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A METHODOICAL STUDY OF DOUBLE ALPHA DECAY IN NUCLEI WITH PROTON NUMBER FROM 60 TO 93 USING CYEM

Using Cubic plus Yukawa plus Exponential Model (CYEM) half-life values of double alpha (2α) decay have been systematically investigated for several radioactive nuclei. In double alpha decay, two identical alpha particles are simultaneously emitted from the unstable parent nuclei. In our realistic model we have used Coulomb plus Yukawa plus Exponential potentials as an interacting barrier for separated fragments and Cubic potential for overlapping region to calculate the half-life time for double alpha decay. In our present work, we have calculated the half-life time of radioactive nuclei with $Z = 60$ to 93 using our CYE Model in two sphere approximation. The calculated half-life time values are compared with available theoretical models such as Analytical Super Asymmetric Fission Model (ASAFM), Semi Empirical Fission formula (Sem-FIS), Coulomb Proximity Potential Model (CPPM) and Modified Generalized Liquid Drop Model (MGLDM). Our results are in well agreement with the results of other available models.

Keywords: radioactive nuclei, cluster decay, double alpha decay.

1. Introduction

Double alpha decay study is one of the most captivating topics in the field of nuclear physics since 1979 [1]. Radioactivity is the term used to describe the spontaneous release of radiation from the nucleus of an unstable atom, leading to either the conversion of the atom into a different element or to its attaining a more stable state. The emitted radiation encompasses alpha, beta, gamma rays and other form of radiations.

Double particle decay occurs when unstable parent nucleus emits two identical particles simultaneously. Double alpha decay [1–22], double beta (2β) decay [23, 24], double gamma (2γ) decay [25–27], dou-

ble proton ($2p$) [28] decay, and double neutron ($2n$) [28] decay are some kinds of double particle emissions. Radioactive nuclei undergo double alpha decay by releasing spontaneously and simultaneously two identical alpha particles. The concept of double alpha decay was initially predicted by Yu.N. Novikov *et al.* in 1979 [1] and they presented the half-life of some isotopes in reference [2]. In 1985, D.N. Poenaru [3] and M. Ivascu used Analytical Super Asymmetric Fission Model (ASAFM) to predict the double alpha decay half-life of $^{219,220}\text{Ac}$, $^{220,221}\text{Th}$, and ^{222}Pa isotopes. After a long interval, V.I. Tretyak [4] calculated the 2α decay half-life of nuclides in natural isotopic composition of elements with atomic number range from $Z = 60$ to 92 using semi-empirical formula in 2021. Subsequently, V.I. Tretyak calculated the first experimental limit on double alpha decay of ^{209}Bi as $T_{1/2} > 2.9 \times 10^{20}$ years using the data taken from the experiment conducted by de Marcillac *et al.* [29].

In the same year, F. Mercier *et al.* [5] performed a microscopic calculation on double alpha decay half-

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lives of ^{212}Po and ^{224}Ra using self consistent framework based on energy density functional. K.P. Santhosh *et al.* [6] studied the nuclear transition of double alpha particles using their Coulomb Proximity Potential Model (CPPM) and Modified Generalized Liquid Drop Model (MGLDM) with incorporating deformation effects and different preformation factors in 2021. Then in 2022, K.P. Santhosh *et al.* [7] predicted the possibility of double alpha decay of even-even isotopes of $Z = 120$ nucleus with $A = 290\text{--}304$ using MGLD model with Q -value dependent preformation factor. In 2022, K.P. Santhosh *et al.* [8] computed the half-life of $^{221\text{--}232}\text{Pu}$ nuclei via double alpha radioactivity utilizing Universal Decay Law (UDL).

In 2022, Deepika Pathak *et al.* [9] studied the spontaneous and simultaneous emission of two alpha particles from a parent nuclei, with atomic number range from $Z = 60$ to 92 and $Z = 118\text{--}126$ (in super heavy region) by employing the interaction potential barrier comprising of the Coulomb and Proximity potentials. At the same year the authors of Ref. [10] calculated the half-life of ^{142}Sm , ^{152}Gd , ^{156}Dy , ^{174}Hf , ^{180}W , $^{184\text{--}186}\text{Os}$, $^{190,192}\text{Pt}$, ^{196}Hg , ^{232}Th , $^{234,238}\text{U}$ isotopes using their interaction potential barrier comprising of the Coulomb and Proximity potentials. Then, V.Yu. Denisov [11] proposed a model to study the decay properties of double alpha decay and predicted the double α decay half-life of various nuclei. Recently in 2023, K.P. Santhosh *et al.* [12] studied the probable chances of double alpha radioactivity in the region $94 \leq Z \leq 101$ using MGLD Model with different preformation factors and Universal Decay Law (UDL). The authors also studied the comparison of double alpha decay and ^8Be decay half-life of ^{229}Pu , ^{230}Am , ^{232}Cm , ^{234}Bk , ^{236}Cf , ^{239}Es , ^{241}Fm , ^{245}Md isotopes utilizing MGLD Model with different preformation factors and Universal Decay Law (UDL). In 2024, Sreelakshmi *et al.* [16], investigated the double alpha decay properties of $^{183\text{--}195}\text{Bi}$ isotopes using Effective Liquid Drop Model (ELDM). Very recently in 2024, some authors studied the double alpha decay phenomenon in atomic nuclei using different methodologies [17–19, 21, 22].

Interested in double alpha decay, we have started our study of its decay properties of double alpha decay from the year 2022. In our previous papers [13–15], we have calculated the half-life values of radioactive isotopes by employing our Cubic plus Yukawa plus Exponential Model in two-sphere approximation

and compared the half-life values with the available other theoretical values. Our current work focuses the probable chances of double alpha decay of various isotopes with atomic number $60 \leq Z \leq 93$, utilizing our well-known Cubic plus Yukawa plus Exponential Model (CYEM) in the two sphere approximation.

2. Description of our Model

In this study, we have used our CYE Model [30], which is mentioned as a realistic model by P.B. Price [31] to explore the double alpha decay characteristics of radioactive nuclei, in which we use a cubic potential in the pre-scission region which is connected by a Yukawa plus Exponential potential in the post scission region. Here the zero-point vibration energy is explicitly included without violating the conservation of energy. The potential for the post scission region is given by

$$V(r) = \frac{Z_1 Z_2 e^2}{r} + V_n(r) - Q \quad r \geq r_t, \quad (1)$$

where $V_n(r)$ is the nuclear interaction energy and written in the form

$$V_n(r) = -D \left[F + \frac{r - r_t}{a} \right] \frac{r_t}{r} \exp \left[\frac{r_t - r}{a} \right] \quad (2)$$

and $r_t = R_1 + R_2$ is the sum of the equivalent sharp-surface radii of the daughter nucleus (R_1) and the emitted fragment (R_2). ‘ D ’ and ‘ F ’ are constants and ‘ a ’ is nuclear radii. The shape of the potential barrier in the overlapping region which connects the ground-state and the contact-point is approximated by a third order polynomial in r suggested by Nix [32] having the form

$$V(r) = -E_v + [V(r_t) + E_v] \left\{ s_1 \left[\frac{r - r_i}{r_t - r_i} \right]^2 - s_2 \left[\frac{r - r_i}{r_t - r_i} \right]^3 \right\}; \quad r_i \leq r \leq r_t, \quad (3)$$

where r_i is the distance between the centres of mass of two portions of a parent nucleus cut by a planar section into two pieces with volume asymmetry of the decay. Let a planar section cut the parent nucleus into two unequal portions with the masses of the heavy and light nuclei of the decay. If h_1 and h_2 are the heights of the heavy and light fragments and R_0 is the radius of the parent nucleus, then

$$r_i = \frac{3}{4} \left[\frac{h_1^2}{R_0 + h_1} + \frac{h_2^2}{R_0 + h_2} \right]$$

Table 1. Logarithmic half-life values for double alpha decay of radioactive nuclei with atomic numbers ranging $Z = 60$ to 93

Parent nuclei	$Q_{2\alpha}$ (MeV)	Log $T_{1/2}$ (years)					Parent nuclei	$Q_{2\alpha}$ (MeV)	Log $T_{1/2}$ (years)				
		CYEM [Present work]	ASAFM [3]	SemFIS [4]	CPPM [6]	MGLDM [6]			CYEM [Present work]	ASAFM [3]	SemFIS [4]	CPPM [6]	MGLDM [6]
¹⁴⁵ Nd	1.4395	143.01	–	–	143.67	140.46	¹⁹⁰ Pt	6.0898	55.52	–	52.43	56.51	52.28
¹⁴⁶ Nd	2.4859	92.03	–	–	90.62	87.65	¹⁹² Pt	4.5671	77.58	–	74.15	78.70	74.14
¹⁴⁸ Nd	1.0115	185.59	–	–	186.40	183.07	¹⁹⁴ Pt	2.8986	119.70	–	–	120.95	116.03
¹⁴⁷ Sm	2.8317	84.27	–	–	84.87	81.74	¹⁹⁵ Pt	2.2633	146.97	–	–	148.30	143.25
¹⁴⁸ Sm	3.8900	60.20	–	58.11	60.72	57.83	¹⁹⁶ Pt	1.1735	237.97	–	–	239.58	239.58
¹⁴⁹ Sm	3.4473	68.87	–	66.67	69.43	66.43	¹⁹⁷ Au	1.9895	165.55	–	–	166.95	161.75
¹⁵⁰ Sm	2.6322	90.28	–	–	90.90	87.72	¹⁹⁶ Hg	4.4615	83.19	–	79.53	84.38	79.59
¹⁵² Sm	0.8194	224.39	–	–	225.49	221.89	¹⁹⁸ Hg	2.9036	124.27	–	–	125.60	120.48
¹⁵¹ Eu	3.5656	68.45	–	66.18	69.04	65.95	¹⁹⁹ Hg	1.9993	167.88	–	–	169.32	164.02
¹⁵³ Eu	1.4089	156.06	–	–	156.87	153.29	²⁰⁰ Hg	1.5291	206.60	–	–	206.16	200.76
¹⁵² Gd	4.1912	58.82	–	56.61	59.40	56.34	²⁰¹ Hg	0.8819	296.97	–	–	281.22	281.22
¹⁵⁴ Gd	2.3701	104.71	–	–	105.43	101.96	²⁰³ Tl	1.0810	264.05	–	–	265.97	260.39
¹⁵⁵ Gd	1.2270	176.78	–	–	177.69	173.95	²⁰⁴ Pb	2.6848	137.70	–	–	139.13	133.76
¹⁵⁶ Dy	3.9575	66.76	–	64.28	67.39	64.09	²⁰⁶ Pb	1.2686	240.82	–	–	242.61	236.96
¹⁵⁸ Dy	1.7940	138.59	–	–	139.46	135.63	²⁰⁹ Bi	3.2922	117.79	–	113.63	119.14	113.79
¹⁶² Er	2.5216	109.28	–	–	110.16	106.26	¹⁹¹ At	15.6012	15.50	–	–	–	12.9459
¹⁶⁴ Er	1.7422	148.13	–	–	149.11	143.25	¹⁹² At	14.9592	17.52	–	–	–	14.8825
¹⁶⁶ Er	0.9137	235.62	–	–	236.97	232.72	¹⁹³ At	14.8412	17.87	–	–	–	15.2201
¹⁶⁹ Tm	1.3365	184.40	–	–	185.52	181.24	¹⁹⁴ At	14.3172	19.63	–	–	–	16.9175
¹⁶⁸ Yb	3.2410	91.01	–	–	91.92	87.95	¹⁹⁵ At	14.1252	20.28	–	–	–	17.5403
¹⁷⁰ Yb	2.5677	112.51	–	–	113.47	109.35	¹⁹⁴ Rn	15.5562	8.89	–	–	9.30	6.26
¹⁷¹ Yb	2.2245	127.06	–	–	128.06	123.87	²¹⁰ Rn	17.7472	1.96	–	–	2.09	–0.32
¹⁷² Yb	1.8627	147.52	–	–	147.63	143.35	²¹¹ Rn	15.7962	7.45	–	–	7.79	4.9
¹⁷³ Yb	1.2116	201.68	–	–	202.91	198.49	²¹² Rn	16.4332	5.39	–	–	5.67	2.93
¹⁷⁴ Yb	0.7904	269.20	–	–	270.89	266.38	²¹⁶ Rn	17.1517	3.32	–	–	3.53	0.97
¹⁷⁵ Lu	2.2652	127.84	–	–	128.88	124.58	²¹⁷ Rn	16.4238	5.34	–	–	5.63	2.89
¹⁷⁶ Lu	1.8290	151.74	–	–	152.78	148.44	²¹⁸ Rn	15.096	9.44	–	–	9.87	6.8
¹⁷⁴ Hf	4.2317	72.90	–	69.91	73.81	69.83	²⁰⁹ Fr	18.4372	1.02	–	–	1.12	–1.24
¹⁷⁶ Hf	3.5650	86.94	–	–	87.90	83.77	²¹⁰ Fr	16.3442	6.67	–	–	7.00	4.11
¹⁷⁷ Hf	3.1927	96.64	–	93.38	97.63	93.42	²¹¹ Fr	17.5072	3.35	–	–	3.55	0.96
¹⁷⁸ Hf	2.8236	108.10	–	–	109.13	104.83	²¹⁵ Fr	15.5228	9.02	–	–	9.44	6.36
¹⁷⁹ Hf	2.4062	124.13	–	–	125.20	120.81	²¹⁶ Fr	16.9914	4.58	–	–	4.84	2.12
¹⁸⁰ Hf	1.8544	153.14	–	–	154.29	149.78	²¹⁷ Fr	17.7238	2.55	–	–	2.73	0.20
¹⁸⁰ Ta	3.5917	88.31	–	85.08	89.32	85.08	²¹⁸ Fr	17.0011	4.48	–	–	4.74	2.03
¹⁸¹ Ta	2.9679	105.66	–	–	106.71	102.34	²¹⁹ Fr	15.6262	8.55	–	–	8.96	5.91
¹⁸⁰ W	4.7695	67.17	–	64.15	68.12	64.03	²⁰⁹ Ra	20.5112	–4.99	–	–	–3.10	–5.06
¹⁸² W	3.8486	84.41	–	–	85.43	81.14	²¹⁰ Ra	21.8522	–6.20	–	–	–4.64	–7.64
¹⁸³ W	3.4801	93.17	–	89.81	94.16	89.84	²¹¹ Ra	19.6012	–2.93	–	–	–1.06	–3.25
¹⁸⁴ W	2.9361	109.01	–	–	110.11	105.62	²¹⁶ Ra	15.9104	8.62	–	–	9.03	5.93
¹⁸⁶ W	2.3370	132.49	–	–	133.66	129.03	²¹⁷ Ra	17.4062	4.20	–	–	4.46	1.73
¹⁸⁵ Re	3.7149	89.46	–	86.08	90.53	86.10	²¹⁸ Ra	17.7493	3.23	–	–	3.46	0.81
¹⁸⁷ Re	2.9926	109.46	–	105.90	110.59	106.00	²¹⁹ Ra	16.9767	5.30	–	–	5.63	2.79
¹⁸⁴ Os	5.4740	60.21	–	57.15	61.17	57.02	²²⁰ Ra	15.7916	8.84	–	–	9.28	6.15
¹⁸⁶ Os	4.2855	79.24	–	75.90	80.25	75.90	²⁰⁹ Ac	22.3392	–7.99	–	–	–6.25	–7.88
¹⁸⁷ Os	4.3941	77.15	–	73.84	78.20	73.82	²¹⁰ Ac	19.8052	–1.24	–	–	–0.72	–2.99
¹⁸⁸ Os	3.7923	89.64	–	–	90.74	86.22	²¹¹ Ac	21.1932	–4.19	–	–	–3.93	–5.84
¹⁸⁹ Os	3.5663	95.11	–	91.60	96.24	91.67	²¹⁷ Ac	16.7362	6.91	–	–	7.28	4.27
¹⁹⁰ Os	2.4919	130.78	–	–	132.01	127.21	²¹⁸ Ac	17.9722	3.43	–	–	3.67	0.96
¹⁹² Os	0.7670	294.53	–	–	281.28	281.28	²¹⁹ Ac	18.3674	2.38	–	–	2.56	0.04
¹⁹¹ Ir	3.7342	93.01	–	89.45	94.10	89.52	²¹⁹ Ac	18.4	2.29	2.1012	–	2.328	–0.120
¹⁹³ Ir	2.0080	158.49	–	–	159.81	154.81	²²⁰ Ac	17.5223	4.57	–	–	4.86	2.04

Parent nuclei	$Q_{2\alpha}$ (MeV)	Log $T_{1/2}$ (years)					Parent nuclei	$Q_{2\alpha}$ (MeV)	Log $T_{1/2}$ (years)				
		CYEM [Present work]	ASAFM [3]	SemFIS [4]	CPPM [6]	MGLDM [6]			CYEM [Present work]	ASAFM [3]	SemFIS [4]	CPPM [6]	MGLDM [6]
^{220}Ac	17.5	4.63	4.1012	–	4.678	2.099	^{223}Pa	17.1722	7.06	–	–	7.47	4.33
^{211}Th	23.3842	–8.64	–	–	–7.68	–9.15	^{231}Pa	10.1922	36.46	–	33.18	37.61	32.88
^{218}Th	17.1222	6.58	–	–	6.95	3.91	^{219}U	17.6142	6.76	–	–	7.17	4.01
^{219}Th	18.3652	3.15	–	–	3.39	0.66	^{234}U	9.6274	41.39	–	–	42.54	37.59
^{220}Th	18.4992	2.77	–	–	2.99	0.30	^{234}U	9.709	40.74	–	37.80	35.72	32.86
^{220}Th	18.5	2.77	2.5011	–	2.801	0.296	^{235}U	8.8913	46.84	–	43.08	48.05	42.94
^{221}Th	17.7862	4.61	–	–	4.91	2.04	^{238}U	7.9416	54.84	–	54.84	56.22	50.90
^{221}Th	17.8	4.57	4.1012	–	4.610	2.005	^{220}Np	18.3302	5.55	–	–	5.92	2.82
^{232}Th	8.1519	50.46	–	46.80	51.75	46.67	^{221}Np	18.9192	3.97	–	–	4.28	1.32
^{218}Pa	17.1442	7.32	–	–	7.74	4.59	^{222}Np	19.9872	1.31	–	–	1.50	–1.19
^{219}Pa	17.8702	5.25	–	–	5.58	2.61	^{223}Np	19.7792	1.76	–	–	1.98	–0.76
^{220}Pa	18.9442	2.41	–	–	2.63	–0.08	^{224}Np	19.0322	3.56	–	–	3.86	0.94
^{221}Pa	19.0742	2.05	–	–	2.25	–0.42	^{225}Np	18.0682	6.06	–	–	6.46	3.31
^{222}Pa	18.1682	4.33	–	–	4.64	1.74	^{226}Np	17.1202	8.73	–	–	9.23	5.85
^{222}Pa	18.1	4.52	4.1012	–	4.546	1.916							

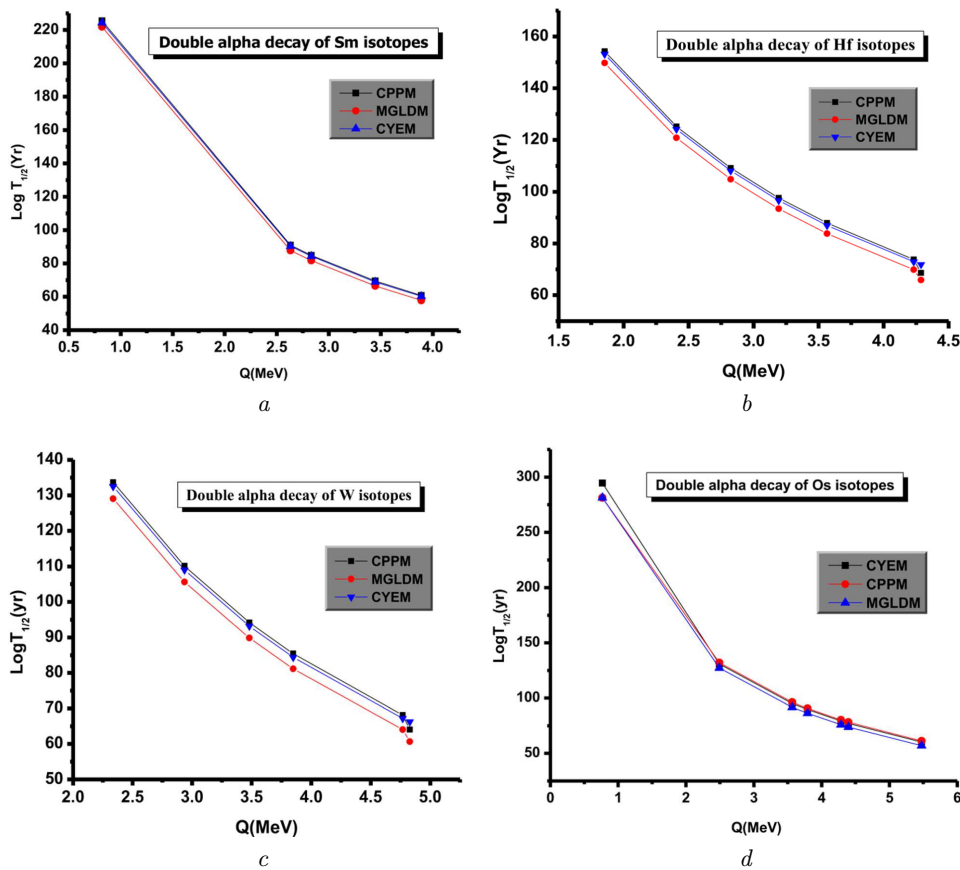


Fig. 1. Variation of logarithm of the half-life with decay energy (Q) for 2α -decay from Sm (a), Hf (b), W (c) and Os (d) isotopes

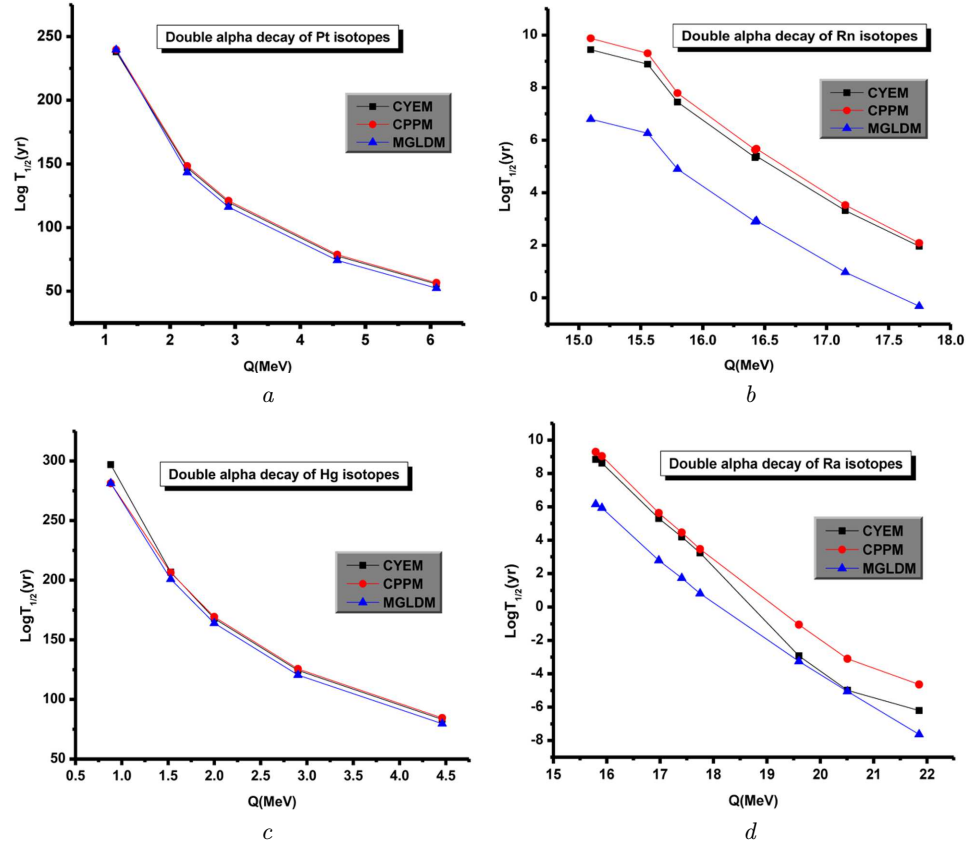


Fig. 2. Variation of logarithm of the half-life with decay energy (Q) for 2α -decay from Pt (a), Rn (b), Hg (c) and Ra (d) isotopes

for calculating the zero – point vibration energy E_v ,

$$E_v = \frac{\pi \hbar}{2} \left[\frac{\left(\frac{2Q}{\mu}\right)^{1/2}}{(C_1 + C_2)} \right] \quad (4)$$

the central radii C_1 and C_2 of the fragments are given by

$$C_i = 1.18 A^{1/3} - 0.48 \quad (i = 1, 2)$$

and

$$\mu = \frac{m_1 m_2}{m_1 + m_2},$$

where μ is the reduced mass of the system and m_1 and m_2 are the mass numbers of the daughter nuclei and the emitted fragment respectively.

2.1. Half-life calculation:

Expressed in seconds, the half-life values are calculated using the formula

$$T = \frac{1.433 \times 10^{-21} (1 + \exp K)}{E_v} \quad (5)$$

the action integral K is given by

$$K = K_L + K_R,$$

where

$$K_L = \frac{2}{\hbar} \int_{r_a}^{r_t} \sqrt{2B_r(r) V(r)} dr,$$

$$K_R = \frac{2}{\hbar} \int_{r_z}^{r_b} \sqrt{2B_r(r) V(r)} dr$$

r_a and r_b are the appropriate zeros of the integrand.

3. Results and Discussion

The focus on the study of double alpha radioactivity is increasing due to the growing number of discovered potential double alpha emitters, making it a crucial tool for exploring nuclear structure. In explaining radioactive decay processes, there exist two kinds of models. They are:

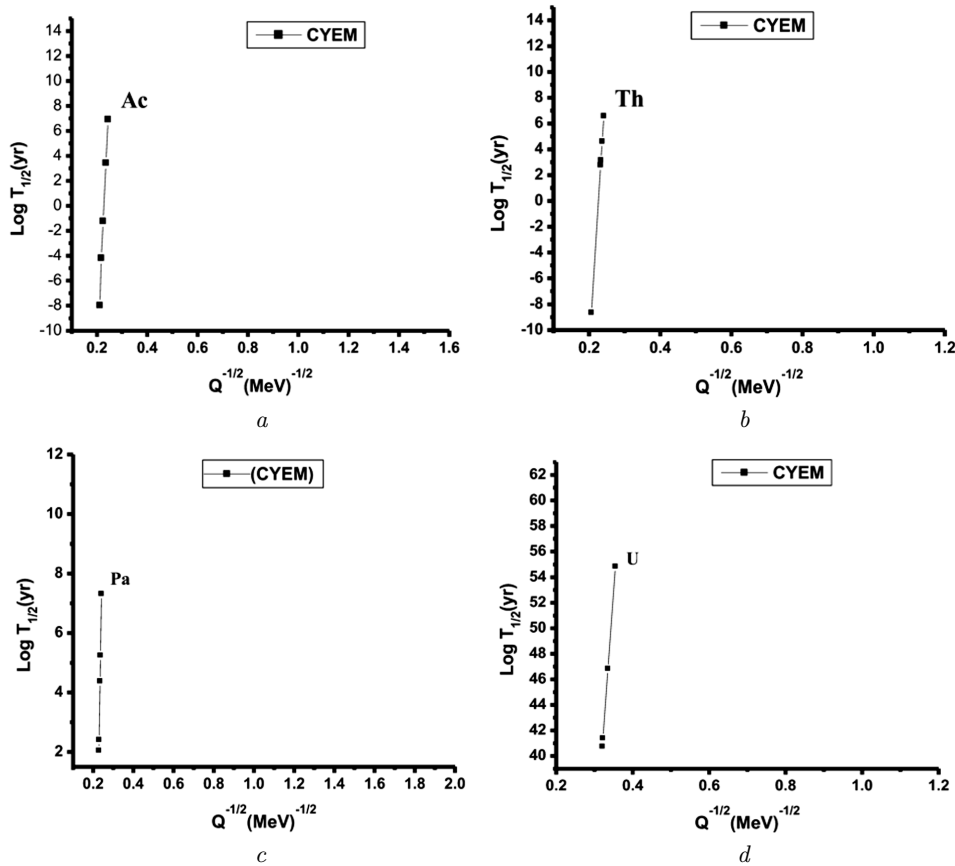


Fig. 3. Geiger-Nuttall plot of 2α -decay for Ac (a), Th (b), Pa (c), and U (d) isotopes

1. Preformed Cluster Model (PCM);
2. Unified Fission Model (UFM).

In Preformed Cluster Model, the cluster is assumed to be pre-born in a parent nucleus before it penetrates the barrier. In Unified Fission Model, the nucleus deforms continuously as it penetrates the nuclear interacting barrier and reaches the scission configuration after running down the Coulomb barrier. Our Cubic plus Yukawa plus Exponential Model serves as a prime example of the Unified Fission Model. Several theoretical and experimental studies have been sug-

Table 2. Half-life values of double alpha decay of ^{224}Ra nuclei

Parent nucleus	$Q_{2\alpha}$ (MeV)	$T_{1/2}$ (days) CYEM
^{224}Ra	12.751 [19]	3.8×10^{22}
	12.19 [20]	1.33×10^{25}

gested for examining the spontaneous double alpha decay phenomenon [1–22].

In our present work, we have predicted the probable chances of double alpha decay with the atomic number range $60 \leq Z \leq 93$ by utilizing our realistic CYE Model [30] in two sphere approximation. Our CYE Model has been utilized to study various phenomena, including the half-life values of alpha decay, cluster decay, spontaneous fission [33–50], superheavy elements [33–41], double alpha decay [13–15], and two-proton decay [42–44].

Table 1 shows the half-life of 2α -decay of $Z = 60$ to 93 isotopes using our CYE Model in the two sphere approximation and compares it with Analytical Super Asymmetric Fission Model (ASAFM) [3], Semi Empirical Fission formula (SemFIS) [4], Coulomb Proximity Potential Model (CPPM) [6] and Modified Generalized Liquid Drop Model (MGLDM) [6]. In Table 1 the first column indicates the parent nuclei with atomic number range from $Z = 60$ to $Z = 93$. The second

column denotes the decay energy ($Q_{2\alpha}$) released during the double alpha decay. Decay energy plays a crucial role in determining the half-life of double alpha decay of radioactive isotopes. The disintegration energy values are taken from the references [3, 4, 6]. The next five columns of Table 1 represent the calculated 2α -decay half-life (in years) of our CYE Model, and other available theoretical models ASAFM, SemFIS, CPPM and MGLDM respectively. Figures 1–3 show the variation of logarithm of half-life of double alpha decay for $Z = 60$ to 93 isotopes. From Figs. 1, 2 it is seen that the calculated half-life values are closer to the available other theoretical results. Additionally we have plotted Geiger Nuttall plot for Ac, Th, Pa, and U isotopes in Fig. 3, and the resultant graph gives the linear output for each nuclei. Furthermore, using our CYE Model in two sphere approximation the 2α decay half-life of ^{224}Ra isotope is calculated for the two different decay energy ($Q_{2\alpha}$) values from Ref. [19] and [20], the resultant half-life values are presented in Table 2.

4. Conclusion

In this present work, we have calculated the double alpha radioactivity half-lives of a selected set of isotopes within the atomic number range from $Z = 60$ to $Z = 93$ using our well-known Cubic plus Yukawa plus Exponential Model in two sphere approximation. The computed half-life values are compared with the available theoretical models namely, Analytical Super Asymmetric Fission Model (ASAFM), Semi Empirical Fission formula (SemFIS), Coulomb Proximity Potential Model (CPPM) and Modified Generalized Liquid Drop Model (MGLDM). From the results presented in Table 1, and Figs 1, 2, it is found that double alpha decay half-life values of these isotopes are in good agreement with the half lives of other theoretical models. Furthermore, we have calculated the half-life of ^{224}Ra isotope using our CYE Model, via double alpha decay and the result is presented in Table 2. We anticipate that our current predictions will prove to have a beneficial impact on future investigations in this particular domain.

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МЕТОДИЧНЕ ДОСЛІДЖЕННЯ ПОДВІЙНОГО АЛЬФА-РОЗПАДУ ЯДЕР ІЗ ЗАРЯДОВИМ ЧИСЛОМ ВІД 60 ДО 93 З ВИКОРИСТАННЯМ СУЕМ

Досліджено періоди напіврозпаду кількох радіоактивних ядер з використанням моделі СУЕМ. У подвійному альфа-розпаді дві альфа-частинки одночасно випромінюються з нестабільних материнських ядер. У нашій реалістичній моделі використовуються потенціали Кулона плюс Юкави плюс експоненційний потенціал як бар'єр для розділених фрагментів та кубічний потенціал для області перекриття, щоб розрахувати час напіврозпаду для подвійного альфа-розпаду. В даній роботі ми розраховали час напіврозпаду радіоактивних ядер із зарядовим числом від $Z = 60$ до 93, використовуючи нашу модель СУЕ у двосферному наближенні. Розраховані значення часу напіврозпаду порівнюються з наявними результатами теоретичних моделей, таких як аналітична суперасиметрична модель поділу (ASAFM), напівемпірична формула поділу (SemFIS), модель кулонівського потенціалу близькодії (CPPM) та модифікована узагальнена модель краплі рідини (MGLDM). Наші результати добре узгоджуються з результатами цих моделей.

Ключові слова: радіоактивні ядра, кластерний розпад, подвійний альфа-розпад.