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BASHAR ALAA ALKOTBE,[1] MAHDI J.S. AL MUSAWI,[2] HUDA H. KASSIM,[1]
ASMAA A. ELBNDAG,[3] M.A. AL-JUBBORI,[4] I. HOSSAIN,[5] FADHIL I. SHARRAD,[1,6]
N. ALDAHAN [1,6]

[1] Departments of Physics, College of Science, University of Kerbala
*(Karbala, Iraq; e-mail: bashar.alaa@uokerbala.edu.iq; huda.kassim1976@gmail.com)*
[2] Deptartment of Medical Physics, College of Medicine, University of Kerbala
*(Karbala, Iraq; e-mail: mahdi.j@uokerbala.edu.iq)*
[3] Department of Physics, Faculty of Science, University of Tripoli
*(Tripoli, Libya; e-mail: somaaf4@gmail.com)*
[4] Departments of Physics, College of Education for Pure Sciences, University of Mosul
*(Mosul, Iraq; e-mail: mushtaq_phy8@yahoo.com)*
[5] Department of Physics, Rabigh College of Science & Arts, King Abdulaziz University
*(Rabigh, Saudi Arabia; e-mail: mihossain@kau.edu.sa)*
[6] College of Health and Medical Technology, University of Alkafeel
*(Najif, Iraq; e-mail: fadhil.altaie@gmail.com, nawrasaldahan@alkafeel.edu.iq)*

# ASSESSMENT OF THE STRUCTURE OF $^{180}$Hg NUCLEUS THROUGH IBM-1 AND IBM-2 MODELS

*We will explain the aspects of $^{180}$Hg nucleus through the interacting boson model (IBM-1) and IBM-2. This nucleus is expected to be typical in the limit of the U(5) symmetry which is deliberate to elucidate the properties of $^{180}$Hg nucleus. A suitable method for fitting is expected to improve the best parameters for a convinced calculated energy level of this nucleus. The intended three energy bands such as ground, gamma, and beta bands in both models are studied and compared with the data obtained earlier by M.A. Al-Jubbori, I. Hossain, F.I. Sharrad, and N. Aldahan. The strengths of quadrupole electromagnetic transitions B(E2) of this nucleus in IBM-1 and IBM-2 models are calculated and matched with reasonable earlier measured data. The potential energy surfaces (PES) of this nucleus for the distortion parameter in the U(5) symmetry in IBM-1 is analyzed. All data on $^{180}$Hg nucleus are well consistent with available measured data.*

*Keywords*: IBM-1, IBM-2, energy level, B(E2), potential energy surface, $^{180}$Hg.

## 1. Introduction

The authors of work [1] reported on the properties of even-even medium-mass nuclei through the Interacting Boson Model (IBM-1). In this model, the nucleus consists of neutrons and protons that are known as nucleons and are not distinguished. In the additional class of models acknowledged as IBM-2 [2, 3] IBM-2, it was considered that the nucleons are distinguished with respect to the inert shells. It was established that each boson ($s$ or $d$-boson) ampule lodge one of two levels: $L = 0$ and $L = 2$. The IBM-1 and IBM-2 are applicable to study the even-even low-lying combined nuclei and are characterized by a stationary number which is recognized as bosons number ($N_b$). In accumulation, these models designed

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three types of vibrational U(5), $\gamma$-soft O(6), rotational SU(3) symmetry from the methodological U(6) assembly [4, 5]. However, many researchers proposed that the nuclei also need three types of intermediate construction that stand at U(5)–O(6), U(5)–SU(3) and SU(3)–O(6) limits [6, 7].

The tiny source of quadruple collectivity and contour existence at a low excitation state in central-shell nuclei close to the double magic nuclei $^{208}$Pb ($Z = 82$, and $N = 126$) shell closures are not entirely recognized yet. The report of the structure of nuclear at the critical point of stage evolutions has captivated widespread curiosity in recent periods. The proton-rich $^{180}$Hg nuclei consist of 80 protons and 100 neutrons, respectively, and they are placed near the proton drip line. The proton-rich nuclei could be studied by fusion evaporation reactions. The configurations of $^{180}$Hg nuclei are $\pi(h_{11/2})^{-4}\nu(i_{13/2})^{-26}$ which indicates 4 proton holes and 26 neutrons holes existed in $^{180}$Hg nucleus and those holes in $^{180}$Hg nucleus are considered rendering to double magic shell closure in $^{208}$Pb nucleus. These nuclear structures are complicated as each hole-protons and hole-nucleons connect by means of every other hole of nucleons. In this case, it is essential to involve definite complex mathematical strategies for diagonalizing the intermediate Hamiltonian in these circumstances. These configurations of this nucleus permit $E2$ transitions in yrast states from excited states to ground state. The enhancements of nuclear structure of even neutron-deficiency in $^{180}$Hg are rarely found in the literature. The studies of excitation levels of three bands and the strengths of decay modes $B(E2)$ in elements with even $A = 180$–$190$ were studied [8–10]. The coexistence in neutron-deficient nuclei in $^{182,184}$Hg was explored in measurements by Siciliano *et al.* [8]. The triaxial deformations of energy surfaces were found in a coexistence configurations in $^{190}$Hg [9]. Garcia–Romas *et al.* [10] clarified the even-even Hg isotopes in $^{182-190}$Hg isotopes by means of the IBM, as well as the configuration mixing and pay excellent consideration to the description of the shape of the nuclei and to its relation to the shape synchronicity phenomena.

In recent times, IBM-1 and IBM-2 models were applied with a SU(3) limit to $^{158}$Gd nucleus [11]. Hossain *et al.* [12] studied O(6) group in $^{108,110,112}$Ru nuclei through the IBM-1 model. Nuclear structure of rare-earth Er–Os nuclei for neutron $N = 100$, 102, 104 belongs to the valence nucleons of double magic nuclei $^{208}$Pb were studied by Al-jubbori *et al.* [13–15]. The nuclear structure of even-even $^{76-82}$Se nuclei through IBM-1 was studied by Sahib *et al.* [16]. The nuclear isobars in $^{186}$W and $^{186}$Os were studied by IBM-1 [17]. All of the previous studies [11–17] suggest to raise the study of the nuclear structure of $^{180}$Hg which gives up more information between the shell of magic number $Z = 82$ to $N = 126$.

At present time, we have chosen to search for the structure of $^{180}$Hg nuclide since. It is neutron-deficient nucleus and belongs to the shell $Z = 82$ and $N = 126$. The goal of this work is to consider the enhancement of nuclear structures of different types of bands and strengths of $B(E2)$ in $^{180}$Hg nucleus basong on both models IBM-1 and IBM-2. Now, this nucleus is assumed to be a harmonic vibrator with U(5) symmetry. The motivation of the current work is required to explore the phenomenological IBM-1 and IBM-2 to explain previously measured data on three types of energy bands and reduced transition strengths of $B(E2)$ for the selected nucleus. These systematic comparative structures of $^{180}$Hg nucleus through present calculations within IBM-1 and IBM-2, as well as previously measured data, will be clarified.

## 2. Theoretical Technique

### *2.1. Interacting Boson Model-1 (IBM-1)*

The protons and neutrons of thenucleus are known as nucleons. The IBM1 typically gives possession on a shortened model space, and this stands for the mathematical clarification of indistinguishable elements by means of creating couples with angular momentum 0 or 2. The equation for the Hamiltonian is given according to IBM-1 [1, 18] as

$$\begin{aligned}
H &= \varepsilon_s(s^\dagger \cdot \tilde{s}) + \varepsilon_d(d^\dagger \cdot \tilde{d}) + \\
&+ \sum_{L=0,2,4} \frac{\sqrt{2L+1}}{2} C_L[[(d^\dagger \times d^\dagger]^{(L)} \times [\tilde{d} \times \tilde{d}]^{(L)}]^{(0)} + \\
&+ \frac{1}{\sqrt{2}} \upsilon_2[[d^\dagger \times d^\dagger]^{(2)} \times [\tilde{d} \times \tilde{s}]^{(2)} + \\
&+ [d^\dagger \times s^\dagger]^{(2)} \times [\tilde{d} \times \tilde{d}]^{(2)}]^{(0)} + \\
&+ \frac{1}{2}\upsilon_0[[d^\dagger \times d^\dagger]^{(0)} \times [\tilde{s} \times \tilde{s}]^{(0)} + [s^\dagger \times s^\dagger]^{(0)} \times [\tilde{d} \times \tilde{d}]^{(0)}]^{(0)} + \\
&+ \frac{1}{2}u_0[[s^\dagger \times s^\dagger]^{(0)} \times [\tilde{s} \times \tilde{s}]^{(0)}]^{(0)} + \\
&+ u_2[[d^\dagger \times s^\dagger]^{(2)} \times [\tilde{d} \times \tilde{s}]^{(2)}]^{(0)}. \qquad (1)
\end{aligned}$$

There are nine terms in the Hamiltonian operator in IBM-1. Two factors give the expression in one-body terms ($L = 0$ and $L = 2$), and $\varepsilon_s$ and $\varepsilon_d$ indicate the energy of the $s$ and $d$ bosons, whereas the additional two-body footings are ($C_0$, $C_1$, $C_4$, $\upsilon_0$, $\upsilon_2$, $u_0u_2$). The $N_b$ is number of bosons which is preserved. In overall, the Hamiltonian operator in Eq. (1) for IBM-1 is specified [19, 20] and simply presented as follows:

$$\hat{H} = \varepsilon\hat{n}_d + a_0\hat{P}\cdot\hat{P} + a_1\hat{L}\cdot\hat{L} + a_2\hat{Q}\cdot\hat{Q} + a_3\hat{T}_3\cdot\hat{T}_3 + a_4\hat{T}_4\cdot\hat{T}_4. \quad (2)$$

Boson energy $\varepsilon(= \varepsilon_d - \varepsilon_s)$, and the detailed forms of operators are given as

$$\left.\begin{aligned}
&\hat{n}_d = d^\dagger \cdot \tilde{d},\\
&\hat{P} = 0.5[(\tilde{d}\cdot\tilde{d}) - (\tilde{s}\cdot\tilde{s})],\\
&\hat{L} = \sqrt{10}\,[d^\dagger \times \tilde{d}]^{(1)},\\
&\hat{Q} = [d^\dagger \times \tilde{s} + s^\dagger \times \tilde{d}]^{(2)} + \chi[d^\dagger \times \tilde{d}]^{(2)},\\
&\hat{T}_r = [d^\dagger \times \tilde{d}]^{(r)}.
\end{aligned}\right] \quad (3)$$

The four operators are specified by the symbols $\hat{n}_d$, $\hat{P}$, $\hat{L}$, $\hat{Q}$, where $\hat{n}_d$ (total number of $d$-bosons), $\hat{P}$ (pairing), $\hat{L}$ (angular momentum), and $\hat{Q}$ (quadruple) operator. The $\hat{T}_r$ denotes hexadecapole and octupole operators for $r = 4$ and 3. The representation $\chi$ denotes the quadruple creation limits 0 and $\pm\frac{\sqrt{7}}{2}$ [21]. The symbols $a_0$, $a_1$, $a_2$, $a_3$, and $a_4$ are the limits for the designate operator $\hat{P}$, $\hat{L}$, $\hat{Q}$; $\hat{T}_r$ giveconnections between the bosons. The PHINT program [22] states on the interactions: $\epsilon = EPS$, $a_0 = 2PAIR$, $a_1 = ELL/2$, and $a_3 = 5OCT$, $CHI = 0$.

The IBM-1 makes three sorts of active regularity, and the eigenvalues are given by [20]

$$\left.\begin{aligned}
&E(n_d\upsilon L) = \varepsilon n_d + \frac{a_1}{12}n_d\,(n_d+4) +\\
&+\left(\frac{a_3}{7} - \frac{a_1}{10} - \frac{3a_4}{70}\right)\upsilon\,(\upsilon+3) +\\
&+\frac{1}{14}\,(a_4 - a_3)\,L\,(L+1)\,, \quad \mathrm{U(5)}\\
&E(\lambda\mu L) = \frac{a_2}{2}\left(\lambda^2 + \mu^2 + \lambda\mu + 3\,(\lambda+\mu)\right) +\\
&+\left(a_1 - \frac{2a_2}{8}\right)L\,(L+1), \quad \mathrm{SU(3)}\\
&E(\sigma\tau L) = \frac{a_0}{4}\,(N-\sigma)\,(N+\sigma+4) +\\
&+\frac{a_3}{2}\tau\,(3+\tau) + \left(a_1 - \frac{a_3}{10}\right)L\,(L+1), \quad \mathrm{O(6)}.
\end{aligned}\right] \quad (4)$$

Here, symbols $\varepsilon$, $a_0$, and $a_2$ indicate U(5), O(6), and SU(3) limits, respectively. The Hamiltonian [1, 19] is familiar for the designs that interrupt the rendering to

$$\hat{H} = a_0\hat{P}\hat{P} + a_1\hat{L}\hat{L} + a_2\hat{Q}\hat{Q}. \quad (5)$$

### *2.2. Interacting Boson Model-2 (IBM-2)*

IBM-2 [23, 24] presents another type of model, where the nuclei consist of protons and neutrons that are distinguished, and the boson number is considered for the existent nucleons exterior to the main closed shells. It is known that $d$ or $s$ bosons live in one of two states for $L = 2$ or $L = 0$, respectively. The applied constructions in even proton- and even neutron-undistinguishable elements are combined under conditions with angular momentum 0 or 2. The symbols $s$ and $d$ show the angular momentum to be zero or two. The notations for neutrons ($\nu$) and protons ($\pi$) indicate $s_\pi$ (proton boson) and $s_\nu$ (neutron boson) with $L = 0$ and $d_\pi$; and $d_\nu$ are proton and neutron bosons with $L = 2$.

In IBM-2, the Hamiltonian has three components and is given by the formula (6):

$$H = H_\nu + H_\pi + V_{\pi\nu}, \quad (6)$$

where $H_\pi$ and $H_\nu$ are the Hamiltonians of proton and neutron bosons, while $V_{\pi\nu}$ is the proton-neutron interaction.

A basic Hamiltonian [25] reads

$$H = \varepsilon\left(\hat{n}_{d\pi} + \hat{n}_{d\nu}\right) + kQ_\pi Q_\nu + V_{\pi\pi} + V_{\nu\nu} + M_{\pi\nu}, \quad (7)$$

where $\varepsilon_\nu\varepsilon_\pi$ are the neutron and proton energies, respectively. It is assumed that ($\varepsilon_\nu = \varepsilon_\pi = \varepsilon$), and the quadrupole operator is

$$Q_\rho = (d^\dagger \times s + s^\dagger \times \tilde{d})^2_\rho + \chi_\rho(d^\dagger \times \tilde{d})^2_\rho \quad \rho = \pi\nu, \quad (8)$$

where the factor $\chi_\rho$ is used to compute the construction of the boson quadruple operators.

The relations $V_{\pi\pi} + V_{\nu\nu}$ suggests $d$-bosons preserving residual $n$-$n$ and $p$-$p$ interactions. We can get equation

$$\hat{V}_{\rho\rho} = \sum_{k=1,2,4}\frac{1}{2}(2L+1)^{\frac{1}{2}}C^\rho_L\left[\left(d^\dagger_\rho \times d^\dagger_\rho\right)^{(L)} \times \times\left(\tilde{d}_\rho \times \tilde{d}_\rho\right)^{(L)}\right]^{(0)}. \quad (9)$$

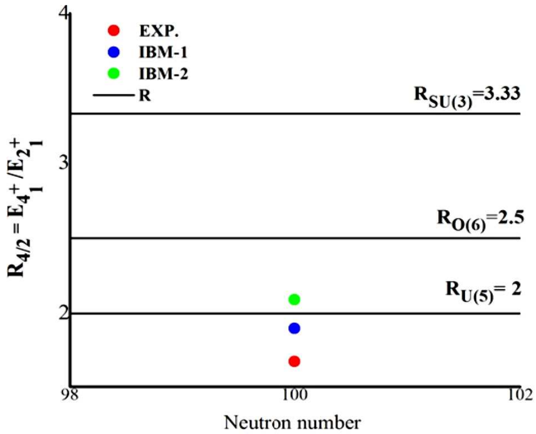


***Fig. 1.*** (Color online) $E(4_1^+)/E(2_1^+)$ for $^{180}$Hg [27–29]. $E(2_1^+)$ indicate 1st $2^+$ level and $E(4_1^+)$ indicate 1st $4^+$ level. Solid line represented U(5), O(6) and SU(3) limit[1]. The previous data Expt., and those in IBM-1 and IBM-2 are indicated by red, blue, and green colors, respectively

*Table 1.* **The ratio $R_{4/2} = E_{4_1^+}/E_{2_1^+}$ for $^{180}$ Hg nuclei for previous Exp. and present IBM-1 & IBM-2**

| Nucleus | $^{180}$Hg | $N_\pi + N_\nu = N$ |
|---|---|---|
| Previous Exp. ($R_{4/2}$) | 1.68 | |
| Present IBM-1 ($R_{4/2}$) | 1.90 | $1 + 9 = 10$ |
| Present IBM-2 ($R_{4/2}$) | 2.09 | |

The last term, the Majorana interactions $M_{\pi\nu}$, can be written as (10)

$$M_{\pi\nu} = \xi_2 (s_\nu^\dagger \times d_\pi^\dagger - d_\nu^\dagger \times s_\pi^\dagger)^2 (s_\nu \times d_\pi^\dagger - d_\nu^\dagger \times s_\pi)^2 - $$

$$- 2 \sum_{k=1,3} \xi_k \left(d_\nu^\dagger \times d_\pi^\dagger\right)^{(k)} \left(\tilde{d}_\nu \times \tilde{d}_\pi\right)^{(k)}. \quad (10)$$

If we possess the confirmation for mixed symmetry states, the Majorana factors are diverse to fix states in the band.

The excitations levels are calculated using code NPBOS [26]. We diagonalizable Hamiltonian (7) and compute the parameters $\varepsilon$, $k$, $x_\pi x_\nu$ and $C_L$ for the best acceptable measured data.

## 3. Results and Discussion

### *3.1. Boson number*

The boson such as $s(L = 0)$ or $d(L = 2)$ is a couple of valence nucleons (protons and neutrons), while the boson number is calculated as the quantity of combined pairs of valence nucleons. The valance nucleons are considered from double magic nuclei. In the current study, the valance nucleons are calculated from double magic nucleus $^{208}$Pb ($Z = 82$, $N = 126$). If $N_p$ is the number of valance protons, and if $N_n$ is the number of valance neutrons, then the total number of bosons $N = (N_p + N_n)/2 = (n_\pi + n_\nu)$. In this study, $^{208}$Pb is involved as the inert core to compute number of bosons of this nucleus. In $^{180}$Hg nucleus far away from the shell closure of $^{208}$Pb, there are 2 holes existed due to the short of valence protons in the shell closure of magic number $Z = 82$ and 18 particles existed due to the short of valence neutrons in the shell closure of magic number $N = 126$. Therefore, the total number of bosons ($N$) of $^{180}$Hg is $(2/2 + 18/2) = 10$, which is presented in Table 1.

### *3.2. $R_{4/2}$ classifications by symmetry*

The collective energies of even-even nuclei are consist of three groups based on the ratio $R_{4/2}$:

$$R_{4/2} = \frac{E\left(4_1^+\right)}{E}\left(2_1^+\right). \quad (11)$$

The symbol $E\left(2_1^+\right)$ is the energy level at $2_1^+$, and $E\left(4_1^+\right)$ is the energy level at $4_1^+$. The energy ratio, $R = E4_1^+/E2_1^+$, shows the symmetry method for the nucleus. Even-even nuclei can be classified according to ratios $R_{4/2}$ [27–29]. The quantity $R_{4/2} = 2.00$ specifies a harmonic vibrator U(5); $R_{4/2} = 2.50$ indicates $\gamma$-unstable O(6) symmetry, and $R_{4/2} = 3.33$ shows an axially symmetric SU(3) rotor [1]. The patterns $E4_1^+$ and $E2_1^+$ designate the measured value of energy levels in $^{180}$Hg [27, 28] at $4_1^+$ (0.706 MeV) and $2_1^+$ (0.434 MeV), respectively, the corresponding IBM-1 energy levels are $4_1^+$ (0.798 MeV) and $2_1^+$ (0.42 MeV), the IBM-2 energy levels are $4_1^+$ (0.703 MeV) and $2_1^+$ (0.335 MeV). We recognized U(5) symmetry for $^{180}$Hg nucleus, since $R_{4/2}$ value for this nucleus is 1.68 [27, 28] and is presented in Table 1. The ratios $E(4_1^+)/\ E(2_1^+)$ for $^{180}$Hg nucleus are shown in Fig. 1. The solid line shows U(5), O(6) and SU(3) symmetry [1]. The data on the previously measured ratios $R_{4/2}$ [27, 28], within IBM-1 and IBM-2 for this nucleus are 1.68, 1.90, and 2.09, respectively and are shown in Fig. 1 with different types of color. It is clearly seen that all points obtained in IBM-1, IBM-2, and previously measured ones are


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*Table 2.* **The parameters used for IBM-1 calculations. All parameters are given in MeV excepted $N$ and CHQ($\chi$)**

| Nucl. | $N_\pi + N_\nu = N$ | $\varepsilon$ | $a_0$ | $a_1$ | $a_2$ | $a_3$ | $a_4$ | CHQ($\chi$) |
|---|---|---|---|---|---|---|---|---|
| $^{180}$Hg | $1+9=10$ | 0.42 | –0.2 | 0.000 | 0.075 | 0.000 | –0.042 | 0.000 |

*Table 3.* **The parameters used for IBM-2 calculations. All parameters are given in MeV**

| Nucl. | $N_\pi + N_\nu = N$ | $\varepsilon_d$ | $\kappa_{\pi\upsilon}$ | $\chi_\pi$ | $\chi_\nu$ | $\xi_2$ | $C_0L_\upsilon$<br>$C_2L_\upsilon$<br>$C_4L_\upsilon$ | $C_0L_\pi$<br>$C_2L_\pi$<br>$C_4L_\pi$ |
|---|---|---|---|---|---|---|---|---|
| $^{180}$Hg | $1+9=10$ | 0.600 | –0.095 | –1.210 | –1.400 | –0.001 | –0.600<br>0.000<br>0.000 | –0.100<br>0.000<br>0.000 |

*Table 4.* **Comparative studies of $g$-band, $\gamma$-band and $\beta$-bands of $^{180}$Hg nucleus in IBM-1 and IBM-2 models and previously measured data**

| $^{180}$Hg | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $g$-band | | | | $\gamma$-band | | | | $\beta$-band | | | |
| $J^\pi$ | IBM-1 | IBM-2 | Exp. | $J^\pi$ | IBM-1 | IBM-2 | Exp. | $J^\pi$ | IBM-1 | IBM-2 | Exp. |
| $0_1^+$ | | 0.000 | 0.000 | $2_2^+$ | 0.915 | 0.925 | 0.925 | $0_2^+$ | 0.64 | 1.147 | 0.419 |
| $2_1^+$ | 0.42 | 0.335 | 0.434 | $3_1^+$ | | 1.240 | 1.430 | $2_3^+$ | | 1.040 | – |
| $4_1^+$ | 0.798 | 0.703 | 0.706 | $4_2^+$ | | 1.349 | – | $4_3^+$ | | 1.575 | – |
| $6_1^+$ | 1.134 | 1.013 | 1.032 | $5_1^+$ | | 1.628 | – | | | | |
| $8_1^+$ | 1.428 | 1.253 | 1.437 | $6_2^+$ | 1.533 | 1.640 | 1.504 | | | | |

near U(5) line, and data in IBM-1 are close to measured ones. Therefore, the IBM-1 calculation is better than that in IBM-2; and $^{180}$Hg nucleus is like to a harmonic vibrator U(5).

### *3.3. Three natural energy levels in $^{180}$Hg nucleus*

Now, we describe the excitation spectra of ground states, gamma and beta bands of the $^{180}$Hg nucleus. In both models in the U(5) limits, we calculated three natural energy bands such as $g$-band, $\beta$ and $\gamma$ bands of $^{180}$Hg [27, 28].

The parameters in IBM-1 and IBM-2 of $^{180}$Hg nucleus are presented in Table 2 and Table 3, respectively. In Table 2, all terms of IBM-1 are specified in MeV; expected $N$ and CHQ($\chi$). In Table 3, all values in IBM-2 are specified in MeV. The systematics comparative study of ground state band, $\gamma$-band, and beta band for $^{180}$Hg nucleus are presented in Table 4. The calculated data from IBM-1 and IBM-2 are consistent with previous experimental data. The most of calculated data in IBM-1 are near measured data. The systematic label schemes of three types of bands in both models, as well as the previous experimental data are mutually compared and are shown in Fig. 2. From the Fig. 2, it is seen that data in IBM-1 at the low lying level are near the measured data compared to IBM-2. Therefore, the data in IBM-1 are better than the data in IBM-2. Actually, the number of parameters in IBM-2 is higher, since it is considered that protons and neutrons are distinguishable, than the number of parameters in IBM-1, where protons and neutrons are not distinguishable. For this, the calculation within IBM-1 is simpler than that in IBM-2.

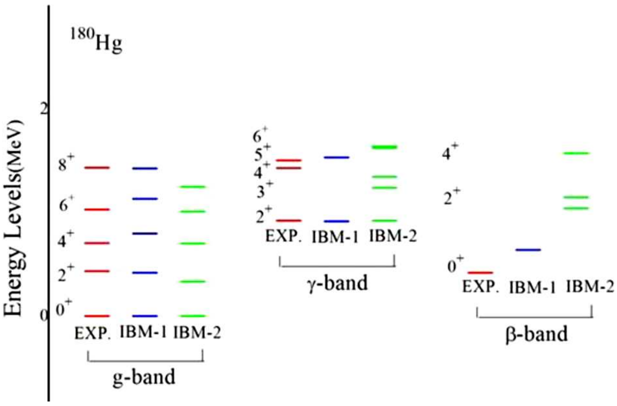


***Fig. 2.*** (Color online) The level scheme of different type of bands: ground ($a$), gamma ($b$) and beta ($c$) in $^{180}$Hg nucleus are mutually compared with presented by IBM-1 and IBM-2 models and previously measured data

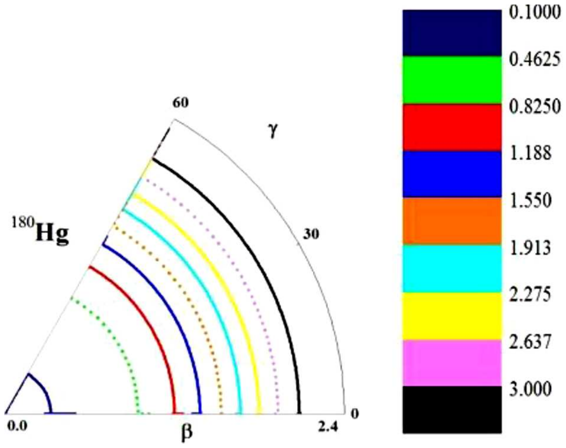


***Fig. 3.*** (Color online) The PES contour plot for $^{180}$Hg nucleus

*Table 5.* **Reduced transition probability $B(E2)\downarrow$ in even $^{180}$Hg nucleus $e_\pi = 0.167$, $e_\nu = 0.184$) in the IBM-2**

| Nucl. | $\alpha$ e · b | $\beta$ e · b | Transition level | $B(E2)$ Exp. $e^2 \cdot b^2$ | $B(E2)$ IBM-1 $e^2 \cdot b^2$ | $B(E2)$ IBM-2 $e^2 \cdot b^2$ |
|---|---|---|---|---|---|---|
| $^{180}$Hg | 0.28 | 0.0 | $2_1^+ \to 0_1^+$ | 0.2958 | 0.78 | 0.4548 |
| | | | $2_2^+ \to 0_1^+$ | – | 0.00 | 0.0270 |
| | | | $4_1^+ \to 2_1^+$ | 1.4931 | 1.41 | 0.8034 |
| | | | $4_2^+ \to 2_1^+$ | – | 2.19 | 0.0031 |
| | | | $4_2^+ \to 2_2^+$ | – | 0.00 | 0.2178 |
| | | | $6_1^+ \to 4_1^+$ | 1.6010 | 1.88 | 1.0793 |
| | | | $8_1^+ \to 6_1^+$ | 2.1586 | 2.19 | 1.2501 |

### *3.4. Electric reduced transition probabilities B(E2)*

We study the strength of the reduced transition $B(E2)$ as one of very important parameters of the decay of the nucleus. The energy levels of excited states of even-even nuclei are given as ($L_i^+ = 2_1^+, 4_1^+, 6_1^+, 8_1^+ ...$) generally decay to the lower lying state $L_f^+ = L_i^+ - 2$ through $E2$ transition. In the present work, the $B(E2)$ data were calculated by means of code PHINT [22]. It was important to calculate the effective charge ($e_B$) from

$$B\left(E2\colon 2_1^+ \to 0_1^+\right) = \frac{\alpha_2^2}{5} N\left(N+4\right) = \frac{e_B^2}{5} N(N+4). \tag{12}$$

In order to find the reduced transition probability, for the effective charge ($\alpha_2$) in IBM-I, we used normalizied experimental data $B(E2; 2_1^+ \to 0_1^+)$ for each isotope using Eq. (12). The value of parameter $\alpha_2^2$ for each isotope was calculated from the measured value of transitions ($2_1^+ \to 0_1^+$). Then it is used to calculate $B(E2)$ values for transitions $4_1^+ \to 2_1^+$, $6_1^+ \to 4_1^+$, $8_1^+ \to 6_1^+$ *etc.* The model wave functions were found through diagonalization of the IBM-2 Hamiltonian. The program NPBEM [26] was useful for assessment of the electromagnetic transition. The $E2$ transition operator [29] is as follows:

$$T^{(E2)} = e_\pi Q_\pi + e_\nu Q_\nu. \tag{13}$$

$Q_\rho$ is the quadrupole operator which is the equivalent to in Hamiltonian (7). The $e_\pi$ and $e_\nu$ are boson effective charges contingent on the boson number $N$, and these effective charges are found by $B(E2\colon 2_1^+ \to 0_1^+)$ suitable to the measured data.

The effective charge $\alpha$ in e.b and $\beta$ in e.b and $B(E2)$ value in $e^2b^2$ by IBM-1, IBM-2, and previous experimental data are presented in Table 5. The effective charge of $^{180}$Hg for $\alpha_2$ is 0.28 e.b, and $\beta$ is 0. The value of $e_\pi = 0.167$, and $e_\nu = 0.184$ in IBM-2. The $B(E2)$ data in IBM-1 and IBM-2 are consistent with the measured data. It is shown that the most of data in IBM-1 are closer to the measured data than those in IBM-2. Therefore, the data from IBM-1 is better than those in IBM-2.

### *3.5. Potential Energy Surface (PES)*

The potential energy surface gives indication for decisive the tiny and regular forms of nuclei. IBM Hamiltonian [30–33] produces the PES plots by means of the Skyrme mean field technique. The IBM-1 energy surface is fashioned by joining the IBM-1 Hamiltonian's expectation value (Eq. (1)) with the coherent state ($|N\beta\ \ \gamma\rangle$) [20]. The creation operators ($b_c^+$) act on a state of boson vacuum $\mid 0\rangle$ to produce the coherent state as follows:

$$|N,\beta,\gamma\rangle = \frac{1}{\sqrt{N!}}\left(b_c^\dagger\right)^N\ |0\rangle, \tag{14}$$

where

$$b_c^\dagger = \frac{1}{\sqrt{1+\beta^2}}\left\{s^\dagger + \beta\left[\cos\gamma(d_0^\dagger) + \right.\right.$$
$$\left.\left. + \sqrt{1/2}\sin\ \gamma(d_2^\dagger + d_{-2}^\dagger)\right]\right\}, \tag{15}$$

then the EPS can be written in terms of $\beta$ and $\gamma$ as

$$E\left(N,\beta,\gamma\right) = \frac{N\varepsilon_d\beta^2}{(1+\beta^2)} +$$
$$+ \frac{N(N+1)}{\left(1+\beta^2\right)^2}\left[\alpha_1\beta^4 + \alpha_2\beta^3\cos 3\gamma + \alpha_3\beta^2 + \alpha_4\right], \tag{16}$$

where $\alpha$'s limits are accompanieding through the coefficients of $C_L$, $v_2$, $v_o$, and $u_o$, as seen from Eq. (1). The factor $\beta$ remarks the total deformation of a nucleus. The nucleus might be sphere-shaped or distorted reliant on either $\beta = 0$ or not. Moreover, the divergence of nucleus symmetry is considered by the $\gamma$ factor. It is known that, when $\gamma = 0$, the nucleus has a prolate shape; when $\gamma = 60$. Figure 3 shows the PES contour, plot for $^{180}$Hg nucleus. The color panel implies the PES standards in MeV.

## 4. Conclusions

The parameters of three types of natural bands (namely, ground, $\gamma$ and $\beta$) and the reduced transition $B(E2)$ strengths for $^{180}$Hg nucleus are calculated within theoretical IBM-1 and IBM-2 models and comparred with the previously measured data and with each other. We have computed energy levels of this nucleus in both models in agreement with measured data. The calculated reduced transition probabilities $B(E2)$ in IBM-1 and IBM-2 are consistent with the experiment. The model IBM-1 is simpler and better than IBM-2. The potential energy surfaces of $^{180}$Hg nucleus are plotted for $\beta$ and $\gamma$ bands and are analyzed in IBM-1. All features of three types for natural bands, energy ratio $R(E(4)/E(2))$, strength of $B(E2)$, and contour map of PES of $^{180}$Hg nucleus in IBM-1 and IBM-2 is obtained. We may conclude that the results of all calculations are reliable and agree with previously measured data for the $^{180}$Hg nucleus which is related to the U(5) symmetry. This theoretical study concerns deformed collective states of a harmonic vibrator U(5), gamma soft O(6) and deformed nuclei O(6) and can be useful for nuclear physicists in the search for 2 enhanced nuclear structures which are not entirely understood yet [n the wide-range shell closures of magic numbers from $Z = 82$ to $N = 126$.

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*Б.А. Алкотбе, М.Дж.С. Аль-Мусаві,*
*Х.Х. Кассім, А.А. Елбндаг, М.А. Аль-Джубборі,*
*І. Хоссейн, Ф.І. Шаррад, Н. Алдахан*

ОЦІНКА СТРУКТУРИ ЯДРА
$^{180}$Hg В МОДЕЛЯХ IBM-1 ТА IBM-2

В моделях із взаємодіючими бозонами IBM-1 та IBM-2 розглянуто властивості ядра $^{180}$Hg, яке відповідає U(5) симетрії. Розраховано параметри трьох енергетичних смуг, інтенсивності квадрупольних електромагнітних переходів і поверхні потенціальної енергії. Результати розрахунків добре узгоджуються з експериментальними даними.

*Ключові слова*: IBM-1, IBM-2, енергетичний рівень, B(E2), поверхня потенціальної енергії, $^{180}$Hg.