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REFLECTANCE AND TRANSMITTANCE OF A LAYERED STRUCTURE FOR A NORMALLY INCIDENT ELECTROMAGNETIC WAVE

Simple analytic expressions have been obtained for the reflection and transmission coefficients in the case of a linearly polarized monochromatic planar electromagnetic wave normally incident on a layered structure consisting of planar dielectric layers. The refractive indices of the layers may be complex-valued. In contrast to the standard transfer-matrix method, in which implicit expressions for the reflectance and transmittance are obtained, the proposed one provides explicit expressions for those parameters in terms of the layers' thicknesses and the coefficients in the Fresnel formulas for the layer interfaces. As an example, the reflection coefficient is determined for the normal incidence of an electromagnetic wave on a semi-infinite periodic structure formed by two continuously alternating dielectric layers.

Keywords: reflectance, transmittance, optics of layered structures, Fresnel equations.

1. Introduction

The optical properties of layered structures, in which layers of different materials alternate, are of considerable interest due to their numerous applications in radioelectronic devices such as antennas and in plasmonics and photoelectronics, e.g., as Bragg mirrors, optical switches and filters, and so forth. The application of liquid crystals, which are sensitive to external electric/magnetic or light fields, as at least one or a few layers [1] opens up wide possibilities for the dynamically controlling of the optical properties of layered structures using weak external quasi-static electric fields. Layered structures with liquid-crystal layers have found practical applications in a wide frequency interval ranging from optical to terahertz frequencies [2].

Obviously, the calculation of the transmission and reflection coefficients for electromagnetic waves is an urgent and primary task for all applications dealing with multi-layer optical structures. For instance, the authors of work [3] proposed a polynomial approach for finding the electromagnetic wave reflectance and transmittance of a layered structure. Recursive relationships for calculating generalized Fresnel coefficients for a layered structure were used in work [4]. The influence of a metal layer on the Fresnel coefficients, in particular, in layered structures, was studied in paper [5]. The generation of the second harmonic at the interfaces in a layered medium was theoretically considered in work [6]. An attempt to obtain the form for the electromagnetic wave transmittance through an anisotropic periodic medium was made in paper [7]. The Fresnel formulas in the case of nonplanar interfaces were derived in work [8].

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Important from the viewpoint of practical application is the case of normal incidence of an electromagnetic wave on a layered structure. In this case, a standard transfer-matrix method has been developed to find the reflectance and transmittance of an electromagnetic wave; see, e.g., work [9]. This method is relatively simple. However, it involves the multiplication of a large number of matrices, if there are a large number of layers. In the case of an infinite layered structure, the number of matrices to be multiplied tends to infinity. This situation can be significantly inconvenient for numerical calculations and can lead to error accumulation. Furthermore, the resulting formulas of the method contain matrix products, which makes the calculation results implicit. Under the condition of normal incidence of an electromagnetic wave on a layered structure, attempts were made to simplify the procedure for obtaining the reflection and transmission coefficients. In particular, the authors of work [10] implicitly coupled the electromagnetic wave reflectance and transmittance of two adjacent layers in the medium. For this purpose, they applied recurrence relationships that related the amplitudes of the electric and magnetic fields in three consecutive layers. The proposed approach does not allow energy losses in the medium to be taken into account, since it is based on the law of energy conservation.

In the present work, explicit expressions are obtained for the reflectance and transmittance of a linearly polarized plane electromagnetic wave in the case of its normal incidence on a layered structure consisting of plane-parallel dielectric layers with given thicknesses and refractive indices. The latter can be complex-valued. The reflection coefficient takes a particularly simple form of an ordinary continued fraction. As is known, the conditions of numerical stability of the calculation of continued fractions are well studied [11]. Therefore, results in such a representation are convenient for numerical calculations, because they allow a simple software implementation. This fact, in particular, makes it possible to obtain reliable results in the case of a semi-infinite layered structure. It should be noted that the proposed approach minimizes the accumulation of errors while numerically calculating the reflectance and transmittance of electromagnetic waves in layered media.

The work is organized as follows. In section 2, the physical model is described, and recurrent relationships for the amplitudes of the electric field of electromagnetic waves propagating forward and backward through a three-layer structure are derived. A general form of the reflection and transmission coefficients is given. In section 3, the reflectance and transmittance of an electromagnetic wave are expressed in terms of the layer thicknesses and the coefficients in the Fresnel formulas for the layer interfaces. The obtained results are verified by applying them to well-studied cases, namely, two semi-infinite media and a layer between two semi-infinite media. In section 4, the proposed approach is used to obtain an analytic expression for the reflectance of an electromagnetic wave from a semi-infinite periodic structure formed by two alternating dielectric layers. Discussion of the results and brief conclusions of the work are given in section 5.

2. Basic Equations and Their Solution

Let us consider a layered structure formed by N alternating homogeneous plane-parallel dielectric layers and bounded between the planes $z = 0$ and $z = L$. The structure under consideration is located on a homogeneous dielectric substrate ($z > L$) with a real refractive index. From the air side ($z < 0$), a plane monochromatic light wave of frequency ω is normally incident on this structure in the positive direction of the axis Oz. The light is considered to be polarized in the direction of the axis Ox, which is oriented along the interface between the air and the layered medium. As the light wave penetrates the dielectric layers, it undergoes a series of successive reflections at the interfaces between them. The schematic diagram of the structure under consideration, together with the propagation directions of the incident, A_i, and reflected, A'_i, electromagnetic waves, is presented in Fig. 1.

Within each layer of the structure under consideration, the vector $\mathbf{E}$ of the light wave electric field strength is a solution of the wave equation $\Delta\mathbf{E} + \frac{\omega^2}{c^2} n^2 \mathbf{E} = 0$, where n is the refractive index of the medium of the corresponding layer. The vector $\mathbf{H}$ of the light wave magnetic field strength is found from the Maxwell equation $\operatorname{rot}\mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t}$. The system is considered to be homogeneous along the coordinate axes Ox and Oy. Then, in accordance with the above, the magnitudes of the electric, E_i, and magnetic, H_i, field strength vectors in the air ($z < 0$) and in the i-th dielectric layer ($d_{i-1} < z < d_i$; here, $d_0 \equiv 0$ and

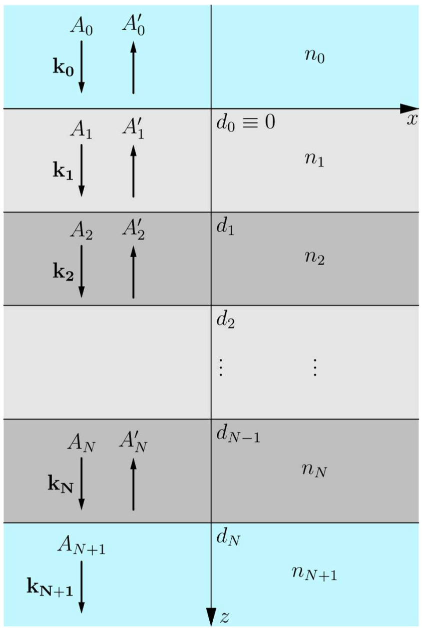

Fig. 1. Structure consisting of N homogeneous plane-parallel dielectric layers. The incident wave propagates along the Oz direction. In each layer, the amplitudes of the waves propagating along and against the Oz-axis are denoted as A and A', respectively

$d_N \equiv L$) have the form

$$E_i(z) = A_i e^{ikn_i z} + A'_i e^{-ikn_i z},$$
$$H_i(z) = \frac{n_i}{c\mu_0}\left(A_i e^{ikn_i z} - A'_i e^{-ikn_i z}\right), \tag{1}$$

where $k = \omega/c$ is the wave number of the electromagnetic wave in vacuum; and the subscript $i = 0, 1, 2, ..., N$ enumerates the media of air and the plane-parallel dielectric layers, with the corresponding refractive indices n_i; see Fig. 1.

At $z \geqslant L$, in the region with a dielectric substrate with the real refractive index n_{N+1}, only the wave that passed through the layered system propagates. The corresponding electric and magnetic field vectors are, respectively,

$$E_{N+1}(z) = A_{N+1} e^{ikn_{N+1} z},$$
$$H_{N+1}(z) = \frac{n_{N+1} A_{N+1}}{c\mu_0} e^{ikn_{N+1} z}. \tag{2}$$

Let us formally introduce a "reflected" wave in the dielectric substrate behind the layered system and denote $A'_{N+1} \equiv 0$.

Obviously, at the interfaces $z = d_i$ (here, $i = 0$, 1, 2, ..., N) between the dielectric layers, the vectors of the electric, $\mathbf{E}_i$, and magnetic, $\mathbf{H}_i$, field strengths of the light wave must satisfy the electrodynamic boundary conditions. From the equality of the tangential components of the electric and magnetic fields at the interfaces, we obtain a system of equations that relates the amplitudes of the incident, A_i, and reflected, A'_i, waves in the neighboring layers,

$$\begin{cases} A_l e^{ikn_l d_l} + A'_l e^{-ikn_l d_l} = \\ = A_{l+1} e^{ikn_{l+1} d_l} + A'_{l+1} e^{-ikn_{l+1} d_l}, \\ n_l\left(A_l e^{ikn_l d_l} - A'_l e^{-ikn_l d_l}\right) = \\ = n_{l+1}\left(A_{l+1} e^{ikn_{l+1} d_l} - A'_{l+1} e^{-ikn_{l+1} d_l}\right). \end{cases} \tag{3}$$

Whence, by expressing the amplitudes A_{l+1} and A'_{l+1} in terms of A_l and A'_l, we have

$$A_{l+1} = a_l e^{i\delta_l} A_l + b_l e^{-i\sigma_l} A'_l, \tag{4}$$

$$A'_{l+1} = b_l e^{i\sigma_l} A_l + a_l e^{-i\delta_l} A'_l, \tag{5}$$

where the following notations were introduced:

$$a_l = \frac{1}{2}\left(1 + \frac{n_l}{n_{l+1}}\right), \quad b_l = \frac{1}{2}\left(1 - \frac{n_l}{n_{l+1}}\right), \tag{6}$$

$$\sigma_l = k(n_l + n_{l+1})d_l, \quad \delta_l = k(n_l - n_{l+1})d_l. \tag{7}$$

Similarly, we express the amplitudes A_l and A'_l in terms of A_{l-1} and A'_{l-1}:

$$A_l = a_{l-1} e^{i\delta_{l-1}} A_{l-1} + b_{l-1} e^{-i\sigma_{l-1}} A'_{l-1}, \tag{8}$$

$$A'_l = b_{l-1} e^{i\sigma_{l-1}} A_{l-1} + a_{l-1} e^{-i\delta_{l-1}} A'_{l-1}. \tag{9}$$

Then from relationships (4), (8), and (9), we can express the amplitude A_{l+1} in terms of A_l and A_{l-1}:

$$A_{l+1} = \alpha_l^- A_l + \beta_l^- A_{l-1}. \tag{10}$$

Similarly, from relationships (5), (8), and (9), we can express the amplitude A'_{l+1} in terms of A'_l and A'_{l-1}:

$$A'_{l+1} = \alpha_l^+ A'_l + \beta_l^+ A'_{l-1}. \tag{11}$$

Here, the following notations were introduced:

$$\alpha_l^{\pm} = a_l e^{\mp i\delta_l} + a_{l-1}\frac{b_l}{b_{l-1}} e^{\pm i(\delta_{l-1} + \sigma_l - \sigma_{l-1})},$$
$$\beta_l^{\pm} = b_l\left[b_{l-1} - \frac{a_{l-1}^2}{b_{l-1}}\right] e^{\pm i(\sigma_l - \sigma_{l-1})}, \tag{12}$$

where $l = 1, 2, ..., N$.

It is easy to see that, from relationship (11), it follows that

$$\frac{A'_{l-1}}{A'_l} = -\frac{\alpha_l^+}{\beta_l^+} + \frac{1}{\beta_l^+} \cdot \frac{1}{\dfrac{A'_l}{A'_{l+1}}}. \tag{13}$$

By successively applying this recurrent formula N times starting from $l = 0$, we find the ratio between the amplitudes of the reflected electromagnetic waves in air and in the first plane-parallel layer,

$$\frac{A'_0}{A'_1} = -\frac{\alpha_1^+}{\beta_1^+} + \frac{1}{\beta_1^+} \cdot \frac{1}{-\dfrac{\alpha_2^+}{\beta_2^+} + \dfrac{1}{\beta_2^+} \cdot \dfrac{1}{\dfrac{\dots}{-\dfrac{\alpha_N^+}{\beta_N^+} + \dfrac{1}{\beta_N^+} \cdot \dfrac{A'_{N+1}}{A'_N}}}}. \tag{14}$$

Taking into account that $A'_{N+1} = 0$ in the substrate behind the layered structure we obtain

$$\beta_1^+ \frac{A'_0}{A'_1} = -\alpha_1^+ + \frac{\beta_2^+}{-\alpha_2^+ + \dfrac{\beta_3^+}{\dfrac{\dots}{-\alpha_{N+1}^+ + \dfrac{\beta_N^+}{\alpha_N^+}}}}. \tag{15}$$

From formula (5) at $l = 0$, we obtain the relationship

$$\frac{A'_0}{A_0} = \frac{\beta_0^+}{-\alpha_0^+ + \dfrac{A'_1}{A'_0}}, \tag{16}$$

where we took into account that $\sigma_0 = \delta_0 = 0$ [see Eqs. (7)] because $d_0 = 0$, and also $\alpha_0^+ = a_0$, $\beta_0^+ = b_0$ [see Eqs. (12)].

Substituting expression (16) into formula (15) gives us the value of the ratio between the amplitudes of the electromagnetic waves reflected from and incident on the layered structure,

$$r = \frac{A'_0}{A_0} = \frac{\beta_0^+}{-\alpha_0^+ + \dfrac{\beta_1^+}{-\alpha_1^+ + \dfrac{\beta_2^+}{\dfrac{\dots}{-\alpha_{N-1}^+ + \dfrac{\beta_N^+}{-\alpha_N^+}}}}}. \tag{17}$$

Whence we get the value of the reflectance $R = |r|^2$ of the electromagnetic wave from the layered structure.

Let us find the form for the transmittance of an electromagnetic wave through a layered medium. From relationship (10), it follows that

$$\frac{A_{l+1}}{A_l} = \alpha_l^- + \beta_l^- \cdot \frac{1}{\dfrac{A_l}{A_{l-1}}}. \tag{18}$$

By successively applying this recurrence relationship l times, we arrive at the ratio between the wave amplitudes in adjacent layers for a wave propagating in the direction of the axis Oz in the form

$$\frac{A_{l+1}}{A_l} = \alpha_l^- + \frac{\beta_l^-}{\alpha_{l-1}^- + \dfrac{\beta_{l-1}^-}{\dfrac{\dots}{\alpha_1^- + \dfrac{\beta_1^-}{\dfrac{A_1}{A_0}}}}}. \tag{19}$$

From formula (4) at $l = 0$, we gat

$$\frac{A_1}{A_0} = \alpha_0^- + \beta_0^- r, \tag{20}$$

where $\alpha_0^- = a_0$, $\beta_0^- = b_0$ [see Eqs. (12)], and taking into account that $\sigma_0 = \delta_0 = 0$ [see Eqs. (7)].

Let us find the ratio between the amplitudes of the wave that has passed through the layered medium and the incident wave:

$$t = \frac{A_{N+1}}{A_0} = \frac{A_{N+1}}{A_N} \cdot \frac{A_N}{A_{N-1}} \cdot \dots \cdot \frac{A_2}{A_1} \cdot \frac{A_1}{A_0}. \tag{21}$$

Using Eqs. (19) and (20), we obtain

$$t = \prod_{l=0}^{N} \Bigg[\alpha_l^- + \frac{\beta_l^-}{\alpha_{l-1}^- + \dfrac{\beta_{l-1}^-}{\dfrac{\dots}{\alpha_1^- + \dfrac{\beta_1^-}{\alpha_0^- + \dfrac{\beta_0^-}{r^{-1}}}}}} \Bigg]. \tag{22}$$

Whence the value of the electromagnetic wave transmittance through a layered medium equals

$$T = \frac{n_{N+1}}{n_0} |t|^2. \tag{23}$$

3. Wave Reflectance and Transmittance

The expressions obtained for the electromagnetic wave reflectance R and transmittance T of a layered medium can be simplified by simultaneously expressing them in terms of quantities that have a direct physical meaning, namely, the thicknesses of the layers $h_l = d_l - d_{l-1}$ and the coefficients in the Fresnel formulas in the case of normal plane-wave incidence on the interface between two media [12],

$$\rho_l = -\frac{b_l}{a_l} = \frac{n_l - n_{l+1}}{n_l + n_{l+1}}. \tag{24}$$

Here, ρ_l is the ratio between the amplitudes of the reflected and incident waves at the interface between two semi-infinite media with the refractive indices n_l and n_{l+1}. Let us rewrite Eq. (17) in the form

$$r = \cfrac{-\cfrac{\beta_0^+}{\alpha_0^+}}{1 - \cfrac{-\cfrac{\beta_1^+}{\alpha_0^+\alpha_1^+}}{1 - \cfrac{-\cfrac{\beta_2^+}{\alpha_1^+\alpha_2^+}}{\cfrac{\cdots}{1 - \cfrac{-\beta_N^+}{\alpha_{N-1}^+\alpha_N^+}}}}}. \tag{25}$$

Let us introduce the notations

$$\xi_0^\pm = -\frac{\beta_0^\pm}{\alpha_0^\pm} = \rho_0, \quad \xi_1^\pm = -\frac{\beta_1^\pm}{\alpha_0^\pm\alpha_1^\pm} = \frac{\rho_1\,(1-\rho_0^2)}{\rho_1 + \rho_0 e^{\mp 2ik_1h_1}}. \tag{26}$$

$$\xi_l^\pm = \frac{-\beta_l^\pm}{\alpha_{l-1}^\pm\alpha_l^\pm} = \frac{\rho_{l-2}\rho_l\,(1-\rho_{l-1}^2)}{\left(\rho_{l-2} + \rho_{l-1}e^{\pm 2ik_{l-1}h_{l-1}}\right)\left(\rho_l + \rho_{l-1}e^{\mp 2ik_lh_l}\right)}, \tag{27}$$

where $l = 2, 3, ..., N$. Then we obtain the following form for the reflectance of the electromagnetic wave: $R = |r|^2$, where

$$r = \cfrac{\xi_0^+}{1 - \cfrac{\xi_1^+}{1 - \cfrac{\xi_2^+}{\cfrac{\cdots}{1 - \xi_N^+}}}}. \tag{28}$$

We can also simplify the expression for t. From relationship (22), it is easy to obtain

$$t = \prod_{l=0}^{N} \alpha_l^- \left(1 - \cfrac{-\cfrac{\beta_l^-}{\alpha_{l-1}^-\alpha_l^-}}{1 - \cfrac{-\cfrac{\beta_{l-1}^-}{\alpha_{l-2}^-\alpha_{l-1}^-}}{\cfrac{\cdots}{1 - \cfrac{\cfrac{-\beta_1^-}{\alpha_0^-\alpha_1^-}}{1 - \cfrac{-\cfrac{\beta_0^-}{\alpha_0^-}}{r^{-1}}}}}}\right). \tag{29}$$

The product $\prod_{l=0}^{N} \alpha_l^-$ can be expressed in terms of the coefficients ρ_l in the Fresnel formulas and the thicknesses h_l of the layers (see Appendix 5),

$$\prod_{l=0}^{N} \alpha_l^- = \sqrt{\frac{n_0}{n_{N+1}}}\, e^{-ik_{N+1}d_N} \prod_{l=1}^{N} e^{ik_lh_l} \cdot \prod_{l=0}^{N} \zeta_l, \tag{30}$$

where

$$\zeta_0 = \frac{1}{\sqrt{1-\rho_0^2}}, \quad \zeta_l = \frac{1}{\sqrt{1-\rho_l^2}}\left(1 + \frac{\rho_l}{\rho_{l-1}} e^{-2ik_lh_l}\right). \tag{31}$$

Then expression (29) for t, taking Eq. (30) and notations (26) and (27) into account, can be rewritten in the form

$$t = \sqrt{\frac{n_0}{n_{N+1}}}\, e^{-ik_{N+1}d_N} \prod_{l=1}^{N} e^{ik_lh_l} \cdot \prod_{l=0}^{N} \zeta_l\eta_l, \tag{32}$$

where

$$\eta_l = 1 - \cfrac{\xi_l^-}{1 - \cfrac{\xi_{l-1}^-}{\cfrac{\cdots}{1 - \cfrac{\xi_1^-}{1 - \cfrac{\xi_0^-}{r^{-1}}}}}}. \tag{33}$$

Whence we obtain the final form for the transmittance of an electromagnetic wave through a layered structure [see formula (23)],

$$T = \prod_{l=1}^{N} \left|e^{ik_lh_l}\right|^2 \cdot \prod_{l=0}^{N} \left|\zeta_l\eta_l\right|^2. \tag{34}$$

Here, we took into account that the refractive index of the substrate located behind the layered structure is a real number so that $|e^{-ik_{N+1}d_N}| \equiv 1$.

Note that the first product in formula (34) differs from unity only in the case of layers with complex refractive indices. In the case of layers with real refractive indices, we have

$$T = \prod_{l=0}^{N} \left|\zeta_l \eta_l\right|^2. \quad (35)$$

Thus, formulas (24), (26), (27), (28), (31), (33), and (34) evaluate the electromagnetic wave reflectance and transmittance of a plane layered structure in the case of normal wave incidence. An especially simple expression is obtained for the reflectance R.

Let us verify the derived formulas by applying them to simple cases, namely, two semi-infinite media ($N = 0$) and a layer between two semi-infinite media ($N = 1$).

In the case $N = 0$, for the reflection coefficient, we have

$$R = |r|^2 = |\xi_0^+|^2 = \rho_0^2, \quad \text{where} \quad \rho_0 = \frac{n_0 - n_1}{n_0 + n_1}, \quad (36)$$

which is a correct result, consistent with the Fresnel formula [12]. For the transmission coefficient, we have

$$T = |\zeta_0(1 - \xi_0^- r)|^2 = \left|\frac{1}{\sqrt{1-\rho_0^2}}(1-\rho_0^2)\right|^2 = 1 - \rho_0^2, \quad (37)$$

as it should be.

In the case $N = 1$ and for an interlayer with a real-valued refractive index, we also obtain correct results: for the reflectance, $R = |r|^2$, where

$$r = \frac{\xi_0^+}{1 - \xi_1^+} = \frac{\rho_0 + \rho_1 e^{2ik_1h_1}}{1 + \rho_0\rho_1 e^{2ik_1h_1}}; \quad (38)$$

and for the transmittance, $T = |t|^2$, where

$$t = \zeta_0\zeta_1(1 - \xi_0^- r)\left(1 - \frac{\xi_1^-}{1 - \xi_0^- r}\right). \quad (39)$$

After some simplifications, we get

$$t = \frac{\sqrt{(1-\rho_0^2)(1-\rho_1^2)}}{1 + \rho_0\rho_1 e^{2ik_1h_1}}. \quad (40)$$

Formulas (38) and (40) agree with the results of work [10].

4. Semi-Infinite Layered Structure

Let us have a semi-infinite layered structure with two alternating dielectric layers characterizing by the refractive indices n_1 and n_2, and the thicknesses h_1 and h_2, respectively. Let an electromagnetic wave fall normally on this structure from the air side. In this case, the fractions in expression (28) for the electromagnetic wave reflectance R become infinite. Then, taking into account the structure periodicity, the coefficients in formula (28) starting from ξ_3 and ξ_4 are repeated,

$$r = \cfrac{\xi_0^+}{1 - \cfrac{\xi_1^+}{1 - \cfrac{\xi_2^+}{1 - \cfrac{\xi_3^+}{1 - \cfrac{\xi_4^+}{1 - \cfrac{\xi_3^+}{1 - \cfrac{\xi_4^+}{\dots}}}}}}}. \quad (41)$$

As a result, the expression for r can be written as follows:

$$r = \cfrac{\xi_0^+}{1 - \cfrac{\xi_1^+}{1 - \cfrac{\xi_2^+}{\gamma}}}, \quad (42)$$

where the value of γ is found from the equation

$$\gamma = 1 - \cfrac{\xi_3^+}{1 - \cfrac{\xi_4^+}{\gamma}} \quad (43)$$

and is equal to

$$\gamma = \frac{1}{2}\left(1 + \xi_4^+ - \xi_3^- + \sqrt{(1 + \xi_4^+ - \xi_3^+)^2 - 4\xi_4^+}\right). \quad (44)$$

Here, the sign "+" in front of the root is chosen based on the requirement that there is one interface between two semi-infinite media in the limiting case $n_1 \to n_2$. Therefore, it must be $r = \xi_0^+$, which is ensured by the appropriate choice of the sign in front of the root.

5. Conclusions

In the presented work, the propagation of a plane, linearly polarized electromagnetic wave through a layered structure consisting of plane-parallel dielectric layers in the case of normal incidence is theoretically investigated. Analytic expressions for the electromagnetic wave reflectance and transmittance of the examined structure are derived. In the case of normal wave incidence on a plane layered structure, finding the reflection and transmission coefficients does not pose any fundamental difficulties; however, the corresponding results obtained by standard methods are rather cumbersome. Instead, the expressions for the electromagnetic wave reflectance and transmittance obtained in this work are relatively simple and make it possible to explicitly express the result in terms of the physical parameters of the layered medium, namely, the thickness of the layers and the coefficients in the Fresnel formulas for the layer interfaces.

The obtained results are also convenient for numerical calculations, because the calculation of a continued fraction can be implemented programmatically using a single cycle. In addition, the numerical stability of continued fractions is well studied. The proposed approach minimizes the accumulation of errors at numerical calculations of the electromagnetic wave reflectance from and transmittance through layered media.

Based on the above approach, a simple analytic expression has been obtained for the electromagnetic wave reflectance from a semi-infinite periodic structure formed by two alternating dielectric layers.

APPENDIX

Let us express the product $\prod_{l=0}^{N}\alpha_l^-$ in terms of the coefficients ρ_l in the Fresnel formulas for the layer interface, and the layers' thicknesses h_l and refractive indices n_l. Using the definition of α_l^-, we have

$$\prod_{l=0}^{N}\alpha_l^- = a_0\prod_{l=1}^{N}\left[a_l e^{i\delta_l} + a_{l-1}\frac{b_l}{b_{l-1}}e^{-i(\delta_{l-1}+\sigma_l-\sigma_{l-1})}\right]. \quad (A.1)$$

Extracting the quantity $a_l e^{i\delta_l}$ from the l-th factor in the product, we obtain

$$\prod_{l=0}^{N}\alpha_l^- = e^{i\sum\limits_{l=1}^{N}\delta_l}\prod_{l=0}^{N}a_l \times$$
$$\times\prod_{l=1}^{N}\left[1+\frac{b_l}{a_l}\frac{a_{l-1}}{b_{l-1}}e^{-i(\delta_{l-1}-\sigma_{l-1}+\delta_l+\sigma_l)}\right]. \quad (A.2)$$

Taking into account that $\rho_l = -\frac{b_l}{a_l}$,

$$\delta_{l-1}-\sigma_{l-1}+\delta_l+\sigma_l = 2kn_lh_l, \quad (A.3)$$

and

$$\sum_{l=1}^{N}\delta_l = -kn_{N+1}d_N + \sum_{l=1}^{N}kn_lh_l, \quad (A.4)$$

we get

$$\prod_{l=0}^{N}\alpha_l^- = e^{-ikn_{N+1}d_N}\prod_{l=1}^{N}e^{ikn_lh_l}\times$$
$$\times\prod_{l=0}^{N}a_l\cdot\prod_{l=1}^{N}\left[1+\frac{\rho_l}{\rho_{l-1}}e^{-2ikn_lh_l}\right]. \quad (A.5)$$

Let us calculate $\prod_{l=0}^{N}a_l$. Taking into account that $\rho_l = \frac{n_l-n_{l+1}}{n_l+n_{l+1}}$, we obtain

$$1-\rho_l^2 = \frac{4n_ln_{l+1}}{(n_l+n_{l+1})^2} = \frac{\frac{n_l}{n_{l+1}}}{\frac{1}{4}\left(1+\frac{n_l}{n_{l+1}}\right)^2} = \frac{n_l}{n_{l+1}}\frac{1}{a_l^2}. \quad (A.6)$$

Whence

$$a_l = \sqrt{\frac{n_l}{n_{l+1}}}\frac{1}{\sqrt{1-\rho_l^2}}. \quad (A.7)$$

Therefore,

$$\prod_{l=0}^{N}a_l = \sqrt{\frac{n_0}{n_{N+1}}}\prod_{l=0}^{N}\frac{1}{\sqrt{1-\rho_l^2}}. \quad (A.8)$$

Substituting Eq. (A.8) into Eq. (A.5), we obtain

$$\prod_{l=0}^{N}\alpha_l^- = \sqrt{\frac{n_0}{n_{N+1}}}e^{-ikn_{N+1}d_N}\prod_{l=1}^{N}e^{ikn_lh_l}\times$$
$$\times\frac{1}{\sqrt{1-\rho_0^2}}\prod_{l=1}^{N}\frac{1}{\sqrt{1-\rho_l^2}}\left[1+\frac{\rho_l}{\rho_{l-1}}e^{-2ikn_lh_l}\right]. \quad (A.9)$$

This expression can be rewritten in the form

$$\prod_{l=0}^{N}\alpha_l^- = \sqrt{\frac{n_0}{n_{N+1}}}e^{-ikn_{N+1}d_N}\prod_{l=1}^{N}e^{ikn_lh_l}\cdot\prod_{l=0}^{N}\zeta_l, \quad (A.10)$$

which corresponds to formula (30).

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КОЕФІЦІЄНТИ ВІДБИВАННЯ ТА ПРОХОДЖЕННЯ ЕЛЕКТРОМАГНІТНОЇ ХВИЛІ, НОРМАЛЬНО ПАДАЮЧОЇ НА ШАРУВАТУ СТРУКТУРУ

Отримано прості аналітичні вирази для коефіцієнтів відбивання та проходження плоскої лінійно поляризованої електромагнітної хвилі у випадку нормального падіння на шарувату структуру з плоскопаралельних діелектричних шарів. Показники заломлення прошарків можуть бути комплексними. На відміну від стандартного методу трансфер-матриці, в якому коефіцієнти відбивання та проходження електромагнітної хвилі отримуються неявно, запропонований метод дає явний вигляд для коефіцієнтів відбивання та проходження через товщини прошарків, а також коефіцієнтів формул Френеля для меж поділу між прошарками. Як приклад застосування отриманих результатів, знайдено коефіцієнт відбивання електромагнітної хвилі нормально падаючої на напівнескінченну періодичну структуру, утворену чергуванням двох діелектричних шарів.

К л ю ч о в і с л о в а: коефіцієнт відбивання, коефіцієнт проходження, оптика шаруватих структур, формули Френеля.