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M.I. KOPP,[1] V.V. YANOVSKY [1,2]

[1] Institute for Single Crystals, Nat. Acad. of Sci. of Ukraine
(60, Nauky Ave., Kharkiv 61072, Ukraine; e-mail: michaelkopp0165@gmail.com)
[2] V.N. Karazin Kharkiv National University
(4, Svobody Sq., Kharkiv 61022, Ukraine; e-mail: yanovsky@isc.kharkov.ua)

THE EFFECT OF THROUGHFLOW AND GRAVITATIONAL MODULATION ON WEAKLY NONLINEAR BIO-THERMAL CONVECTION IN A POROUS MEDIUM LAYER

We investigate the impact of periodically varying gravitational fields and a throughflow on the bio-thermal Darcy–Brinkman convection within a porous medium layer saturated with a Newtonian fluid containing gyrotactic microorganisms. The study includes an examination of two types of a throughflow: one directed against the gravity field and another one along it. We assume that the gravitational modulation has a small amplitude, quantified as a second-order smallness in the dimensionless parameter ϵ, which represents the supercritical parameter of the Rayleigh number. For weakly nonlinear convection, a Ginzburg–Landau (GL) equation with a periodic coefficient is derived in the third order in ϵ. To analyze the heat and mass transfer, we numerically solve the GL equation. The numerical results reveal that the vertical throughflow in the bio-thermal convection exhibits a dual nature, allowing for both augmentation and a reduction of the heat and mass transfers. We investigate the influence of variations in the Vadasz number, Peclet number, bioconvective Peclet number, frequency, and amplitude of modulation on the heat and mass transfer. The effects of these parameters are depicted graphically, illustrating that higher values of the Vadasz and Peclet numbers, as well as increased modulation amplitude, positively impact the heat and mass transfer. In addition, a comparative analysis of modulated and non-modulated systems shows a significant effect of the modulation on the stability of systems.

K e y w o r d s: bio-thermal convection, gravity modulation, throughflow, gyrotactic microorganism, Ginzburg–Landau amplitude equation.

1. Introduction

Studying a fluid flow through porous materials has practical implications in a variety of domains, including soil mechanics, oil production, fluid engineering, and groundwater hydrology, as well as industrial filtration. A new field of the study, bio-thermal convection in porous media, is gaining the increasing interest. It is important to conduct a theoretical research to examine the interaction between the bioconvection and the natural convection. To move forward with the study of bioconvection, it is essential to understand the dynamics of this significant interaction. Several researchers have developed mathematical models for the thermal convection in both fluid and a porous medium. Several authors have contributed to the study of the thermal instability in fluid layers and porous media, including Chandrasekhar [1], Drazin and Reid [2], and Vafai [3]. Ingham and Pop [4] and Nield and Bejan [5] have written comprehensive monographs on the subject. In addition, Vadasz [6] has provided a detailed review of the fluid flow and heat transfer in rotating porous media. These works analyze and discuss various aspects and challenges associated with the thermal instability in such systems.

Bioconvection in porous media is a recently emerging area of research that has been gaining atten-

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tion. The term "bioconvection" refers to the formation of convective patterns due to the presence of self-propelled microorganisms that have a greater density than the surrounding fluid medium. This phenomenon accounts for the movement of bacteria and algae, which have a higher density than water. The migration of bacteria is caused by gravity forces (gyrotactic microorganisms), oxygen concentration gradients (oxytactic microorganisms), light radiation (phototaxis microorganisms), nutrition gradients (chemotaxis microorganisms), and other variables. The concentration of self-propelled microorganisms can range from 10^7 cm^{-3} in a low concentration regime to 10^{11} cm^{-3} in a turbulent regime containing nearly densely packed microorganisms.

The focus of this paper is on gravitactic microorganisms; those that move in water against the direction of gravity. When these gyrotactic bacteria swim in a certain direction, they increase the density of the base fluid. Bioconvection occurs due to the unstable density stratification that arises, when microbiological organisms are denser than their surrounding fluid. The accumulation of these organisms causes the upper layer to be denser than the region below, leading to the instability and various flow patterns [7]. The bioconvection of gravitactic microorganisms was extensively studied by researchers. Childress *et al.* [8] were the first to develop a comprehensive theory and mathematical model for this phenomenon. Hill *et al.* [9] later presented a theoretical bioconvective model that specifically focused on gravitactic microorganisms. Pedley *et al.* [10] analyzed the stability of bioconvection involving gyrotactic microorganisms within a shallow layer of a regular fluid using a linear stability theory. The studies identified the necessary conditions for the initiation of a bioconvective flow.

Several publications have explored the effects of gyrotactic microorganisms on fluid flows in bounded porous media. Nield, Kuznetsov, and Avramenko [11, 15] have made significant contributions to understanding the biological process dynamics in porous media. In their work, Kuznetsov and Avramenko [11] established that the system remains stable, and the bioconvection does not occur, if the permeability is below a critical value. Conversely, when the permeability exceeds the critical value, the bioconvection can develop. They further investigated [12] the occurrence of the bioconvection in a horizontal layer filled with a saturated porous medium. Critical Rayleigh numbers were determined for different values of the Peclet number, gyrotaxis number, and cell eccentricity. The influence of a vertical flow on the onset of the bioconvection [13] in a suspension of gyrotactic microorganisms in a porous medium was studied in another publication. A linear analysis was used to obtain an equation for the critical Rayleigh number. It was demonstrated that the vertical throughflow stabilizes the system. A continuum model of thermobioconvection was presented in [14], focusing on oxytate bacteria in a porous medium. This study examined the effect of the heating of microorganisms from below on the stability of a horizontally layered fluid saturated with a porous medium. A relationship between the critical value of the Rayleigh number and the thermal Rayleigh number was obtained by using the Galerkin method to solve the linear stability problem. Avramenko [15] developed a nonlinear theory of bioconvection for gyrotactic microorganisms in a layer of ordinary liquid based on the Lorenz approach. His work delineated the boundaries of various hydrodynamic regimes observed in two-dimensional bioconvection.

Dmitrenko's study [16] offers a comprehensive review of the main aspects of the bioconvection in nanofluids and porous media. The study presents a mathematical model based on the Darcy's law for porous media. Sharma and Kumar [17] conducted research on the effects of high-frequency vertical vibrations on the onset of the bioconvection in a dilute solution of gyrotactic microorganisms using analytic and numerical methods. Their findings showed that high-frequency, low-amplitude vertical vibrations and the bioconvection Peclet number have a stabilizing effect on the system. Kushwaha *et al.* [18] conducted a more detailed analysis of the stability of vibrational systems. The analysis focused on shallow layers filled with randomly swimming gyrotactic microorganisms. Garg *et al.* [19] recently studied the stability of the thermo-bioconvection flow of a Jeffery fluid containing gravitactic microorganisms in an anisotropic porous medium.

Over a last few decades, the Darcy–Brinkman model has been extensively used in researches related to porous media. In particular, Zhao *et al.* [20] expanded its application by studying the biothermal convection [21] in a highly porous medium, considering a suspension of gyrotactic microorganisms. They conducted a stability analysis to examine the behavior of biothermal convection under the influence of heating from below. Kopp *et al.* [22] used the Darcy–Brinkman model to investigate biothermal instability in a porous medium saturated by a water-based nanofluid containing gyrotactic microorganisms in the presence of a vertical magnetic field. They found that an increase in the concentration of gyrotactic microorganisms enhances the onset of magnetic convection. Moreover, as shown in their study, spherical gyrotactic microorganisms are more effective in contributing to the development of biothermal instability. Additionally, Kopp and Yanovsky [23] studied the impact of the rotation effect, specifically the Coriolis force, on biothermal convection in a layer of porous medium saturated with a suspension containing gyrotactic microorganisms

Controlling heat and mass transfer is crucial in engineering and technical applications. In order to manipulate convective processes, various methods are used, such as external parametric or modulation effects on the system. Understanding the impact of modulation on convection is important to comprehend how external disturbances or parameter changes can affect flow and transport phenomena in the system. Temperature modulation, gravity modulation, rotation modulation, and magnetic field modulation are some of the commonly used techniques for modulation. In this study, we will use a convection control method that involves the modulation of the gravity field. Before presenting our detailed rationale for selecting gravity modulation, we will provide a concise overview of relevant literature that explores the use of gravity modulation in diverse convective systems.

The technique of using gravity modulation to improve the stability of a heated fluid layer that is heated from below was first introduced by Gresho and Sani in their study [24]. Since then, many researchers have explored the effects of gravity modulation on the onset of convection. Malashetty and Begum extended these investigations in their study [25] by considering additional physical conditions and non-Newtonian fluids. They examined the impact of small amplitude gravity modulation on the initiation of convection in both fluid layers and fluid-saturated porous layers. Kiran [26] conducted studies on the nonlinear thermal instability in a porous medium saturated with viscoelastic nanofluid under gravitational modulation. Over the years, Kiran *et al.* conducted several studies [27, 29] to investigate the impact of gravity modulation on Rayleigh–Bénard convection (RBC) and Darcy convection. Their focus was on the effect of g-jitter on RBC in nanofluids [30], and they used the Ginzburg–Landau (GL) model to carry out nonlinear analysis. They also calculated the thermal and concentration Nusselt numbers, taking into account various physical parameters. Additionally, Manjula *et al.* [31] studied the combined effects of gravity modulation and rotation on thermal instability in a horizontal layer of a nanofluid.

Kopp and Yanovsky [32] were the first to explore the use of s gravity modulation in controlling the development of the bio-thermal convection in a layer of porous media that is saturated with a Newtonian fluid and contains gyrotactic microorganisms. In their work, they developed a weakly nonlinear theory of bio-thermal convection with finite amplitude, whose evolution is described by the Ginzburg–Landau (GL) equation. They considered the case of slowly moving microorganisms, which corresponds to a low bioconvective Peclet numbers, and assumed the concentration of microorganisms in the liquid layer to be approximately constant. The numerical analysis of the GL equation revealed that the spherical shape of microorganisms enhances the efficiency of the heat transfer process.

In this study, as well as in article [32], we investigate the influence of the gravitational modulation on the development of the biothermal convection in a layer of porous medium saturated with the Newtonian fluid containing gyrotactic microorganisms. However, unlike article [32], we do not place restrictions on the speed of movement of microorganisms and also consider the stratification of the concentration of microorganisms. In addition, we take the throughflow at the layer boundaries into account. The research is focused on investigating the impact of a throughflow and the gravitational field modulation using the Ginzburg–Landau (GL) model. The study of s weakly nonlinear biothermal convection in porous media with gyrotactic organisms under the influence of a throughflow and the modulation of the gravitational field has potential applications in various fields such as environmental science, biotechnology, medicine, geoscience, and materials science.

2. Problem Statements and Basic Equations

Consider an infinite horizontal layer of a porous medium filled with a Newtonian fluid that contains gyrotactic microorganisms. The porous layer has a thickness h and is heated from below, while being cooled from above, as given in Fig. 1. The temperature difference at the lower and upper boundaries is designated as ΔT. The Cartesian reference system is chosen so that the z-axis is directed vertically upward and the x-axis is horizontal. Gravity acts vertically downward with an acceleration that varies according to the periodic law in time $\mathbf{g}(t) = \mathbf{e}g_0(1 + \epsilon^2\delta\cos(\omega_g t))$, where δ and ω_g are the amplitude and frequency of the gravitational modulation, respectively. Designation ϵ is a small dimensionless parameter that will be explained later on. In this paper, we consider a vertical flow in two possible directions. In the first case, the flow is directed vertically upward, i.e., opposite to the gravity field. In the second case, the flow is directed vertically downward, i.e., parallel to the direction of the gravity field. In the case of a dilute suspension of floating microorganisms, we assume that the liquid is incompressible and the porous matrix is not capable of absorbing microorganisms. Additionally, we use the Darcy–Brinkman model with the Boussinesq approximation. Under these assumptions, the equations of continuity, momentum, heat balance, and for the concentration of microorganisms have the following form [32]:

$$\nabla\,\mathbf{V}_{\mathrm{D}} = 0, \tag{1}$$

$$\frac{\rho_0}{\varepsilon}\frac{\partial \mathbf{V}_{\mathrm{D}}}{\partial t} = -\nabla P + \widetilde{\mu}\nabla^2\mathbf{V}_{\mathrm{D}} - \frac{\mu}{K}\mathbf{V}_{\mathrm{D}} - \\ - \mathbf{e}g(t)\rho_0(1-\beta(T-T_0)) - \mathbf{e}g(t)(\delta\rho)\mathcal{V}n, \tag{2}$$

$$(\rho c)_m\frac{\partial T}{\partial t} + (\rho c)_f\mathbf{V}_{\mathrm{D}}\,\nabla T = k_m\nabla^2 T, \tag{3}$$

$$\frac{\partial n}{\partial t} = -\mathrm{div}\left(n\mathbf{V}_{\mathrm{D}} + nW_c\hat{\mathbf{l}} - D_m\nabla n\right), \tag{4}$$

$$g(t) = g_0(1+\epsilon^2\delta\cos(\omega_g t)). \tag{5}$$

Here $\mathbf{V}_{\mathrm{D}} = (u, v, w)$ is the Darcy velocity, which is related to the fluid velocity $\mathbf{V}$ as $\mathbf{V}_{\mathrm{D}} = \varepsilon_p\mathbf{V}$, ε_p is the porosity of the porous medium, K is the permeability of the porous medium, ρ_0 is the fluid's density at the reference temperature, P is the pressure, β is the thermal expansion coefficient, g is the gravitational acceleration, $\mathbf{e} = (0, 0, 1)$ is a unit vector in the direction of the axis z, $\widetilde{\mu}$ is the Brinkman effective viscosity, μ is the viscosity of fluid, $(\rho c)_f$ is the heat capacity of fluid, $(\rho c)_m$ is the effective heat capacity, k_m is the effective thermal conductivity, n is the concentration of microorganisms, $\delta\rho$ is the of the densities difference of microorganisms and a base fluid: $\rho_m - \rho_f$, $\mathcal{V}$ is the average volume of a microorganism, and D_m is the diffusivity of microorganisms. We assumed that random motions of microorganisms are simulated by the diffusion process; $W_c\hat{\mathbf{l}}(t)$ is the average microorganism swimming velocity, abd (W_c is a constant). The unit vector $\hat{\mathbf{l}}(t)$ represents the direction of movement of the microorganisms, and it is a time-periodic quantity due to the modulation of the gravitational field.

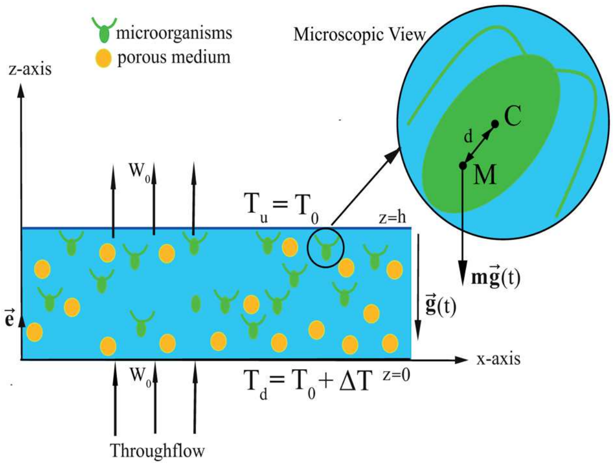

Fig. 1. Geometry of the problem. A schema of a gyrotactic microorganism: a cell of the green alga *Chlamydomonas reinhardtii.* The center of buoyancy is denoted C and is offset by a distance d from the center of mass M

We assume a constant temperature and throughflow velocity $\mathbf{W}_0$ on the boundaries. The boundary conditions are

$$w = W_0,\quad \frac{dw}{dz} = 0,\quad T = T_d,\quad \mathbf{J}\cdot\mathbf{e} = 0,\quad \text{at } z = 0, \tag{6}$$

$$w = W_0,\quad \frac{dw}{dz} = 0,\quad T = T_u,\quad \mathbf{J}\cdot\mathbf{e} = 0,\quad \text{at } z = h, \tag{7}$$

where $\mathbf{J} = n\mathbf{V}_{\mathrm{D}} + nW_c\,\hat{\mathbf{l}}(t) - D_m\nabla n$ is the flux of microorganisms.

To analyze the problem, we have introduced the following non-dimensional parameters:

$$\begin{aligned}
&(x^*, y^*, z^*) = \frac{(x, y, z)}{h},\ \mathbf{V}_{\rm D}^* = \mathbf{V}_{\rm D}\frac{h}{\alpha_m},\\
&t^* = \frac{t\alpha_m}{h^2\widetilde{\sigma}},\ T^* = \frac{T - T_u}{T_d - T_u} = \frac{T - T_u}{\Delta T},\\
&P^* = \frac{PK}{\mu\alpha_m},\ \widetilde{\sigma} = \frac{(\rho c)_m}{(\rho c)_f},\ n^* = n\mathcal{V},\\
&\omega_g^* = \omega_g\frac{h^2\widetilde{\sigma}}{\alpha_m},\quad \alpha_m = k_m/(\rho c)_f,
\end{aligned} \tag{8}$$

where α_m is the coefficient of thermal diffusivity.

Equations (1)–(4) in their non-dimensionless form (after omitting the asterisks) can be expressed as:

$$\nabla\cdot\mathbf{V}_{\rm D} = 0, \tag{9}$$

$$\begin{aligned}
&\frac{1}{\mathcal{V}_a}\frac{\partial\mathbf{V}_{\rm D}}{\partial t} = -\nabla P + D_a\nabla^2\mathbf{V}_{\rm D} - \mathbf{V}_{\rm D} -\\
&-\mathbf{e}f_m\frac{{\rm R}_b}{L_b}n + \mathbf{e}f_m{\rm Ra}T,
\end{aligned} \tag{10}$$

$$\frac{\partial T}{\partial t} + (\mathbf{V}_{\rm D}\nabla)T = \nabla^2 T, \tag{11}$$

$$\frac{1}{\widetilde{\sigma}}\frac{\partial n}{\partial t} = -\nabla\left(n\mathbf{V}_{\rm D} + \frac{\rm Pe}{L_b}n\,\widehat{\mathbf{1}}(t) - \frac{1}{L_b}\nabla n\right), \tag{12}$$

where $f_m = 1 + \epsilon^2\delta\cos(\omega_g t)$. The non-dimensional boundary conditions are given as

$$\begin{aligned}
&w = {\rm Pe}_0,\quad \frac{dw}{dz} = 0,\quad T = 1,\\
&n({\rm Pe}_0 L_b + {\rm Pe}) = \frac{dn}{dz},\quad \text{at}\quad z = 0,
\end{aligned} \tag{13}$$

$$\begin{aligned}
&w = {\rm Pe}_0,\quad \frac{dw}{dz} = 0,\quad T = 0,\\
&n({\rm Pe}_0 L_b + {\rm Pe}) = \frac{dn}{dz},\quad \text{at}\quad z = 1.
\end{aligned} \tag{14}$$

In Eqs. (9)–(12), we introduced notations for dimensionless parameters of the following form:

$$\mathcal{V}_a = \frac{\varepsilon(\rho c)_m\widetilde{\mu}}{\rho_0 k_m D_a}$$

is the modified Vadasz number,

$$D_a = \frac{\widetilde{\mu}K}{\mu h^2}$$

is the Darcy number,

$${\rm R}_b = \frac{g(\delta\rho)hK}{\mu D_m}$$

is the bioconvection Rayleigh–Darcy number,

$$L_b = \frac{\alpha_m}{D_m}$$

is the bioconvection Lewis number,

$${\rm Ra} = \frac{\rho_0 ghK\beta\Delta T}{\mu\alpha_m}$$

is the Rayleigh–Darcy number,

$${\rm Pe}_0 = \frac{W_0 h}{\alpha_m}$$

is the Peclet number,

$${\rm Pe} = \frac{W_c h}{D_m}$$

is the bioconvection Peclet number.

We assume that the basic state of the fluid does not depend on time, and the quantities in this state are given by

$$\mathbf{V}_b = (0, 0, {\rm Pe}_0),\ P = P_b(z),\ T = T_b(z),\ n = n_b(z), \tag{15}$$

We will obtain the steady profiles of the temperature $T_b(z)$, concentration of microorganisms $n_b(z)$, and pressure distribution $P_b(z)$ in the basic state by solving the following equations:

$$\frac{d^2T_b}{dz^2} - {\rm Pe}_0\frac{dT_b}{dz} = 0, \tag{16}$$

$$\frac{dn_b}{dz} = n_b({\rm Pe}_0 L_b + {\rm Pe}), \tag{17}$$

$$\frac{dP_b}{dz} = -\frac{{\rm R}_b}{L_b}n_b + {\rm Ra}T_b. \tag{18}$$

After integrating Eq. (16) and applying the boundary conditions (13)–(14), we can determine the temperature distribution $T_b(z)$:

$$T_b(z) = \frac{e^{{\rm Pe}_0 z} - e^{{\rm Pe}_0}}{1 - e^{{\rm Pe}_0}}. \tag{19}$$

Next, we obtain a solution for n_b, which matches the result in [12]:

$$n_b(z) = n_b(0)e^{({\rm Pe}_0 L_b + {\rm Pe})z}, \tag{20}$$

where $n_b(0)$ is the value of the number density at the bottom of the layer. The constant $n_b(0)$ is found as

$$n_b(0) = \frac{\langle n\rangle(\mathrm{Pe}_0 L_b + \mathrm{Pe})}{e^{(\mathrm{Pe}_0 L_b + \mathrm{Pe})} - 1}, \quad \langle n\rangle = \int_0^1 n_b(z)\,dz. \tag{21}$$

As shown below, the explicit form of the pressure P_b is unnecessary.

The perturbations on the basic state with a small amplitude are supposed to be in the form:

$$\begin{aligned} &\mathbf{V}_\mathrm{D} = \mathbf{V}_b + \mathbf{V}'(u', v', w'),\\ &T = T_b + T', \quad n = n_b + n',\\ &P = P_b + P', \quad \widehat{\mathbf{l}}(t) = \mathbf{e} + \widehat{\mathbf{m}}'(t), \end{aligned} \tag{22}$$

After reviewing prior research [7], [11], we will consider the gravity modulation and write the equation governing a perturbation of the unit vector indicating the direction of swimming of microorganisms:

$$\begin{aligned} &\widehat{\mathbf{m}}'(t) = \widetilde{\mathcal{B}}_0(1 - \epsilon^2\delta\cos(\omega_g t))\zeta\mathbf{i} -\\ &- \widetilde{\mathcal{B}}_0(1 - \epsilon^2\delta\cos(\omega_g t))\xi\mathbf{j} + 0\cdot\mathbf{e}. \end{aligned} \tag{23}$$

Here,

$$\widetilde{\mathcal{B}}_0 = (\mu\alpha_\perp/\rho_0 g_0 d)(\alpha_m/h^2) = \mathcal{B}_0(\alpha_m/h^2),$$

$\mathbf{i}$ and $\mathbf{j}$ are the unit vectors in the x- and y-directions, respectively. The dimensionless parameter $\mathcal{B}_0$ represents the reorientation of microorganisms under the influence of a gravitational moment relative to viscous resistance at the absence of a modulation. In Eq. (23), the parameters ζ and ξ in the x- and y-components of vector $\widehat{\mathbf{m}}'$ are

$$\begin{aligned} &\zeta = -(1-\alpha_0)\frac{\partial w'}{\partial x} + (1+\alpha_0)\frac{\partial u'}{\partial z},\\ &\xi = (1-\alpha_0)\frac{\partial w'}{\partial y} - (1+\alpha_0)\frac{\partial v'}{\partial z}. \end{aligned} \tag{24}$$

α_0 is the cell eccentricity which is given by the following equation [4, 7]:

$$\alpha_0 = \frac{r_{\max}^2 - r_{\min}^2}{r_{\max}^2 + r_{\min}^2}, \tag{25}$$

where $r_{\max}$ and $r_{\min}$ are the semi-major and semi-minor axes of the spheroidal cell.

If we substitute expressions (22) into Eqs. (9)–(12), the resulting equations for $\mathbf{V}', T', n'$ are:

$$\nabla\times\mathbf{V}' = 0 \tag{26}$$

$$\frac{1}{\mathcal{V}_a}\frac{\partial\mathbf{V}'}{\partial t} = -\nabla P' + D_a\nabla^2\mathbf{V}' - \mathbf{V}' - \mathbf{e}f_m\frac{\mathrm{R}_b}{L_b}n' + \mathbf{e}f_m\mathrm{Ra}T', \tag{27}$$

$$\frac{\partial T'}{\partial t} + \mathrm{Pe}_0\frac{\partial T'}{\partial z} + w'\frac{dT_b}{dz} + (\mathbf{V}'\nabla)T' = \nabla^2 T', \tag{28}$$

$$\begin{aligned} &\frac{1}{\widetilde{\sigma}}\frac{\partial n'}{\partial t} = -\nabla(\mathbf{V}'n') - w'\frac{dn_b}{dz} - \left(\mathrm{Pe}_0 + \frac{\mathrm{Pe}}{L_b}\right)\frac{\partial n'}{\partial z} +\\ &+ \frac{1}{L_b}\nabla^2 n' + \mathrm{PeG}_0 n_b(1 - \epsilon^2\delta\cos(\omega_g t))\Lambda, \end{aligned} \tag{29}$$

where

$$\Lambda = (1+\alpha_0)\frac{d^2 w'}{dz^2} + (1-\alpha_0)\left(\frac{\partial^2 w'}{\partial x^2} + \frac{\partial^2 w'}{\partial y^2}\right),$$

$\mathrm{G}_0 = D_m\mathcal{B}_0/h^2$ is a dimensionless orientation parameter in the absence of modulation [5].

In the two-dimensional flow model, we define the velocities using the stream function, denoted by ψ:

$$u' = \frac{\partial\psi}{\partial z}, \quad w' = -\frac{\partial\psi}{\partial x}. \tag{30}$$

After substituting (30) into Eqs. (27)–(29), we obtain the following dimensionless governing equations (without the primes):

$$\left(\frac{1}{\mathcal{V}_a}\frac{\partial}{\partial t} + 1 - D_a\nabla_\perp^2\right)\nabla_\perp^2\psi = f_m\frac{\mathrm{R}_b}{L_b}\frac{\partial n}{\partial x} - f_m\mathrm{Ra}\frac{\partial T}{\partial x}, \tag{31}$$

$$\mathrm{Pe}_0\frac{\partial T}{\partial z} - \frac{\partial\psi}{\partial x}\frac{dT_b}{dz} - \nabla_\perp^2 T = -\frac{\partial T}{\partial t} + \frac{\partial(\psi, T)}{\partial(x,z)}, \tag{32}$$

$$\begin{aligned} &\mathrm{PeG}_0(2 - f_m)n_0\widehat{\alpha}\frac{\partial\psi}{\partial x} - \frac{\partial\psi}{\partial x}\frac{dn_b}{dz} + \left(\mathrm{Pe}_0 + \frac{\mathrm{Pe}}{L_b}\right)\frac{\partial n}{\partial z} -\\ &- \frac{1}{L_b}\nabla_\perp^2 n = -\frac{1}{\widetilde{\sigma}}\frac{\partial n}{\partial t} + \frac{\partial(\psi, n)}{\partial(x,z)}, \end{aligned} \tag{33}$$

$$\nabla_\perp^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}, \ \widehat{\alpha} = \nabla_\perp^2 + \alpha_0\left(\frac{\partial^2}{\partial z^2} - \frac{\partial^2}{\partial x^2}\right).$$

To study time-periodic convective phenomena, we introduce two time scales: a fast scale (t_0) and a slow scale (τ). Then the time derivative in Eqs. (31)–(33) can be represented as [33]:

$$\frac{\partial}{\partial t} \to \frac{\partial}{\partial t_0} + \epsilon^2\frac{\partial}{\partial\tau}. \tag{34}$$

After substituting (34) into system (31)–(33), the resulting equations can be written in the form:

$$\left(\frac{1}{\mathcal{V}_a}\frac{\partial}{\partial t_0}+1-D_a\nabla_\perp^2\right)\nabla_\perp^2\psi-f_m\frac{\mathrm{R}_b}{L_b}\frac{\partial n}{\partial x}+f_m\mathrm{Ra}\frac{\partial T}{\partial x}=$$
$$=-\frac{\epsilon^2}{\mathcal{V}_a}\frac{\partial}{\partial\tau}\nabla_\perp^2\psi, \tag{35}$$

$$\left(1\frac{\partial}{\partial t_0}+\mathrm{Pe}_0\frac{\partial}{\partial z}-\nabla_\perp^2\right)T-\frac{\partial\psi}{\partial x}\frac{dT_b}{dz}=$$
$$=-\epsilon^2\frac{\partial T}{\partial\tau}+\frac{\partial(\psi,T)}{\partial(x,z)}, \tag{36}$$

$$\mathrm{PeG}_0(2-f_m)n_0\widehat{\alpha}\frac{\partial\psi}{\partial x}-\frac{\partial\psi}{\partial x}\frac{dn_b}{dz}+\left(\mathrm{Pe}_0+\frac{\mathrm{Pe}}{L_b}\right)\frac{\partial n}{\partial z}-$$
$$-\frac{1}{L_b}\nabla_\perp^2 n+\frac{1}{\widetilde{\sigma}}\frac{\partial n}{\partial t_0}=-\frac{\epsilon^2}{\widetilde{\sigma}}\frac{\partial n}{\partial\tau}+\frac{\partial(\psi,n)}{\partial(x,z)}, \tag{37}$$

The above system of equations (35)–(37) is solved using stress-free, isothermal, and iso-concentration boundary conditions:

$$\psi=\nabla_\perp^2\psi=T=n=0 \quad \text{on} \quad z=0 \quad \text{and} \quad z=1. \tag{38}$$

3. The Finite-Amplitude Equation

Let us begin to study the weakly nonlinear regime of oscillatory convection by introducing a small perturbation parameter ϵ, which shows the deviation from the critical state of the convection occurrence. Then all variables in Eqs. (35)–(36) can be represented as a series in powers of ϵ as

$$\begin{aligned}&\mathrm{Ra}=\mathrm{Ra}_c+\epsilon^2\mathrm{Ra}_2+\epsilon^4\mathrm{Ra}_4+...,\\&\mathbf{X}=\epsilon\mathbf{X}_1+\epsilon^2\mathbf{X}_2+\epsilon^3\mathbf{X}_3+...,\end{aligned} \tag{39}$$

where $\mathbf{X}=(\psi,T,n)$, Ra_c represents the critical Rayleigh number at which the convection initiates in the absence of a gravity modulation. Substituting (39) into equations (35)–(36), we will solve it for different orders in ϵ.

3.1. First-order system

At the lowest order, we have

$$\left(\frac{1}{\mathcal{V}_a}\frac{\partial}{\partial t_0}+1-D_a\nabla_\perp^2\right)\nabla_\perp^2\psi_1-\frac{\mathrm{R}_b}{L_b}\frac{\partial n_1}{\partial x}+\mathrm{Ra}_c\frac{\partial T_1}{\partial x}=0, \tag{40}$$

$$\left(\frac{\partial}{\partial t_0}+\mathrm{Pe}_0\frac{\partial}{\partial z}-\nabla_\perp^2\right)T_1-\frac{\partial\psi_1}{\partial x}\frac{dT_b}{dz}=0, \tag{41}$$

$$\mathrm{PeG}_0n_b\widehat{\alpha}\frac{\partial\psi_1}{\partial x}-\frac{\partial\psi_1}{\partial x}\frac{dn_b}{dz}+\left(\mathrm{Pe}_0+\frac{\mathrm{Pe}}{L_b}\right)\frac{\partial n_1}{\partial z}-$$
$$-\frac{1}{L_b}\nabla_\perp^2 n_1+\frac{1}{\widetilde{\sigma}}\frac{\partial n_1}{\partial t_0}=0. \tag{42}$$

Equations (40)–(42) describe the linear regime of bioconvection. To check the possibility of the existence of oscillating modes in the system, we will look for solutions to Eqs. (40)–(42) in the following form:

$$\begin{aligned}&\psi_1=\left(A(\tau)e^{i\omega t_0}+A^*(\tau)e^{-i\omega t_0}\right)\sin k_cx\sin\pi z,\\&T_1=\left(\Theta(\tau)e^{i\omega t_0}+\Theta^*(\tau)e^{-i\omega t_0}\right)\cos k_cx\sin\pi z,\\&n_1=\left(\Pi(\tau)e^{i\omega t_0}+\Pi^*(\tau)e^{-i\omega t_0}\right)\cos k_cx\sin\pi z,\end{aligned} \tag{43}$$

where $A^*(\tau),\Theta^*(\tau),\Pi^*(\tau)$ are complex conjugate amplitudes. Solutions of the general form (43) are expressed in terms of unknown slow-time functions and the satisfy boundary conditions (38). Substituting solutions (43) into Eqs. (40)–(42), we obtain the following relations between the oscillation amplitudes:

$$\Theta(\tau)=-\frac{k\theta_0A(\tau)}{i\omega+a^2},\quad \Pi(\tau)=\frac{kL_b\widetilde{\sigma}\widetilde{G}A(\tau)}{i\omega L_b+a^2\widetilde{\sigma}}, \tag{44}$$

where

$$\theta_0=\frac{4\pi^2}{4\pi^2+\mathrm{Pe}_0^2},\ \widetilde{G}=\frac{4\pi^2\langle n\rangle}{4\pi^2+(\mathrm{Pe}+\mathrm{Pe}_0L_b)^2}\times$$
$$\times\left(\mathrm{Pe}G_0(a^2+\alpha_0(\pi^2-k^2))+\mathrm{Pe}+\mathrm{Pe}_0L_b\right),\ a^2=k_c^2+\pi^2.$$

The amplitude $A(\tau)$ remains unknown for now. From the Eq. (40), it is easy to find an expression for the critical Rayleigh number Ra_c:

$$\mathrm{Ra}_c=\frac{(i\omega+a^2)a^2}{k^2\theta_0}\left(1+D_aa^2+\frac{i\omega}{\mathcal{V}_a}\right)-$$
$$-\frac{R_b\widetilde{G}(i\omega+a^2)}{L_b(i\omega L_b+a^2\widetilde{\sigma})\theta_0}. \tag{45}$$

In the cases of stationary convection with $\omega=0$ and small Peclet numbers $\mathrm{Pe}\to0,\mathrm{Pe}_0\to0$, the expression in (45) matches the result of paper [7]. The expression for Ra_c can be represented as the sum of the real and imaginary parts:

$$\mathrm{Ra}_c=\mathrm{Ra}^{(r)}+i\omega\mathrm{Ra}^{(i)}.$$

In the case of an oscillatory mode of convection $\omega \neq$ $\neq 0$ ($\mathrm{Ra}^{(i)} = 0$), the critical Rayleigh–Darcy number for the oscillatory instability is found from (45) in the following form:

$$\mathrm{Ra}_{osc} = \frac{a^2}{k^2\theta_0}\left(a^2(1 + D_a a^2) - \frac{\omega^2}{\mathcal{V}_a}\right) - \\ - \frac{R_b\widetilde{G}}{\theta_0}\frac{\left(a^4 + \omega^2\frac{L_b}{\widetilde{\sigma}}\right)}{\left(a^4 + \omega^2\frac{L_b^2}{\widetilde{\sigma}^2}\right)}. \tag{46}$$

The frequency of oscillations must satisfy the following equation: пїЅ

$$\omega^2 = \frac{\widetilde{\sigma}(\widetilde{\sigma} - L_b)}{L_b^2}\frac{k^2\mathcal{V}_a R_b\widetilde{G}}{\mathcal{V}_a(1 + D_a a^2) + a^2} - a^4\frac{\widetilde{\sigma}^2}{L_b^2}. \tag{47}$$

It is possible to have an oscillatory instability, if the inequality $\widetilde{\sigma} > L_b$ is met. However, for the parameters of our problem ($\widetilde{\sigma} \approx 1.4$, $\alpha_m = 0.143\times10^{-6}$ m^2/s, $D_m = 5 \times 10^{-8}$ m^2/s), this inequality does not hold. Therefore, we will move on to the study of the stationary convection regime.

3.2. Second-order system

In this order, nonlinear effects begin to appear on the right-hand sides of the Eqs. (36)–(37), which represent the interaction between the fluid movement, temperature, and diffusion of microorganisms. A system of equations in this order can be written as follows:

$$\left(1 - D_a\nabla_\perp^2\right)\nabla_\perp^2\psi_2 - \frac{\mathrm{R}_b}{L_b}\frac{\partial n_2}{\partial x} + \mathrm{Ra}_c\frac{\partial T_2}{\partial x} = 0, \tag{48}$$

$$\left(\mathrm{Pe}_0\frac{\partial}{\partial z} - \nabla_\perp^2\right)T_2 - \frac{\partial\psi_2}{\partial x}\frac{dT_b}{dz} = \frac{\partial\psi_1}{\partial x}\frac{\partial T_1}{\partial z} - \\ - \frac{\partial T_1}{\partial x}\frac{\partial\psi_1}{\partial z}, \tag{49}$$

$$\mathrm{PeG}_0 n_b\widehat{\alpha}\frac{\partial\psi_2}{\partial x} - \frac{\partial\psi_2}{\partial x}\frac{dn_b}{dz} + \left(\mathrm{Pe}_0 + \frac{\mathrm{Pe}}{L_b}\right)\frac{\partial n_2}{\partial z} - \\ - \frac{1}{L_b}\nabla_\perp^2 n_2 = \frac{\partial\psi_1}{\partial x}\frac{\partial n_1}{\partial z} - \frac{\partial n_1}{\partial x}\frac{\partial\psi_1}{\partial z}. \tag{50}$$

The second-order solutions, accounting for the boundary conditions (38), can be expressed using the first-order solutions.

$$\psi_2 = 0, \quad T_2 = -\frac{k_c^2\theta_0}{8\pi a^2}A^2(\tau)\sin 2\pi z,$$

$$n_2 = \frac{k_c^2\widetilde{G}L_b^2}{8\pi a^2}A^2(\tau)\sin 2\pi z. \tag{51}$$

The horizontally averaged Nusselt number $\mathrm{Nu}(\tau)$ for the stationary mode of convection can be evaluated by the following expression:

$$\mathrm{Nu}(\tau) = 1 + \frac{\left[\frac{k_c}{2\pi}\int\limits_0^{2\pi/k_c}\left(\frac{\partial T_2}{\partial z}\right)dx\right]_{z=0}}{\left[\frac{k_c}{2\pi}\int\limits_0^{2\pi/k_c}\left(\frac{dT_b}{dz}\right)dx\right]_{z=0}} = \\ = 1 + \frac{k_c^2\pi^2\left(e^{\mathrm{Pe}_0} - 1\right)}{a^2(4\pi^2 + \mathrm{Pe}_0^2)\mathrm{Pe}_0}A^2(\tau). \tag{52}$$

By analogy with (52), we find a quantitative characteristic of the mass transfer (Sherwood number Sh) of the concentration of microorganisms:

$$\mathrm{Sh}(\tau) = 1 + \frac{k_c^2\pi^2\left(e^{(\mathrm{Pe}+\mathrm{Pe}_0 L_b)} - 1\right)L_b^2A^2(\tau)}{a^2(\mathrm{Pe} + \mathrm{Pe}_0L_b)^2(4\pi^2 + (\mathrm{Pe} + \mathrm{Pe}_0L_b)^2)} \times \\ \times\left(\mathrm{Pe}G_0(a^2 + \alpha_0(\pi^2 - k^2)) + \mathrm{Pe} + \mathrm{Pe}_0L_b\right). \tag{53}$$

Once the expression for the amplitude $A(\tau)$ is derived, we will conduct evaluations of the heat and mass transfer quotients. It i's worth noting that the impact of the gravity modulation is significant only at the third order in ϵ, as can be seen from the asymptotic expansion in Eq. (39).

3.3. Third-order system

At the third order, we have

$$\left(1 - D_a\nabla_\perp^2\right)\nabla_\perp^2\psi_3 - \frac{\mathrm{R}_b}{L_b}\frac{\partial n_3}{\partial x} + \mathrm{Ra}_c\frac{\partial T_3}{\partial x} = N_{31}, \tag{54}$$

$$\left(\mathrm{Pe}_0\frac{\partial}{\partial z} - \nabla_\perp^2\right)T_3 - \frac{\partial\psi_3}{\partial x}\frac{dT_b}{dz} = N_{32}, \tag{55}$$

$$\mathrm{PeG}_0 n_b\widehat{\alpha}\frac{\partial\psi_3}{\partial x} - \frac{\partial\psi_3}{\partial x}\frac{dn_b}{dz} + \left(\mathrm{Pe}_0 + \frac{\mathrm{Pe}}{L_b}\right)\frac{\partial n_3}{\partial z} - \\ - \frac{1}{L_b}\nabla_\perp^2 n_3 = N_{33}, \tag{56}$$

where

$$N_{31} = \left(\frac{a^2}{\mathcal{V}_a}\frac{\partial A(\tau)}{\partial\tau} - \mathrm{Ra}_c\frac{k_c^2\theta_0}{a^2}A(\tau)\delta\cos(\Omega\tau) - \\ - \mathrm{Ra}_2\frac{k_c^2\theta_0}{a^2}A(\tau) - \mathrm{R}_b\frac{k_c^2\widetilde{G}}{a^2}A(\tau)\delta\cos(\Omega\tau)\right) \times \\ \times \sin k_cx\sin\pi z,$$

$$N_{32} = \left(\frac{k_c\theta_0}{a^2}\frac{\partial A(\tau)}{\partial\tau} - \frac{k_c^3\theta_0 A(\tau)^3}{4a^2}\cos 2\pi z\right)\times$$
$$\times\ \cos k_c x \sin\pi z,$$
$$N_{33} = \left(-\frac{k_c\widetilde{G}L_b}{\widetilde{\sigma}a^2}\frac{\partial A(\tau)}{\partial\tau} + \frac{k_c^3\widetilde{G}L_b^2}{4a^2}A^3(\tau)\cos 2\pi z - \right.$$
$$\left. - k_c \mathrm{PeG}_0 n_b(a^2+\alpha_0(\pi^2-k^2))A(\tau)\delta\cos(\Omega\tau)\right)\times$$
$$\times\ \cos k_c x\sin\pi z.$$

We use the Fredholm alternative [34] to ensure the existence of a third-order solution, and obtain the Ginzburg–Landau equation for the stationary mode of convection with time-periodic coefficients in the following form:

$$K_1\frac{\partial A}{\partial\tau} - K_2(\tau)A + K_3A^3 = 0, \tag{57}$$

where K_1, K_2, K_3 are coefficients:

$$\begin{aligned} K_1 &= \frac{a^2}{\mathcal{V}_a} + \mathrm{Ra}_c\frac{k_c^2\theta_0}{a^4} + \mathrm{R}_b\frac{k_c^2L_b}{a^4\widetilde{\sigma}}\widetilde{G},\\ K_2(\tau) &= \frac{k_c^2\theta_0}{a^2}\mathrm{Ra}_c\left(\frac{\mathrm{Ra}_2}{\mathrm{Ra}_c}+\delta\cos(\Omega\tau)\right) + \\ &+ \mathrm{R}_b\frac{4\pi^2k_c^2\langle n\rangle(\mathrm{Pe}+\mathrm{Pe}_0L_b)}{a^2(4\pi^2+(\mathrm{Pe}+\mathrm{Pe}_0L_b)^2)}\delta\cos(\Omega\tau),\\ K_3 &= \frac{k_c^4}{8a^4}(\mathrm{Ra}_c\theta_0 + \mathrm{R}_bL_b^2\widetilde{G}). \end{aligned} \tag{58}$$

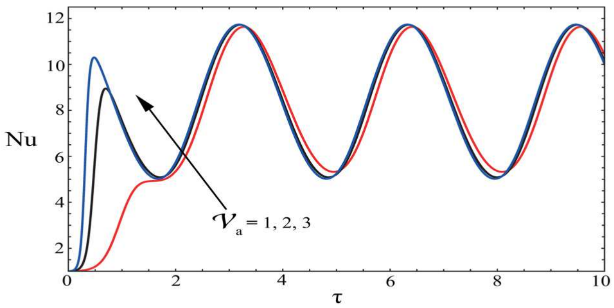

Fig. 2. Dependence of the Nusselt number Nu on the time τ for $\mathcal{V}_a$ variations

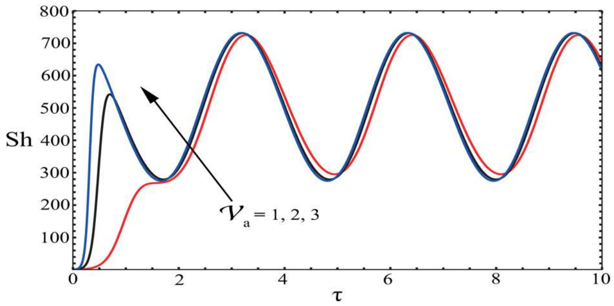

Fig. 3. Dependence of the Sherwood number Sh on the time τ for $\mathcal{V}_a$ variations

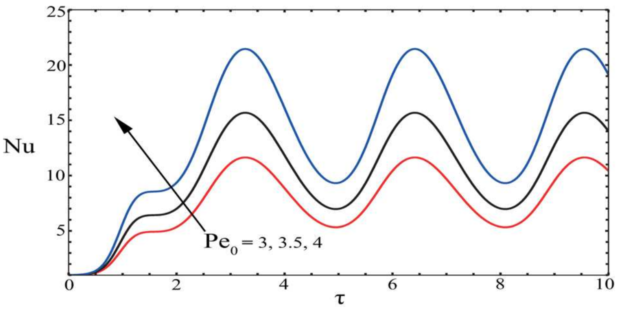

Fig. 4. Dependence of the Nusselt number Nu on the time τ for positive Pe_0 variations

When there is no throughflow and a low movement speed of microorganisms, the GL equation! (57) was obtained by Kopp and Yanovsky [32]. The expressed in Eq. (57) is non-autonomous, which makes obtaining an analytic solution challenging. Therefore, we have used Mathematica's built-in function NDSolve to solve it numerically. We have assumed that Ra_2 is approximately equal to Ra_c, as our focus is on the nonlinearity near the critical state of convection. In the weakly nonlinear theory of convective instability, the relative deviation ϵ^2 of the Rayleigh number Ra from its critical value Ra_c serves as a small expansion parameter. The equation is solved with the initial condition $A(0) = A_0$, where A_0 represents the chosen initial amplitude of convection. For the unmodulated case, Eq. (57) can be solved analytically, and its solution is as follows:

$$\widetilde{A}(\tau) = \frac{A_0}{\sqrt{\frac{K_3}{K_2}A_0^2 + \left(1 - A_0^2\frac{K_3}{K_2}\right)\exp\left(-\frac{2\tau K_2}{K_1}\right)}}, \tag{59}$$

where $\widetilde{A}(\tau)$ represents the convection amplitude in the unmodulated case, and K_1 and K_3 have the same expressions as given in (58), while $K_2 = k_c^2\theta_0\mathrm{Ra}_2/a^2$.

4. Results and Discussion

In order to analyze the non-autonomous GL equation (57), we conducted a numerical analysis in the Mathematica software environment. The initial amplitude was set to $A(0) = 0.5$, and the gravitational modulation strength was set to (δ, Ω). The results of the numerical calculations are depicted in Figures 2–9, showing graphical representations of the heat Nu

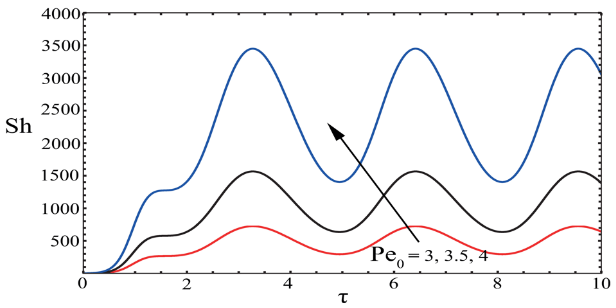

Fig. 5. Dependence of the Sherwood number Sh on the time τ for positive Pe_0 variations

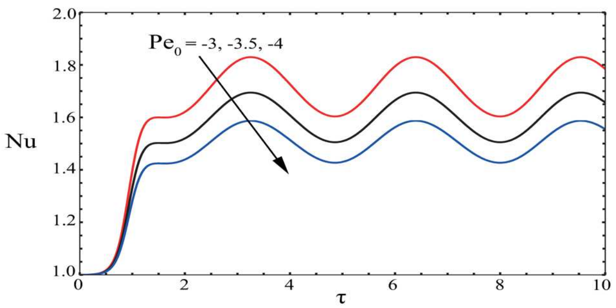

Fig. 6. Dependence of the Nusselt number Nu on the time τ for negative Pe_0 variations

and mass Sh transfer as a function of the dimensionless time parameter τ. By varying the parameters of the mixed fluid, such as $\mathcal{V}_a, \mathrm{Pe}_0$, and Pe, as well as the modulation parameters (δ, Ω), we studied their impact on the heat and mass transfer characteristics. We assume that the fluid's viscosity is not high enough to hinder the exploration of convection and heat transfer phenomena. Due to the moderate Vadasz number $\mathcal{V}_a$, the thermal diffusivity and kinematic viscosity of the fluid are balanced in a high-porosity medium. We will study the behavior of the system under small perturbations, where the influence of gravity field modulation is not overly dominant. Assuming a low frequency of the gravity modulation, we can conclude that lower frequencies maximize the impact of the gravity modulation on the system's behavior. Another important subject is vertical throughflow into consideration for either increasing or decreasing heat and mass transfer.

We start with research to investigate the influence of Vadasz number $\mathcal{V}_a$ variations on heat and mass transfer, while keeping other parameters constant at $\mathrm{Pe}_0 = 3, \mathrm{Pe} = 1, \Omega = 2, \delta = 0.3$. Observing Figs 2 and 3, it is evident that an increase in the Vadasz number $\mathcal{V}_a$ leads to a temporary surge in heat and concentration transfer. This suggests that systems with higher Vadasz numbers tend to exhibit more efficient heat and mass transfer characteristics. Therefore, the Vadasz number $(\mathcal{V}_a)$ is a crucial factor in enhancing the heat and concentration transport, particularly at low time values. It is worth noting tha, since the Vadasz number is proportional to the Prandtl

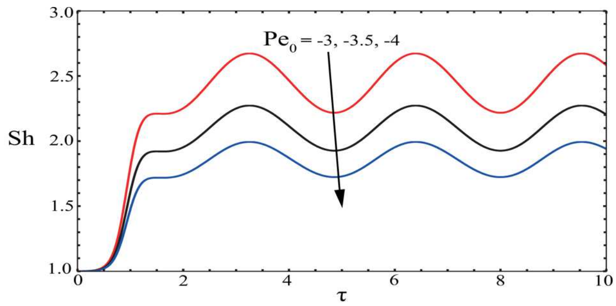

Fig. 7. Dependence of the Sherwood number Sh on the time τ for negative Pe_0 variations

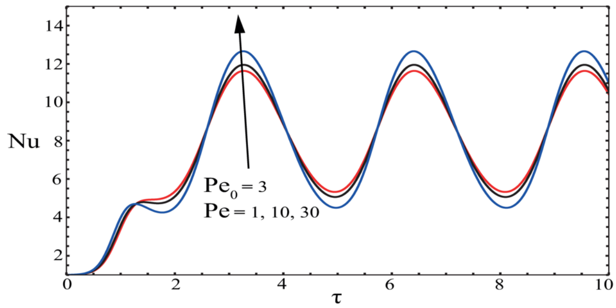

Fig. 8. The variation of the Nusselt number (Nu) with respect to time (τ) is examined for different bioconvective Peclet numbers (Pe) while keeping the Peclet number at a positive value $(\mathrm{Pe}_0 = 3)$

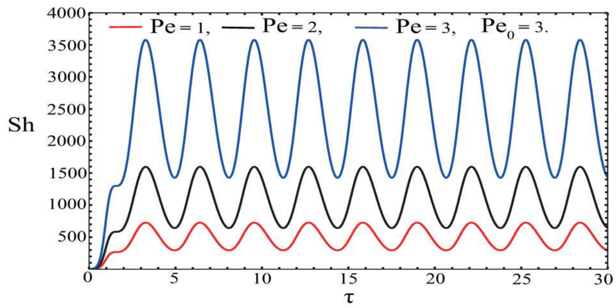

Fig. 9. The variation of the Sherwood number (Sh) with respect to the time (τ) is examined for different bioconvective Peclet numbers (Pe) while keeping the Peclet number at a positive value $(\mathrm{Pe}_0 = 3)$

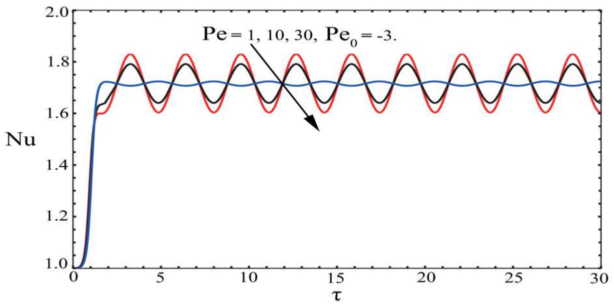

Fig. 10. The variation of the Nusselt number (Nu) with respect to the time (τ) is examined for different bioconvective Peclet numbers (Pe) while keeping the Peclet number at a negative value ($\mathrm{Pe}_0 = -3$)

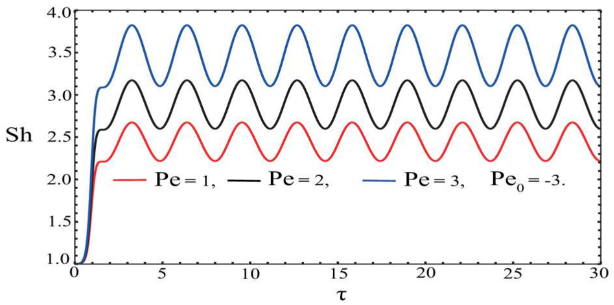

Fig. 11. The variation of the Sherwood number (Sh) with respect to time (τ) is examined for different bioconvective Peclet numbers (Pe) while keeping the Peclet number at a negative value ($\mathrm{Pe}_0 = -3$)

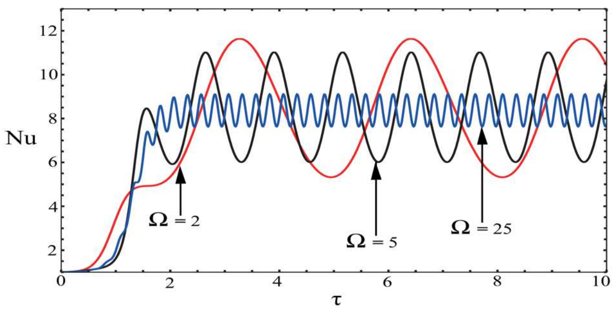

Fig. 12. Dependence of the Nusselt number Nu on the time τ for Ω variations

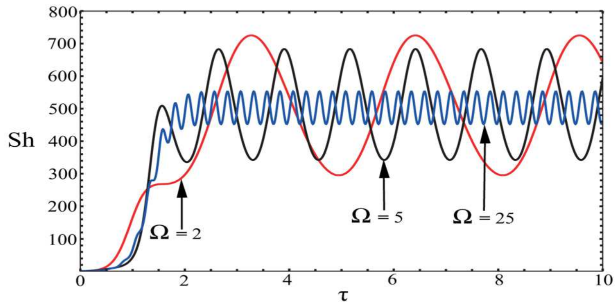

Fig. 13. Dependence of the Sherwood number (Sh) on the time τ for Ω variations.

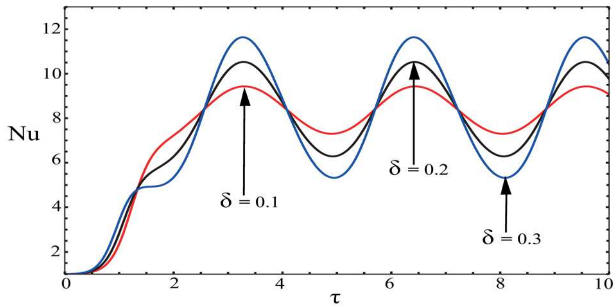

Fig. 14. Dependence of the Nusselt number Nu on the time τ for δ variations

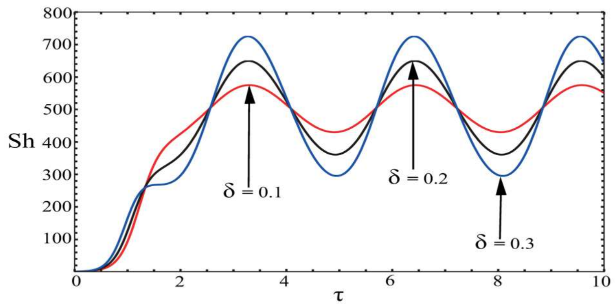

Fig. 15. Dependence of the Sherwood number Sh on the time τ for δ variations

number, a similar trend has been reported in previous studies by Kiran *et al.* [26, 30], as well as Kopp and Yanovsky [32]. Figures 4, 5 and 6, 7 show the the effect of the values of a throughflow on the heat and mass transfer in the bio-thermal convection. As can be seen from Figs 4, 5, with positive values of the Peclet number $\mathrm{Pe}_0 > 0$, the Nusselt and Sherwood numbers increase. At positive Peclet numbers, the throughflow is directed against the gravity, as is the direction of movement of microorganisms. In this case, the convective flows increase, leading to an increase in the heat and mass transfer. For negative Peclet numbers $\mathrm{Pe}_0 < 0$, the throughflow is directed along the direction of the gravity. In this case, as shown in Figs 6, 7, with an increase in the absolute values of the Peclet number, the Nusselt and Sherwood numbers decrease. This behavior can be explained by the fact that the throughflow directed

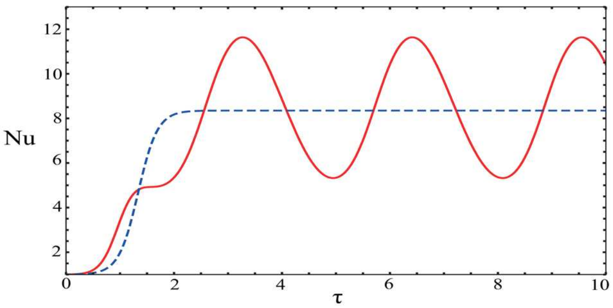

Fig. 16. Variations of the Nusselt numbers $\mathrm{Nu}(\tau)$ in the absence of $\delta = 0$ (dashed line) and the presence of $\delta = 0.3, \Omega = 2$ (solid line) modulation of the gravity field

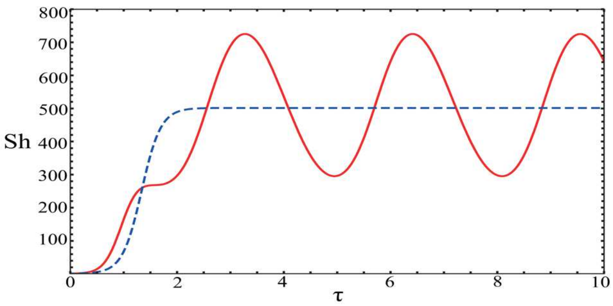

Fig. 17. Variations of the Sherwood number Sh in the absence of $\delta = 0$ (dashed line) and the presence of $\delta = 0.3, \Omega = 2$ (solid line) modulation of the gravity field

along the force of gravity prevents the movement of microorganisms and, as a result, leads to a decrease in the heat and mass transfer.

Let us now clarify the question of how a change in the swimming speed of microorganisms (increase in the bioconvective Peclet number Pe) affects the heat and mass transfer in the system in the presence of a throughflow. Figures 8 and 9 illustrate that as the bioconvective Peclet number increases Pe in the scenario of throughflow opposing the gravity $\mathrm{Pe}_0 = 3$, there is a notable augmentation in both the heat and mass transfer. In contrast, when the throughflow aligns with the gravity direction (refer to Figs 10, 11), we observe a dampening effect on the heat transfer modulation, coupled with an increase in mass transfer.

In Figs 12, 13, we illustrate the influence of the modulation frequency (Ω). Specifically, at lower modulation frequencies, corresponding to the low-frequency case ($\Omega = 2$), a higher heat and mass transfer is achieved compared to the higher vibrational rates ($\Omega = 5$ and $\Omega = 25$).

Figures 14, 15 demonstrate the effect of the modulation amplitude (δ) on the heat and mass transfer within the system. The study encompasses an interval of δ values from 0.1 to 0.3, carefully selected to augment heat transfer. Importantly, these experiments are conducted with the fluid devoid of solid particles. It is worth emphasizing that the modulation frequency (Ω) exerts a diminishing effect on the heat and mass transfer. This observation aligns with previous findings by Gresho and Sani [24]m as well as Kopp *et al.* [35] in the context of ordinary fluids. These results underscore the importance of employing a low-frequency g-jitter to optimize the transport process and enhance the heat transfer in the system.

Equation (59) provides an analytic expression for the amplitude of convection in the unmodulated case. By utilizing this amplitude, we present a comparison between the modulated system and the unmodulated one in Figs 16, 17. The graphs illustrate a sudden increase in $\mathrm{Nu}(\tau)$ and $\mathrm{Sh}(\tau)$ for low values of the time parameter τ, stabilizing for higher values of τ. However, in the case of the modulated system, both $\mathrm{Nu}(\tau)$ and $\mathrm{Sh}(\tau)$ exhibit oscillatory behavior.

The study of gravitational modulation and vertical throughflow in highly porous media is important for the external control over the heat and mass transfer.

5. Conclusions

We have employed the Darcy–Brinkman model to formulate a weakly nonlinear theory investigating the combined influences of the gravity modulation and the throughflow on the bio-thermal convection within a porous medium saturated with a Newtonian fluid containing gyrotactic microorganisms. Our analysis is grounded on perturbation theory, specifically focusing on the small supercriticality parameter ϵ, representing a deviation from the critical Rayleigh number. Within our analysis, we consider the modulated gravity field's small amplitude as second order in ϵ. We ascertain that, at the first order in ϵ, the parametric modulation does not significantly affect the convection development, aligning with predictions from linear theory. Moreover, for the given problem parameters, we established that the heat transfer predominantly occurs through stationary convection without oscillatory movements. However, delving into the third order of ϵ, we derive a nonlinear Ginzburg–Landau equation with a time-periodic coefficient. Through a numerical analysis, we extract several insights from these results. These conclusions provide valuable understanding regarding the influence of conclusio conclusio the gravity modulation on the bio-thermal convection in porous media, particularly considering the effect of a vertical throughflow. Based on our findings, we summarize the following key conclusions:

1. An increase in the values of the parameters $\mathcal{V}_a$ leads to a short-term surge in the heat and mass transfer.

2. The heat and mass transfer intensify, when there is a throughflow opposing the gravity $\mathrm{Pe}_0 > 0$, whereas they diminish, when the throughflow coincides with the gravitational direction $\mathrm{Pe}_0 < 0$.

3. Raising the modulation frequency Ω causes a reduction in the variations of the Nusselt numbers $\mathrm{Nu}(\tau)$, consequently suppressing the heat and mass transfer.

4. Augmenting the modulation amplitude δ improves the heat and mass transfer.

Thus, the use of the gravity field modulation contributes to the effective control over the process of bio-thermal convection containing gyrotactic microorganisms. In summary, it is pertinent to investigate the impact of manipulating factors beyond gravity that induce the movement in microorganisms. For instance, thermotactic microorganisms demonstrate a notable responsiveness to temperature adjustments, while magnetotactic microorganisms react to changes in magnetic fields. Additionally, microorganisms exhibiting chemotaxis are sensitive to alterations in chemical concentration gradients. All these challenges outlined will be thoroughly addressed in subsequent research endeavors.

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М.Й. Копп, В.В. Яновський

ВПЛИВ НАСКРІЗНОГО
ПОТОКУ І ГРАВІТАЦІЙНОЇ МОДУЛЯЦІЇ
НА СЛАБКОНЕЛІНІЙНУ БІОТЕРМІЧНУ
КОНВЕКЦІЮ У ПОРИСТОМУ ШАРІ СЕРЕДОВИЩА

У цьому дослідженні вивчається вплив періодично змінних гравітаційних полів та наскрізного потоку на біотермічну конвекцію Дарсі–Брінкмана в шарі пористого середовища, насиченого ньютонівською рідиною, що містить гіротактичні мікроорганізми. Дослідження включає аналіз двох типів течії: спрямованого проти напряму поля сили тяжіння та вздовж нього. Ми припускаємо, що амплітуда гравітаційної модуляції є невеликою та має другий порядок малості за безрозмірним параметром ϵ, який представляє надкритичний параметр числа Релея. Для слабконелінійної конвекції ми отримали рівняння Ґінзбурґа–Ландау (ҐЛ) з періодичним коефіцієнтом у третьому порядку за ϵ. Для аналізу тепломасоперенесення ми чисельно розв'язуємо рівняння ҐЛ. Числові результати показують, що вертикальний наскрізний потік при біотермічній конвекції має подвійну природу, дозволяючи як збільшувати, так і зменшувати тепло- та масоперенесення. Досліджено вплив змін числа Вадасза, числа Пекле, біоконвективного числа Пекле, частоти та амплітуди модуляції на тепло- та масоперенесення. Вплив цих параметрів зображено графічно, що показує, що вищі значення чисел Вадасза та Пекле, а також збільшена амплітуда модуляції позитивно впливають на тепло- та масоперенос. Окрім того, порівняльний аналіз модульованих та немодульованих систем показує суттєвий вплив модуляції на стійкість систем.

Ключові слова: біотеплова конвекція, гравітаційна модуляція, наскрізний потік, гіротактичний мікроорганізм, амплітудне рівняння Ґінзбурґа–Ландау.