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RECIPROCALITY OF ELECTRICAL CONDUCTANCE AND POLARIZATION IN THEIR FREQUENCY DEPENDENCE

This paper presents the electrical polarization and conductance, which can be considered separately only at a constant voltage, while, in the alternating electrical field, they are two sides of the same process, namely, forced by electrical field inertial motion of partially bound and relatively free charged particles. To describe this process, the complex permittivity $\varepsilon^*(\omega) = \varepsilon'(\omega) - i\varepsilon''(\omega)$ and the complex conductivity $\sigma^*(\omega) = \sigma'(\omega) + i\sigma''(\omega)$ can be used equally that is shown by the examples of dielectrics, semiconductors, and metals. As the frequency increases, the delay of polarization leads to an increase in the conductivity, while the delay in the electronic conductivity of a metal can be described by the negative permittivity. This possibility has been explored using the relaxation and the resonance models with the examples of dielectrics, semiconductors, and metals. It is shown that the semiconductor "conductivity", supposedly arising instead of the delayed polarization is not adequate to the classical understanding of this parameter and can be considered only as the effective conductivity. The physical mechanisms of these transformations are explained.

K e y w o r d s: electrical conductivity, electrical polarization, permittivity dispersion, conductivity dispersion.

1. Introduction

While studying the mechanisms of electrical polarization in the dielectrics, in order to simplify the task, it is usually accepted that the electrical conductivity of a material should not to be taken into account. In turn, while studying various processes conditioned by the electrical conductivity in semiconductors and conductors, as a rule, no attention is paid to their electrical polarization [1–3]. In the most cases, these assumptions are justified. However, there are rare, but,

nonetheless, important cases where the interdependence of intrinsic polarity and conductivity becomes the essential physical phenomenon. For example, one should not neglect the intrinsic polarity, while semiconductor non-polar crystals such as Si or Ge are characterized by non-direct band gap in electron energy spectrum, whereas polar semiconductor crystals such as $A^{III}B^V$ (possessing piezoelectric symmetry) and $A^{II}B^{VI}$ (with pyroelectric symmetry) are crystals characterized by the direct band gap.

It is appropriate to note that two important properties of dielectrics – manifestation of polarization and very low conduction – are largely interdependent. Firstly, the electrons or holes, appearing in a dielectric as a result of various activation processes, usually become less mobile, because their own electrical field polarizes the surrounding nano-sized areas in the dielectric. So, they are forced to move together

with these polarized regions (creating polarons). Consequently, even such a small amount of free electrons, which appear in a dielectric due to the thermal activation of impurities, may not cause any appreciable charge transfer, just because local polarization clusters arising around charge carriers, which reduces their mobility in the electrical field [3].

Secondly, a small concentration of charge carriers and their low mobility, in turn, are responsible for the long-time existence of an electrostatic field in the dielectrics. In conductors, this field is screened by free charge carriers (in metals, for example, the screening radius is close to the interatomic distance). Thus, the electrical polarization contributes to the emergence and existence of a relatively stable state in dielectrics with low electronic conduction. However, this stability may be broken in a dielectric by its heating or by high-intensity irradiation, particularly, by the coherent optical (laser) irradiation. At that, charge carriers are generated in a very high concentration, shielding the electrical field. So, the dielectric can be converted into a conductive medium. The stability of the non-conducting state of dielectrics may be compromised also by a strong electrical field, which accelerates free electrons (or holes) up to the energy, at which they can no longer be "captured" by polarization of nano-sized surroundings. These fast electrons cause the percussive ionization in the dielectric, resulting in growing the number of free electrons that ultimately gives rise to the electrical breakdown, and insulator changes into a conductor.

In fact, this or that connection of polarization and conductivity always takes place. The point is that, in any dielectrics, the applied electrical field has an impact on always existing charged particles, which might be as bound so relatively free. At that, in the permanent field, the reaction of these charges looks like electrical polarization and electrical conduction. They can be considered separately and characterized by the relative permittivity ε and the conductivity σ. However, in the alternating electrical field $E(\omega)$, the electrically induced charged particles displace and after forced to move, which leads to both the active and reactive electrical current, that usually is described by the complex permittivity $\varepsilon^*(\omega) = \varepsilon'(\omega) - i\varepsilon''(\omega)$, but also can be depicted by the complex conductivity $\sigma^*(\omega) = \sigma'(\omega) + i\sigma''(\omega)$. An unambiguous relationship between these complex parameters can be established. When the electrical re-

action of a substance is described by the impact of the applied electrical field, both of these descriptions are equivalent. However, most used parameter is the complex permittivity, although, as will be seen later, the complex conductivity sometimes might be more informative.

Using various examples, we consider the general cases of different descriptions of electrical response to an alternating electrical field acting on a substance.

2. Inertia of Electrical Polarization Mechanism

All physical processes are limited by a speed of their completion. As for the electrical polarization, which is described by the permittivity, the least inertial is the elastic displacement of electronic shells relative to cores ($\sim 10^{-16}$ s), more inertial is the ionic lattice polarization ($\sim 10^{-13}$ s) and various mechanisms of thermal (relaxation) polarization (10^{-4}–10^{-9} s). In all cases, a mutually conditioned change in the permittivity and electrical absorption (characterized by the effective conductivity) is observed.

The relationship between the complex parameters $\varepsilon^*(\omega)$ and $\sigma^*(\omega)$ can be established on the basis of the Lorentz–Maxwell's equations. The interaction of electrical and magnetic fields E and H in a matter is described by:

$$
\begin{aligned}
&\operatorname{rot} E = -\partial B/\partial t, \quad \operatorname{rot} H = j + \partial D/\partial t, \\
&\operatorname{div} D = \rho, \quad \operatorname{div} B = 0,
\end{aligned}
\tag{1}
$$

where B is the the magnetic induction, j is the current density, $D = \varepsilon_0 \varepsilon E$ is the dielectric displacement, and ρ is the density of electrical charges. The relationship between the magnetic field H and the magnetic induction B is defined as $B = \mu_0 \mu^* H$, where μ_0 is the magnetic permeability of vacuum, while $\mu^* = \mu' - i\mu''$ is the complex magnetic permeability with the real and the loss parts. In the case under consideration, i.e., for the diamagnetics and weak paramagnetics, it can be set $\mu^* = \mu' = 1$ and $\mu'' = 0$. Therefore, any magnetic processes will not be taken into account in future considerations [1].

Therefore, below, only the electrical polarization and the electrical conduction will be considered. Both of these processes occur due to the electrical field effect on the movement of charged particles in a substance. So, in AC, the polarization and conduction are usually interdependent. To be described in a sinusoidal electrical field: $E(\omega) = E_0 \exp(i\omega t)$, both the
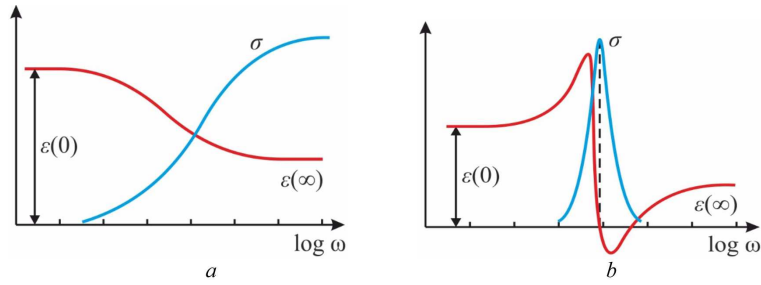
Fig. 1. Comparison of frequency dependence of the permittivity and effective conductivity for two main models describing the polarization dispersion:relaxation (*a*), resonance (*b*)

dielectric displacement and the conduction should be represented by complex parameters.

In the Maxwell–Lorentz equations (1), the second one ($\mathrm{rot}\,H = j + \partial D/\partial t$) shows that the current density j and the time derivative of the electrical displacement $\partial D/\partial t$ are additive quantities. That is why the current density $j = \sigma^* E$ and derivative $\partial D/\partial t = i\omega\varepsilon_0\varepsilon^* E$ would be represented as equivalent functions, if the complex values will be used to describe them. When these functions are presented by real and imaginary parts, the complex conductivity $\sigma^*(\omega)$ is connected to the complex permittivity $\varepsilon^*(\omega)$:

$$\sigma^*(\omega) = \sigma(\omega) + i\sigma(\omega) = i\omega\varepsilon_0\varepsilon^*(\omega),$$
$$\varepsilon^*(\omega) = \varepsilon(\omega) - i\varepsilon(\omega), \tag{2}$$

where ε_0 is the dielectric permittivity of vacuum. Hence, in a sinusoidal electrical field, the complex conductivity and complex dielectric function are interconnected. It follows that the real and imaginary parts of the dielectric permittivity can be expressed through the components of the complex conductivity:

$$\varepsilon'(\omega) = \sigma''(\omega)/(\varepsilon_0\omega),\ \varepsilon''(\omega) = \sigma'(\omega)/(\varepsilon_0\omega). \tag{3}$$

Similarly, the real and imaginary parts of the complex conductivity can be expressed through the components of the complex permittivity:

$$\sigma'(\omega) = \varepsilon_0\omega\varepsilon''(\omega), \quad \sigma'(\omega) = \varepsilon_0\omega\varepsilon'(\omega). \tag{4}$$

It should be noted that the formally established connections between $\sigma^*(\omega)$ and $\varepsilon^*(\omega)$ are expedient to be explained by the microscopic mechanisms of substance responses, as from the side of polarization processes, which means the separation of bound electrical charges, so from the side of the electrical conduction that means the transfer of non-bound charges [4].

In principle, any of the aforementioned complex parameters can describe completely the frequency dependence of the electrical response of a substance to the alternating voltage. Nevertheless, exactly the frequency dependence of the permittivity is traditionally represented in most researches. However, it should be noted that, in some cases, while processing the experimental data, the more informative parameter turned out the frequency-dependent conductivity; for example, to determine accurately the natural frequency of a damped oscillator: it corresponds exactly to the $\sigma'(\omega)$ maximum, as will be shown below.

Relaxational and resonant changes of the electrical response parameters with increasing the frequency are an inevitable process due to inertia of particles involved in the polarization (at that, it is assumed that a constant-voltage conductivity is so small that it can be neglected). The delay of polarization looks like the electrical charge transfer, i.e., as if the effective conductivity appearance. It can both increase or decrease during a frequency growth in accordance with the physical nature of the polarization and depending on the properties of a dielectric. Two typical cases of the permittivity and effective conductivity interdependent changing with frequency are shown in Fig. 1.

The relaxation dispersion of the permittivity, Fig. 1, *a*, consists in $\varepsilon'(\omega)$ gradual decrease from the initial value of $\varepsilon(0)$ to the end value of $\varepsilon(\infty)$, at that, the relaxing polarization mechanism gives contribution to the permittivity $\Delta\varepsilon = \varepsilon(0) - \varepsilon(\infty)$. The relaxation polarization in dielectric spectra (permittivity and absorbed energy power) are shown in Fig. 2.

The dynamic properties of the relaxation polarization can be described by Debye formula that also accounts for the contribution $\varepsilon(\infty)$ of rapid processes
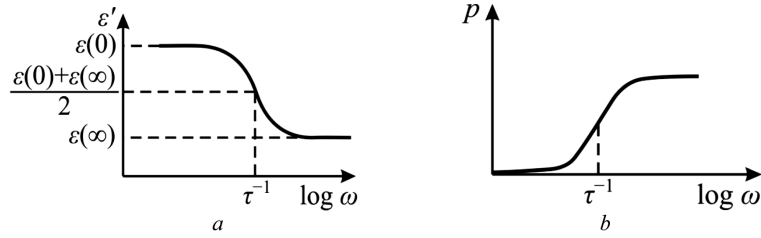
Fig. 2. Relaxation polarization in dielectric spectra: permittivity (a); absorbed energy power (b)
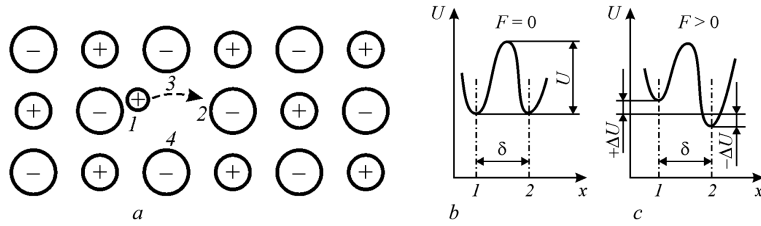

Fig. 3. Model of the ionic relaxation polarization: a – small positive ion of an impurity is localized nearby one (*1*) of four possible negative ions, and it has ability to jump in position (*2*); b – in the absence of an external field positions *1* and *2* equally likely; c – external electrical field stimulates jumps $1 \rightarrow 2$

of polarization:

$$\varepsilon^*(\omega) = \varepsilon' - i\varepsilon'' = \varepsilon(\infty) + [\varepsilon(0) - \varepsilon(\infty)]/(1 + i\omega\tau), \quad (5)$$

$$\sigma'(\omega) = \varepsilon_0 \omega \varepsilon''(\omega) = \varepsilon_0 \tau \omega^2 [\varepsilon(0) - \varepsilon(\infty)]/(1 + \omega^2 \tau^2),$$

where τ is the relaxation time, $\varepsilon(0)$ is the initial permittivity.

Hence, it follows that the effective conductivity σ'_{ef} in the dispersion region increases with the frequency, as the relaxation contribution to the permeability $\Delta\varepsilon$ decreases: Fig. 1, a. Eventually, the effective conductivity gradually reaches a constant value $\sigma'_{\text{ef}} = \varepsilon_0 \Delta\varepsilon/\tau$.

This dependence of the effective conductivity in dielectrics and wide-gap semiconductors are observed usually in a broad frequency range (10^{-4}–10^8 Hz), being typical 0f quite different structures and various chemical compositions [4].

This means that directed by the electrical field thermally the activated motion of partially bound charged particles (whose localizations are determined by a set of potential barriers) gives rise to both the polarization and effective conduction.

An ionic crystal can serve as a convincing example of the relaxation polarization delay. Most of ions in a crystal (their concentration is $n \sim 10^{22}$ cm^{-3}) are located at their lattice sites; this position is rather

stable and can not be disturbed by an electrical field applied to a crystal, which causes only a slight shift of the ionic sub-lattices (that constitutes the quasi-elastic polarization). However, any crystal, almost inevitably, contains a certain concentration ($n_0 \ll n$) of impurities or structural defects; so, charged particles are loosely bound in crystal lattice.

The ionic polarization arising from Frenkel defects (weakly constrained between site ions) is shown in Fig. 3, a. Thermally activated local hopping is possible only for weakly bounded ions that remain localized in vicinities of structural defects. However, at every moment only a part of these n_0 ions is involved in the mechanism of thermal polarization. At the thermal chaotic motion, weakly bounded ions overcome the average potential barrier U, separating two (or more) possible locations of these ions. Obviously, the temporal localization of hopping ions is maintained only at low temperatures: $U \gg k_{\text{B}}T$.

In Fig. 3, b the equiprobable positions of ions in potential minima are denoted as *1* and *2*. Some ions overcome the potential barrier and jump, for example, from position *1* to position *2* (or vice versa). In the case of high barriers, when frequency grows, the charged particles do not have enough time (during a quarter of applied voltage period) to reach the places of their next localization, and, therefore, they con-
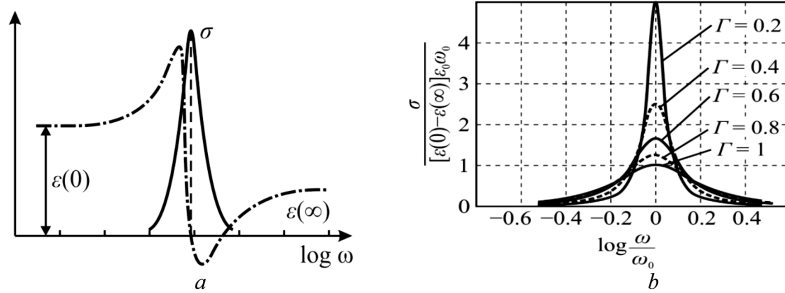
Fig. 4. Permittivity (ε) and the effective conductivity (σ) frequency dependence for the resonance mechanism of polarization (*a*) and the conductivity maximum dependence on the damping coefficient (*b*)

tinuously follow in phase with the changing electrical field contributing to the conductivity. For this reason, the contribution of their motion to the polarization is ceased, resulting in a permittivity reduction: $\varepsilon'(\omega)$ decreases accompanied by n increase in $\varepsilon''(\omega)$, i,e., the conductivity increases: $\sigma'(\omega) = \varepsilon_0 \omega \varepsilon''(\omega)$. It looks as if the polarization turned into a some effective conductivity. Very slight increase in the effective conductivity $\sigma'(\omega)$ in a broad frequency range, Fig. 1, *a*, can be explained by an essential difference in the heights of potential barriers and sizeable distinction in lengths of free paths of charged particles.

Resonant dispersion of the permittivity, Fig. 1, *b*, characterized by the fact that the derivative $d\varepsilon'/d\omega$ at first is positive but then,, at the resonance point, it changes the sign to a negative value, but, after the anti-resonance, the derivative $d\varepsilon'/d\omega$ again becomes positive. Therefore, the permittivity passes through the maximum and minimum. The dielectric dispersion in ionic crystals depends on the difference between frequencies of longitudinal ω_{LO} and transverse ω_{TO} optical vibrations of a lattice. It is characterized by the Lorentz dispersion equation [1]:

$$\varepsilon(\omega) = \varepsilon(\infty) + [\varepsilon(0) - \varepsilon(\infty)]/[1 - (\omega/\omega_{TO})^2],$$

$$\varepsilon(0) - \varepsilon(\infty) = nq^2/\varepsilon_0 m\omega_{TO}^2 = nq^2/[c - (nq^2/3\varepsilon_0)]. \quad (6)$$

Here n is the concentration of ionic couples and m is their reduced mass. The Ionic (far infrared) polarization appreciably affects dielectric properties of crystals: the permittivity is high for the high q (charge of ions) and low c (elastic coefficient of ions coupling). An effective conductivity $\sigma'_{ef} = \varepsilon_0 \omega \varepsilon''(\omega)$ while resonance dispersion is character-

ized by formula:

$$\sigma(\omega) = \{[\varepsilon(0) - \varepsilon(\infty)]\omega_0 \varepsilon_0/\Gamma\}\{[\Gamma^2(\omega/\omega_0)^2]/$$

$$/[(1 - (\omega/\omega_0)^2 + \Gamma^2(\omega/\omega_0)^2]\},$$

This yields the presence of a maximum $\sigma'_{max} = \varepsilon_0 \Delta\varepsilon\omega_0/\Gamma$, locating exactly at the resonance frequency ω_0 of the oscillator describing this dispersion, where Γ is the relative damping factor and $\Delta\varepsilon = \varepsilon(0) - \varepsilon(\infty)$ is the dielectric contribution of the resonant polarization mechanism (Fig. 4).

When the resonant dispersion occurs, in its initial stage, the dielectric losses and the effective conductivity increase due to the weakening of elastic bonds between ions of a lattice, when the resonance is approached. As a result, at the same value of the electrical field, the mutual shift of ions increases critically; at that, the permittivity shows a maximum. At the resonance frequency, elastic bonds between ions no longer have enough time to manifest itself to rapidly changing alternating electric field. So, the system of cations and anions looks like "electrolyte" (so, the maximum of effective conductivity is seen).

Then the phase of a mutual displacement of ions changes in such a way that, in the external fast alternating field, an anti-resonance occurs, at which the dielectric contribution become negative, while the effective conductivity decreases. The negative value of the permittivity means that, in given frequency range,S the dynamic elastic displacement of ionic sublattices moves in antiphase with the acting electric field.

With a further increase of the frequency, the ionic lattice eventually cannot respond to the rapid change of the electrical field, so, the contribution to the permittivity from the ionic polarization $\Delta\varepsilon = \varepsilon(0) - \varepsilon(\infty)$

disappears, but the electronic (optical) polarization of ionic shells still establishes and provides a certain value of $\varepsilon(\infty)$.

Summing up, it should be noted that, in the case of gradual increase of the frequency (that means a more rapid changing of the electrical field), the lag in a displacement of bound charges and the inertia of free charge carriers begins to affect. So, at sufficiently high frequencies, their usual mode of movement becomes impossible.

When the frequency increases, the electrically induced displacement of bound charges, which at lower frequencies contributes to the polarization, at higher frequencies is converted into the effective conduction. In dielectrics and wide-gap semiconductors, the relaxation and resonant dispersion models (both leading to the appearance of the effective conductivity) are considered separately.

However, when the conductivity dispersion in metals, resulting in the negative effective permittivity, the relaxation and resonance phenomena are should be considered simultaneously that complicates the problem.

3. Inertia of Electrical Conduction Mechanisms

When discussing various aspects of the conduction, it should be noted that the time-lag of the electronic conductivity does not seen noticeably even in the infrared frequency region. Thus, the normal (described by zone theory) electronic charge transfer in the metals and alloyed semiconductors does not lead to any frequency dependence of the conductivity over the whole frequency range used in the electronics (up to terahertz). Nevertheless, when the frequency of the electromagnetic field extremely increases, the movement of electrons also manifests their inertia, and the frequency dependence of the electronic conductivity becomes seen in the optical frequency range as in the metals so in other conductors (allayed semiconductors, graphite, nanotubes, graphene, *etc.*)

However, the dispersion of the electronic conductivity in the conductors has a quite another character as compared to the effective conductivity conditioned by a delay of the polarization in dielectrics. In the above-discussed cases (Fig. 1), an increase of the effective $\sigma(\omega)$ was explained by the time-delay of different mechanisms of polarization. Eventually, this ef-

fective conductivity decreases in proportion to the increasing frequency.

In what follows, the quite another mechanisms of $\sigma(\omega)$ decrease will be discussed, which become apparent, as a rule, in the highest frequency region. In this case, the inertia of free charge carriers begins to affect. That is why, their movement in phase with rapidly changing electrical field becomes no longer possible. At that, highly mobile electrons in metals and semiconductors, being affected by a very high-frequency electrical field, demonstrate the plasma resonance and the conductivity dispersion caused by the electronic gas. In the metals, it is seen in the ultraviolet frequency range, while, in the semiconductors, the less dense electronic plasma leads to the resonance phenomena and the conductivity dispersion in the optical frequencies.

When charge carries show their inertia, a gradual decrease of the conductivity is seen in ultraviolet wavelength range (above 10^{16} Hz) that is confirmed by the reflection coefficient frequency dependence. Eventually, the shielding effect of the polarization by free electrons gradually disappears;, so, the electronic shells of cation lattice in the metal lattice becomes noticeable as a contribution to the permittivity.

Simplified model can describe the dispersion of the conductivity from the point of view of relaxation processes. In the absence of an external field, all chaotic motions of electrons are equally probable, and there is no electrical current. But, in the case of $E > 0$, some average directed velocity of electrons appears. The equation of such motion of electronic "gas" should describe the drift velocity v dependence on the electrical field under conditions of electron collisions which slow down the charge transfer. If density of electrons is n, than the induced current is $j = -nev = \sigma E$. Therefore, by drift velocity determination, it is possible to find the regularities for the conductivity.

Suppose that, under the action of a constant electrical field E_0, the average drift velocity $\langle v \rangle_0$ establishes in time, Fig. 5, *a*. This velocity depends on the average free path l of electrons (dependent on crystal lattice perfection). In any case, the drift velocity in metals is small ($v < 1$ m/s) in comparison with the Fermi speed of electrons chaotic movement ($v \sim 10^6$ m/s).

Although the motion of an individual electron is difficult to trace analytically, the derivation of equation for the "average" electron motion can be made,
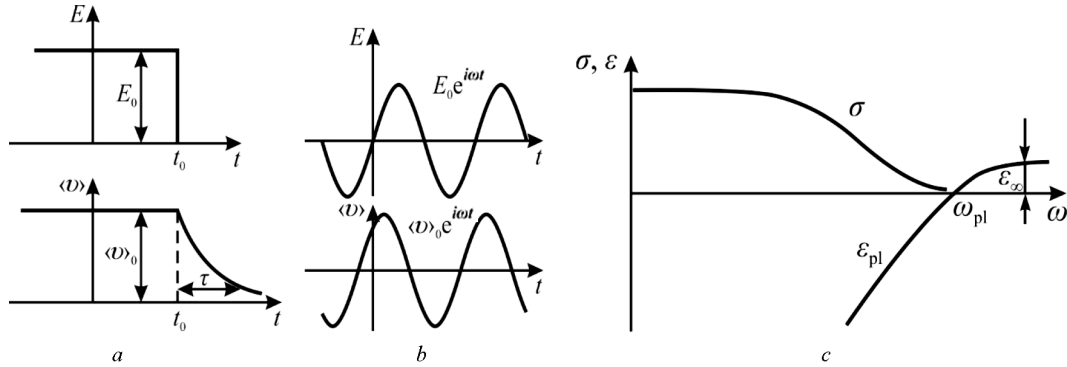
Fig. 5. Time-dependence of the drift velocity of free electrons under the influence of constant (*a*) and alternating (*b*) electrical field *E*; frequency dependence of conductivity (*c*)

if we consider the movement of an electron in a small time interval dt, where t is the time after the last collision of the electron.

At the initial moment, the speed of the electron is v_0, but, after that, under the action of the force eE, during a free path time τ, it acquires the additional velocity $v = -eE\tau/m$, which is proportional to the applied field: $v = \mu E$ where μ is the mobility of drifting electrons.

Because the acceleration $a = eE/m$ works over a time interval τ, an increment in the velocity is $a \cdot \tau$, i.e., a desired value. Since many electrons after their collisions have arbitrarily directed velocities v_0, they do not contribute to the average drift velocity:

$$v = -eE\tau/m, \quad j = \sigma E = ne = ne\mu E = (ne^2\tau/m)E,$$
$$\sigma = ne^2\tau/m, \quad \mu = e\tau/m.$$

As is seen, the formula for the conductivity includes all known quantities except τ; therefore, by measuring conductivity, one can find the relaxation time of the electronic gas in a metal. At normal temperatures for metals, parameter $\tau \sim 10^{-14}$ s. The average free path of electrons l (a distance between their collisions) is defined as $l = v_0\tau$; at that, by considering that the average speed of the chaotic motion of electrons is $\sim 10^6$ m/s, the free length is estimated at ~ 10 microns, i.e., thousands of times higher than crystal lattice parameter.

Relations (1) describe the charge transfer in a conductor without taking the dynamics of this process into account, but this is very important in the case of a rapidly changing electromagnetic field $E(t)$. The point is that, due to the collisions, a part of electrical energy, acquired by electron along its free path, is converted into the thermal energy of lattice vibrations and slows down the electrons reaction to $E(t)$.

It should be noted that if a free electron moves in the electrical field, its acceleration is proportional to the field. However, in the case of the drift of electrons, there is bo their acceleration, but the velocity is proportional to the applied electrical field. That is why the conductivity is proportional not to the first power of electron's charge ($\sigma \sim e$), but to its square ($\sigma \sim e^2$).

Next, we suppose that, at moment $t = t_0$ the electrical field is turned-off, Fig. 5, *a*. As a result, the collisions of electrons lead to a decrease in their drift velocity $\langle v \rangle(t) \to 0$. This means that, in a new dynamic equilibrium state (not disturbed by an external field), any drift of electrons is absent. The principle of non-equilibrium thermodynamics states that the rate of recovery of any equilibrium state is proportional to the magnitude of deviation from this state, i.e., $dv/dt = -kv$.

The dimension of proportionality coefficient k is a value inverse to the time, so it can be denoted as $k = \tau^{-1}$, i.e., the relaxation time, which, in the metal, is very small ($\tau \approx 10^{-14}$ s) and, during the electrical field switching, it is difficult to notice it.

However, despite the extreme smallness of the response time, the dynamics of almost free movement of electrons still can be estimated by the indirect measurement of τ.

In this case, one needs to explore metals behaviour at a very high frequencies, as already shown in Fig. 5, *c*: by studying of $\sigma(\omega)$ variance, i.e., frequency dependence of conductivity, it is possible experimentally evaluate τ for different metals.

The differential equation, describing gradual decrease of drift velocity when approaching to the equilibrium state is $dv(t)/dt = -(1/\tau)v(t)$. The solution of this equation results in the description of time-dependent gradual decay of drift velocity:

$$dv(t)/dt = -v(t)/\tau, \quad dv(t)/v(t) = -dt/\tau,$$
$$v(t) = v_0 \exp(-t/\tau),$$

which depicts the relaxation-type decrease of electrons drift velocity, and, accordingly, the electronic conductivity: $\sigma \sim \langle v \rangle$, caused by the collisions of electrons (at first approximation, without taking into account the inertia of this process due to a small but finite mass of moving electrons).

The frequency dependence of conductivity $\sigma(\omega)$ can be found, if the alternating electrical field $E(\omega) = E_0 \exp(i\omega t)$ acts in a model under consideration (the system of electrons, fast moving chaotically with collisions, at that slowly drifting under the external influence). The drift velocity forcedly is similar to driving the field time-dependence: $v(\omega) = v_0 \exp(i\omega t)$, while the conductivity is determined by the ratio $v(t)/E(t)$ as it is determined by formula $\sigma = nev/E$.

To find the frequency dependence of the conductivity, it is necessary to solve the first-order inhomogeneous differential equation. It was obtained, when considering the mechanism of electron drift, assuming that the acting force eE is the alternating electrical field:

$$m\{dv(t)/dt\} = -mv(t)/\tau + eE_0 \exp(i\omega t),$$

where all three components have the dimension of a force. Considering only the stationary solution of this equation (excluding transient solutions), we obtain:

$$(1 + i\omega\tau)mv_0 = e\tau E_0,$$

insofar as $\sigma = nev_0/E_0$. The final result is:

$$\sigma^*(\omega) = \sigma(0)/(1 + i\omega\tau).$$

The fact that the conductivity is a complex value follows from the considered model. The real and imaginary parts of the conductivity $\sigma^*(\omega) = \sigma'(\omega) + i\sigma''(\omega)$ are determined as follows:

$$\sigma'(\omega) = \sigma(0)/(1 + \omega^2\tau^2),$$
$$\sigma''(\omega) = -\sigma(0)\omega\tau/(1 + \omega^2\tau^2), \quad \sigma(0) = ne^2\tau/m. \tag{7}$$

In the dispersion region. The conductivity decreases with frequency, Fig. 5, *c*. This drop in the conductance also means an increase in losses, since the resistivity increases.

Far from the region of a dispersion, when $\omega\tau \ll 1$. The real part of the conductivity $\sigma'(\omega) \approx \sigma(0)$ remains practically constant, while its imaginary part, remaining rather small, else increases linearly with the frequency: $|\sigma''(\omega)| \approx \sigma(0)\omega\tau$.

The study of the reflection coefficient in the terahertz frequency range (below the plasma frequency) shows that a negative permittivity of highly conductive metals is close to several thousands, while, in the lower-conductive (usually, a magnetic) metals, it equals several hundreds.

The obtained relations (7) allow us not only to explain a decrease of the conductivity with the frequency rise, but also to describe the frequency dependence of the effective permittivity, which, as can be seen from Fig. 3, *c*, takes a negative value in the dispersion region and at lower frequencies:

$$\varepsilon'(\omega) = \sigma''(\omega)/(\varepsilon_0\omega) = -\sigma(0)\tau/[\varepsilon_0(1 + \omega^2\tau^2)],$$
$$\varepsilon''(\omega) = \sigma'(\omega)/(\varepsilon_0\omega) = \sigma(0)/[\varepsilon_0\omega(1 + \omega^2\tau^2)]. \tag{8}$$

As seen in Fig. 5, *b*, the changing of the real part of the effective permittivity and its negative sign: $\varepsilon\prime(\omega) = -\sigma''(\omega)/(\varepsilon_0\omega)$ are due to a delay in the electron drift motion.

During electron motion in their free path (between the collisions), very high frequency field changes its direction, and the electron, without experienced collision, is forced to change the direction of its movement during its free run. At the same time, the phase of its movement always lags behind an phase in the changing field (this peculiarity is also a consequence of the spatial dispersion, at which the length of an EM wave in a conductor becomes commensurate with the free path of electrons).

The electronic plasma concept expands the understanding of the conductivity dispersion in terms of some resonance process. Plasma can be considered as a subsystem, which interacts with the crystal lattice that facilitates plasma properties.

It is pertinent to note that plasma is made up of positive and negative charge carriers; at that, plasma can be charged (like the electronic plasma in metals) or neutral (like the electron-hole plasma in semiconductors). The density of electrons in charged
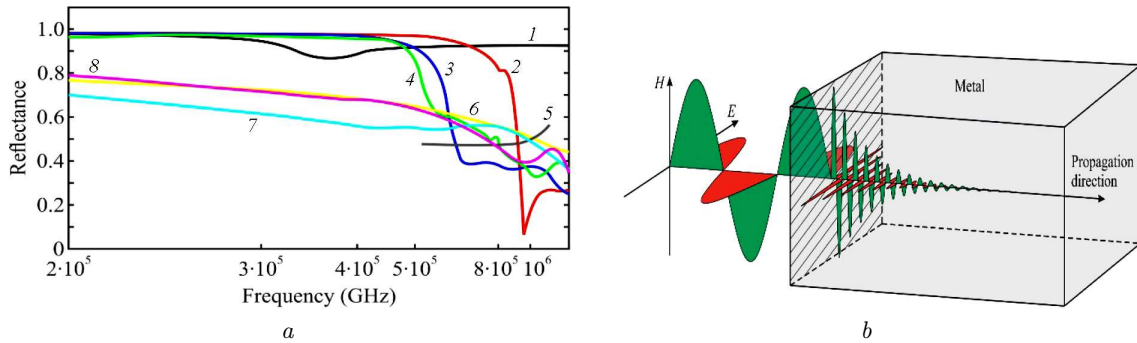
Fig. 6. EM wave interaction with a metal: *a* – reflectance spectra of metals: aluminum (*1*), silver (*2*), gold (*3*), copper (*4*), beryllium (*5*), cobalt (*6*), iron (*7*) and nickel (*8*); *b* – wavelength reduction and attenuation in a metal

plasma usually is close to 10^{22} cm^{-3}, while, in a neutral plasma, charge carries density might be 10^{15}–10^{18} cm^{-3}.

A characteristic property of plasma is the collective excitations (plasma oscillations), which, at sufficiently high frequency, lag behind the rapidly changing external field, which leads to the dispersion of the conductivity. Plasma frequency (ω_{pl}) of electrons in the conductors (which determines the quasi-particle plasmon) is important parameter for metals and semiconductors. It characterizes dynamic properties of the almost-free gas of electrons.

It is generally accepted that, at a plasma frequency, the permittivity vanishes ($\varepsilon'(\omega) = 0$) crossing the abscissa axis, but then increases reaching a value $\varepsilon_\propto$ (in different metals $\varepsilon_\propto = 4$–8). Exactly this behavior of the free electron gas in the metals in a very-high-frequency EM field requires the introduction of the plasma frequency concept.

To estimate the plasma frequency, the spatial dispersion can be considered, but a simpler model can also be applied. Suppose that a returning force $\delta F = = exE$ arises, when a group of n electrons shifts from their equilibrium position onto some distance x relatively to the positively charged "non-moving" ionic lattice. This displacement can be induced by the external electrical field eE/ε_0. At that, the force of inertia $F_{in} = Ma = nmdx^2/dt^2$ opposes the disturbing electrical force $F_{el} = -nex\,eE/\varepsilon_0$. The solution of this differential equation $F_{in} + F_{el} = = 0$ leads to the frequency of plasma oscillations: $\omega_{pl} = ne^2/m\varepsilon_0)^{1/2}$.

Therefore, the returning force by its action on the displaced electrons causes their oscillations around equilibrium position. The experiment indicates that

the electronic gas in metals really demonstrates its inertia at frequencies near 10^{16} Hz: at the higher frequencies, electrons have no time to follow fast changes of the electromagnetic field. The frequency dispersion of the complex conductivity can be expressed in terms of the plasma frequency:

$$\sigma^*(\omega) = \sigma(0)/(1 + i\omega\tau), \qquad (9)$$

where $\sigma(0) = ne^2\tau/m = \omega_{pl}^2\tau$. Thus, the conductivity $\sigma(\omega)$ of metals looks like practically constant value $\sigma(0)$, Fig. 5, *a*, which defines the ohmic losses of a conductor in the broad frequency range including the terahertz range. With a further increase in the frequency, a smooth decrease in the conductivity up to zero is observed, accompanied by the negative effective permittivity.

Above the plasma frequency (in the ultraviolet range), a positive permittivity $\varepsilon_{uv} = \varepsilon_\propto$ is determined, which is due to the elastic displacements of non-collectivized electrons (bound electronic shells inside ionic cores).

The existence of plasma oscillations is confirmed by the study of metals reflection spectra, Fig. 6, *a* [5]. These investigations indicate that the process of $\sigma(\omega)$ dispersion not nearly, as a simple elementary theory predicts, because the manifestation of resonance processes is obvious. It is possible that the electron shells of different cations affect the spectrum in a different ways (Au^{+1} repeats Rn shell, Cu^{+2}–Kr shell, Ag^{+1}–Xe shell, and Al^{+3}–Ar shell).

But, in the 3*d* and 4*f* metals, the electronic configurations of ions contain the uncompensated spins, which, by their magnetic moments, affect the dynamics of a drift motion of the electronic gas. That is why,

the EM waves reflection and the conductivity of ferromagnetics much less than in the high-conductivity metals.

EM waves in a metal, in the ideal case of a superconductor, cannot, they are completely reflected. But, in the optical range, any superconductivity no longer manifests itself, because the photons energy is much higher than the energy of Cooper pairs.

Therefore, in a real metal, despite the dominant reflection of the EM wave, it, nevertheless, partially penetrates into the conductor. From the microscopic point of view, the absorption of a part of the configuration incident EM radiation occurs mainly because the electrons of s metal, excited by the EM radiation as during their transition to the higher energy levels above the Fermi level. So, at their subsequent return in the equilibrium state (accompanied by the secondary emission of EM waves), interact with the ionic lattice exciting its oscillations, i.e., convert some of EM energy into a heat.

From the macroscopic point of view, the phase velocity of an EM wave, penetrating into a conductor, is greatly reduced in comparison with its velocity in a vacuum, so that the length of the EM wave in the conductor becomes hundreds of times shorter than in vacuum, Fig. 6, *b*. A reduction of the EM wavelength in a substance occurs by $|\varepsilon^*|^{1/2}$ times. Therefore, this occurs due to a large effective permittivity that corresponds in electrodynamics to the large conductivity dispersion.

Without a doubt, due to the huge step in the impedance of "vacuum–metal" interface, as well as through a very small phase velocity of EM waves in a conductor, any EM wave falling on the conductor (even in case of grazing incidence) refracts practically in the direction perpendicular to conductor's surface. At that, the transmittance of a part of the EM radiation incident on s conductor is possible only in the case of a very thin metal layer.

The analysis shows that the intensity of an incident SM wave decreases exponentially, while it propagates through a metal, Fig. 6, *b*, that leads to a much lower intensity of the transmitting wave. This happens, because the metal strongly damp the initial intensity of TM waves.

Thus, the frequency dispersion of the electronic conductivity of conductors has the mixed relaxation and plasma-resonant natures, which leads to negative effective permittivity.

4. Conclusions

When the electrical field frequency increase, one can observe the frequency dispersion of both the permittivity due to a delay in the reaction of bound charges and the conductivity, due to an inertia in the number of free charge carriers.

Usually, these dispersions are described separately: by the complex permittivity $\varepsilon^*(\omega) = \varepsilon'(\omega) - i\varepsilon''(\omega)$ and the complex conductivity $\sigma^*(\omega) = \sigma'(\omega) + i\sigma''(\omega)$. But, formally these parameters can be used as equivalent. This possibility has been explored using the relaxation and the resonance models with the examples of dielectrics, semiconductors, and metals.

It is shown that the "conductivity", supposedly arising instead of delayed polarization, is not adequate to the classical understanding of this parameter and can be considered only as the effective conductivity. In the dielectrics and wide-gap semiconductors, relaxation and resonant dispersion models (both leading to the appearance of the effective conductivity) are considered separately.

But, in the conductors, the frequency dispersion of the conductivity is observed as the relaxation and resonance phenomena sly. It resulting in the "negative permittivity" that is obviously not adequate to the usual understanding of this concept and should be considered as the effective parameter.

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ВЗАЄМНІСТЬ ЕЛЕКТРОПРОВІДНОСТІ
ТА ПОЛЯРИЗАЦІЇ В ЇХ ЧАСТОТНІЙ
ЗАЛЕЖНОСТІ

У цій статті представлено електричну поляризацію та провідність, які можна розглядати окремо лише при постійній напрузі, тоді як у змінному електричному полі вони є двома сторонами одного процесу, а саме інерційного руху під дією електричного поля частково зв'язаних та відносно вільних заряджених частинок. Для опису цього процесу можна

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однаково використовувати комплексну діелектричну проникність $\varepsilon^*(\omega) = \varepsilon'(\omega) - i\varepsilon''(\omega)$ і комплексну провідність $\sigma^*(\omega) = \sigma'(\omega) + i\sigma''(\omega)$, що показано на прикладах діелектриків, напівпровідників і металів. Зі збільшенням частоти, затримка поляризації приводить до збільшення провідності, тоді як затримка електронної провідності металу може бути описана негативною діелектричною проникністю. Ця можливість була досліджена за допомогою моделей релаксації та резонансу на прикладах діелектриків, напівпровідників і металів. Показано, що "провідність", яка нібито виникає замість сповільненої поляризації, не є адекватною класичному розумінню цього параметра і може розглядатися лише як ефективна провідність. Пояснені фізичні механізми цих перетворень.

К л ю ч о в і   с л о в а: електропровідність, електрична поляризація, дисперсія діелектричної проникності, дисперсія електропровідності.