CALCULATION OF THE ANISOTROPY PARAMETER FOR GALAXY CLUSTERS

This work has analyzed the observational data on galaxy clusters from SDSS DR8. Based on the observational data on the distribution of a surface density, the values of the anisotropy parameter are determined for 31 clusters. A classification of galaxy clusters based on the velocity anisotropy parameters is proposed, according to which the clusters are divided into three groups. The relationship between the anisotropy parameter and the main physical characteristics of galaxy clusters has been studied.

1. Introduction

Galaxy clusters are the most massive gravitationally bound formations in the Universe (see, for example, [1–5]). They also include hot ionized intergalactic gas and dark matter. The masses of galaxy clusters are $10^{13} - 10^{15}$ solar mass, with the bulk of the mass usually coming from dark matter. Intergalactic gas takes the second place in terms of mass, and the galaxies themselves take only the third place. The characteristic size of massive galaxy clusters is about 10 million light years. The velocity of movement of galaxies within a cluster is usually 1000–2000 km/s. The masses of galaxy clusters are determined by measuring the velocity spread of individual galaxies relative to the average value or by determining the temperature and distribution of hot gas filling the cluster.

Many observational studies have been published on galaxy clusters. Here, we present the main works. The article [1] provides a compiled catalog of 2712 rich galaxy clusters discovered during a sky survey by the Palomar Observatory. 1682 clusters were selected from the catalogue, meeting certain criteria for inclusion in a homogeneous statistical sample. The authors of the study made the following conclusions: (1) the richness distribution function of clusters $N(n)$ increases rapidly with decreasing $n$; (2) the data do not allow a straightforward conclusion that the spatial density of cluster centers varies with distance; (3) galactic dimming of approximately a few tenths of a magnitude exists at high northern galactic latitudes of around 300 galactic longitude; (4) there is a highly significant non-random surface distribution of clusters, both when clusters at all distances and when clusters at different distances are considered.
In their work [6], the authors studied rich galaxy clusters. They showed that several rich clusters are moving through the Universe at velocities of thousands of kilometers per second. The high velocity of these galaxies and their dense distribution in the Universe mean that they are bound together by gravitational forces much greater than those that can be explained by their apparent mass, i.e., the mass represented by the objects visible on photographic plates. They conclude that the giant galaxy M87, located near the center of a large cluster in the constellation Virgo, may have a black hole at its core with a mass equal to five billion suns.

Authors of the works [2] compiled a catalog consisting of 4073 rich clusters of galaxies. Each cluster includes at least 30 members. The data were collected during a survey of the photographic plates of the clusters. Also included is a revised catalog including Bautz–Morgan types [7] and redshifts. The work [8] explored rich galaxy clusters that are fundamental components of the large-scale structure observed in the Universe. Comparisons of high-z clusters and low-z clusters show that they have recently undergone significant evolution, both in the dynamics and subpopulations of the galaxies. However, there is an evolutionary gap of \( \approx 2 \) billion years during an important transition epoch between these two extremes in the present-day observations of the clusters. The LARC study will fill this gap using a combination of high-resolution X-ray images, optical images, and spectroscopic data of the clusters in the redshift range \( 0.07 \leq z \leq 0.16 \). Based on such data, the authors will conduct a detailed analysis of the dynamics of clusters and the evolutionary history of the galaxies included in them.

In the work [9] is published a catalog representing the distribution by morphological classes of about 6000 galaxies in 55 rich clusters. In the work [10], the results of a study of the approximate members of 351 Abel clusters with an apparent magnitude of \( 10^m \) are presented and a catalog of 115 new clusters and 17 new superclusters is published. The work [11] carried out a spectroscopic analysis of 286 rich clusters previously identified based on observations with the DuPont telescope at the Las Campanas Observatory. In the work [12], the radioactive members of the Abel cluster were studied, and a catalog of 150 clusters was compiled according to X-ray and radio parameters. The work [13] processed images from the SDSS-III project and identified 132684 galaxy clusters, with more than 95 percent of these clusters having masses \( M_{200} > 10^{14} M_\odot \). In the work [14], the dynamic properties of 2092 rich galaxy clusters based on X-ray, optical, and radio data were studied. In [15], the masses of 1191 clusters were determined, and, in addition, the data given in [14] have been updated and/or supplemented. In [16], the author gave the main physical characteristics of 275 rich clusters, including their masses.

The statistical properties of clusters impose strong constraints on the cosmology of galaxies [17–18]. Statistical studies of galaxy clusters place constraints primarily on cosmological parameters [17] through the mass function [19–24]. These clusters are also important for studying the evolution of galaxies in dense environments [25–27] and, as a natural telescope for studying high-redshift galaxies [28–31]. In the ΛCDM cosmological model, galaxy clusters are created by density perturbations on large scales [32]. The classification of these objects has not yet been fully determined as well. It should be noted that the analysis of observational data on galaxy clusters has not yet been carried out, and the corresponding theory of their formation has not been developed. More specifically, we note that there is no extended list-catalogue of rich galaxy clusters and a statistical analysis of all known galaxy clusters in order to find their common physical properties; the classification of galaxy clusters according to individual physical parameters has not been carried out. The study of galaxy clusters is also of great interest due to the fact that these objects can be used as indicators of physical state and evolution.

2. Calculation of Anisotropy Parameter

At present, when the observational base for many astrophysical objects, including galaxy clusters, has become quite rich, it makes sense to study the problem of the evolution of galaxy clusters. Probably, it is the fact that the clusters, at first glance, are very similar to each other due to their almost spherical shape with very weak symmetry. But, it is physically clear that these clusters must differ in internal structure, degree of velocity distribution, etc. It should be noted that theoretical aspects of the origin and evolution of galaxy clusters require the creation of an observational database. Therefore, we analyzed observational
data for these objects and found surface densities for several galaxy clusters. For this, we used SDSS DR8 databases.

From the analysis of observational data to study the evolution of galaxy clusters, we need to find some of their parameters. One of them, basically, could be the velocity anisotropy parameter. Different authors define it in different ways: as a simple ratio of the average values of the multipliers of the kinetic energy of radial and transversal movements $\langle T_r / T_\perp \rangle$, and as $1 - \sigma_r^2 / \sigma_\perp^2$, where $\sigma_r^2$ and $\sigma_\perp^2$ are the radial and transversal multipliers of velocity dispersion. In this work, we take the following value as the velocity anisotropy parameter

$$ A = \frac{(2 \Pi^2 - T^2)}{\Pi^2}. $$

This value was introduced for the first time in [33]. In (1) $\Pi^2$ and $T^2$ are the radial and transversal dispersions of residual velocities. Just like in [33–34], we assume it is constant for the entire system as a whole, although, in reality, it should depend on the distance inside the cluster. This parameter interests us primarily because of its possible dependence on the other physical characteristics of galaxy clusters, although it is also of independent interest. It is essential that the anisotropy parameter is convenient for analyzing the stage of evolution of galaxy clusters, because each stage is characterized by a certain structure and distribution of density and velocity. With the transition from an early stage to a later one, the structure becomes smoother and tends to have a more isotropic velocity distribution (as opposed to the radially elongated distribution in the early stages). The value of the velocity anisotropy parameter $A = 0$ corresponds to a spherical velocity distribution (late stage of the evolution), $A = 2$ corresponds to a radially elongated distribution (early stage of system’s evolution).

To determine the velocity anisotropy parameter in galaxy clusters in view of observational data on the apparent surface density, we will consider a stationary model with spherical symmetry. From the hydrodynamic equation, it is easy to derive

$$ \frac{d}{dr} \left[ \frac{r^2}{D} \frac{d}{dr} \left( \frac{D}{\Pi^2} \right) + \left( \frac{2 \Pi^2 - T^2}{r} \right) \right] = -4 \pi G D r^2, \tag{2} $$

eliminating the potential in the hydrodynamic equation with use of the Poisson equation. In (2), the dimensionless density $D$ is introduced. If we move to a new variable $x = 4 \pi G r^2/\Pi^2$, then Eq. (2) will take a relatively simple form:

$$ \frac{d}{dx} \left( x^2 \frac{d D}{D} \right) = -D x^2 - A. \tag{3} $$

Solving Eq. (3) numerically, we obtain the dependences of $D$ and $D' = \frac{dD}{dx}$ on $x$, and the dimensionless density $D$ clearly depends on the value of the anisotropy parameter. We solve Eq. (3) for various values of $A \in [0; 2]$. Using the known values of $D$ and $D'$, we can proceed to the surface density, i.e., to the projection of the spatial density onto the picture plane. Next, we calculate the surface density using the expression

$$ F(r) = -A \int_{r/\alpha}^{\infty} \sqrt{x^2 - (r/\alpha)^2} D'(x) \, dx. \tag{4} $$

By obtaining the corresponding values of the surface density $F(r)$ for given values of the anisotropy parameter and comparing them with the observed surface densities for specific systems, we select the optimal value of $A$. The scale factors are determined by minimizing the difference between the observed and theoretical densities. Based on the physical nature of this problem, it is convenient to use a fairly proven symplectic method for minimizing the following function:

$$ f = \sum_{i=1}^{N} \left[ -A \int_{r_i/\alpha}^{\infty} \sqrt{x^2 - (r_i/\alpha)^2} D'(x) \, dx - F_0 \right]^2 \tag{5} $$

according to parameters $\alpha$, $A$, and $F_0$. Here, $N$ is the number of circular zones on photographic plates (or images) in which the number of stars was counted. Thus, the value of the anisotropy parameter that gives the minimum $f$ is taken as the most probable value of $A$.

Using the above method, we calculated the values of the anisotropy parameter $A$ for 31 galaxy clusters, within the simplex method for minimizing the squared difference between the theoretical and observed density functions. The calculation results are given below in Table 1.
From Table 1, it can be seen that, for 5 galaxy clusters, the anisotropy parameter values are close to zero, i.e., about 16 percent of the clusters have a nearly spherical velocity distribution and are likely to be in a later stage of evolution. Note that it is specifically for the found values of the anisotropy parameter that the corresponding theoretical results obtained are in good agreement with the observational surface density functions. For 4 clusters, the values of the velocity anisotropy parameter are close to 2. Therefore, the velocity distribution in these clusters corresponds to a radially elongated one, which may perhaps be due to their nonstationarity in a regular field. The remaining values of the anisotropy parameter for galaxy clusters turned out to be close to 1.

Table 1. Anisotropy parameter values for galaxy clusters

<table>
<thead>
<tr>
<th>No.</th>
<th>Names of galaxy clusters</th>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>J001051.4 + 290940</td>
<td>0.08</td>
</tr>
<tr>
<td>2</td>
<td>J002016.1 + 000446</td>
<td>0.00</td>
</tr>
<tr>
<td>3</td>
<td>J002712.5 + 190405</td>
<td>0.88</td>
</tr>
<tr>
<td>4</td>
<td>J002809.9 + 244744</td>
<td>1.50</td>
</tr>
<tr>
<td>5</td>
<td>J004118.5 + 252609</td>
<td>0.60</td>
</tr>
<tr>
<td>6</td>
<td>J004511.7 + 084111</td>
<td>0.48</td>
</tr>
<tr>
<td>7</td>
<td>J023127.6 + 065856</td>
<td>1.12</td>
</tr>
<tr>
<td>8</td>
<td>J023952.7 - 013419</td>
<td>1.44</td>
</tr>
<tr>
<td>9</td>
<td>J083057.3 + 655031</td>
<td>0.88</td>
</tr>
<tr>
<td>10</td>
<td>J085007.9 + 360414</td>
<td>0.72</td>
</tr>
<tr>
<td>11</td>
<td>J090912.7 + 105829</td>
<td>1.24</td>
</tr>
<tr>
<td>12</td>
<td>J091609.0 - 002226</td>
<td>0.56</td>
</tr>
<tr>
<td>13</td>
<td>J091753.4 + 514338</td>
<td>1.32</td>
</tr>
<tr>
<td>14</td>
<td>J092048.3 + 302818</td>
<td>0.72</td>
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<tr>
<td>15</td>
<td>J094951.8 + 170711</td>
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<tr>
<td>16</td>
<td>J100226.8 + 203102</td>
<td>0.76</td>
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<td>17</td>
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<td>18</td>
<td>J111450.3 - 121351</td>
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<td>19</td>
<td>J112358.8 + 212850</td>
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<td>20</td>
<td>J113129.5 - 010208</td>
<td>0.84</td>
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<td>J133238.4 + 503336</td>
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<td>30</td>
<td>J164325.4 + 132236</td>
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<td>31</td>
<td>J212823.4 + 013536</td>
<td>0.80</td>
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</tbody>
</table>

Next, we are interested in the question: is it possible to classify galaxy clusters according to the velocity anisotropy parameter? For this purpose, we have compiled Table 2.

The following parameters are given here: the magnitude of the brightest member in the X-ray range – \( r_m \), [mag], the richness of clusters – \( \langle R_L \rangle \) [13], the photometric redshift – \( z \), the radius of the cluster core – \( r_0 \), [arcmin].

First of all, comparing the values of the velocity anisotropy parameter, we can notice that they can be unambiguously divided into three classes (see Tabl. 3).

Note that there are no clusters in the interval 1.5 < \( A \) < 2.0, and this requires a future search for this range.
additional data on the apparent density for galaxy clusters. Nevertheless, it is interesting to compare the average values of the main physical parameters of the three classes of these clusters.

In Table 4, the average values of a number of characteristics of the indicated groups of galaxy clusters are given: photometric redshift – ⟨z⟩, the value of the brightest member in the X-ray range – ⟨r_m⟩, radius within which the average cluster density is 200 times higher than the critical density of the Universe – ⟨r_200⟩, [Mpc], cluster richness – ⟨R_L+⟩, cluster core radius – ⟨r_0⟩.

We also note the results of calculations of the values of the correlation coefficient of the anisotropy parameter with well-known data on the other physical characteristics, according to which two-dimensional or multidimensional classifications can be basically made. Unfortunately, these coefficient values are very small, and it is not possible to create such classifications. For example, correlation coefficients were found between the following parameters: 0.205 for A = R_L+, 0.320 for A = r_200/0.357 A = r_0, -0.086 for A = z, -0.089 for A = r_m.

### Table 3. Values of the velocity anisotropy parameter

<table>
<thead>
<tr>
<th>Classes</th>
<th>A</th>
<th>Quantity of clusters</th>
</tr>
</thead>
<tbody>
<tr>
<td>RGC 1</td>
<td>0.0 &lt; A &lt; 0.5</td>
<td>5</td>
</tr>
<tr>
<td>RGC 2</td>
<td>0.5 &lt; A &lt; 1.0</td>
<td>18</td>
</tr>
<tr>
<td>RGC 3</td>
<td>1.0 &lt; A &lt; 1.5</td>
<td>8</td>
</tr>
</tbody>
</table>

### Table 4. Average values of the characteristics of the three groups

<table>
<thead>
<tr>
<th>Classes</th>
<th>⟨z⟩</th>
<th>⟨r_m⟩</th>
<th>⟨r_200⟩</th>
<th>⟨R_L+⟩</th>
<th>⟨r_0⟩</th>
</tr>
</thead>
<tbody>
<tr>
<td>RGC 1</td>
<td>0.22</td>
<td>16.57</td>
<td>1.87</td>
<td>112.17</td>
<td>0.77</td>
</tr>
<tr>
<td>RGC 2</td>
<td>0.27</td>
<td>16.97</td>
<td>2.02</td>
<td>145.39</td>
<td>0.59</td>
</tr>
<tr>
<td>RGC 3</td>
<td>0.25</td>
<td>16.46</td>
<td>2.10</td>
<td>163.13</td>
<td>0.26</td>
</tr>
</tbody>
</table>

3. Conclusions
An analysis of observational data on galaxy clusters from SDSS DR8 database was performed. Based on observational data, the number of galaxies is calculated, and their surface density distributions in galaxy clusters are found. Using these data, the values of the velocity anisotropy parameter are determined for 31 galaxy clusters. According to the velocity distribution calculation results, 5 galaxy clusters have spherical symmetry. In addition, a classification of galaxy clusters based on velocity anisotropy parameters has been developed, and it has been shown that they can be divided into three groups. The relationships between the anisotropy parameter and the main physical characteristics of galaxy clusters have been studied.

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Розрахунок параметрів анізотропії для скупчення галактик

У цій роботі проаналізовано дані спостережень за скупченнями галактик із SDSS DR8. На основі спостережуваних даних з розподілу поверхневої густини визначено величину параметра анізотропії $\sigma_8$. Запропоновано класифікацію скупчень галактик на основі параметрів анізотропії швидкості, згідно з якою скупчення поділяються на три групи. Досліджено зв'язок між параметрами анізотропії та основними фізичними характеристиками скупчень галактик.

Ключові слова: галактики, скупчення галактик, поверхнева густина, параметри анізотропії, еволюція.