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MAGNETOGENESIS IN NON-LOCAL MODELS DURING INFLATION

The generation of magnetic fields during the inflation in an electromagnetic model with a non-local form factor in Maxwell's action is studied. The equations of motion for the electromagnetic field are derived and solved. It is found that the conformal symmetry breaking due to the non-local form factor does not lead to the generation of magnetic fields during the inflation in the absence of an interaction with the inflaton field. If such a coupling takes place, then the presence of the form factor inhibits the generation of primordial magnetic fields compared to the case where the non-local form factor is absent.

Keywords: magnetogenesis, non-local models.

1. Introduction

The quest for quantum gravity is one of the driving forces behind research in the modern fundamental physics. It is well known that the general relativity is a non-renormalizable theory. Non-local theories of gravity provide an attractive possibility to regularize UV divergences and formulate a consistent theory. It is commonly believed that such a theory should resolve the singularities of black hole solutions in the general theory of relativity and shed light on the beginning and initial conditions of the Big Bang [1–5].

The need for non-local theories to regularize high-energy divergences arises from the unitarity problem encountered in regularizations with a finite number of higher order derivatives. Indeed, although the addition of higher derivatives to the quantum field action regularizes UV divergences, these derivatives produce ghost states connected with the Ostrogradskii instability [6] endangering unitarity. Non-local form factors with entire functions of the d'Alembertian $\Box = g^{\mu\nu}\nabla_\mu\nabla_\nu$ avoid this problem, because such models with infinite number of derivatives do not produce new poles in the propagators of fields and, therefore, do not generate new physical degrees of freedom. It is noticeable also that vertices of the exponential form $e^{\Box}$, which is an entire function, appear in the string field theory [7]. The corresponding non-local gravity theories were studied in [8, 9].

It is worth adding that the non-locality in quantum field models appears not only as a means to regularize UV divergences, but also in the derivation of effective field theories accounting for the quantum corrections of heavy particles. The same was applied to the quantum matter radiative corrections in semiclassical gravity (see, e.g., [10]). It was suggested in [11] that the vacuum polarization effects during the inflation could be relevant for the generation of cosmological-scale magnetic fields.

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Recently, magnetic fields with an extremely large coherence length measured in megaparsecs were detected in cosmic voids through the gamma-ray observations of distant blazars [12–16]. Such an extremely large coherence length suggests that these fields have a cosmological origin. The inflation can easily provide such a large coherence length for generated magnetic fields. However, the conformal symmetry of Maxwell's action should be broken, otherwise, fluctuations of the electromagnetic field cannot be enhanced in the conformally flat Friedmann–Lemaitre–Robertson–Walker (FLRW) background [17]. In inflationary magnetogenesis studies, this breaking is usually taken in the form of the kinetic or axion coupling of the electromagnetic field with the inflaton field [18–21]. Since non-local theories introduce an additional dimensional parameter, they necessarily break the conformal symmetry of Maxwell's action as well. Certainly, it would be very interesting, if this breaking is sufficient to generate a magnetic field of an appropriate strength during the inflation without the need for a kinetic or axion coupling. This question provides the main motivation for the study in the present paper.

The paper is organized as follows. The magnetogenesis in a non-local electromagnetic model is considered in Sec. 2. The obtained results are summarized in Sec. 3. Throughout the paper, we use the units with $\hbar = c = 1$.

2. Non-local Electromagnetic Model

As we mentioned above, non-local models utilize form factors to ensure the convergence at high momenta. In our analysis, we consider the exponential form factor $e^{\Box/M^2}$, where M is the regularizing mass parameter whose natural value is the Planck mass M_p. Then the corresponding Maxwell's action takes the form

$$S = \int \sqrt{-g}\, d^4x \times \times \left[-\frac{1}{4} g^{\mu\alpha} g^{\nu\beta} F_{\mu\nu} e^{\Box/M^2} F_{\alpha\beta} + j_\mu A^\mu\right], \tag{1}$$

where $g_{\mu\nu}$ is the spacetime metric, $F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$ is the strength tensor of the electromagnetic field A_μ, and j_μ is the electric current of charged matter fields. Clearly, in view of the presence of the dimensional factor M in the form factor, this regularized Maxwell's action is not conformally symmetric which implies that electromagnetic fields, in principle, could be produced in an expanding FLRW background with scale factor $a(t)$, whose metric is given by $g_{\mu\nu} = \mathrm{diag}\,(1, -a^2, -a^2, -a^2)$, even in the absence of the interaction with charged matter fields.

Since M is assumed to be larger than any other parameter in the model, including the Hubble constant H, the role of the conformal symmetry breaking due to the term $e^{\Box/M^2}$ for the magnetogenesis could be determined by approximating this non-local form factor with its first two terms in the Taylor expansion $e^{\Box/M^2} \approx 1 + \Box/M^2$. Then we have

$$S = \int \sqrt{-g}\, d^4x \times \times \left[-\frac{1}{4} F_{\mu\nu} \left(1 + \frac{g^{\sigma\rho}\nabla_\sigma\nabla_\rho}{M^2}\right) F^{\mu\nu} + j_\mu A^\mu\right] \tag{2}$$

and obtain the following equations of motion for the electromagnetic field:

$$\nabla_\mu \left(1 + \frac{g^{\sigma\rho}\nabla_\sigma\nabla_\rho}{M^2}\right) F^{\mu\nu} + j^\nu = 0. \tag{3}$$

Further, it is convenient to rewrite the above equation as follows:

$$\left(1 + \frac{g^{\sigma\rho}\nabla_\sigma\nabla_\rho}{M^2}\right) \nabla_\mu F^{\mu\nu} - - \frac{1}{M^2} [g^{\sigma\rho}\nabla_\sigma\nabla_\rho, \nabla_\mu] F^{\mu\nu} + j^\nu = 0, \tag{4}$$

where $[g^{\sigma\rho}\nabla_\sigma\nabla_\rho, \nabla_\mu]$ is the commutator of the d'Alembertian and the covariant derivative.

Further,

$$-[\nabla_\sigma\nabla_\rho, \nabla_\mu] = \nabla_\sigma[\nabla_\mu, \nabla_\rho] + [\nabla_\mu, \nabla_\sigma]\nabla_\rho \tag{5}$$

and, for the commutator of a covariant derivatives, we have

$$[\nabla_\mu, \nabla_\sigma]\phi_{\mu_1\ldots\mu_k} = -\sum_{i=1}^{k} R^\lambda{}_{\mu_i\mu\sigma} \phi_{\mu_1\ldots\mu_{i-1}\lambda\mu_{i+1}\ldots\mu_k}.$$

Since

$$R_{\lambda\mu_i\mu\sigma} = H^2(g_{\lambda\mu} g_{\mu_i\sigma} - g_{\lambda\sigma} g_{\mu_i\mu})$$

for the de Sitter space [22], where H is related to the Hubble constant in an inflationary expanding Universe, we find, for the two commutators in Eq. (5),

$$g^{\sigma\rho}[\nabla_\mu, \nabla_\sigma]\nabla_\rho F^{\mu\nu} = -H^2 \nabla_\mu F^{\mu\nu},$$
$$g^{\sigma\rho}\nabla_\sigma[\nabla_\mu, \nabla_\rho] F^{\mu\nu} = 2H^2 \nabla_\mu F^{\mu\nu}.$$

Then Eq. (4) takes the form

$$\left(1+\frac{g^{\sigma\rho}\nabla_\sigma\nabla_\rho}{M^2}\right)\nabla_\mu F^{\mu\nu}+\frac{H^2}{M^2}\nabla_\mu F^{\mu\nu}+j^\nu=0 \quad (6)$$

or, equivalently,

$$\left(1+\frac{H^2}{M^2}\right)\nabla_\mu F^{\mu\nu}+\frac{\Box}{M^2}\nabla_\mu F^{\mu\nu}+j^\nu=0. \quad (7)$$

Defining

$$\nabla_\mu F^{\mu\nu}=\frac{1}{\sqrt{-g}}\frac{\partial(\sqrt{-g}F^{\mu\nu})}{\partial x^\mu}=f^\nu, \quad (8)$$

Eq. (7) implies the following equation for f^ν:

$$\left(1+\frac{H^2}{M^2}\right)f^\nu+\frac{\Box}{M^2}f^\nu+j^\nu=0. \quad (9)$$

Further,

$$\Box f^\nu=g^{\alpha\beta}\nabla_\alpha\nabla_\beta f^\nu=g^{\alpha\beta}\nabla_\alpha\left(\frac{\partial f^\nu}{\partial x^\beta}+\Gamma^\nu_{\rho\beta}f^\rho\right)=$$
$$=\frac{1}{\sqrt{-g}}\partial_\alpha(\sqrt{-g}g^{\alpha\beta}\partial_\beta f^\nu)+\frac{1}{\sqrt{-g}}\partial_\alpha(\sqrt{-g}g^{\alpha\beta}\Gamma^\nu_{\sigma\beta})f^\sigma+$$
$$+2\Gamma^\nu_{\sigma\alpha}g^{\alpha\beta}\partial_\beta f^\sigma+g^{\alpha\beta}\Gamma^\nu_{\sigma\alpha}\Gamma^\sigma_{\rho\beta}f^\rho, \quad (10)$$

where $\Gamma^\nu_{\alpha\beta}$ is the Christoffel symbol

$$\Gamma_{\nu,\alpha\beta}=\frac{1}{2}\left(\frac{\partial g_{\nu\alpha}}{\partial x^\beta}+\frac{\partial g_{\nu\beta}}{\partial x^\alpha}-\frac{\partial g_{\alpha\beta}}{\partial x^\nu}\right). \quad (11)$$

In the FLRW background and in the conformal time $\eta=\int^t dt'/a(t')$, the metric has the simple form $g_{\mu\nu}=a^2\eta_{\mu\nu}$, where $\eta_{\mu\nu}$ is the Minkowski spacetime metric $\eta_{\mu\nu}=\mathrm{diag}\,(1,-1,-1,-1)$. During the inflation, the scale factor is given by $a=-1/(H\eta)$, and the function f^ν defined in Eq. (8) equals

$$f^\nu=\frac{1}{\sqrt{-g}}\frac{\partial(\sqrt{-g}F^{\mu\nu})}{\partial x^\mu}=\frac{\eta^{\nu\sigma}}{\sqrt{-g}}\eta^{\alpha\beta}\partial_\alpha\partial_\beta A_\sigma,$$

where $\eta_{\alpha\beta}$ is the Minkowski spacetime metric, $A^\nu=\eta^{\nu\sigma}A_\sigma$ is the electromagnetic field potential, and the Coulomb gauge $A_0=0$ and $\mathrm{div}\mathbf{A}=0$ was used. Thus, we have

$$f^\nu=H^4\eta^4\eta^{\alpha\beta}\partial_\alpha\partial_\beta A^\nu=$$
$$=H^4\eta^4\begin{cases}0 & \text{for}\quad \nu=0,\\ \eta^{\alpha\beta}\partial_\alpha\partial_\beta A^i & \text{for}\quad \nu=i,\quad i=1,2,3.\end{cases}$$

By using Eqs. (10) and (11), one may check that $\Box f^\nu=0$ for $\nu=0$. Then, for the vanishing current $j^\nu=0$, the equations of motion (9) take the form

$$\left(1+\frac{H^2}{M^2}\right)f^i+\frac{\Box}{M^2}f^i=0, \quad (12)$$

where $f^i=H^4\eta^4\eta^{\alpha\beta}\partial_\alpha\partial_\beta A^i$. Further, by using Eq. (11) and $g_{\mu\nu}=a^2\eta_{\mu\nu}$, we find that Eq. (10) equals

$$\Box f^i=\frac{1}{a^4}\left(\frac{(\partial_\eta^2 a^2)}{2}+2(\partial_\eta a^2)\partial_\eta+a^2\eta^{\mu\nu}\partial_\mu\partial_\nu\right)\frac{D^i}{a^4},$$

where $D^i=\eta^{\alpha\beta}\partial_\alpha\partial_\beta A^i$. Then Eq. (12) gives

$$D^i+\frac{H^2}{M^2}\left(4\eta\partial_\eta D^i+\eta^2\eta^{\alpha\beta}\partial_\alpha\partial_\beta D^i\right)=0. \quad (13)$$

In the Coulomb gauge, only two transverse polarizations of the electromagnetic field remain. Then the electromagnetic vector-potential operator can be decomposed over the set of creation/annihilation operators as follows:

$$\hat{\mathbf{A}}(\eta,\mathbf{x})=\int\frac{d^3\mathbf{k}}{(2\pi)^{3/2}}\sum_{\lambda=\pm}\Big\{\boldsymbol{\epsilon}_\lambda(\mathbf{k})\hat{b}_{\lambda,\mathbf{k}}A_\lambda(\eta,\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}+$$
$$+\boldsymbol{\epsilon}^*_\lambda(\mathbf{k})\hat{b}^\dagger_{\lambda,\mathbf{k}}A^*_\lambda(\eta,\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x}}\Big\}, \quad (14)$$

where $\boldsymbol{\epsilon}_\lambda(\mathbf{k})$ is a set of two transverse circular polarization vectors, which satisfy the following conditions:

$$\begin{aligned}&\mathbf{k}\cdot\boldsymbol{\epsilon}_\lambda(\mathbf{k})=0,\quad \boldsymbol{\epsilon}^*_\lambda(\mathbf{k})=\boldsymbol{\epsilon}_{-\lambda}(\mathbf{k}),\\ &[i\mathbf{k}\times\boldsymbol{\epsilon}_\lambda(\mathbf{k})]=\lambda k\boldsymbol{\epsilon}_\lambda(\mathbf{k}).\end{aligned} \quad (15)$$

The creation/annihilation operators satisfy the standard commutation relations

$$[\hat{b}_{\lambda,\mathbf{k}},\,\hat{b}^\dagger_{\lambda',\mathbf{k}'}]=\delta_{\lambda\lambda'}\delta^{(3)}(\mathbf{k}-\mathbf{k}'). \quad (16)$$

Substituting decomposition (14) into Eq. (13), we obtain the equation governing the evolution of the mode function A_λ

$$(\partial_\eta^2+\mathbf{k}^2)A+\frac{H^2}{M^2}\left[4\eta\partial_\eta+\eta^2(\partial_\eta^2+\mathbf{k}^2)\right](\partial_\eta^2+\mathbf{k}^2)A=0, \quad (17)$$

where, for simplicity, we suppress index λ in the mode function. Making the change of the variable $z=k\eta$, we get

$$(\partial_z^2+1)A+\frac{H^2}{M^2}\left[4z\partial_z+z^2(\partial_z^2+1)\right](\partial_z^2+1)A=0. \quad (18)$$

The form factor $e^{\Box/M^2}$ is an entire function of the d'Alembertian. This ensures that the photon propagator has only two poles, i.e., there are only two electromagnetic modes at the given momentum. Clearly, there are two solutions to Eq. (18)

$$A_\pm = C_\pm e^{\pm iz},$$

which describe the usual free electromagnetic modes in the absence of any non-local form factor and conformal symmetry breaking in the free electromagnetic sector. Any other solution to Eq. (18) is spurious and is related to the expansion of the form factor into a Taylor series and retaining only its first two terms. Therefore, we conclude that the non-local form factor does not affect the free evolution of the electromagnetic field during the inflation. In other words, the form of Eq. (18) with two operator-valued multipliers (∂_z^2+1) acting on A means that the inclusion of the non-local form factor does not eliminate or modify the solutions for free electromagnetic fields in the expanding FLRW Universe.

It is interesting to determine how the non-local form factor affects the inflationary magnetogenesis in models, where the electromagnetic field interacts with the inflaton field φ. In the pseudoscalar inflation [20], this interaction in Maxwell's action (1) is described by the current of the following form:

$$j^\nu = \frac{I'(\varphi)}{2\sqrt{-g}}\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}\partial_\mu\varphi,$$

where $\varepsilon^{\mu\nu\alpha\beta}$ is the totally antisymmetric Levi-Civita tensor, and $I(\varphi)$ is a function of the coupling of the electromagnetic field with the inflaton field φ. Maxwell's action with the non-local form factor (1) yields the following equations of motion for the electromagnetic field:

$$\nabla_\mu e^{\Box/M^2}F^{\mu\nu} + j^\nu = 0. \tag{19}$$

It is difficult to find explicit solutions to the above equation. However, we could find qualitatively how the presence of a non-local form factor affects the magnetogenesis. The form factor $e^{\Box/M^2}$ equals approximately 1 for the eigenvalues of the d'Alembertian less than M^2 and rapidly increases for the eigenvalues larger than M^2. Therefore, to satisfy the above equation for the given j^ν, one would expect that $F^{\mu\nu}$ should be smaller in the case where the form factor $e^{\Box/M^2}$ is present. This means that the presence of a non-local form factor results in the suppressed magnetogenesis in inflationary models.

3. Conclusion

The generation of magnetic fields in a non-local electromagnetic model with the form factor in Maxwell's action in the form of the exponential of the d'Alembertian during the inflation is studied. Solving the equations of motion for the electromagnetic field, it is found that the conformal symmetry breaking induced by the non-local form factor does not lead to the generation of magnetic fields in an inflationary expanding Universe.

Adding the interaction with the inflaton field allows one to generate primordial magnetic fields. Comparing the magnetic field generation in the models of the pseudoscalar inflation without and with a non-local form factor shows that the presence of a form factor inhibits the generation of primordial magnetic fields.

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МАГНІТОГЕНЕЗ У НЕЛОКАЛЬНИХ МОДЕЛЯХ ПІД ЧАС ІНФЛЯЦІЇ

Досліджено генерацію магнітних полів під час інфляції в електромагнітній моделі з нелокальним формфактором у дії Максвелла. Отримано відповідні рівняння руху для електромагнітного поля та знайдено їх розв'язки. Виявлено, що порушення конформної симетрії завдяки нелокальному формфактору не призводить до генерації магнітних полів під час інфляції за відсутності взаємодії з полем інфлатону. Якщо ж така взаємодія має місце, то наявність формфактора пригнічує генерацію первинних магнітних полів порівняно з випадком, де нелокальний формфактор відсутній.

Ключові слова: магнітогенез, нелокальні моделі.