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SEMI-SYMMETRIC METRIC GRAVITY [1]

We will study a geometric extension of general relativity, which is based on a connection with a special type of torsion. This connection satisfies that its torsion tensor is fully determined by a vectorial degree of freedom, and it was first introduced by Friedmann and Schouten. We explore its physical implications by presenting three cosmological models within the considered geometric extension of GR, and compare the predictions of the models with those of ΛCDM and the observational data of the Hubble function. Our results show that the geometry envisioned by Friedmann could explain the observational data for the Hubble function without the need of dark energy.

Keywords: cosmological models, semi-symmetric metric gravity, general relativity, torsion tensor, Hubble function.

1. Introduction

General Relativity [1] (GR) has been highly successful in explaining numerous observed phenomena such as the perihelion shift of Mercury, light deflection, Shapiro time delay, Nortvedt effect, gravitational waves [2, 3]. However, it is widely known that the theory faces both observational and theoretical challenges, including issues such as dark matter, dark energy, the naturalness problem and the Hubble tension. Therefore, over the past few decades, a variety of extensions to GR, known as modified theories of gravity, have been developed to address these issues. These extensions can be broadly categorized into three groups:

1. Modifications where the action $f(R) = R$ is changed.
2. Modifications where the geometry is changed.
3. Modifications where both the geometry and action are changed.

In this paper, we study an extension of general relativity, in which we modify the underlying connection, hence passing to non-Riemannian geometry. In this setting, an affine connection is characterized by three quantities: the Riemann, non-metricity and torsion tensors. There are eight possibilites, as depicted in Fig. 1.

The extension we propose is based on a special type of Riemann–Cartan geometry, which we call *semi-symmetric metric geometry*, where non-metricity vanishes, but torsion is non-zero and takes a special form. This geometry was first introduced by J. Schouten and Alexander Friedmann [4], the founder of modern cosmology, in 1924 shortly before his death. In the physics literature, this connection has been widely ignored, despite its physical relevance: Schouten noted in his book that if a person moves on the Earth's surface always facing a specific point, this displacement is semi-symmetric and metric [5]. On the other hand, the mathematical properties of semi-symmetric metric connections have been thoroughly studied [6, 7]. Therefore, the aim of this paper is to explore the physical and cosmological implications of the geometry envisioned by Friedmann.

2. Geometric Preliminaries

In this section, we provide the geometric preliminaries needed to formulate semi-symmetric metric gravity. To begin, we establish our conventions and define

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[1] This work is based on the results presented at the XII Bolyai–Gauss–Lobachevskii (BGL-2024) Conference: Non-Euclidean Geometry in Modern Physics and Mathematics.

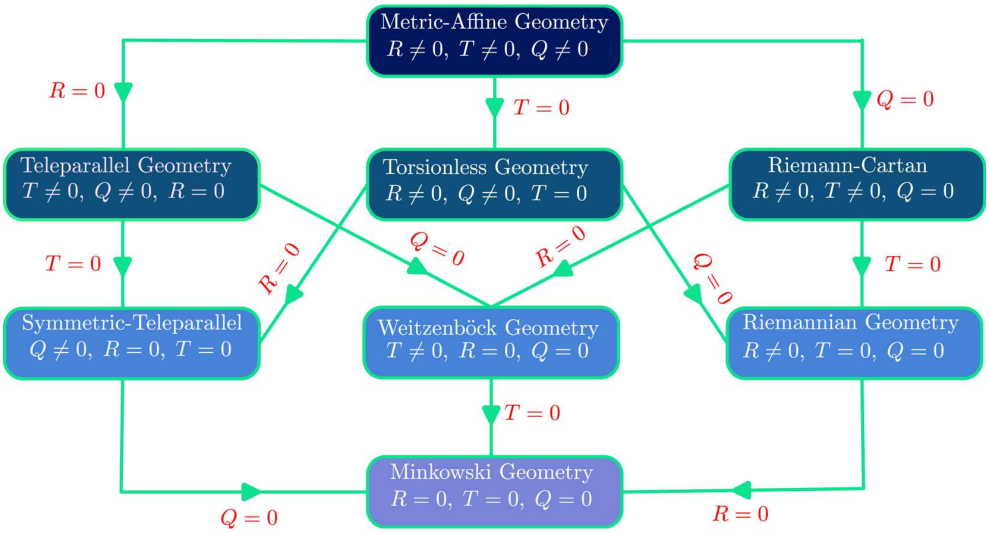


Fig. 1. The eight possible classes of geometries described by torsion, non-metricity and the Riemann tensor

the torsion as

$$T^\mu{}_{\nu\rho} = 2\Gamma^\mu{}_{[\rho\nu]}. \tag{1}$$

For the semi-symmetric metric connection introduced by Friedmann and Schouten, the torsion tensor can be expressed as [4]

$$T^\mu{}_{\nu\rho} = \pi_\rho \delta^\mu_\nu - \pi_\nu \delta^\mu_\rho \ \text{ for some one-form } \ \pi_\rho. \tag{2}$$

Substituting this form of torsion into the general decomposition

$$\Gamma^\mu{}_{\nu\rho} = \gamma^\mu{}_{\nu\rho} + \frac{1}{2} g^{\lambda\mu}(-Q_{\lambda\nu\rho} + Q_{\rho\lambda\nu} + Q_{\nu\rho\lambda}) - \\ - \frac{1}{2} g^{\lambda\mu}(T_{\rho\nu\lambda} + T_{\nu\rho\lambda} - T_{\lambda\rho\nu}) \tag{3}$$

yields the Christoffels symbols

$$\Gamma^\mu{}_{\nu\rho} = \gamma^\mu{}_{\nu\rho} - \pi^\mu g_{\rho\nu} + \pi_\nu \delta^\mu_\rho. \tag{4}$$

The Riemann tensor of the semi-symmetric metric connection

$$Riem^\mu{}_{\nu\rho\sigma} = \Gamma^\lambda{}_{\nu\sigma}\Gamma^\mu{}_{\lambda\rho} - \Gamma^\lambda{}_{\nu\rho}\Gamma^\mu{}_{\lambda\sigma} + \partial_\rho \Gamma^\mu{}_{\nu\sigma} - \partial_\sigma \Gamma^\mu{}_{\nu\rho} \tag{5}$$

can be decomposed using the Riemann tensor $\overset{\circ}{Riem}{}^\mu{}_{\nu\rho\sigma}$ of the Levi-Civita connection as [6]

$$Riem^\mu{}_{\nu\rho\sigma} = \overset{\circ}{Riem}{}^\mu{}_{\nu\rho\sigma} - S_{\sigma\nu}\delta^\mu_\rho + S_{\rho\nu}\delta^\mu_\sigma - \\ - g_{\sigma\nu} S_{\rho\lambda} g^{\lambda\mu} + g_{\rho\nu} S_{\sigma\lambda} g^{\lambda\mu}, \tag{6}$$

where the tensor $S_{\nu\sigma}$ is given by

$$S_{\nu\sigma} = \overset{\circ}{\nabla}_\nu \pi_\sigma - \pi_\nu \pi_\sigma + \frac{1}{2} g_{\nu\sigma} \pi_\lambda \pi^\lambda. \tag{7}$$

The Ricci tensor and scalar can be straightforwardly obtained and they read

$$R_{\nu\sigma} = \overset{\circ}{R}_{\nu\sigma} - 2S_{\sigma\nu} - g_{\nu\sigma} S_{\lambda\beta} g^{\lambda\beta}, \\ R = \overset{\circ}{R} - 6 S_{\beta\lambda} g^{\beta\lambda}. \tag{8}$$

3. Semi-Symmetric Metric Gravity

In this section, we outline the gravitational theory built upon the previously introduced semi-symmetric metric connection. First of all, we postulate that the gravitational field equations are given by

$$R_{(\nu\sigma)} - \frac{1}{2} R g_{\nu\sigma} = 8\pi T_{\nu\sigma}, \tag{9}$$

where $R_{\nu\sigma}$ and R denote the Ricci curvature and scalar of the semi-symmetric metric connection, respectively, and $T_{\nu\sigma}$ denotes the matter energy-momentum tensor. Substituting the formulas provided in (8) into (9) we obtain

$$\overset{\circ}{R}_{\nu\sigma} - \frac{1}{2} g_{\nu\sigma} \overset{\circ}{R} - S_{\sigma\nu} - S_{\nu\sigma} + 2 g_{\sigma\nu} S_{\lambda\beta} g^{\lambda\beta} = 8\pi T_{\nu\sigma}. \tag{10}$$

To obtain a complete post-Riemannian expansion of the Einstein equation, we also express S in terms of

π using (7). After some algebraic manipulations, we readily find

$$\overset{\circ}{R}_{\nu\sigma} - \frac{1}{2} g_{\nu\sigma} \overset{\circ}{R} - \overset{\circ}{\nabla}_\sigma \pi_\nu - \overset{\circ}{\nabla}_\nu \pi_\sigma + 2\pi_\sigma \pi_\nu + + 2 g_{\sigma\nu} \overset{\circ}{\nabla}_\lambda \pi^\lambda + g_{\nu\sigma} \pi^\rho \pi_\rho = 8\pi T_{\nu\sigma}. \tag{11}$$

As mentioned in the introduction, our goal is to explore cosmological implications of the semi-symmetric metric connection. To this end, we evaluate the field equations (11) for an isotropic, homogeneous and spatially flat FLRW metric

$$ds^2 = -dt^2 + a^2(t)\delta_{ij} dx^i dx^j. \tag{12}$$

As matter we consider a perfect fluid described by the energy-momentum tensor

$$T_{\nu\sigma} = \rho u_\nu u_\sigma + p(u_\nu u_\sigma + g_{\nu\sigma}). \tag{13}$$

The problem is taken into account in a comoving frame, in which the four-velocity is given by

$$u_\nu = (-1, 0, 0, 0) \iff u^\nu = (1, 0, 0, 0). \tag{14}$$

Finally, as we are in a highly symmetric case, we choose

$$\pi_\nu = (-\omega(t), 0, 0, 0) \iff \pi^\nu = (\omega(t), 0, 0, 0). \tag{15}$$

Living with these assumptions, the Friedmann equations take the form

$$3H^2 = 8\pi\rho - 3\omega^2 + 6H\omega, \tag{16}$$

$$2\dot{H} + 3H^2 = -8\pi p + 4H\omega - \omega^2 + 2\dot{\omega}, \tag{17}$$

where we have denoted $H = \dot{a}/a$.

It is easily seen that the Friedmann equations of GR are recovered in the limit $\omega \to 0$. Hence, we interpret the additional terms as an effective geometric type dark energy

$$\rho_{\rm eff} = \frac{1}{8\pi}\left(6H\omega - 3\omega^2\right), \quad p_{\rm eff} = -\frac{1}{8\pi}\left(4H\omega - \omega^2 + 2\dot{\omega}\right). \tag{18}$$

We can, therefore, rewrite the Friedmann equations as

$$3H^2 = 8\pi(\rho + \rho_{\rm eff}), \tag{19}$$

$$2\dot{H} + 3H^2 = -8\pi(p + p_{\rm eff}). \tag{20}$$

Hence, the continuity equation takes the form

$$\dot{\rho} + 3H(\rho + p) + \dot{\rho}_{\rm eff} + 3H(\rho_{\rm eff} + p_{\rm eff}) = 0. \tag{21}$$

An equivalent form is also given by

$$3H(\rho + p) + \frac{3}{8\pi}\left[\frac{d}{dt}\left(2H\omega - \omega^2\right) + + H\left(2H\omega - 2\omega^2 - 2\dot{\omega}\right)\right] = 0. \tag{22}$$

To simplify the formalism, we introduce a set of dimensionless variables, according to

$$H = H_0 h, \quad \tau = H_0 t, \quad \omega = H_0 \Omega, \quad \rho = \frac{3H_0^2}{8\pi} r, \quad p = \frac{3H_0^2}{8\pi} P. \tag{23}$$

In these notations, the Friedmann equations can be rewritten as

$$h^2 = r + r_{\rm eff}, \tag{24}$$

$$2\frac{dh}{d\tau} + 3h^2 = -3(P + P_{\rm eff}), \tag{25}$$

with

$$r_{\rm eff} = 2h\Omega - \Omega^2, \quad P_{\rm eff} = -\frac{1}{3}\left(4h\Omega - \Omega^2 + 2\frac{d\Omega}{d\tau}\right), \tag{26}$$

where $\rho_{\rm eff} = (3H_0^2/8\pi) r_{\rm eff}$, and $p_{\rm eff} = (3H_0^2/8\pi) P_{\rm eff}$.

To directly compare with observations, we also introduce the redshift variable

$$1 + z = 1/a, \quad \text{which implies} \quad \frac{d}{d\tau} = -(1+z)h(z)\frac{d}{dz}. \tag{27}$$

The evolution equations in the redshift representation read

$$h^2(z) = r(z) + 2h(z)\Omega(z) - \Omega^2(z), \quad 2(1+z)h(z)\frac{dh(z)}{dz} + 3h^2(z) = -3P(z) + + 4h(z)\Omega(z) - \Omega^2(z) - 2(1+z)h(z)\frac{d\Omega}{dz}. \tag{28}$$

4. Cosmological Models

In this section, we construct three cosmological models by imposing equations of state between the matter pressure and matter energy density, and the effective pressure and effective matter density, respectively. We then compare our findings with the standard ΛCDM model. In what follows, we will consider, for matter, a pressureless dust, that is, we set $P = 0$.

4.1. Analytical cosmological model

Given the assumption $P = 0$, we only have to consider equations of state which relate the effective components. For the first model, we assume that both the effective pressure and effective density are constants, but they differ. Mathematically, this can be formulated as

$$3\omega\left(2H - \omega\right) = \Lambda, \;\; 4H\omega - \omega^2 + 2\dot{\omega} = \frac{2}{3}k\Lambda, \tag{29}$$

where k and Λ are non-zero constants and k is assumed to be positive. By eliminating H from the equations of state, we obtain, for ω, the differential equation

$$2\dot{\omega} + \omega^2 + 2(1-k)\frac{\Lambda}{3} = 0, \tag{30}$$

which admits an analytical solution

$$\omega(t) = \sqrt{\frac{2(k-1)\Lambda}{3}}\tanh\left[\frac{\sqrt{(k-1)\Lambda}}{\sqrt{6}}\left(t-t_0\right)\right], \tag{31}$$

where t_0 is an arbitrary constant of integration. Then, using this expression, we obtain for the Hubble function

$$H(t) = \frac{\sqrt{\Lambda}}{2\sqrt{6(k-1)}}\tanh\left[\frac{\sqrt{(k-1)\Lambda}\,(t-t_0)}{\sqrt{6}}\right]\times$$
$$\times\left\{\coth^2\left[\frac{\sqrt{(k-1)\Lambda}\,(t-t_0)}{\sqrt{6}}\right] + 2(k-1)\right\}. \tag{32}$$

The matter density obtained from the first Friedmann equation $\rho = 3H^2 - \Lambda$ takes the form

$$8\pi\rho(t) = \frac{\Lambda}{8(k-1)}\left\{\coth\left[\sqrt{\frac{(k-1)\Lambda}{6}}\left(t-t_0\right)\right] -\right.$$
$$\left.- 2(k-1)\tanh\left[\sqrt{\frac{(k-1)\Lambda}{6}}\left(t-t_0\right)\right]\right\}^2. \tag{33}$$

Similarly, the pressure can be expressed as

$$8\pi p(t) = \frac{\Lambda}{24(k-1)}\left\{(4k-7)\mathrm{csch}^2\times\right.$$
$$\times\left[\sqrt{\frac{(k-1)\Lambda}{6}}\left(t-t_0\right)\right] + 4(k-1)^2\mathrm{sech}^2\times$$
$$\left.\times\left[\sqrt{\frac{(k-1)\Lambda}{6}}\left(t-t_0\right)\right] + 4k^2 - 4k - 3\right\}. \tag{34}$$

The scale factor of this cosmological model is given by

$$a(t) = a_0\sinh^{2(k-1)}\left[\sqrt{\frac{(k-1)\Lambda}{6}}\left(t-t_0\right)\right]\times$$
$$\times\cosh^{\sqrt{6}}\left[\sqrt{\frac{(k-1)\Lambda}{6}}\left(t-t_0\right)\right]. \tag{35}$$

4.2. Cosmological model with linear equation of state for dark energy

For the second cosmological model, we assume that the dark components satisfy a linear equation of state

$$P_{\rm eff}(z) = -\sigma(z)r_{\rm eff}(z) + \lambda, \tag{36}$$

where λ is a constant and $\sigma(z)$ is given by the CPL parametrization

$$\sigma(z) = \sigma_a + \sigma_0\frac{z}{1+z}. \tag{37}$$

In this case, the evolution equations of the Universe are given by

$$-2(1+z)h(z)\frac{dh(z)}{dz} + 3h^2(z) = 3\lambda +$$
$$+ 3\sigma(z)\;\left[2h(z)\Omega(z) - \Omega^2(z)\right], \tag{38}$$

$$-2(1+z)h(z)\frac{d\Omega(z)}{dz} = 2\left[3\sigma(z) - 2\right]h(z)\Omega(z) +$$
$$+\left[1 - 3\sigma(z)\right]\Omega^2(z) + 3\lambda. \tag{39}$$

The system of equations (38) and (39) has to be solved numerically with the initial conditions $h(0) = 1$, and $\Omega(0) = \Omega_0$. Nevertheless, the closure relation determines the initial condition Ω_0, given the present day matter density through the formula

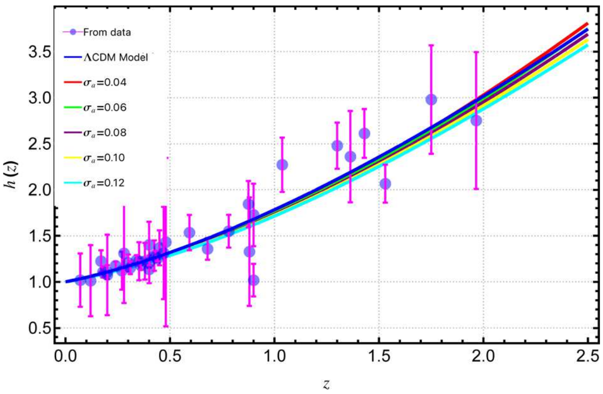
$$\Omega_0 = 1 + \sqrt{r(0)}. \tag{40}$$

In the following, we will numerically integrate differential equations (38) and (39) and compare the predictions of the theory with the ΛCDM framework, in which the Hubble function is given by

$$H = H_0\sqrt{\frac{\Omega_m}{a^3} + \Omega_\Lambda} = H_0\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}, \tag{41}$$

where $\Omega_m = \Omega_b + \Omega_{\rm DM}$, with $\Omega_b = \rho_b/\rho_{\rm cr}$, $\Omega_{\rm DM} = \rho_{\rm DM}/\rho_{\rm cr}$, and $\Omega_\Lambda = \Lambda/\rho_{\rm cr}$, where $\rho_{\rm cr}$ is the critical density of the Universe. The deceleration parameter is given by the relation

$$q(z) = \frac{3(1+z)^3\Omega_m}{2\left[\Omega_\Lambda + (1+z)^3\Omega_m\right]} - 1. \tag{42}$$



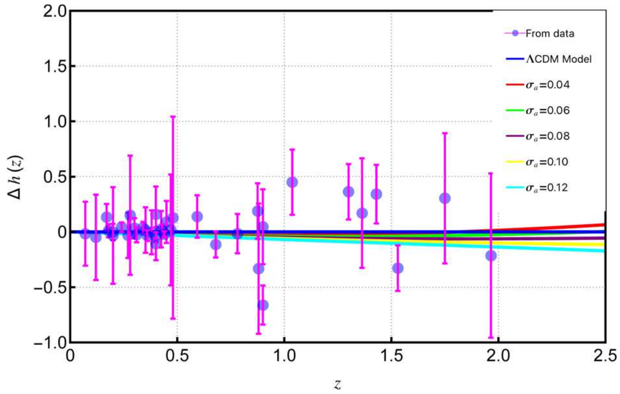
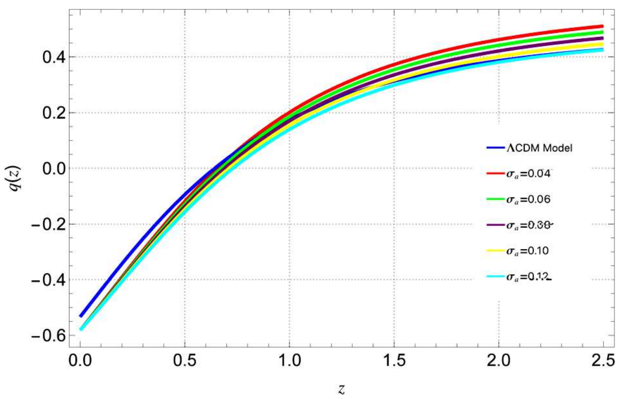


Fig. 2. Variations as a function of the redshift z of the dimensionless Hubble function (left panel) and of the difference between the dimensionless Hubble function and of the ΛCDM (right panel). For the linear equation of state, the parameters are given by $\lambda = 0.79$, $r(0) = 0.311$, $\sigma_0 = -0.10$ and different values of σ_a



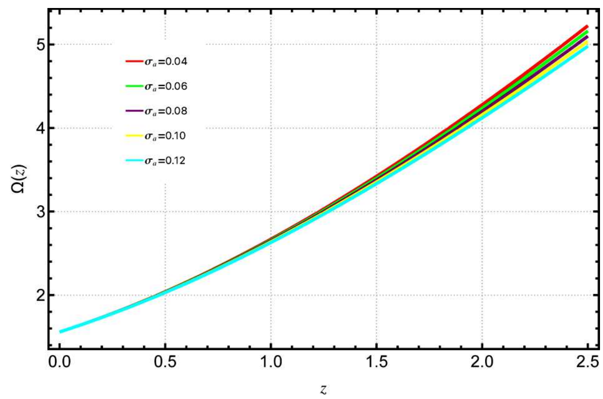


Fig. 3. Variations as a function of the redshift z of the deceleration parameter $q(z)$ (left panel) and of the torsion vector $\Omega(z)$ (right panel) for the linear equation of state model with parameters $\lambda = 0.79$, $r(0) = 0.311$, $\sigma_0 = -0.10$ and different values of σ_a

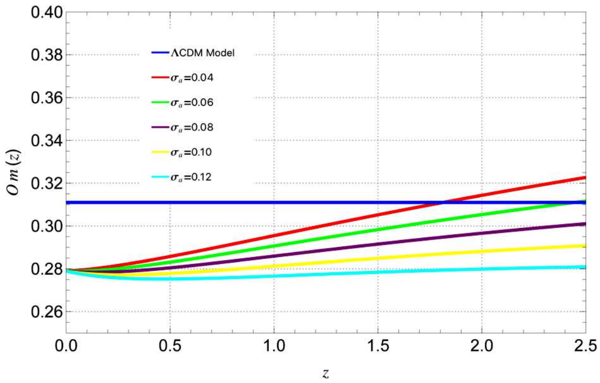


Fig. 4. Behavior of the function $Om(z)$ for Model II, for $\lambda = 0.79$, $r(0) = 0.311$, $\sigma_0 = -0.10$, and different values of σ_a

As for the parametrization, we use the numerical values $\Omega_m = 0.3075$, $H_0 = 67.1$, $\Omega_\Lambda = 0.6911$ [8]. The data is taken from [9].

The variations of the Hubble function and of the difference between the present model's Hubble function and the one of ΛCDM are depicted in Fig. 2. It can be seen that, for the considered range of parameters, the linear equation of state model can reproduce well both the observational data and the predictions of the standard ΛCDM model. However, at higher redshifts $z > 2$, the predictions of the present model differ from the ΛCDM and depend on the initial conditions.

The variations of the deceleration parameter $q(z)$ and torsion vector $\Omega(z)$ are depicted in Fig. 3

As it can be seen from Fig. 3, in the interval $0 < z < 0.5$, there seem to be small deviations, our model predicting a slightly smaller deceleration parameter than the standard ΛCDM paradigm. From the same figure we deduce that the torsion is an increasing function of the redshift and takes positive

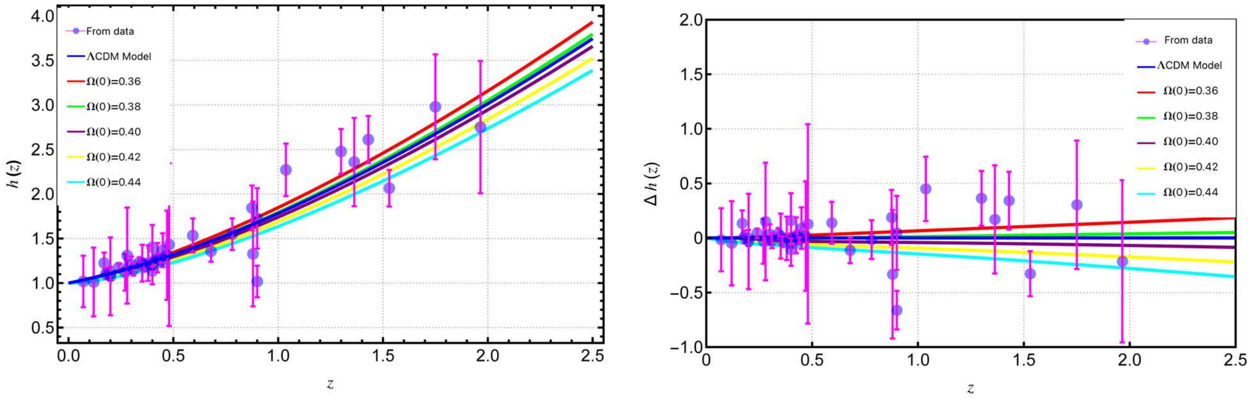


Fig. 5. Variations of the dimensionless Hubble function $h(z)$(left panel), and of the difference $\delta h(z)$ (right panel) for the polytropic model with $K = -2$ and several distinct initial conditions Ω_0

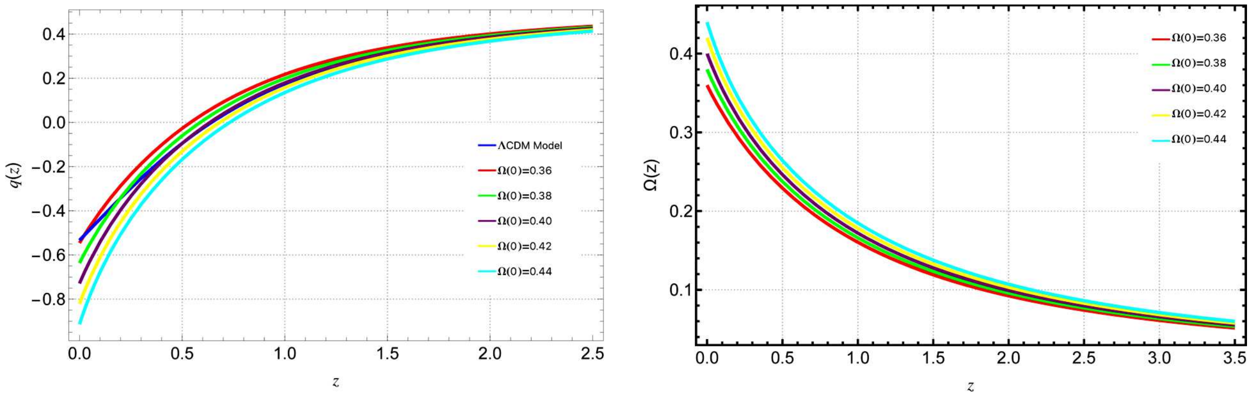


Fig. 6. Variations as a function of the redshift z of the deceleration parameter $q(z)$ and of the torsion vector $\Omega(z)$ for the polytropic model with $K = -2$ and several distinct initial values $\Omega(0)$

values during the cosmological evolution. The parameter σ_a slightly changes $\Omega(z)$ only for larger values $z > 2$.

The behavior of the function $Om(z)$ is represented in Fig. 4. The $Om(z)$ diagnostic differs from the ΛCDM drastically, indicating a possibility of transition between phantom-like and quintessence-like evolutions.

4.3. Cosmological model with polytropic equation of state for dark energy

As a third cosmological model, we consider a polytropic equation of state

$$P_{\text{eff}} = K r_{\text{eff}}^2, \tag{43}$$

where K is a constant. In this case, the cosmological evolution of the Universe is governed by

$$-2(1+z)h(z)\frac{dh(z)}{dz} + 3h^2(z) - 4h(z)\Omega(z) + \\ + \Omega^2(z) + 2(1+z)h(z)\frac{d\Omega(z)}{dz} = 0, \tag{44}$$

$$\frac{1}{3}\left[4h(z)\Omega(z) - \Omega^2(z) - 2(1+z)h(z)\frac{d\Omega}{dz}\right] + \\ + K\left[2h(z)\Omega(z) - \Omega^2(z)\right]^2 = 0. \tag{45}$$

The above system has to be integrated with the initial conditions $h(0) = 1$ and $\Omega(0) = \Omega_0$. After integration, we obtain the matter energy density from the closure relation

$$r(z) = h^2(z) - 2h(z)\Omega(z) + \Omega^2(z). \tag{46}$$

The variations as the function of the redshift z of the dimensionless Hubble function $h(z)$ and the difference $\delta H(z)$ are represented for $K = -2$ and different values of $\Omega(0)$ in Fig. 5.

As can be seen from Fig. 5, the polytropic model describes very well both the observational data of the

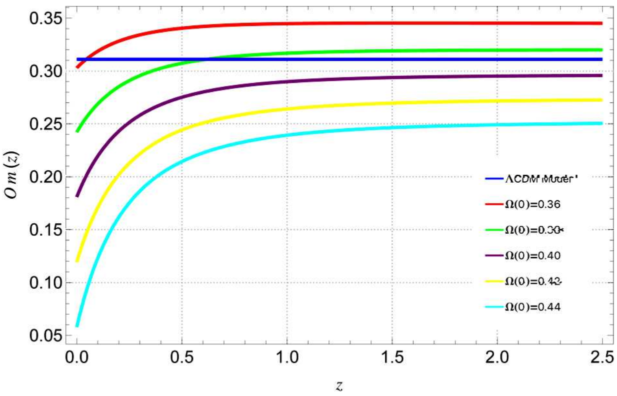


Fig. 7. Behavior of the function $Om(z)$ for the polytropic model with $K = -2$ and different values of $\Omega(0)$

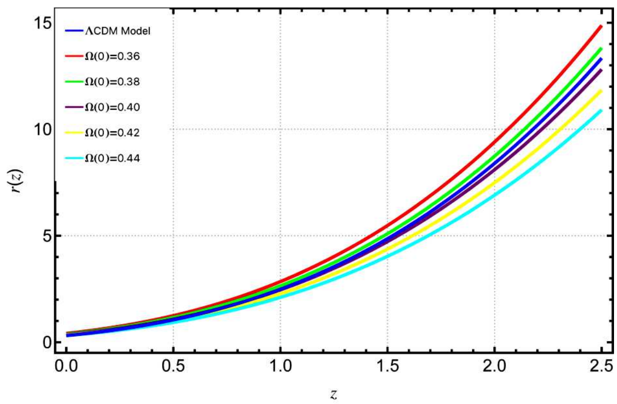


Fig. 8. Behavior of the matter density $r(z)$ as a function of the redshift z for the polytropic model with $K = -2$ and different values of $\Omega(0)$

Hubble function and the ΛCDM model. However, at redshifts $z > 1.5$, some small deviations from ΛCDM appear.

The evolution of the deceleration parameter $q(z)$ and of the torsion vector $\Omega(z)$ are depicted in Fig. 6.

From Fig. 6 we can see that small deviations appear in the redshift range $0 < z < 1.5$ in the case of deceleration parameter, our model predicting slightly smaller values. From $z > 1.5$, the predictions of our model and the ΛCDM basically coincide. The torsion vector is a decreasing function of the redshift and takes positive values on the interval $0 < z < 3.5$. The initial value chosen does not affect the high-redshift behaviour of the torsion vector.

The $Om(z)$ diagnostic function of the polytropic model is shown on Fig. 7. This differs significantly from the ΛCDM $Om(z)$ function at lower redshifts, as it is a monotonically increasing functions instead of a constant. However, at large redshifts, it asymptotically converges to a constant value.

The present day matter density of the polytropic model can be seen in Fig. 8. The predictions of this model and the ΛCDM paradigm basically coincide up to $z \simeq 1$.

5. Summary and Conclusions

In the present paper, we have investigated a geometric extension of general relativity, by including the torsion of a special type, first introduced by Friedmann in 1924. We have written down the Einstein field equations in a post-Riemannian expansion, highlighting the effects of torsion. To see the physical relevance of the torsion terms, we considered three cosmological models, where the effective dark energy and pressure were related by three different equations of state. We have qualitatively shown that our model is able to reproduce the predictions of the standard ΛCDM framework, and to explain the observational data, without the need of the dark energy.

Nevertheless, it is important to mention that our analysis is qualitative in nature, and a detailed comparison with larger datasets including an MCMC analysis to constrain the parameters of the models are needed for quantitative predictions.

Further prospects of the current work include, but are not limited to:

1. Studying spherically symmetric solutions of the field equations, either in vacuum, or in the presence of matter. Either black hole models or stellar structures could be explored in semi-symmetric metric gravity.

2. Finding the non-relativistic limit of our field equations and considering if the torsion could account for the galaxy rotation curves, which is usually attributed to dark matter.

Altogether, we conclude the work with the idea of that the Friedmann's envisioned geometry could be a plausible alternative of standard general relativity, in which dark energy has a purely geometric origin.

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НАПІВСИМЕТРИЧНА МЕТРИЧНА ҐРАВІТАЦІЯ

У цій роботі ми вивчаємо геометричне розширення загальної теорії відносності, яке базується на зв'язку з особливим типом кручення. Цей зв'язок полягає у тому, що його тензор кручення повністю визначається векторним ступенем вільності, і він був вперше введений Фрідманом і Схоутеном. Ми досліджуємо його фізичні наслідки, представляючи три космологічні моделі в рамках розглянутого геометричного розширення загальної теорії відносності, і порівнюємо прогнози моделей з передбаченнями ΛCDM-моделі (з холодною темною матерією і темною енергією), а також даними спостережень для функції Хаббла. Наші результати показують, що геометрія, передбачена Фрідманом, може пояснити дані спостережень для функції Хаббла без потреби в темній енергії.

Ключові слова: космологічні моделі, напівсиметрична метрична ґравітація, загальна теорія відносності, тензор кручення, функція Хаббла.