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COHERENCE, BROKEN SYMMETRY AND NONDISSIPATIVE MOTION OF A QUANTUM OSCILLATOR

Using the example of a quantum oscillator, the connection between the dynamical coherent state with the phase symmetry breaking and the existence of the non-dissipative motion is considered. In multiparticle systems of interacting particles similar states manifest themselves as superfluidity and superconductivity.

Keywords: quantum oscillator, coherent states, broken phase symmetry, anomalous and normal averages, pair correlations, superfluidity, superconductivity.

1. Introduction

For understanding the properties of multiparticle systems, exactly solvable problems play an important role, in particular, the ideal gas and harmonic quantum oscillator models. The dynamical coherent state (DCS) of a quantum oscillator was considered as early as by Schrödinger at the dawn of quantum mechanics [1, 2]. A quantum oscillator can be considered as a simplest model of a solid body [3]. The representation of coherent states (CS) is widely used in the study of various quantum systems [4–7].

In this paper, we consider a harmonic quantum oscillator in the dynamical coherent state. Attention is paid to the fact that the peculiarity of such a state is that it has a nondissipative internal motion with a non-zero average momentum, and at the same time the symmetry with respect to the phase transformation proves to be broken. It is noted that the transition to a coherent state with broken phase symmetry leads to the existence of nondissipative flows, which in Bose systems manifest themselves as superfluidity, and in charged Fermi systems as superconductivity.

2. “Normal” and Coherent States of a Quantum Oscillator

The Hamiltonian of a quantum oscillator [2]

$$H = \frac{p^2}{2M} + \frac{M\omega^2 x^2}{2} \tag{1}$$

can be represented in terms of non-self-adjoint creation a^+ and annihilation a operators, satisfying the commutation relation $\left[a, a^+\right] \equiv aa^+ - a^+a = 1$. The coordinate and momentum operators are determined through these operators by the relations

$$x = \sqrt{\frac{\hbar}{2M\omega}}\,(a^+ + a), \qquad p \equiv -i\hbar\frac{d}{dx} = i\sqrt{\frac{M\hbar\omega}{2}}\,(a^+ - a), \tag{2}$$

and the Hamiltonian (1) takes the known form

$$H = \hbar\omega\left(a^+a + \frac{1}{2}\right). \tag{3}$$

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It is invariant with respect to the phase transformation

$$a \to a' = ae^{i\alpha}, \quad a^+ \to a'^+ = a^+e^{-i\alpha} \tag{4}$$

or transformation of the coordinate and momentum operators

$$\begin{aligned} x \to x' &= -\frac{p}{M\omega}\sin\alpha + x\cos\alpha, \\ p \to p' &= p\cos\alpha + M\omega x\sin\alpha, \end{aligned} \tag{5}$$

where α is an arbitrary real number. The eigenstates of the Hamiltonian (3) $H|n\rangle = \varepsilon_n|n\rangle$ with energy $\varepsilon_n \equiv \hbar\omega(n + 1/2)$ are characterized by integers $n = 0, 1, 2, \ldots$. The time dependence of vectors in states with fixed energy has the form $|n,t\rangle = e^{-i\frac{\varepsilon_n}{\hbar}t}|n\rangle$. The actions of the operators a^+, a on the state vector are given by the known relations $a^+|n\rangle = \sqrt{n+1}\,|n+1\rangle$, $a|n\rangle = \sqrt{n}\,|n-1\rangle$, $a^+a|n\rangle = n|n\rangle$. The ground (vacuum) state $|0\rangle$ is defined as a solution of the equation $a|0\rangle = 0$, and the vectors of excited states of the oscillator are found as a result of the action of powers of the operator a^+ on the ground state $|n\rangle = \frac{(a^+)^n}{\sqrt{n!}}|0\rangle$. In the coordinate representation, the wave functions of the oscillator are expressed through the Hermite polynomials $H_n(x)$ [2]:

$$\varphi_n(x) = \left(\frac{M\omega}{\pi\hbar}\right)^{1/4}\frac{1}{\sqrt{2^n n!}}\,e^{-\frac{M\omega}{2\hbar}x^2}H_n\!\left(x\sqrt{\frac{M\omega}{\hbar}}\right). \tag{6}$$

In the stationary states of the oscillator, the average values of the coordinate and momentum are equal to zero $\overline{x} \equiv \langle n|x|n\rangle = 0$, $\overline{p} \equiv \langle n|p|n\rangle = 0$, and the product of coordinate and momentum fluctuations $I \equiv \sqrt{\left(\overline{x^2} - \overline{x}^2\right)\left(\overline{p^2} - \overline{p}^2\right)}$ for the n-th level is $I_n = \hbar\left(n + \frac{1}{2}\right)$. The minimum value of the product of coordinate and momentum fluctuations is achieved in the ground state $I_0 = \hbar/2$. Such states of the oscillator with a certain energy will be called "normal".

An arbitrary time-dependent state vector can be decomposed over the complete system of eigenvectors of the oscillator

$$\big|\Phi(t)\big\rangle = \sum_{n=0}^{\infty} C_n\,e^{-i\frac{\varepsilon_n}{\hbar}t}\,|n\rangle. \tag{7}$$

Let us consider such a specific state in which the expansion coefficients in (7) have the form

$$C_n = \frac{\chi^n}{\sqrt{n!}}\,e^{-\frac{|\chi|^2}{2}}, \quad \sum_{n=0}^{\infty}\big|C_n\big|^2 = 1, \tag{8}$$

where χ is an arbitrary complex number that is independent of time. In the case of expansion coefficients (8), the probability of finding the oscillator in the state $|n\rangle$ is determined by the Poisson distribution $\big|C_n\big|^2 = e^{-|\chi|^2}\frac{|\chi|^{2n}}{n!}$. Then the state (7) takes the form

$$\big|\Phi_\chi(t)\big\rangle = e^{-\frac{|\chi|^2}{2}}\sum_{n=0}^{\infty}\frac{\chi^n}{\sqrt{n!}}\,e^{-i\frac{\varepsilon_n}{\hbar}t}|n\rangle. \tag{9}$$

At $\chi = 0$ (9) coincides with the wave function of the oscillator in the ground state. The vector (9) is an eigenstate of the annihilation operator $a|\Phi_\chi(t)\rangle = \chi(t)|\Phi_\chi(t)\rangle$ with the time-dependent eigenvalue $\chi(t) \equiv \chi e^{-i\omega t}$. Note that usually one considers the stationary coherent states [4–7]:

$$|\chi\rangle \equiv \big|\Phi_\chi(0)\big\rangle = e^{-\frac{|\chi|^2}{2}}\sum_{n=0}^{\infty}\frac{\chi^n}{\sqrt{n!}}\,|n\rangle. \tag{10}$$

The dynamical coherent state can be obtained by the action of the operator $U(t) = \exp\!\left[-i\omega t\left(a^+a + \frac{1}{2}\right)\right]$ on the stationary CS. It is important to emphasize that the stationary CS is not a solution of the stationary Schrödinger equation, whereas the DCS (9) is an exact solution of the nonstationary Schrödinger equation. Consideration of the nonstationary coherent states is fundamentally important in the study of systems in which nondissipative flows may exist.

In the coordinate representation the function (9) takes the form

$$\begin{aligned} \Phi_\chi(x,t) &= \left(\frac{M\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{|\chi|^2}{2}}\,e^{-i\frac{\omega t}{2}} \times \\ &\times e^{-\frac{M\omega}{2\hbar}x^2}\sum_{n=0}^{\infty}\frac{\chi(t)^n}{\sqrt{2^n n!}}\,H_n\!\left(x\sqrt{\frac{M\omega}{\hbar}}\right). \end{aligned} \tag{11}$$

Using the formula for a sum of Hermite polynomials

$$\sum_{k=0}^{\infty}\frac{t^k}{k!}\,H_k(x) = e^{2xt-t^2}, \tag{12}$$

we obtain the representation of function (11) in the form

$$\begin{aligned} \Phi_\chi(x,t) &= \left(\frac{M\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{|\chi|^2}{2}}\,e^{-i\frac{\omega t}{2} + \frac{\chi^2(t)}{2}} \times \\ &\times e^{-\frac{M\omega}{2\hbar}\left(x - \chi(t)\sqrt{\frac{2\hbar}{M\omega}}\right)^2}. \end{aligned} \tag{13}$$

This function is a wave packet that does not diffuse with time.

3. Difference between Symmetries of "Normal" and Coherent States

As noted, the Hamiltonian (3) is symmetric with respect to the phase transformations (4). If the oscillator is in states with fixed energy $\varepsilon_n \equiv \hbar\omega\big(n+\frac{1}{2}\big)$, then the averages of only the phase-invariant operators can be nonzero $\langle n|a^+a|n\rangle = n$, while the averages of the phase-noninvariant operators $\langle n|a|n\rangle = \langle n|a^+|n\rangle = 0$ and $\langle n|a^2|n\rangle = \langle n|a^{+2}|n\rangle = 0$ are equal to zero. Thus, the symmetry of the averages in the normal states coincides with the symmetry of the Hamiltonian.

In the coherent states, both the normal averages from the phase-invariant creation and annihilation operators and the anomalous averages from the phase-noninvariant operators turn out to be nonzero:

$$\begin{aligned}
&\big\langle\Phi_\chi(t)\big|a^+a\big|\Phi_\chi(t)\big\rangle = |\chi|^2,\\
&\big\langle\Phi_\chi(t)\big|a^+\big|\Phi_\chi(t)\big\rangle = \chi^*(t),\\
&\big\langle\Phi_\chi(t)\big|a\big|\Phi_\chi(t)\big\rangle = \chi(t),\\
&\big\langle\Phi_\chi(t)\big|a^{+2}\big|\Phi_\chi(t)\big\rangle = \chi^{*2}(t),\\
&\big\langle\Phi_\chi(t)\big|a^2\big|\Phi_\chi(t)\big\rangle = \chi^2(t).
\end{aligned} \tag{14}$$

In this case, the symmetry of the anomalous averages turns out to be lower than the symmetry of the Hamiltonian (3). Such states are called states with spontaneously broken phase symmetry.

The average values of the coordinate and momentum in the coherent state, in contrast to the normal state, are nonzero and depend on time:

$$\begin{aligned}
&\overline{x}(t) = \big\langle\Phi_\chi(t)\big|\,x\big|\Phi_\chi(t)\big\rangle = \sqrt{\frac{\hbar}{2M\omega}}\,\big(\chi^*(t)+\chi(t)\big),\\
&\overline{p}(t) = \big\langle\Phi_\chi(t)\big|\,p\,\big|\Phi_\chi(t)\big\rangle = i\sqrt{\frac{M\hbar\omega}{2}}\,\big(\chi^*(t)-\chi(t)\big).
\end{aligned} \tag{15}$$

It follows that $\dot{\overline{x}}(t) = \overline{p}(t)/M$, $\dot{\overline{p}}(t) = -M\omega^2\,\overline{x}(t)$, and the average values of both coordinate and momentum satisfy the classical oscillator equation $\ddot{\overline{x}}(t)+\omega^2\overline{x}(t) = 0$, $\ddot{\overline{p}}(t)+\omega^2\overline{p}(t) = 0$. In contrast to the stationary state in the normal state of the oscillator, where the average momentum is zero, in the dynamical coherent state the average momentum is nonzero and oscillates in time. Therefore, in the dynamical coherent state there exists a nondissipative internal motion, which is analogous to states of multiparticle systems with nondissipative flows. The wave function of the coherent state (13) can be expressed through the averages $\overline{x}(t), \overline{p}(t)$:

$$\begin{aligned}
&\Phi_\chi(x,t) = \left(\frac{M\omega}{\pi\hbar}\right)^{1/4} e^{-i\left(\frac{\omega t}{2}+\frac{\overline{p}(t)\overline{x}(t)}{2\hbar}\right)}\times\\
&\times e^{i\frac{\overline{p}(t)}{\hbar}x}\,e^{-\frac{M\omega}{2\hbar}(x-\overline{x}(t))^2}.
\end{aligned} \tag{16}$$

In this form, this wave function was first obtained by Schrödinger [1, 2].

The averages over the coherent state of the squares of coordinate and momentum have the form

$$\begin{aligned}
&\overline{x^2}(t) = \big\langle\Phi_\chi(t)\big|\,x^2\big|\Phi_\chi(t)\big\rangle =\\
&= \frac{\hbar}{2M\omega}\big(\chi(t)^{*2}+\chi(t)^2+2|\chi|^2+1\big),\\
&\overline{p^2}(t) = \big\langle\Phi_\chi(t)\big|\,p^2\big|\Phi_\chi(t)\big\rangle =\\
&= -\frac{M\hbar\omega}{2}\big(\chi(t)^{*2}+\chi(t)^2-2|\chi|^2-1\big).
\end{aligned} \tag{17}$$

Taking into account (15) and (17), we find for the uncertainty relation

$$I \equiv \sqrt{\big(\overline{x^2}-\overline{x}^2\big)\big(\overline{p^2}-\overline{p}^2\big)} = \frac{\hbar}{2}. \tag{18}$$

As was already shown by Schrödinger [1, 2], the uncertainty I (18) in the coherent state is minimal. For the normal state of the oscillator, the uncertainty is minimal only in the ground state.

In the coherent state, the energy of the oscillator is not precisely determined. The quantum-mechanical average of energy in the dynamical coherent state is independent of time and can be expressed through the averages of the squares of coordinate and momentum (17)

$$\begin{aligned}
&\varepsilon(\chi) = \hbar\omega\left(\big\langle\Phi_\chi(t)\big|a^+a\big|\Phi_\chi(t)\big\rangle+\frac{1}{2}\right) =\\
&= \hbar\omega\left(|\chi|^2+\frac{1}{2}\right) = \frac{\overline{p^2}(t)}{2M}+\frac{M\omega^2}{2}\,\overline{x^2}(t).
\end{aligned} \tag{19}$$

Thus, the dynamical coherent state is a state with a constant average energy, in which there exists an undamped motion and the phase symmetry is broken.

4. Discussion. Conclusions

The main feature of superfluid and superconducting systems is the possibility of stationary flows of mass or charge in them, which can exist for arbitrarily long time without attenuation. The transition from the normal state to the superfluid or superconducting state is a phase transition. According to the general theory of phase transitions [8], a phase transition to an ordered state should be accompanied by the appearance of a new characteristic – the order parameter. For a long time it was unclear what the order parameter is in superconducting or superfluid transitions. Even before the discovery of superfluidity, L.V. Shubnikov [9] suggested a hypothesis that the phase transition He I – He II is accompanied by an ordering similar to the transition from a liquid to a crystalline state. However, after superfluidity was discovered [10, 11], it became unclear how such ordering could support nondissipative flows. The correct form of the order parameter in the framework of the phenomenological description was proposed by Ginzburg and Landau [12]. They used the complex macroscopic wave function as an order parameter for superconductors. Such an order parameter allows one to construct an expression for the macroscopic density of the nondissipative flow, similar to the way the probability flux density is constructed in quantum mechanics. Since the macroscopic wave function, like the wave function in quantum mechanics, is defined up to an arbitrary phase factor, then such a state is called a state with broken phase symmetry. In a simple version for superconductors, a microscopic justification of the Ginzburg–Landau theory on the basis of the BCS theory [13] was given by Gorkov [14]. He showed that the existence of superconducting properties is associated with the appearance of the anomalous averages. The phenomenological Ginzburg–Landau approach was extended by Ginzburg and Pitaevskii to a superfluid Bose liquid [15]. The development of this theory was continued in works [16, 17]. It should be noted that in the well-known Landau theory of superfluidity [18, 19] the violation of phase symmetry is also implicitly taken into account through the introduction of the superfluid density associated with the modulus of the complex order parameter, and the superfluid velocity determined by the phase gradient. In Bose systems, a simple modeling of superfluidity at zero temperature leads to the Gross–Pitaevskii equation [20, 21]. The macroscopic wave function in this theory is the coherent state [22].

For the macroscopic complex wave function to exist, the system must have nonzero anomalous averages of the form $\langle a_{k_1\sigma_1} a_{k_2\sigma_2}\rangle$ or $\langle a_{k\sigma}\rangle$ that violate the phase symmetry. In this case, the macroscopic wave function of the superconductor in the s-state will be of the form

$$\Psi(\mathbf{r}_1, \mathbf{r}_2) \sim \sum_{\mathbf{k}_1, \mathbf{k}_2} e^{-i(\mathbf{k}_1\mathbf{r}_1 + \mathbf{k}_2\mathbf{r}_2)} \langle a_{\mathbf{k}_1\uparrow} a_{\mathbf{k}_2\downarrow}\rangle,$$

and the macroscopic wave function of the Bose system of particles with zero spin – of the form

$$\Psi(\mathbf{r}) \sim \sum_{\mathbf{k}} e^{-i\mathbf{k}\mathbf{r}} \langle a_{\mathbf{k}}\rangle.$$

At present, as can already be seen from the cited works, it is quite clear that in any physical system where the phenomena of superfluidity or superconductivity exist, the phase symmetry must be necessarily violated. At the same time, of course, each system also has properties that are specific to it. Nevertheless, there has appeared and continues to appear a large number of works devoted to this problem, where the violation of the phase symmetry is absent. In this case, there cannot be states with equilibrium nondissipative flows [23].

However, the problem of establishing at the microscopic level the connection between the coherence of the state, the violation of the phase symmetry and the existence of the nondissipative flows in multiparticle Fermi and Bose systems continues to remain relevant. In this paper, using a simple example of a quantum harmonic oscillator, the connection is traced between the violation of the phase symmetry and the existence of macroscopic motion in the dynamical coherent state. It is shown that, in contrast to the states of the oscillator with a fixed energy, in which the average momentum is equal to zero, in the dynamical coherent state it is different from zero and oscillates according to the equation for a classical oscillator. At the same time, in the DCS the phase-noninvariant anomalous averages also turn out to be different from zero.

Note that the energy in the considered DCS is determined only by the phase-invariant average (19). In more complex systems of interacting particles, energy and other observable quantities are determined not only by normal averages, but also by anomalous averages. Thus, coherent states, not being eigenfunctions of some Hermitian operator, contribute to observable quantities. An example of this is the gap in the spectrum of quasiparticle excitations of a superconductor. Being a measurable quantity, it is determined by the anomalous average. Thus, in states with broken phase symmetry, the notion of a quantum-mechanical observable quantity has a more general character than in normal systems.

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КОГЕРЕНТНІСТЬ, ПОРУШЕНА
СИМЕТРІЯ ТА НЕДИСИПАТИВНИЙ
РУХ КВАНТОВОГО ОСЦИЛЯТОРА

На прикладі квантового осцилятора розглянуто зв'язок між динамічним когерентним станом із порушенням фазової симетрії та існуванням недисипативного руху. У багаточастинкових системах взаємодіючих частинок подібні стани проявляються як надплинність та надпровідність.

Ключові слова: квантовий осцилятор, когерентні стани, порушена фазова симетрія, аномальні та нормальні середні, парні кореляції, надплинність, надпровідність.