https://doi.org/10.15407/ujpe69.9.684

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INSTABILITY AND DISTURBANCE OF FERROMAGNETIC PENDULUM OSCILLATIONS AT MAGNETIC-ORIENTATION PHASE TRANSITION INDUCED BY MAGNETIC FIELD

Nonlinear effects of magnetization and magnetic phase transition on the stability and dynamics of a pendulum made of soft-magnetic ferromagnet have been considered. The pendulum is a beam, with its longitudinal dimension being much larger than the transverse dimensions. It has been shown that the magnetization of the pendulum affects its stability and can lead to a critical change in the pendulum equilibrium state in a magnetic field directed perpendicularly (transversely) to the pendulum. The oscillating system loses its rigidity in the critical field, and the eigenfrequency of mechanical pendulum oscillations tends to zero. The critical character of the influence of the magnetic field on the pendulum occurs due to the magnetic-field-induced orientational magnetic phase transition in the ferromagnetic material of the pendulum, which is accompanied by a change in its magnetic state symmetry. An alternating magnetic field together with a stationary magnetic field induces forced mechanical oscillations of the pendulum if the stationary field strength is larger than a threshold value. If the stationary field is less than the critical one, the alternating magnetic field can cause the parametric resonance of the mechanical oscillations of the pendulum.

Keywords: magnetic pendulum, eigenfrequency, orientational magnetic phase transition, parametric resonance, forced oscillations.

1. Introduction

The study of oscillating systems (pendulums) containing magnets and those affected by external magnetic fields are of great interest. They serve as objects to research the manifestations of nonlinear dynamics [1–6], which can be easily monitored in a real-time mode. Such pendulums contain rigid magnets subjected to the force action of an external magnetic field created by a permanent magnet or a current-carrying coil [7, 8].

Free oscillations of a non-magnetic physical pendulum in the absence of a magnetic field take place under the action of the gravity force moment, which is applied to the center of mass of the pendulum's body. As a result, the pendulum performs a periodic (oscillating) rotational motion around the horizon-

Citation: Kalita V.M., Reshetniak S.O., Ryabchenko S.M. Instability and disturbance of ferromagnetic pendulum oscillations at magnetic-orientation phase transition induced by magnetic field. *Ukr. J. Phys.* **69**, No. 9, 684 (2024). https://doi.org/10.15407/ujpe69.9.684.

tal axis [9]. If the pendulum is made of a magnetic material, the external magnetic field also affects the pendulum oscillations,. If the magnetic field is non-uniform or if the pendulum magnetization direction does not coincide with the field direction, then there arises a torque, which affects the rigidity and dynamics of the oscillating system [1–5].

In this work, we consider the influence of an external magnetic field on a magnetic pendulum, which can induce a magnetic phase transition in the ferromagnet from which the pendulum is produced. The nonlinearity caused by the phase transformation can lead to a critical change in the pendulum dynamics. It is to be expected that, as a result of the magnetic phase transition induced by the magnetic field, the equilibrium conditions for the pendulum, its eigenfrequency, and the disturbance character of the mechanical oscillations of the pendulum by the magnetic field may change. We also will consider a non-coercive magnetic pendulum in a uniform magnetic field that induces an orientational magnetic phase transition in the pendulum ferromagnet.

In the course of orientational magnetic phase transitions, a spontaneous change takes place in the orientation of the order parameter (magnetization vector) [10, 11] in magnetics. The orientational phase transition occurs as a result of the competition between the interactions, for example, due to the competition between the external magnetic field and the magnetic anisotropy field, as is observed in uniaxial ferromagnets when the magnetic field is directed perpendicularly to the easy axis of the ferromagnet [12–14]. If a uniaxial ferromagnet is in a low field whose strength is lower than the strength of the magnetic anisotropy field, there emerges an angular phase, a low-symmetry state with the magnetization vector directed non-collinearly with the magnetic field strength vector. But, if the magnetic field exceeds the magnetic anisotropy field, the magnetization vector becomes co-directional with the intensity of the vector magnetic field and oriented along the ferromagnet symmetry axis; this is a high-symmetry magnetic state. Such a change in the states of uniaxial ferromagnet occurs in a critical manner as a magnetic-field-induced phase transition of the second kind [11] between the low-symmetry and high-symmetry magnetic states.

Orientational magnetic phase transitions manifest themselves in the dynamic properties of magnets, when monitoring the fluctuations of the magnetic moment vector [15]. The oscillation and frequency conditions change, if the direction of the magnetic field [16–18] or, which is the same, the orientation of the specimen with respect to the magnetic field changes. In the case where a magnetic field acts on the pendulum, a completely different situations is observed, because the matter concerns mechanical oscillations of the pendulum rather than high-frequency oscillations of the magnetization vector of the ferromagnetic substance of the the pendulum. Therefore, the influence of a magnetic field on the pendulum and a possible orientational magnetic phase transition in the latter is an interesting problem, because the following questions arise. How does some change in the orientation of the magnetization vector affect the pendulum oscillations? Does such a change have a critical behavior? In other words, is this a critical phenomenon for mechanical oscillations of the pendulum?

In the literature [19–25], it is reported on that a magnetic field can induce a critical bending of a soft-magnetic highly elastic beam with a fixed end. If a beam is subjected to a critical bending by a magnetic field, the symmetry of the magnetic state of the beam and the symmetry of the beam shape change simultaneously [26], and the bending itself is accompanied by a non-uniform rotation of the beam sections at various angles [19]. The beam ineity and a change in the beam shape symmetry make it difficult to unambiguously determine whether the critical bending of the beam occurs only due to the orientational magnetic phase transition induced by the magnetic field. In the case of magnetic pendulum, the orientational magnetic phase transition can also be accompanied by a critical rotation, as it occurs at bending, but without changing the shape symmetry.

In this work, we will consider a homogeneous pendulum made of a soft-magnetic beam whose longitudinal size is much larger than its transverse size. The applied magnetic field is uniform and oriented horizontally, i.e., perpendicularly (transversely) to the long axis of the beam in the undisturbed state. We will show that the stationary magnetic field can affect the stability of the pendulum. It will be found that the magnetic field can induce a critical rotation of the pendulum, which occurs as a result of the orientational magnetic phase transition. However, unlike the case of uniaxial ferromagnet, the transverse field, when magnetizing the pendulum, induces a transi-

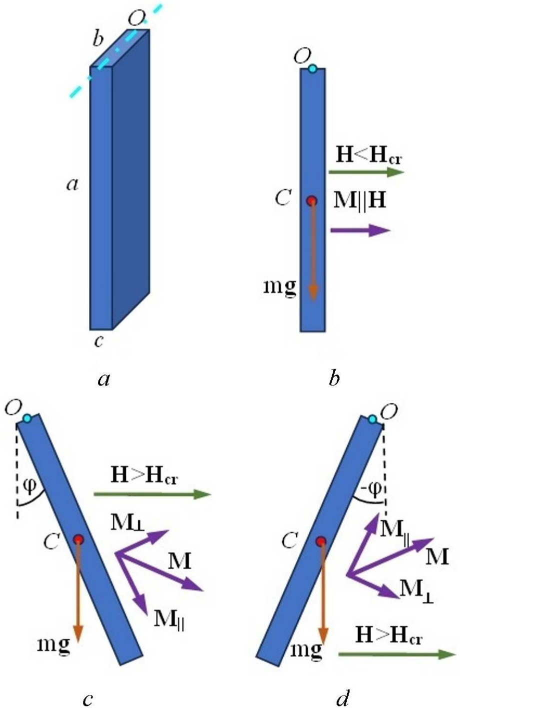

Fig. 1. Images of pendulums that have the shape of a rectangular parallelepiped (*a*) or a cylinder (*b*); (*b*–*d*) pendulum states in a transverse ($\mathbf{H} \perp m\mathbf{g}$) DC magnetic field. O is the axis of pendulum rotation, C is the beam's center of mass

tion from a high-symmetry magnetic phase into a low-symmetry one. The dependence of the pendulum stability on the field strength and the possibility of critical transition affect the eigenfrequencies of pendulum oscillations. It will be shown that, in the subcritical region, a low AC magnetic field in combination with the DC one can disturb the parametric resonance of mechanical pendulum oscillations, which are similar to the swing's oscillations. We will also demonstrate that if the DC field becomes larger than the threshold value, the AC magnetic field can induce forced mechanical oscillations of the pendulum.

The considered problem concerning the influence of a magnetic field on the stability and oscillations of a physical pendulum and the obtained results can be useful when analyzing the influence of a magnetic field on magnetic particles in an elastomer matrix [27, 28]. Particles in the matrix rotate due to the action of the torque created by the magnetic field and the competing torque created by the elastically deformed matrix. The action of the latter torque is similar to the action of the moment of gravity force [28], and it is directed opposite to the vector of the particle rotation angle. However, the task of describing the effect of a magnetic field on a physical pendulum is much simpler as compared to the task of a magnetized particle in an elastomer matrix. It is so, because the deformation of the elastomer matrix is non-uniform, when the particle rotates [27]. Such inhomogeneities are absent in the pendulum case.

2. Critical Rotation of the Beam

In Fig. 1, the images of two pendulums (magnetized beams) are depicted. The longitudinal dimensions of the beams are much larger than their transverse dimensions. One beam is a rectangular parallelepiped with the edges $a \gg b > c$; the other is a cylinder of height a and radius r ($a \gg 2r$).

The rectangular beam (Fig. 1, *a*) has a thin rod at the maximum of its upper face. The rod is fixed in the middle of the edge a and oriented along the edge b. It has a negligibly small mass and a negligibly small moment of inertia. The right and left ends of the rod are fixed in frictionless bearings, so that the beam can rotate around the rod. In Fig. 1, *a*, the rod is shown as a blue dash-dotted line and marked as the O-axis of pendulum rotation. Therefore, the beam can oscillate by rotating around this axis like a physical pendulum. The pendulum is in a uniform horizontal magnetic field, which is perpendicular to the O-axis and, therefore, to the long axis of the beam (when the pendulum is in the equilibrium state in the absence of magnetic field).

Figure 1 illustrates two examples of pendulums – a rectangular plate and a cylinder – because formally the descriptions of the states of both pendulums are identical irrespective of whether the pendulum is a rectangular parallelepiped or a cylinder. In the experiment, it is easier to fabricate a rectangular parallelepiped. At the same time, conditions for the uniformity of the internal magnetic field are better realized for the cylindrical pendulum. Hence, there are no shape advantages between these pendulums.

Three states of a magnetized pendulum are possible.

(i) See Fig. 1, *b*. The pendulum does not deviate from the initial state (the vertical orientation of the beam). The gravitational force equals mg, where m is the beam mass, and g is the free-fall acceleration. It is directed along the beam axis and does not create a torque. The beam magnetization is directed along the field direction, $\mathbf{M} \parallel \mathbf{H}$, and the vector product $\mathbf{M} \times \mathbf{H} = 0$, so the pendulum is in equilibrium.

(ii) See Fig. 1, *c*. The pendulum is deflected by an angle $\varphi \neq 0$ from the gravitational field direction, i.e., the gravitational force is directed at an angle φ with respect to the beam. The beam magnetization $\mathbf{M}$ is non-collinear with $\mathbf{H}$, and its components are $M_\parallel = \chi_\parallel H \sin\varphi$ and $M_\perp = \chi_\perp H \cos\varphi$, where $\chi_\parallel$ and $\chi_\perp$ are the longitudinal and transverse, respectively, components of magnetic susceptibility. Additionally, $\chi_\parallel > \chi_\perp$, because the long axis of the beam is the axis of easy magnetization.

(iii) See Fig. 1, *d*. This is the state with the opposite rotation of the pendulum by the angle $(-\varphi)$ and the longitudinal component of magnetization $\mathbf{M}_\parallel$ directed upwards along the beam.

The energy of the pendulum can be written in the form

$$W(\varphi, H) = \frac{1}{2} mga(1 - \cos\varphi) - \frac{1}{2}\Delta\chi H^2 V \sin^2\varphi, \quad (1)$$

where $\Delta\chi = \chi_\parallel - \chi_\perp$, and $V = abc$ or $V = \pi r^2 a$ is the beam volume. The energy $W(\varphi, H)$ is an even function of H and φ. States (ii) and (iii) have the same energy. The rotation and magnetization in Fig. 1, *c* are equivalent to the rotation and magnetization in Fig. 1, *d*.

The pendulum equilibrium condition is determined from the equation

$$\frac{dW(\varphi, H)}{d\varphi} = \frac{V}{2} \sin\varphi\, (\rho g a - 2\Delta\chi H^2 \cos\varphi) = 0, \quad (2)$$

where ρ is the density of the beam substance. Equation (2) satisfies the condition of mechanical equilibrium, i.e., the gravitational torque is compensated by the torque acting on the beam due to its magnetization.

Equation (2) has two solutions. For the first solution, we have $\sin\varphi = 0$, which corresponds to the equilibrium state of the pendulum with $\varphi_e = 0$, i.e., when the beam is not deflected The second solution corresponds to the equilibrium state in which the beam is deflected, i.e., $\varphi_e \neq 0$; namely,

$$\cos\varphi_e = \frac{\rho g a}{2\Delta\chi H^2}. \quad (3)$$

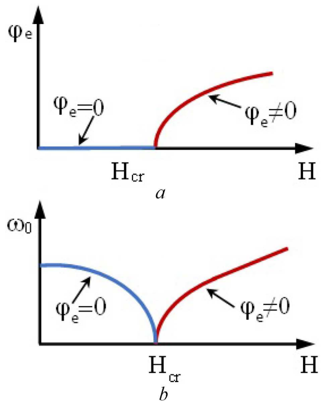

Fig. 2. Field dependence of the pendulum deflection angle φ_e in the magnetic field H (*a*). The field dependence of the pendulum eigenfrequency ω_0 (*b*)

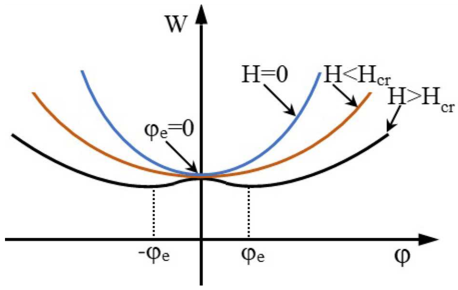

Fig. 3. Dependences of the energy W on the pendulum rotation angle φ for various DC magnetic fields. The equilibrium deflection angle $\varphi_e = 0$ if $H < H_{\rm cr}$, and $\varphi_e \neq 0$ if $H > H_{\rm cr}$

This solution holds, when the field strength becomes stronger than the critical one,

$$H > H_{\rm cr} = \sqrt{\frac{\rho g a}{2\Delta\chi}}. \quad (4)$$

The dependence $\varphi_e(H)$ of the rotation angle on the field strength is shown in Fig. 2, *a*. It has a critical character in a vicinity of the threshold field $H_{\rm cr}$, when $H > H_{\rm cr}$ and $H/H_{\rm cr} \to 1$. Then

$$\varphi_e \approx \sqrt{2(H/H_{\rm cr} - 1)}.$$

If $H < H_{\rm cr}$, the pendulum beam is in a highly symmetric magnetic state, with the magnetization being directed along the heavy axis of the beam, and there is no longitudinal magnetization, $M_\parallel = 0$. If $H > H_{\rm cr}$, the pendulum beam retains its shape and is deflected by the field into one of two possible energy minima (see Fig. 3). The latter arise owing to the spontaneous emergence of the longitudinal component of the beam magnetization, when the beam undergoes an orientational magnetic phase transition from a high-symmetry state into a low-symmetry one.

In the field $H > H_{\rm cr}$ and $H \to H_{\rm cr}$, the field dependence of $M_\parallel$ in a vicinity of the threshold field has a critical index of 1/2,

$$M_\parallel = \chi_\parallel H \sin\varphi_e = \pm\chi_\parallel H\sqrt{1-\frac{H_{\rm cr}^2}{H^2}} \approx$$
$$\approx \pm\chi_\parallel\sqrt{2H_{\rm cr}}\sqrt{H-H_{\rm cr}}. \quad (5)$$

The derivative of the longitudinal component of the beam magnetization is equal to infinity at the critical point, $\left(dM_\parallel/dH\right)_{H=H_{\rm cr}} = \pm\infty$, which corresponds to the critical behavior of magnetization at the orientational magnetic phase transition.

A magnetic field directed perpendicularly to the easy axis of a uniaxial ferromagnet induces the orientational magnetic phase transition from a low-symmetry magnetic state into a high-symmetry one. The inverse process occurs in a pendulum: a transverse magnetic field induces an orientational magnetic phase transition from a high-symmetry magnetic state into a low-symmetry one.

3. Eigenfrequency of Beam Oscillations

The eigenfrequency ω_0 of the pendulum is determined from the relationship $\omega_0^2 = k/I$, where $I = ma^2/3$ is the moment of inertia of the beam, and k is the rigidity coefficient of the oscillating system, which is equal to the second derivative of the energy (1) in the pendulum equilibrium position,

$$k = \left.\frac{d^2W}{d\varphi^2}\right|_{\varphi=\varphi_e}.$$

If the field $H < H_{\rm cr}$, the value of the coefficient k in the pendulum state with $\varphi_e = 0$ depends on the magnetic field,

$$k = \left.\frac{d^2W}{d\varphi^2}\right|_{\varphi=0} = \frac{1}{2}mga - \Delta\chi H^2 V. \quad (6)$$

So, if the pendulum is deflected, then the value of the coefficient k is mainly determined by the action of gravitation, and the magnetic field reduces the value of the coefficient k.

The eigenfrequency of pendulum oscillations in a transverse DC field lower than the threshold, $H < H_{\rm cr}$, is determined from the expression

$$\omega_0(H<H_{\rm cr}) = \sqrt{\frac{3}{2}\frac{g}{a}\left(1-\frac{H^2}{H_{\rm cr}^2}\right)} =$$
$$= \omega_0(H=0)\sqrt{1-\frac{H^2}{H_{\rm cr}^2}}, \quad (7)$$

where $\omega_0^2(H=0) = \frac{3}{2}\frac{g}{a}$ is the square of the eigenfrequency in the field absence.

The eigenfrequency (7) decreases as the magnetic field grows, and vanishes at the critical point $H = H_{\rm cr}$ (see Fig. 2, *b*). The magnitude of the eigenfrequency derivative (7) tends to infinity at the critical point, $(d\omega_0/dH)_{H=H_{\rm cr}} \to -\infty$.

If $H > H_{\rm cr}$, the field dependence of the coefficient k changes,

$$k = \left.\frac{d^2W}{d\varphi^2}\right|_{\varphi_e\neq 0} = \Delta\chi V H^2 - \frac{m^2g^2a^2}{4\Delta\chi V H^2} =$$
$$= \Delta\chi V H^2\left(1-\frac{H_{\rm cr}^4}{H^4}\right). \quad (8)$$

For the deflected pendulum, when the field becomes stronger than the threshold value, the magnetic field dominates over gravity when determining the pendulum stability.

Using Eq. (8), we find the following expression for the pendulum eigenfrequency at $H > H_{\rm cr}$:

$$\omega_0(H>H_{\rm cr}) = \sqrt{\frac{3\Delta\chi H^2}{\rho a^2}\left(1-\frac{H_{\rm cr}^4}{H^4}\right)}. \quad (9)$$

The eigenfrequency (9) of the pendulum vanishes at the critical point, and its derivative tends to infinity at this point.

At $H \gg H_{\rm cr}$, the eigenfrequency increases proportionally to the magnetic field magnitude (see Fig. 2, *b*). This occurs as a result of the beam magnetization and rotation. As H increases, the pendulum rotates toward the field direction, the $M_\parallel$ component of the beam magnetization increases, i.e., the magnitude of the vector **M** increases, and this vector becomes more inclined to the beam axis, which is the axis of easy magnetization. As a result, the value of k increases.

4. Excitation of Beam Oscillations by an AC Magnetic Field

Let the pendulum beam undergo, besides the action of a DC field **H**, also the action of a periodic DC magnetic field $\mathbf{h}$, which is collinear to the field $\mathbf{H}$ ($\mathbf{H} \parallel \mathbf{h}$) and has the amplitude h_a and the frequency ω, i.e., $\mathbf{h} = \mathbf{h}_a e^{i\omega t}$. The influence of the AC field depends on the beam state. If the pendulum is not deflected by the DC field and is magnetized along the field $\mathbf{H}$, $H < H_{\rm cr}$, then the variable component of the magnetic field can excite the beam oscillations only due to the field dependence of the beam eigenfrequency, i.e., due to the parametric resonance. In the deflected state, when the DC component of the field is greater than the threshold value, $H > H_{\rm cr}$, and the magnetization is not collinear with the magnetic field strength vector, then the AC field component will induce forced oscillations.

4.1. Forced oscillations

Let us expand energy (1) in a series in a small pendulum deflection angle $\Delta\varphi = \varphi - \varphi_e$, where φ_e is the beam rotation angle induced by the DC component of the field, provided that the field increment ΔH is small:

$$W(\varphi, H) = \frac{1}{2} mga(1 - \cos\varphi_e) - $$
$$- \frac{1}{2}\Delta\chi H^2 V \sin^2\varphi_e + $$
$$+ \left.\frac{\partial W}{\partial\varphi}\right|_{H,\varphi_e} \Delta\varphi + \left.\frac{\partial W}{\partial H}\right|_{H,\varphi_e} \Delta H + $$
$$+ \frac{1}{2}\left.\frac{\partial^2 W}{\partial\varphi^2}\right|_{H,\varphi_e} (\Delta\varphi)^2 + \left.\frac{\partial W}{\partial\varphi\partial H}\right|_{H,\varphi_e} \Delta\varphi\Delta H + $$
$$+ \frac{1}{2}\left.\frac{\partial^2 W}{\partial H^2}\right|_{H,\varphi_e} (\Delta H)^2. \qquad (10)$$

We take into account that, in the equilibrium state, Eq. (2) holds, and the second derivative with respect to the angle is equal to the coefficient k. We also suppose that $\Delta H \ll H$ and neglect the last term in Eq. (10). Let us also assume that $\Delta H = h_a e^{i\omega t}$, where h_a is the amplitude of the AC field. On the basis of those approximations, we obtain that if the DC component is larger than the threshold value, $H > H_{\rm cr}$, then, in the linear approximation for the small values of the pendulum rotation angle induced by the DC magnetic field, two torques act on the pendulum. One of them is proportional to the deflection angle,

$$N_\varphi = -\Delta\chi V H^2 \left(1 - \frac{H_{\rm cr}^4}{H^4}\right) \Delta\varphi = -k\Delta\varphi,$$

and is directed opposite to the deflection angle vector, and the other is the forcing torque

$$N_h = \Delta\chi V H \sin 2\varphi_e h_a e^{i\omega t}.$$

The differential equation for forced beam oscillations [6, 29] at $H > H_{\rm cr}$ can be written in the form

$$\frac{d^2\Delta\varphi}{dt^2} + \omega_0^2(H > H_{\rm cr})\Delta\varphi =$$
$$= \Delta\chi \frac{V}{I} H \sin 2\varphi_e h_a e^{i\omega t}. \qquad (11)$$

From Eq. (11), we obtain that the amplitude of forced oscillations induced by the magnetic field can be written in the form

$$\Delta\varphi_{\max} = \frac{V}{I} \frac{\Delta\chi H \sin 2\varphi_e}{\omega_0^2(H > H_{\rm cr}) - \omega^2} h_a =$$
$$= \frac{2\Delta\chi \frac{3}{\rho a^2} \frac{H_{\rm cr}^2}{H} \sqrt{1 - \frac{H_{\rm cr}^2}{H^2}}}{\frac{3\Delta\chi H^2}{\rho a^2}\left(1 - \frac{H_{\rm cr}^4}{H^4}\right) - \omega^2} h_a. \qquad (12)$$

As one can see from Eq. (12), the amplitude of the forced mechanical oscillations of the pendulum depends on the DC component of the field, because the eigenfrequency is field-dependent. It should be taken into account that the numerator in Eq. (12) also depends on the parameter H.

4.2. Parametric resonance

If the pendulum beam is not deflected by a stationary field, i.e., $\varphi_e = 0$, then no forced oscillations induced by a low DC field directed in parallel to the DC field can arise, because $N_h = 0$; for such a beam orientation, the beam magnetization is parallel to the field. However, we must take into account that if $H < H_{\rm cr}$, the eigenfrequency of pendulum oscillations depends on the magnitude of the applied magnetic field. If, besides the DC field, an AC magnetic field is also in action (let it look like $h = h_a \cos\omega t$), then, for $\varphi_e = 0$ the expression for the eigenfrequency of pendulum oscillations can be written in the form

$$\omega_0^2 = \omega_0^2(H = 0)\left(1 - \frac{(H + h_a\cos\omega t)^2}{H_{\rm cr}^2}\right) \approx$$
$$\approx \omega_0^2(H = 0)\left(1 - \frac{H^2 + 2Hh_a\cos\omega t}{H_{\rm cr}^2}\right) =$$
$$= \omega_0^2(H = 0)\left(1 - \frac{H^2}{H_{\rm cr}^2} - \frac{2Hh_a\cos\omega t}{H_{\rm cr}^2}\right) =$$

$$= \omega_0^2(H)\left(1 - \frac{2Hh_a}{H_{\rm cr}^2 - H^2}\cos\omega t\right). \qquad (13)$$

By substituting expression (13) into the equation for the eigenoscillations of the pendulum, we arrive at the Mathieu equation [29]

$$\frac{d^2\Delta\varphi}{dt^2} + \omega_0^2(H)(1 - \delta\cos\omega t)\Delta\varphi = 0, \qquad (14)$$

where the parameter $\delta = 2h_aH/(H_{\rm cr}^2 - H^2)$. In Eq. (14), we took into account that if $\varphi_e = 0$, the equality $\Delta\varphi_e = \varphi$ holds.

From Eq. (14), we have that if $h_a \ll H$, the parametric resonance effect can be observed. The fundamental frequency at which the parametric resonance is observed will be twice the eigenfrequency of pendulum oscillations, $\omega \approx 2\omega_0(H)$ [29].

In the case $H = 0$, the value of the parameter δ in the Mathieu equation is proportional to the squared amplitude of the AC magnetic field, $\delta \sim h_a^2$, and the fundamental resonance frequency $\omega \approx \omega_0(H)$.

Note that the frequency features in the behavior of the magnetic pendulum were considered in the approximation of small pendulum deflection angles.

5. Conclusions

The stability of a physical pendulum made of soft-magnetic ferromagnet in a transverse magnetic field has been described. It is shown that if the pendulum magnetization is not collinear with the magnetic field, the torque caused by the magnetic field changes the pendulum equilibrium position. If the pendulum magnetization is collinear with the magnetic field, the equilibrium position of the pendulum is not disturbed, but the rigidity of the oscillating system changes: its magnitude decreases with an increase in the magnetic field strength. Under the influence of a DC magnetic field, the equilibrium position of the pendulum changes, if the magnetic field becomes greater than the threshold value. The threshold character of a change in the equilibrium position of the pendulum occurs due to the orientational magnetic phase transition in the ferromagnet from whichi9 the pendulum is made. At the critical point, a phase transition takes place from a high-symmetry magnetic state into a low-symmetry one.

A weak DC magnetic field, being added to a DC one, induces forced mechanical oscillations of the pendulum if the constant field component exceeds the critical value. If the DC field is less than the critical one, then the DC component of the magnetic field cannot induce forced mechanical oscillations of the pendulum. However, if the DC field is less than the threshold value, the induction of a parametric resonance for the mechanical oscillations of the pendulum is possible by applying a DC magnetic field. For oscillations with finite (not small) deflection angles, new effects in the behavior of the magnetic pendulum as a strongly nonlinear system should be expected.

As it was noted in Introduction, the loss of pendulum stability in a magnetic field is similar to the magnetic-field-induced critical rotation of a magnetic particle in an elastomer matrix. In paper [26], the perturbation of forced mechanical oscillations of a particle under the action of a DC magnetic field was also described, but the parametric resonance effect was not analyzed. Based on the results of our present research, we may assume that a parametric resonance induced by a DC magnetic field is also possible for mechanical vibrations of a particle in an elastomer matrix.

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Received 05.07.24.
Translated from Ukrainian by O.I. Voitenko

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НЕСТІЙКІСТЬ ТА ЗБУРЕННЯ КОЛИВАНЬ ПРИ ІНДУКОВАНОМУ МАГНІТНИМ ПОЛЕМ ОРІЄНТАЦІЙНОМУ МАГНІТНОМУ ФАЗОВОМУ ПЕРЕХОДІ У ФІЗИЧНОМУ МАЯТНИКУ

Розглянуто ефекти нелінійного впливу намагнічування та магнітного фазового переходу на стійкість та динаміку маятника, виготовленого з магнітом'якого феромагнетика, який має форму балки, поздовжній розмір якої набагато більший за її поперечні розміри. Показано, що намагнічування маятника впливає на стійкість та може призвести до критичної зміни рівноваги маятника в перпендикулярному (поперечному) до маятника магнітному полі. В критичному полі відбувається втрата жорсткості коливальної системи, а частота власних механічних коливань маятника прямує до нуля. Критичний характер впливу магнітного поля на маятник пов'язаний із індукованим магнітним полем орієнтаційним магнітним фазовим переходом у феромагнетику маятника, який супроводжується зміною симетрії його магнітного стану. Змінне магнітне поле, додане до стаціонарного магнітного поля, індукує вимушені механічні коливання маятника за умови, що стаціонарне поле більше за порогову величину. Коли стаціонарне поле менше від критичного, то змінне магнітне поле може спричинити параметричний резонанс механічних коливань маятника.

Ключові слова: магнітний маятник, власна частота, орієнтаційний магнітний фазовий перехід, параметричний резонанс, вимушені коливання.