $\nu = 0.85$, $b_l = 0.15$, $b_h = 0.4$, $u_{i0} = 0$, $u_{j0} = 0$, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{20}$: when $V = V_{dl}\,(> V_S) = 1.65$, 1.7, 1.75, 1.8 (*a*), when $V = V_{dl}\,(> V_S) = 1.85$, 1.9, 1.95, 2.0 (*b*)

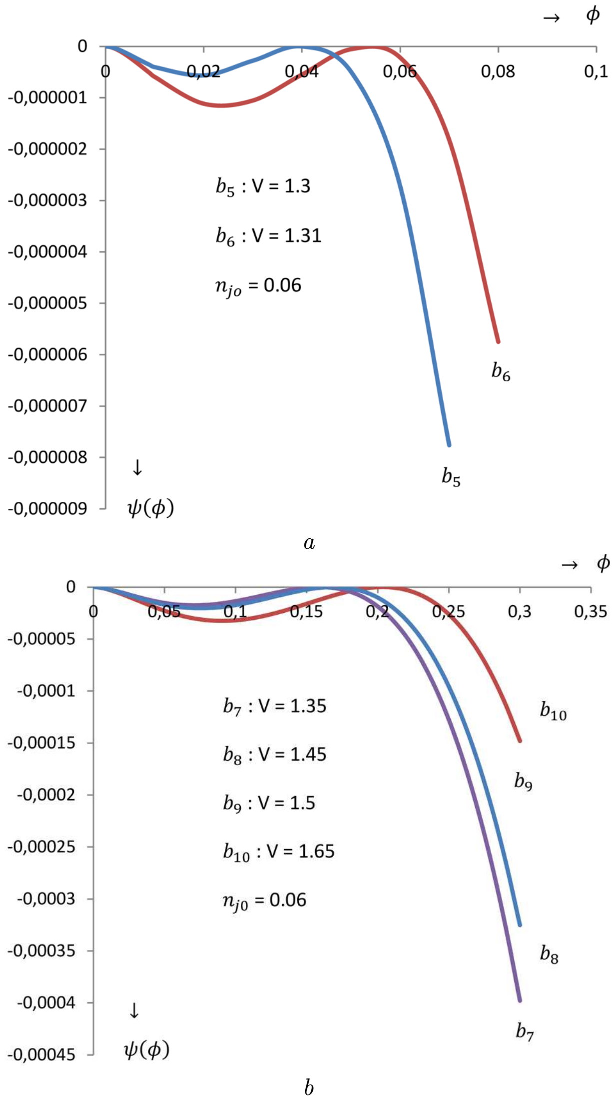



***Fig. 5.*** Sagdeev potential function $\psi(\phi)$ as a function of $\phi$ for small amplitude slow mode double layers under the variation of $V = V_{dl} (> V_S)$ for $n_{jo} = 0.06$, $n_{io} = 0.89$, $Q = 1.9$, $\chi = 0.17$, $\sigma_p = 0.41$, $\beta_1 = 0.25$, $Z = 1$, $\mu = 0.15$, $\nu = 0.85$, $b_l = 0.15$, $b_h = 0.4$, $u_{i0} = 0$, $u_{j0} = 0$, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{20}$: when $V = V_{dl} (> V_S) = 1.3$, 1.31 (*a*), when $V = V_{dl} (> V_S) = 1.35$, 1.45, 1.5, 1.65 (*b*)

Fig. 4, *b* shows the small amplitude slow mode compressive double layers obtained from the Sagdeev potential function $\psi(\phi)$ as a function of $\phi$ with varying $V = V_{dl} (> V_S)$ for $n_{j0} = 0.05$, $n_{i0} = 0.88$, $u_{i0} = 0$, $u_{j0} = 0$, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{20}$ and $Q = 1.9$ with $V = V_{dl} (> V_S) = 1.85$, 1.90, 1.95 and 2.0 denoted respectively by the curves $s_5$, $s_6$, $s_7$ and $s_8$. The slow mode compressive double layers are observed for the above values of $V = V_{dl} (> V_S)$ where the positive ion density $(n_i)$ will not be complex valued. Again the amplitudes of the respective double layers decrease for higher values of $V = V_{dl} (> V_S)$.

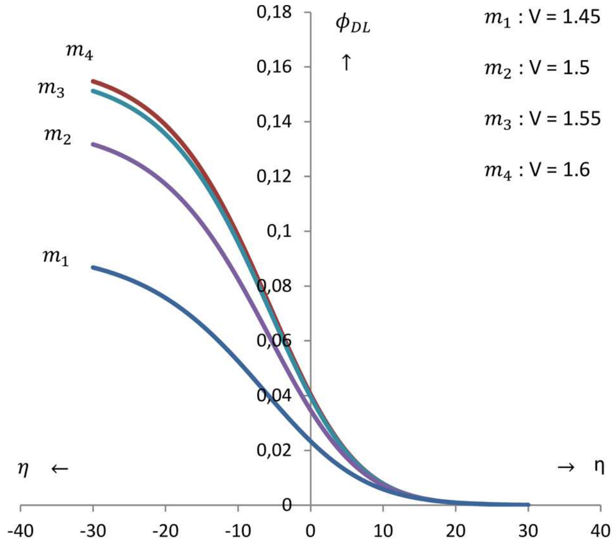



***Fig. 6.*** Small amplitude slow mode double layer solutions ($\phi_{\mathrm{DL}}$) as functions of $\eta$ with varying $V = V_{dl} (> V_S)$ for $n_{jo} = 0.05$, $n_{io} = 0.88$, $Q = 1.9$, $\chi = 0.17$, $\sigma_p = 0.41$, $\beta_1 = 0.25$, $Z = 1$, $\mu = 0.15$, $\nu = 0.85$, $b_l = 0.15$, $b_h = 0.4$, $u_{i0} = 0$, $u_{j0} = 0$, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{20}$ when $V = V_{dl} (> V_S) = 1.45$, 1.5, 1.55, 1.6

Fig. 5, *a* represents the small amplitude slow mode compressive double layers obtained from the Sagdeev potential function $\psi(\phi)$ as a function of $\phi$ with varying $V = V_{dl} (> V_S)$ for another concentration of negative ions $n_{j0} = 0.06$ which is slightly greater than the previous value $n_{j0} = 0.05$, for $u_{i0} = 0$, $u_{j0} = 0$, $n_{io} = 0.89$, $\chi = 0.17$, $\sigma_p = 0.41$, $\mu = 0.15$, $\nu = 0.85$, $b_l = 0.15$, $b_h = 0.4$, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{20}$, $\beta_1 = 0.25$, $Z = 1$ and $Q = 1.9$ with $V = V_{dl} (> V_S) = 1.3$ and 1.31 denoted respectively by the curves $b_5$ and $b_6$.

Fig. 5, *b* shows the small amplitude slow mode compressive double layers obtained from the Sagdeev potential function $\psi(\phi)$ as a function of $\phi$ with varying $V = V_{dl} (> V_S)$ for another concentration of negative ion $n_{j0} = 0.06$ which is slightly greater than the previous value $n_{j0} = 0.05$ for $u_{i0} = 0$, $u_{j0} = 0$, $n_{io} = 0.89$, $\chi = 0.17$, $\sigma_p = 0.41$, $\mu = 0.15$, $\nu = 0.85$, $b_l = 0.15$, $b_h = 0.4$, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{20}$, $\beta_1 = 0.25$, $Z = 1$ and $Q = 1.9$ with $V = V_{dl} (> V_S) = 1.35$, 1.45, 1.50 and 1.65 denoted respectively by the curves $b_7$, $b_8$, $b_9$ and $b_{10}$.

In Fig. 6, the small amplitude slow mode compressive double layer solutions $\phi_{\rm DL}$ as functions of $\eta$ are shown with varying $V = V_{dl}\,(> V_S)$ for $u_{i0} = 0$, $u_{j0} = 0$, $n_{j0} = 0.05$, $n_{i0} = 0.88$, $\chi = 0.17$, $\sigma_p = 0.41$, $\mu = 0.15$, $\nu = 0.85$, $b_l = 0.15$, $b_h = 0.4$, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{20}$, $\beta_1 = 0.25$, $Z = 1$ and $Q = 1.9$ when $V = V_{dl}\,(> V_S) = 1.45$, 1.5, 1.55 and 1.6 denoted respectively by the curves $m_1$, $m_2$, $m_3$ and $m_4$. As $V = V_{dl}\,(> V_S)$ gradually increases, the respective curves intersect the $\phi_{\rm DL}$-axis at higher values than the previous ones and it attains the maximum value when $V = V_{dl}\,(> V_S)$ equals 1.6.

Fig. 7, *a* shows the small amplitude slow mode compressive double layer solutions $\phi_{\rm DL}$ as functions of $\eta$ with varying $V = V_{dl}\,(> V_S)$ for $u_{i0} = 0$, $u_{j0} = 0$, $n_{j0} = 0.05$, $n_{i0} = 0.88$, $\chi = 0.17$, $\sigma_p = 0.41$, $\mu = 0.15$, $\nu = 0.85$, $b_l = 0.15$, $b_h = 0.4$, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{20}$, $\beta_1 = 0.25$, $Z = 1$ and $Q = 1.9$ when $V = V_{dl}\,(> V_S) = 1.65$, 1.7, 1.75 and 1.8 denoted respectively by the curves $m_5$, $m_6$, $m_7$ and $m_8$. The curve $m_8$ for $V = V_{dl}\,(> V_S) = 1.8$ intersects the $\phi_{\rm DL}$-axis at higher values than the defined curves $m_5$ at $V = V_{dl}\,(> V_S) = 1.65$, $m_6$ at $V = V_{dl}\,(> V_S) = 1.7$ and $m_7$ at $V = V_{dl}\,(> V_S) = 1.75$. As $V = V_{dl}\,(> V_S)$ increases from 1.65 to 1.8, double layer amplitude decreases.

In Fig. 7, *b*, the small amplitude slow mode compressive double layer solutions $\phi_{\rm DL}$ as functions of $\eta$ are shown with varying $V = V_{dl}\,(> V_S)$ for $u_{i0} = 0$, $u_{j0} = 0$, $n_{j0} = 0.05$, $n_{i0} = 0.88$, $\chi = 0.17$, $\sigma_p = 0.41$, $\mu = 0.15$, $\nu = 0.85$, $b_l = 0.15$, $b_h = 0.4$, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{20}$, $\beta_1 = 0.25$, $Z = 1$ and $Q = 1.9$ when $V = V_{dl}\,(> V_S) = 1.85$, 1.9, 1.95 and 2.0 denoted respectively by the curves $n_1$, $n_2$, $n_3$ and $n_4$. It is further noted that the amplitudes of the double layers decrease when $V = V_{dl}\,(> V_S)$ is increasing from 1.85 to 2.0.

In Fig. 8, *a*, the small amplitude slow mode compressive double layer solutions $\phi_{\rm DL}$ as functions of $\eta$ are shown with varying $V = V_{dl}\,(> V_S)$ for $u_{i0} = 0$, $u_{j0} = 0$, $n_{j0} = 0.05$, $n_{i0} = 0.88$, $\chi = 0.17$, $\sigma_p = 0.41$, $\mu = 0.15$, $\nu = 0.85$, $b_l = 0.15$, $b_h = 0.4$, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{20}$, $\beta_1 = 0.25$, $Z = 1$ and $Q = 1.9$ when $V = V_{dl}\,(> V_S) = 1.29$ and 1.3 denoted respectively by the curves $e_7$ and $e_8$. The curve $e_8$ intersects the $\phi_{\rm DL}$-axis at larger values from the origin than the curve $e_7$.

Fig. 8, *b* shows the small amplitude slow mode compressive double layer solutions $\phi_{\rm DL}$ as functions of $\eta$ with varying $V = V_{dl}\,(> V_S)$ for $u_{i0} = 0$, $u_{j0} = 0$, $n_{j0} = 0.05$, $n_{i0} = 0.88$, $\chi = 0.17$, $\sigma_p = 0.41$, $\mu = 0.15$, $\nu = 0.85$, $b_l = 0.15$, $b_h = 0.4$, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{20}$, $\beta_1 = 0.25$, $Z = 1$ and $Q = 1.9$ when $V = V_{dl}\,(> V_S) = 1.35$ and 1.4 denoted respec-

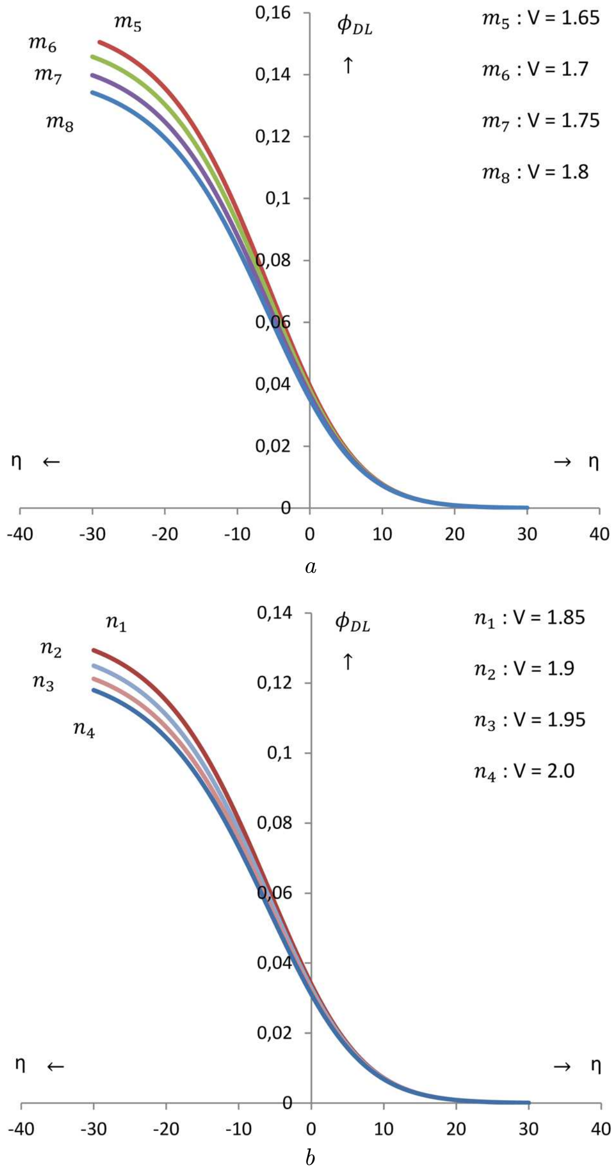
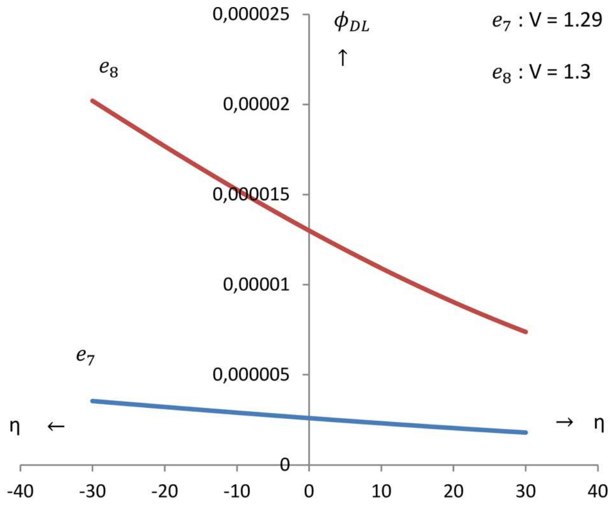



***Fig. 7.*** Small amplitude slow mode double layer solutions ($\phi_{\rm DL}$) as functions of $\eta$ with varying $V = V_{dl}\,(> V_S)$ for $n_{jo} = 0.05$, $n_{io} = 0.88$, $Q = 1.9$, $\chi = 0.17$, $\sigma_p = 0.41$, $\beta_1 = 0.25$, $Z = 1$, $\mu = 0.15$, $\nu = 0.85$, $b_l = 0.15$, $b_h = 0.4$, $u_{i0} = 0$, $u_{j0} = 0$, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{20}$: when $V = V_{dl}\,(> V_S) = 1.65$, 1.7, 1.75, 1.8 (*a*), when $V = V_{dl}\,(> V_S) = 1.85$, 1.9, 1.95, 2.0 (*b*)



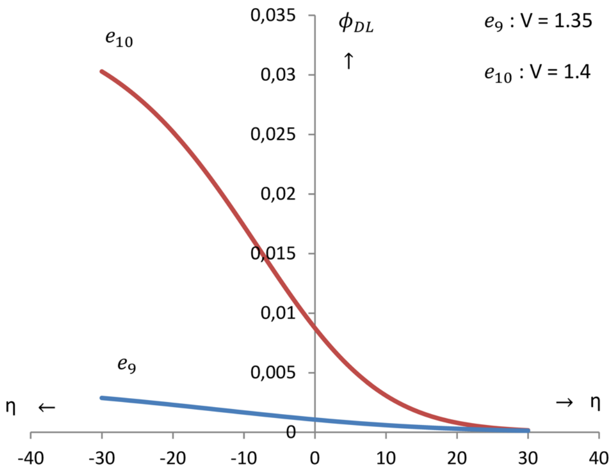




***Fig. 8.*** Small amplitude slow mode double layer solutions ($\phi_{\mathrm{DL}}$) as functions of $\eta$ with varying $V = V_{dl}\,(> V_S)$ for $n_{jo} = 0.05$, $n_{io} = 0.88$, $Q = 1.9$, $\chi = 0.17$, $\sigma_p = 0.41$, $\beta_1 = 0.25$, $Z = 1$, $\mu = 0.15$, $\nu = 0.85$, $b_l = 0.15$, $b_h = 0.4$, $u_{i0} = 0$, $u_{j0} = 0$, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{20}$: when $V = V_{dl}\,(> V_S) = 1.29$, 1.3 ($a$), when $V = V_{dl}\,(> V_S) = 1.35$, 1.4 ($b$)

tively by the curves $e_9$ and $e_{10}$. The curve $e_{10}$ in this figure intersects the $\phi_{\mathrm{DL}}$-axis at higher values than the curve $e_9$.

## 6. Conclusions

In this paper, the author studies the large amplitude slow mode compressive solitons and small amplitude slow mode compressive double layers in two-temperature non-isothermal electron plasmas containing warm adiabatic positive and negative ions and warm isothermal positrons using the Sagdeev pseudopotential method. This model is applicable to the plasma sheet boundary layer of the Earth's magnetosphere and auroral plasmas. In this model, the slow mode is found when the thermal speeds of positive and negative ion species are different. For the plasma composition ($Ar^+$, $SF_6^-$), the present author investigates the large amplitude slow mode compressive solitons and small amplitude slow mode compressive double layers using the Sagdeev pseudopotential function $\psi(\phi)$, first-order ($\phi_1$) as well as second-order ($\phi_2$) solitary wave solutions supported by the allowed slow mode phase velocities ($V_S$) which satisfy the inequality $\sqrt{\frac{3\sigma_i}{n_{io}}} < V_S < \sqrt{\frac{3\sigma_j}{Qn_{jo}}}$ for the arbitrarily chosen set of parameters. It is important to point out that there also exists upper values of $V\,(= V_S)$ for solitons where the ion number density should not become complex valued but the larger value of $V\,(> V_S)$ compared with the upper limit of the soliton value of $V\,(= V_S)$, lead to the formation of double layers. The small amplitude slow mode double layer profiles obtained from the Sagdeev potential function $\psi(\phi)$ and double layer solutions $\phi_{\mathrm{DL}}$ under the variation of the different values of $V_S$ show the different characteristics of these nonlinear wave structures.

In this paper, some important observations are made for slow mode structures:

1. When ion densities are real and $Q > 1$, $\sigma_j > \sigma_i$, large amplitude slow mode compressive solitons are observed for the inequality $\sqrt{\frac{3\sigma_i}{n_{io}}} < V_S < \sqrt{\frac{3\sigma_j}{Qn_{jo}}}$.

2. Large amplitude compressive slow mode solitons are observed from $V\,(= V_S) = 0.93$ to $0.97$ when $n_{jo} = 0.05$, $\sigma_i = \frac{1}{100}$, $\sigma_j = \frac{1}{30}\,(\sigma_j > \sigma_i)$, $Q = 1.9$, $\chi = 0.17$ and from $V\,(= V_S) = 1.01$ to $1.20$ when $n_{jo} = 0.05$, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{20}\,(\sigma_j > \sigma_i)$, $Q = 1.9$, $\chi = 0.17$. The values of $V\,(= V_S)$ differ due to the changes in the temperatures of positive and negative ions.

3. The existence domain of large amplitude slow mode compressive solitons is found for $0.93 \le V_S \le 0.97$ when $n_{jo} = 0.05$, $\sigma_i = \frac{1}{100}$, $\sigma_j = \frac{1}{30}\,(\sigma_j > \sigma_i)$, $Q = 1.9$ and it will be again found for $1.01 \le V_S \le 1.20$ when $n_{jo} = 0.05$, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{20}\,(\sigma_j > \sigma_i)$, $Q = 1.9$. The same large amplitude slow mode compressive solitons is not found for $V_S \ge 1.21$ when $n_{jo} = 0.05$, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{20}\,(\sigma_j > \sigma_i)$, $Q = 1.9$ and this is possible due to the change of the temperature of positive and negative ions.

4. Small amplitude slow mode compressive double layers are found from $V = V_{dl} (> V_S) = 1.3$ to 1.65 when $n_{jo} = 0.06$, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{20}$ $(\sigma_j > \sigma_i)$, $Q = 1.9$, $\chi = 0.17$, $\sigma_p = 0.41$ and again the same is found from $V = V_{dl} (> V_S) = 1.65$ to 2.0 when $n_{jo} = 0.05$, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{20}$ $(\sigma_j > \sigma_i)$, $Q = 1.9$, $\chi = 0.17$, $\sigma_p = 0.41$

5. No solitons are found for $V > V_{dl}$ where $V$ is the soliton velocity and $V_{dl}$ is the double layer velocity for $n_{jo} = 0.05$, $\sigma_i = \frac{1}{30}$, $\sigma_j = \frac{1}{20}$ $(\sigma_j > \sigma_i)$, $Q = 1.9$, $\chi = 0.17$, $\sigma_p = 0.41$. This indicates that the solitons velocity range is terminated by the occurrence of the double layers.

The slow mode nonlinear wave structure help in understanding the complex dynamics of plasmas in astrophysical systems or laboratory experiments. The study of slow mode solitons and double layers helps interpret observations in space plasmas, plasma technology, fusion research and laboratory experiments. Slow mode solitons and double layers are associated with stability and energy transport processes, wave propagation and particle acceleration occurring both in fundamental plasma physics and practical applications.

Future work of the author is to investigate the large amplitude compressive slow mode double layers in magnetized plasmas with two-temperature non-isothermal electrons.

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ВПЛИВ ФАЗОВОЇ ШВИДКОСТІ ПОВІЛЬНОЇ МОДИ НА НЕЛІНІЙНІ ХВИЛЬОВІ СТРУКТУРИ У ДВОТЕМПЕРАТУРНІЙ НЕІЗОТЕРМІЧНІЙ ПЛАЗМІ

У беззіткненнєвій, ненамагніченій нерелятивістичній плазмі, що складається з теплих адіабатичних позитивних і негативних іонів, теплих ізотермічних позитронів і двотемпературних неізотермічних електронів, методом псевдопотенціалу Сагдєєва досліджено нелінійні хвильові структури повільної моди. "Дисперсійне співвідношення" виводиться з базового набору нормалізованих рівнянь для рідини у припущенні, що розв'язки мають вигляд поодинокої хвилі, а потім аналітично визначається фазова швидкість повільного режиму ($V_S$). Вплив нормалізованої фазової швидкості ($V_S$) на солітони повільної моди великої амплітуди й подвійні шари повільної моди малої амплітуди всебічно проаналізовано за допомогою профілів потенціальної функції Сагдєєва $\psi(\phi)$, розв'язків поодинокої хвилі першого ($\phi_1$) і другого ($\phi_2$) порядку та розв'язків подвійного шару $\phi_{\rm DL}$ у двотемпературній неізотермічній електронній плазмі.

*Ключові слова*: фазова швидкість у повільному й швидкому режимах, неізотермічна плазма, псевдопотенціал Сагдєєва, стискальні солітони повільного режиму та подвійні шари.