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PHYSICAL ASPECTS OF 2014 NOBEL PRIZE IN PHYSIOLOGY OR MEDICINE: 2. THE FIRST PRINCIPLE AND UNIVERSALITY CLASS FOR GRID CELLS IN THE BRAIN

The main purpose of this review article is to use the fluctuation theory of phase transitions for studying the process of the emergence of hexagonal grid cells in the brain (2014 Nobel Prize in Physiology or Medicine). Particular attention is paid to the application of the Feynman's classification of three stages of the study of natural phenomena for: 1) a brief description of the experimental stage of the discovery of the hexagonal structures of grid cells in human and animal brains; 2) the theoretical stage of research on the hexagon formation in the physical system of Benard cells, as well as the neurophysiological system of grid cells, discovered by Edward Mozer and May-Britt Mozer; 3) the most important stage, which allows one to formulate the first principle of the emergence of grid cells in the brain and, generally speaking, the first principle for the hexagon formation in different objects of inanimate and living nature. Our original theoretical findings are the following: (a) Polyakov's conformal invariance hypothesis is violated for a system of grid cells in the brain; (b) the system of grid cells in the brain belongs to the universality class including the 3D Ising model in a magnetic field, as well as a real classical liquid-vapor system;(c) to formulate the first principle for a reliable theoretical justification of the emergence of hexagonal grid cells in the brain, it is necessary to use the fluctuating part of Gibbs thermodynamic potential (the Ginzburg–Landau Hamiltonian) for a system with chemical (biochemical) reactions.

Keywords: first principle, universality class, grid cells, hexagons in human brain, conformal invariance hypothesis, Ginzburg–Landau Hamiltonian.

1. Introduction

This article is a continuation of the review publications [1, 2], which were devoted to the formulation of Richard Feynman's classification of three stages in the study of natural phenomena and the application of this classification to the hexagonal grid cells in the brain. Norwegian neurophysiologists Edward Moser and May-Britt Moser for the discovery of grid cells [3], as well as American and British neurophysiologist John O'Keefe for the discovery of place cells [4], were awarded the Nobel Prize in Physiology or Medicine in 2014 "for their discoveries of cells that constitute a positioning system in the brain" [5].

In our previous article [1], a number of examples of the application of "The Feynman's classification of the three stages of studying natural phenomena" were considered. As one of such examples, the following provisions related to the problem of the formation of grid cells in the brain were discussed: experimental studies of the emergence of hexagonal structures in the brain, generation and propagation of an action potential along the axon in the model by Alan Hodgkin and Andrew Huxley (the 1963 Nobel Prize in Physiology or Medicine) [6], physical parameters of the human brain and its medical applications in hyperthermia, and Edward Moser's idea about a certain

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analogy between grid cells in the brain and Aleksei Abrikosov's vortex lattice in superconductors of the 2^{nd} kind (the 2003 Nobel Prize in Physics) [7, 8].

Here, the main objectives of this review article will be aimed at using the fluctuation theory of phase transitions to formulate the first principle in the sense of Feynman's classification, which can explain the appearance of hexagonal structures of grid cells, as well as the class of universality for the system of grid cells in the brain. To achieve these goals, we shall investigate the synergetic similarity of two systems using the methods of theoretical physics: one physical system, representing the inanimate world, and the other medical system, representing the living world. More precisely speaking, hexagonal structures will be considered, at first glance, in completely different systems, namely: (*a*) the physical system of Benard cells in a viscous liquid with a vertical temperature gradient [9–15] and (*b*) the neurophysiological system of grid cells in the brain providing the spatial orientation of humans and animals [3–5, 16, 17]. We recall the established fact that the disruption in the functioning of the system of grid cells in the brain can be a genetic cause of Alzheimer's disease in its early stages [17].

The study of systems mentioned above will be based on the fluctuation models, which describe the process of formation of ordered structures in the form of hexagonal grid cells in the brain [2, 18–20]. The implementation of the well-known synergetic principle "order through fluctuations" [21] requires the creation and application of such fluctuation models that use the basic principles of theoretical and experimental studies of phase transitions and critical phenomena such as the fluctuation Ginzburg–Landau Hamiltonian [22] for a chemically reacting system [18-20], the renormalization-group approach in the theory of phase transitions [23, 24], the methods of field theory in statistical physics [25–27], the method of collective variables in the theory of phase transitions [28–31], the concept of universality classes [32–40], the hypotheses of scaling [32–38] and conformal [41, 42] invariance, and the Haken's theoretical results for hexagons in Benard cells [43, 44].

The structure of this review article is as follows. Section 2 discusses the Benard–Rayleigh effect in the sense of the first (experimental) stage of Feynman's classification (subsection 2.1), as well as the theoretical explanation for the emergence of Benard cells as the second (theoretical) stage of Feynman's classification (subsection 2.2). Subsection 2.3 contains an overview of the theoretical results by Hermann Haken, using the generalized Ginzburg–Landau equations and giving an explanation of the formation of hexagonal Benard cells, which, in our opinion, is close to the main third stage of Feynman's classification. Section 3 is devoted to obtaining the main (including original) results of this review article, namely: the explanation of the theoretical provisions and ideas of the theory of phase transitions and critical phenomena, leading to a consistent formulation of the first principle and the universality class for the emergence of the hexagonal grid cells in the brain. Section 3 includes the following subsections: the method of collective variables for studying the properties of the 3D Ising model in an external magnetic field (subsection 3.1); the conception of universality classes and clarifying the question about the universality class to which the system of grid cells in the brain should be attributed- (subsection 3.2), the Polyakov's conformal invariance hypothesis and its realization in the lattice-gas model (subsection 3.3); the violation of the Polyakov's conformal invariance hypothesis in the system of grid cells and the real classical liquid-vapor system (subsection 3.4); the fluctuation model that uses the Ginzburg–Landau Hamiltonian for the chemically reacting system of brain neurons and the formulation of the first principle according to the Feynman's classification that explains the formation of hexagonal grid cells in the brain, as well as, in a broader sense, the occurrence of hexagons in living and inanimate nature (subsection 3.5). The last Section 4 contains the main conclusions of our review article.

2. Benard–Rayleigh Effect and Haken's Explanation of Hexagons in Benard Cells

In our previous articles [1, 2], Feynman's classification was illustrated by a number of examples. The first of these examples is the phenomenon of light refraction at the boundary of two media, considered by Richard Feynman in his famous lecture course [45] describing the three stages of studying the light refraction phenomenon, namely: 1) the experimental stage associated with the experiments of Claudius Ptolemy on the refraction of light at the air-water boundary, 2) the theoretical stage associated with the establishment of the law of refraction by Willibrord Snellius, and 3) the final stage of the formulation of the first principle in the light refraction phenomenon, associated with the "Principle of the least time" of Pierre de Fermat.

The second example of the three stages of the cognition of natural phenomena, which was considered earlier in [1], concerned the laws of conservation of energy, momentum, and angular momentum. As is known, the formulation of the first principles for these laws at the third stage of the Feynman's classification is connected with the use of Emmy Noether's mathematical theorem on variational invariants [46], as well as with a direct proof of the connection between the conservation laws and the symmetry properties of space and time, established in the classic monograph by Lev Landau and Evgeniy Lifshits [47].

The purpose of this article is to use Feynman's classification to explain the three stages of cognition for the phenomenon of hexagonal grid cells in the human and animal brains. To achieve this goal, a synergetic analogy between Edvard Moser and May-Britt Moser grid cells [3–5] and Benard cells [9–15] will be used. Below in this section, all three stages of cognition in accordance with the classification of Richard Feynman will be considered for hexagonal Benard cells.

2.1. Experimental discovery of Benard cells as the first stage of Feynman's classification

Hexagons in Benard cells are observed in viscous fluids with temperature gradient and were first discovered experimentally by Henri Benard in 1900 [9], being the first stage of studying this phenomenon in the sense of Feynman's classification. The emergence of hexagonal Benard cells is usually associated with the effect of hydrodynamic instability that arise (a) in viscous liquids, (b) in the presence of a temperature gradient vector directed vertically downward (in other words, when the liquid is heated from below) [10]. At a certain relationship between the temperature gradient, which is a control parameter, the thickness of the liquid layer, as well as the coefficients of dynamic viscosity, thermal conductivity, thermal expansion, and isobaric heat capacity, in a sufficiently thin horizontal liquid layer located perpendicular to the directions of the temperature gradient and the gravity force vectors, regular hexagonal structures can arise. In the central part of these hexagons, the hotter liquid near the lower surface begins to rise vertically upward, while the colder liquid near the upper surface descends near the edges of the hexagons.

Another obvious reason, additional to the described above Benard–Rayleigh hydrodynamic instability effect [9, 10], is the Marangoni–Gibbs convection effect [11, 12], which gives a different explanation for the formation of hexagonal structures. The reason for this effect is that the presence of a vertical downward temperature gradient vector with $T_1 > T_2$ leads to the appearance of an oppositely directed surface tension gradient vector with $\sigma(T_1) < \sigma(T_2)$. As a result, the effect of hydrodynamic instability and convective flows arises that causes the movement of fluid from the layers with a lower value $\sigma(T_1)$ to the upper layers with higher values $\sigma(T_2)$.

2.2. Theoretical explanaition for the emergence of Benard cells as the second stage of Feynman's classification

The theoretical explanation for the emergence of Benard cells was proposed apparently for the first time in 1916 by Lord Rayleigh (John William Strutt), the 1904 Nobel Prize laureate in Physics, in his article [10]. The appearance of hexagonal Benard cells, or the Benard–Rayleigh effect [9, 10], is associated with a condition for hydrodynamic instability and convective flows, which is additional to conditions (a) and (b) mentioned in subsection 2.1, namely: (c) when the Rayleigh number

$$\mathrm{Ra} = \rho^2 h^3 \Delta T C_p g \beta / \eta \lambda, \tag{1}$$

becomes equal or slightly greater than its critical value $\mathrm{Ra}_{\mathrm{cr}}$ [13–15]. In (1), ρ is the fluid density; h is a thickness of the fluid layer; $\Delta T = T_1 - T_2$ is the temperature difference of the lower T_1 and upper T_2 surfaces of the fluid layer; C_P is the heat capacity at constant pressure; g is the acceleration due to gravity; β is the coefficient of thermal expansion; η is the dynamic viscosity; λ is the coefficient of thermal conductivity which is related to the coefficient of temperature diffusivity χ by the relation $\chi = \lambda/\rho C_P$.

In the sense of the second stage of Feynman's classification, the hydrodynamic theory of convective instability and formation of Benard hexagonal cells was developed in books [13–15]. To determine a critical value of the Rayleigh number $\mathrm{Ra}_{\mathrm{cr}}$, we will further use the results presented in the books by Lev Landau, Evgeniy Lifshits [14] and Werner Ebeling [15]. It turns out to be convenient to reduce the system of appropriately dimensioned hydrodynamic equations, linearized with respect to small perturbations of temperature τ, pressure Δp, and velocity υ_i, $i = x, y, z$, to the following partial differential equation for the temperature variable $\tau(x, y, z)$:

$$\Delta^3\tau = \mathrm{Ra}\,\Delta_2\tau, \tag{2}$$

where $\Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$ and $\Delta_2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ are, correspondingly, the three-dimensional and two-dimensional Laplace operators.

Representing the temperature deviation $\tau(x, y, z)$ in a layer of a viscous liquid located between two horizontal surfaces $z = 0$ and $z = h$ with temperature T_1 and T_2 in the form $\tau(x, y, z) = f(z)\exp[i(k_x x + k_y y)]$, where k_x and k_y are the components of the wave vector $\mathbf{k}$ in the x, y plane, one obtains the following ordinary differential equation for a function $f(z)$ depending only on the vertical coordinate z of the liquid layer:

$$\left(\frac{d^2}{dz^2} - k^2\right)^3 f + \mathrm{Ra}\frac{k^2}{h^4} f = 0. \tag{3}$$

Finding the solution of the differential equation (3) as $f(z) = C\exp(\mu z/h)$, the sixth-order algebraic characteristic equation with respect to μ_n ($n = 1 \dots 6$) was obtained

$$(\mu^2 - a^2)^3 = -(\mathrm{Ra})\,a^2, \tag{4}$$

where a is the coefficient of thermal expansion. Such algebraic equation has the following roots [15]:

$$\{\mu_n\} = \{\pm ir_0, \pm r, \pm r*\}, r_0 = a\sqrt{\tau - 1},$$
$$r = a\sqrt{1 + (1/2)\tau(1 + i\sqrt{3}}, \tag{5}$$
$$r^* = a\sqrt{1 + (1/2)\tau(1 - i\sqrt{3}}.$$

After the substitution of the found solutions (5) into the boundary conditions and without touching on further details of the corresponding calculations, one may obtain such results. For the boundary conditions in the form of free surfaces, the critical value of the Rayleigh number $\mathrm{Ra}_{\mathrm{cr}} = 657.5$, while, for the boundary conditions in the form of two rigid horizontal surfaces, the critical value of the Rayleigh number turns out to be equal $\mathrm{Ra}_{\mathrm{crit}} = 1708$. It should be pointed out, as was indicated in the book of L.D. Landau and E.M. Lifshits [14], that the last result regarding the Rayleigh number $\mathrm{Ra}_{\mathrm{crit}} = 1708$ was obtained by H. Jeffrey in 1908, i.e. eight years ahead of Rayleigh's article [10]. It was established that hexagons in the form of Benard cells arised only at $\mathrm{Ra} \geq \mathrm{Ra}_{\mathrm{cr}}$, i.e., if the Rayleigh number Ra was only slightly greater than its critical value $\mathrm{Ra}_{\mathrm{crit}}$. With a further increase in Ra, cylindrical structures became more preferable than hexagonal structures.

2.3. Haken's theoretical explanation of hexagonal Benard cells

For the case of hexagonal structures in Benard cells, Hermann Haken proposed an explanation, being close to the 3rd stage of the Feynman's classification and using the generalized Ginzburg–Landau equations [38, 39].

The consequences obtained by H. Haken from these equations are as follows: (*a*) formula (8.153) from [43] for the potential function $\Phi(A_k)$ which is actually equivalent to the Ginzburg–Landau Hamiltonian for the Fourier components of the order parameter; (*b*) the following kinetic equation for the order parameter $\psi(x, t)$ (see formula (9.5.15) from [44]):

$$\partial\psi/\partial t = \lambda\psi + A\psi^2 - B\psi^3, \tag{6}$$

where λ, A, and B are the coefficients (see its connection with the parameters of the Ginzburg–Landau Hamiltonian in paragraphs 8.13 and 9.4, 9.5, correspondingly, in Haken's monograpfs [43, 44]).

As noted by H. Haken in [44], the numerical solution of Eq. (6) at the zero coefficient A, found by Greenside, Cochrane Jr. and Schraer, does not lead to the appearance of hexagons. At the same time, in his unpublished studies, H. Haken received hexagonal structures at $A \neq 0$. Below in Section 3, we will consider, in more details, the issue of a rigorous theoretical justification of the appearance of a cubic nonlinearity in the Ginzburg–Landau Hamiltonian and a quadratic nonlinearity in the kinetic equation of the form (6) for the order parameter. It is the inclusion of these nonlinear terms that leads to the hexagonal structures based on fluctuation models of self-organization processes describing the formation of ordered structures near critical (bifurcation) points.

Here, we would like to emphasize the fact that the presence of a quadratic term $A\psi^2$ in (6) is equivalent to introducing an odd cubic term of the following form into the Ginzburg–Landau Hamiltonian $H_{\mathrm{GL}}[\psi_{\mathbf{k}}]$ for the Fourier components $\psi_{\mathbf{k}}$ of the order parameter:

$$A \sum_{\mathbf{k},\mathbf{k}',\mathbf{k}''} \psi_{\mathbf{k}}\psi_{\mathbf{k}'}\psi_{\mathbf{k}''}\delta_{\mathbf{k}+\mathbf{k}'+\mathbf{k}'',0}. \tag{7}$$

The summation in (7) over the wave vectors should be carried out over an equilateral triangle under the conditions $\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3=0$ and $|\mathbf{k}_1|=|\mathbf{k}_2|=|\mathbf{k}_3|=$ $=k_0=\mathrm{const}$. Obviously, these conditions lead to a coordinate lattice consisting of equilateral triangles, which, in turn, creates equilateral hexagons.

3. The First Principle and the Universality Class of the Emergence of Hexagonal Grid Cells in the Brain

In this section, we will sequentially consider the main provisions that are necessary to formulate the first principle of the formation of hexagonal grid cells in the brain from the point of view of the proposed Feynman's classification. This first principle of the nonlinear interaction of order-parameter fluctuations in living and inanimate systems, which will be proposed here to explain the hexagonal structures of grid cells in the brain, uses fluctuation models of the processes of self-organization and the formation of ordered structures in chemically reacting systems [2, 18–20].

As was already mentioned in Introduction, the main provisions underlying the 3$^{\mathrm{rd}}$ stage of the study of hexagons in the neurophysiological system of grid cells in accordance with the Feynman's classification are the following: the method of collective variables in the theory of phase transitions and critical phenomena; the concept of universality classes; the Polyakov's conformal invariance hypothesis and its violation in systems belonging to the universality class of 3D Ising model in an external magnetic field; the fluctuation model based on the Ginzburg–Landau Hamiltonian for chemically reacting systems.

3.1. The method of collective variables in the theory of phase transitions

The method of collective variables in the theory of phase transitions was developed by I.R. Yukhnovskii and his co-authors [28–31, 48]. Using this method and the Jacobian of transformation (see [48] and references there) from microscopic variables to collective variables, the authors of these works consistently obtained the fluctuation Ginzburg–Landau Hamiltonian. This Hamiltonian was commonly postulated in the renormalization group approach in the theory of phase transitions, for which the American physicist Kenneth Wilson received the Nobel prize in Physics in 1982.

The starting point in the method of collective variables for describing second-order phase transitions is the following expression for the Hamiltonian:

$$H = -\frac{1}{2}\sum_{\mathbf{i},\mathbf{j}} \Phi(r_{\mathbf{ij}})\sigma_{\mathbf{i}}\sigma_{\mathbf{j}} - h\sum_{l}\sigma_{\mathbf{l}}, \tag{8}$$

describing magnetic systems in a three-dimensional Ising model in an external magnetic field. In formula (8), the following designations are used: $\Phi(r_{ij})$ is the interparticle interaction potential, $r_{ij}=|\mathbf{r_i}-\mathbf{r_j}|$ is the distance between the sites $\mathbf{i}$ and $\mathbf{j}$ of the lattice, which, for definiteness, will be assumed to be cubic, the variables $\sigma_{\mathbf{l}}=\pm1$, and $h=\mu_{\mathrm{B}}H'$, where μ_{B} is the Bohr magneton, H' is the magnetic field intensity.

To simplify further calculations, the interparticle interaction potential is chosen in the form

$$\Phi(r_{ij}) = A\exp(-r_{ij}/b), \tag{9}$$

where $A=\mathrm{const}$, b is a quantity describing the interaction radius.

Further, a fundamentally important transition from microscopic individual variables $\sigma_{\mathbf{l}}$ to N nodal collective variables $\rho_{\mathbf{k}}$ in the statistical sum reads

$$Z = \int \exp\left(\frac{1}{2}\sum_{\mathbf{k}} \beta\Phi(k)\,\rho_{\mathbf{k}}\rho_{-\mathbf{k}}\right) J_h(\rho)(d\rho)^N, \tag{10}$$

where $J_h(\rho)$ is the corresponding Jacobian of the transition in variables $\sigma_{\mathbf{l}}$ to variables $\rho_{\mathbf{k}}$ [48]. Omitting further, generally speaking, rather complicated calculations of integral (10) and using the explicit form of the Jacobian $J_h(\rho)$, it can be shown that the final expression for the partition function Z of the three-dimensional Ising model, which describes the phase transition of a magnetic system in an external magnetic field, contains the following contributions. The first contribution corresponds to the Gaussian approximation (or the Ornstein–Zernike approximation) containing the squares of the Fourier components of collective variables. The second contribution corresponds to the cubic nonlinearity and contains the products of three Fourier components of collective variables. The next contribution corresponds to the biquadratic nonlinearity (or the approximation of the biquadratic distribution in the terminology used in the method of collective variables), which is characterized by the presence of the product of four Fourier components of collective variables. Obviously, all these combined contributions correspond to the Ginzburg–Landau fluctuation Hamiltonian. Note that going beyond the biquadratic distribution approximation allows one to consider higher even and odd contributions from nonlinear terms containing higher products of the Fourier components of collective variables. In addition, one should pay attention to the fact that the coefficients in terms containing odd degrees of the Fourier components of collective variables are proportional to the external field h. For $h \to 0$, all odd contributions disappear.

3.2. The concept of universality classes near bifurcation (critical) points

Let us now consider the concept of universality classes which is essential for the study of systems, generally speaking, of different nature, demonstrating the same behavior of physical properties in the vicinity of bifurcation (critical) points, as well as points (lines) of second-order phase transitions.

For ***bulk systems*** with linear dimensions L significantly exceeding the correlation radius (correlation length) ξ of the characteristic order parameter, the concept of a universality class includes the following main features which must be the same: 1) the same spatial dimension d, 2) the same number n of order-parameter components (for example, $n = 1$ for the scalar order parameter in the Ising model, $n = 2$ for the two-component order parameter in the plane rotator model, $n = 3$ for the vector order parameter in the Heisenberg model), 3) the same type (short- or long-range) of intermolecular interaction in comparison with the intermolecular distance, 4) the same symmetry of the fluctuation Ginzburg–Landau Hamiltonian, or fluctuation part of the Gibbs thermodynamical potential [32–40].

For ***spatially bounded systems***, for which $L \approx \xi$, to this list of identical features, the following should be added: 5) the same type of geometry of the system or, in other words, the lower crossover dimension d_{cros} ($d_{\mathrm{cros}} = 2$ for a slitlike, plane-parallel pore, $d_{\mathrm{cros}} = 1$ for a cylindric pore, $d_{\mathrm{cros}} = 0$ for a bounded sphere or cube), 6) the same type of boundary conditions (hygrophylic, hydrophobic, partially wetting), 7) the same physical properties under consideration [2, 33, 40, 49–51].

To explain additional conditions for universality classes in spatially limited systems, such features should be taken into account. In bulk systems, when a strong inequality $L \gg \xi$ is satisfied, the only critical values of thermodynamic parameters exist for a certain substance (for example, a single critical temperature $T_{\mathrm{C}} = 647.3$ K for water [52]). When critical values of these thermodynamic parameters are reached in bulk liquids, physical properties such as isothermal compressibility, isobaric and isochoric heat capacities, sound speed, self-diffusion coefficient, *etc.*, take on infinite or zero values in an approximation that does not account for the effects of spatial and temporal dispersion. In spatially limited systems, when such a condition $L \leq \xi$ is valid, the physical properties of the same substances (for example, the same water) are known to drastically change their critical behavior. The numerical values of the physical properties mentioned above become finite and non-zero at certain extremum points. It turns out that the shifts of the critical values of such thermodynamic parameters of bounded systems as the critical temperature $T_{\mathrm{C}}(L)$, the critical density $\rho_{\mathrm{C}}(L)$, the critical pressure $P_{\mathrm{C}}(L)$ with respect to the values of the same critical parameters $T_{\mathrm{C}}(\infty)$, $\rho_{\mathrm{C}}(\infty)$, $P_{\mathrm{C}}(\infty)$ in bulk systems cease to be universal and are different for each physical property.

To illustrate what has been said, consider the following example [53]. The shift of the critical temperature $T_{\mathrm{C}}(L)$ in a bounded system with a linear dimension L relative to the critical temperature $T_{\mathrm{C}}(\infty)$ in a bulk system is described by the following universal formula:

$$\Delta T_{\mathrm{C}} = T_{\mathrm{C}}(\infty) - T_{\mathrm{C}}(L) = KL^{-1/\nu(L)}. \qquad (11)$$

Here, $\nu(L)$ is the critical index of the temperature dependence of the correlation radius $\xi(L)$ in bounded liquids, which should depend on the linear size L of the system in the direction of its spatial limitation, when considering the effects of the dimensional crossover [54]; and K is a nonuniversal coefficient, the appearance of which in formula (11) is associated with the presence of nonuniversal parameters of a particular substance in the condition of an extremum for a certain physical property.

3.3. Polyakov's conformal invariance hypothesis of the theory of phase transitions

To rigorously substantiate the Haken's statement that Benard cells have a hexagonal geometry only in the presence of a quadratic nonlinearity in the kinetic equation (6) for the order parameter, one should turn to the conformal invariance hypothesis, first formulated by A.M. Polyakov in [41]. The physical meaning of this fundamental hypothesis of the theory of phase transitions and critical phenomena is reduced to the orthogonality (statistical independence) of anomalously fluctuating quantities (for example, order parameters) ϕ_1 and ϕ_2, which have different scale dimensions $\Delta_{\phi_1} \neq \Delta_{\phi_2}$. In other words, for such statistically independent quantities, the pair correlator $\langle\phi_1\phi_2\rangle$ approaches a zero value: $\langle\phi_1\phi_2\rangle = 0$. As an example of the application of the conformal invariance hypothesis, let us consider the behavior of the pair correlation function of order parameter fluctuations and energy fluctuations near critical (bifurcation) points for two different systems: 1) a real classical liquid-vapor system and 2) a magnet in the lattice-gas model.

For the first case of a classical fluid, V.L. Pokrovskii showed in [42] that the pair correlator $\langle\phi_1\phi_2\rangle$ turns out to be nonzero and is characterized by the following temperature dependence in the vicinity of the critical isochore:

$$\langle\phi_1\phi_2\rangle = \langle\Delta N\Delta E\rangle_{V=V_c} \sim \langle\Delta\rho\Delta E\rangle_{V=V_c} \sim$$

$$\sim \tau^{-(\alpha+\gamma)/2} \approx \tau^{-0.674}. \qquad (12)$$

Here, ϕ_1 and ϕ_2 are the fluctuations of the order parameters, being the deviations of the number of particles $\phi_1 = \Delta N = (N - N_c)/N_c$ or the deviations of the density $\phi_1 = \Delta\rho = (\rho - \rho_c)/\rho_c$), and the deviations of the energy $\phi_2 = \Delta E = (E - E_c)/E_c$ from its critical values N_c, ρ_c, E_c, correspondingly; $\tau = (T - T_c)/T_c$ is the temperature deviation from the critical value T_c, $\alpha = 0.110$ and $\gamma = 1.237$ are the critical exponents, the numerical values of which are taken from the articles of M.A. Anisimov and his coauthors [36, 55–57]. The result obtained means that, for a classical liquid, the conformal invariance hypothesis is not fulfilled, and fluctuations of the number of particles ΔN (or density $\Delta\rho$) and energy fluctuations ΔE are not orthogonal, being statistically dependent quantities. Moreover, the pair correlator $\langle\Delta N\Delta E\rangle \sim \langle\Delta\rho\Delta E\rangle$ turns out to be an anomalously fluctuating quantity, as the critical temperature is approached. Note that the account for the effects of spatial dispersion (spatial nonlocality of fluctuations) and/or temporal dispersion (temporal nonlocality of fluctuations) should provide a natural physical result – the finiteness of this quantity directly at the very point of the phase transition [58].

For the second system of the lattice-gas model, the situation turns out to be significantly different [41, 42, 59]. Fluctuations of the number of particles (or density) and energy fluctuations are statistically independent at the critical isochore, which corresponds to the zero value of the pair correlator

$$\langle\phi_1\phi_2\rangle = \langle\Delta N\Delta E\rangle_{V=V_c} \sim \langle\Delta\rho\Delta E\rangle_{V=V_c} = 0. \qquad (13)$$

In other words, for the fluctuating quantities given in (13) in the lattice-gas model, Polyakov's conformal invariance hypothesis is satisfied [36].

The considered lattice-gas model is isomorphic (or belongs to the same universality class) to the three-dimensional (3D) Ising model in the absence of a magnetic field ($h = 0$). Therefore, the analog of formula (25) for the 3D Ising model at $h = 0$ is the following relation [41]:

$$\langle\phi_1\phi_2\rangle = \langle\Delta M\Delta E\rangle_{h=0} = 0, \qquad (14)$$

where it was accounted for that the role of a fluctuating quantity ϕ_1 for the Ising model is played by fluctuations of the magnetization ΔM. Formula (14) means that the fluctuations of the magnetization ΔM and energy fluctuations ΔM are statistically independent. Therefore, in accordance with the conformal invariance hypothesis, such fluctuations for the 3D Ising model at $h = 0$ are orthogonal. Note again that the vanishing of the paired correlator is an approximate result which assumes the neglect of the effects of spatial and/or temporal dispersion. As indicated in [58], the terms describing the effects of spatial and/or temporal dispersion should naturally be added: 1) to those quantities (for example, the diffusion coefficient, the speed of sound, a pair correlator for a magnet in the lattice-gas model, *etc.*), which vanish at critical (bifurcation) points or points (lines) of second-order phase transitions; 2) to the inverse values of those quantities (for example, susceptibility, isobaric and isochoric heat capacities, *etc.*) which anomalously increase with approaching the critical (bifurcation) points or points (lines) of second-order phase transitions.

3.4. The violation of the Polyakov's conformal invariance hypothesis in real classical fluids and its consequences

Let us return to the connection between the Polyakov's conformal invariance hypothesis and Haken's statement that hexagonal structures in Benard cells are observed only in the presence of a quadratic nonlinearity $A\psi^2$ in the kinetic equation (6). As was noted in Section 2, this nonlinearity in (6) is completely equivalent to the presence of a cubic nonlinearity of the form (7) in the fluctuation Ginzburg–Landau Hamiltonian. In turn, the appearance of the odd (cubic) term (7) in the Ginzburg–Landau Hamiltonian is directly related to the Polyakov's conformal invariance hypothesis or, more precisely, is a direct consequence of the violation of the conformal invariance hypothesis in real classical fluids.

Thus, it can be argued that the two systems considered above – the real classical liquid and the magnetic in the lattice-gas model – belong, generally speaking, to different universality classes due to the different symmetries of the Ginzburg–Landau Hamiltonians (the fluctuation part of the Gibbs thermodynamic potentials). Hexagons arise only in those systems located near critical (bifurcation) points for which the Ginzburg–Landau Hamiltonian is not invariable with respect to the inversion of the order parameter $\phi \leftrightarrow -\phi$, and an odd (cubic) nonlinearity arises in the Ginzburg–Landau Hamiltonian.

In our previous article [1], we discussed the physical properties of the human brain matter using the information contained in [60–64]. It turned out that, according to some parameters (average density, relative dielectric permittivity), the gray matter of the brain is close to water. The discovery of hexagonal grid cells in the entorhinal cortex of brain by Edward Moser and May-Britt Moser in 2005 makes the system of grid cells in the brain even closer to a real liquid-vapor system. Indeed, the main and fundamental explanation for this fact lies in that both of these systems belong to the same universality class of the 3D Ising model in an external magnetic field, for which the Polyakov's conformal invariance hypothesis is violated.

It should be noted that the violation of the Polyakov's conformal invariance hypothesis is also a reason for the appearance of a singularity of the temperature derivative for the so-called "diameter of the coexistence curve (CC)" of the liquid-vapor system. As shown in a number of theoretical works (see, for example, [34, 36, 65–67], the diameter of the CC, defined as the half-sum of the average densities on the liquid and gas (vapor) branches of the CC $\rho_d(\tau) = (\rho_{\text{liquid}} + \rho_{\text{vapor}})/2$, is characterized by the same singularity of its temperature derivative as the isochoric heat capacity C_V near the critical point of the classical liquid-vapor system:

$$d\rho_d/dT \sim C_V \sim |\tau|^{-\alpha}. \tag{15}$$

The first experimental confirmation of the singularity of the diameter was obtained at the Department of Molecular Physics of the Taras Shevchenko Kyiv National University in [68], and then confirmed in a number of other experimental studies (see, for examples, [69–72]). Table illustrates simultaneously (*a*) the similarity (isomorphism) between the system of grid cells in the brain and a real liquid-vapor system, as well as (*b*) the principal difference in the physical properties of a real liquid-vapor system and a lattice-gas model.

The comparison of the physical properties of three different systems, presented in Table, allows us to draw the following conclusion: *the violation of the Polyakov's conformal invariance hypothesis is the reason for the appearance of hexagons in the system of grid cells in the brain, as well as the singularity of the temperature derivative of the density diameter along the CC in a real classical liquid-vapor system.*

3.5. Ginsburg–Landau Hamiltonian and fluctuation model for systems with chemical reactions. The first principle for grid cells in the brain

To theoretically describe the hexagonal grid cells in the brain, one should write the fluctuating part of the Gibbs thermodynamic potential $\Delta G(\xi)$ in a system with chemical (biochemical) reactions, using the Ginsburg–Landau Hamiltonian in the following gen-

Comparison of grid cells system, real liquid-vapor system and lattice-gas model

System	Universality class	Odd terms in Ginzburg–Landau Hamiltonian	Conformal invariance Polyakov's hypothesis	Singularity of diameter temperature derivative	Hexagons in Benard cells and grid cells in the brain
Grsd cells in the brain	3D Ising model in magnetic field	Present	Not realized	–	Present
Real liquid-vapor system	3D Ising model in magnetic field	Present	Not realized	Present	Present
Lattice-gas model	3D Ising model without magnetic field	Absent	Realized	Absent	Absent

eralized form [2, 18–20]:

$$\Delta G(\xi) = \int d\mathbf{r}[(1/2)\lambda\xi^2 + (1/2)C_2(\nabla\xi)^2 + \\ + (1/3)C_3\xi^3 + (1/4)C_4\xi^4 - A^*\xi]. \qquad (16)$$

The choice of the Gibbs thermodynamic potential $G(T, p, N_i)$ is determined by the real conditions of isothermality and isobaricity of living objects. In (16), ξ is the reaction coordinate (or the degree of completeness) which is the order parameter of a thermodynamical system with chemical reaction, being determined by the formula:

$$d\xi = dN_i/\nu_i, \qquad (17)$$

where dN_i is a change in the number of moles of an i-component, while ν_i are the stoichiometric coefficients of the chemical reaction.

The conjugated quantity (in the thermodynamic sense) for the reaction coordinate ξ is the affinity

$$A^* = \sum_i \nu_i\mu_i, \qquad (18)$$

depending on the chemical potentials μ_i of all components of a chemical reaction.

Formula (16) involves not only the Gaussian quadratic term $\lambda\xi^2$ and the forth-order nonlinear fluctuation contribution $C_4\xi^4$, being usually accepted for the standard form of the Ginsburg–Landau Hamiltonian, but also the cubic nonlinear interaction $C_3\xi^3$ of the order-parameter fluctuations in the chemically reacting system. Such odd term appears in (16) due to the violation of Polyakov's conformal invariance hypothesis [41]. Note also that the last term in (16), containing the product of the reaction coordinate and the affinity, has the meaning of the interaction of the order parameter ξ with the external field A^*. It should be noted that some examples of the interaction mechanisms of order parameter fluctuations in systems with chemical (biochemical) reactions were considered in the kinetic nonlinear models studied in [73–85].

As a result of calculating the functional derivative of the fluctuation part of the Gibbs thermodynamic potential $\Delta G(\xi)$ with respect to the order parameter ξ, we obtain the following kinetic differential equation for the coordinate (degree of completeness) $\xi(\mathbf{r})$ of the biochemically reacting system of grid cells:

$$\frac{\partial\xi}{\partial t} = -\Gamma(\lambda\xi - C_2\Delta\xi + C_3\xi^2 + C_4\xi^3 - A^*). \qquad (19)$$

The nonlinear kinetic equation (19) for the order parameter ξ and the Ginzburg–Landau Hamiltonian (the fluctuation part of the thermodynamic Gibbs potential) (16) for a chemically reacting system of neurons in the brain have the same functional form as the nonlinear kinetic equation for the order parameter (9.5.15) from [44] and expression (8.153) for the potential function from [43] in the Haken's theory describing the occurrence of hexagonal Benard cells. Therefore, all statements by Hermann Haken about the formation of hexagons in Benard cells can be almost completely transferred to the system of grid cells in the brain.

Thus, conducted research allows us to formulate the following first principle: *the formation of hexagonal structures in the system of grid cells in the brain occurs due to the nonlinear interaction of fluctuations in the degree of completeness (coordinate) of chemically interacting systems of neurons*. In addition, an obligatory condition for the nonlinear interaction of fluctuations of the order parameter in the system of grid cells in the brain is the violation of the Polyakov's conformal invariance hypothesis, a direct consequence of which is the appearance of a term cubic in fluctuations of the degree of completeness in the Ginzburg–Landau Hamiltonian.

4. Conclusions

The main purpose of this review article is to apply the fluctuation theory of phase transitions to studying the process of the appearance of hexagonal grid cells as a necessary component of the positional system in the brain of humans and animals (2014 Nobel prize in Physiology or Medicine). Three important conclusions necessarily follow from the studies conducted in this article:

1. The use of the Ginzburg–Landau Hamiltonian and the associated nonlinear kinetic equation for the order parameter of a chemically reacting system means accepting the following hypothesis. The neurophysiological system of grid cells in the brain, in which hexagons appear near the points of structural bifurcation transitions, actually belongs to the same universality class as the real liquid-vapor system near the critical point, as well as the 3D Ising model in the presence of an external magnetic field near the second-order phase transition.

2. As a direct consequence of this hypothesis, the first principle of the formation of hexagonal structures in the system of grid cells must be associated with the nonlinear interaction of fluctuations of the degree of completeness (coordinate) of chemical (biochemical) reactions in the brain.

3. A rigorous theoretical substantiation of this first principle for the emergence of hexagons requires the mandatory involvement of the fact that the Polyakov's conformal invariance hypothesis for fluctuations of the order parameters is violated for the system of grid cells in the brain, and, in a more general case, fluctuations of the corresponding order parameters in systems of living and inanimate nature near their bifurcation (critical) points and boundary of stability.

1. A.V. Chalyi, K.A. Chalyy, E.V. Zaitseva, A.A. Kryshtopa. Feynman's classification of natural phenomena and physical aspects of 2014 Nobel prize in physiology and medicine. *Ukr. J. Phys.* **67**, 736 (2022).
2. A.V. Chalyi. Synergetic dialogue "physics-medicine": Hexagones in living and inanimate nature. *J. Mol. Liq.* **329**, 114249 (2021).
3. T. Hafting, M. Fyhn, S. Molden, M.-B. Moser, E.I. Moser. Microstructure of a spatial map in the entorhinal cortex. *Nature* **436**, 801 (2005).
4. J. O'Keefe. Place units in the hippocampus of the freely moving rat. *Exp. Neurol.* **51**, 78 (1976).
5. The Nobel Prize in Physiology or Medicine 2014 to J.O'Keefe, M.-B. Moser, E.I. Moser. NobelPrize.org. *Nobel Prize Outreach AB 2023* [URL: https://www.nobelprize.org/prizes/medicine/2014/summary/].
6. J. Eccles, A.L. Hodgkin, A.F. Huxley. *Nobel Lectures, Physiology or Medicine* (Elsevier Publishing Company, 1972).
7. A.A. Abrikosov, V.L. Ginzburg, A.J. Leggett. The Nobel Prize in Physics 2003. [URL: https://www.nobelprize.org/prizes/physics/2003/summary/].
8. A.A. Abrikosov. On the magnetic properties of superconductors of the second group. *J. Exp. Theor. Phys.* **5**, 1174 (1957).
9. H. Benard. Les tourbillans cellulaires dans une nappe liquide. *Revue Generale des Sciences, Pares et Appliquees* **11**, 1261 (1900).
10. Lord Rayleigh. On convection currents in a horizontal layer of fluid, when the higher temperature is on the under side. *The London, Edinburg, and Dublin Philosophical Magazine and Journal of Science* **32**, 529 (1916).
11. C. Marangoni. *On the Expansion of a Droplet of a Liquid Floating on the Surface of Another Liquid* (Fratelli Fusi, 1869).
12. J.W. Gibbs. On the equilibrium of heterogeneous substances. Part II. *Transactions of the Connecticut Academy of Arts and Sciences* **3**, 343 (1878).
13. S. Chandrasekhar. *Hydrodynamic and Hydromagnetic Stability* (Clarendon, 1961) [ISBN: 978-0198512370].
14. L.D. Landau, E.M. Lifshits. *Fluid Mechanics. 2nd edition* (Elsevier, 2012).
15. W. Ebeling. *Struturbildung bei irreversible Prozessen. Einführung in die Theorie dissipativer Strukturen* (Teubner-Verlag, 1976).
16. J. Jacobs, C.T. Weidemann, J.F. Miller *et al.* Direct recording of grid-like neuronal activity in human spatial navigation. *Nat. Neorosci.* **16**, 1188 (2013).
17. L. Kunz, T.N. Schroder, H. Lee, *et al.* Reduced grid-cell-like representations in adults at genetic risk for Alzheimer's disease. *Science* **350**, 430 (2015).
18. A.V. Chalyi, A.G. Lebed. *Non-Homogeneous Liquids near the Critical Point and the Boundary of Stability and Theory of Percolation in Ceramics* (Harwood Acad. Publ., 1993) [ISBN: 3718652196].
19. A.V. Chalyi, I.V. Nezhinskii, V.M. Sysoev. The fluctuation mechanism of the formation of spatio-temporal structures in chemically active systems. In: *Proceedings of II International Workshop on Nonlinear and Turbulent Processes in Physics* (Gordon and Breach, 1984).
20. A.V. Chalyi, Ya.V. Tsekhmister, K.A. Chalyy. *Ordering and Self-Organization Processes in Fluctuation Models of Open Systems* (Bogomolets National Medical University, 2001).
21. G. Nicolis, I. Prigogine. *Self-Organization in Nonequilibrium Systems: From Dissipative Structures to Order through Fluctuations* (Wiley, 1977) [ISBN: 978-0471024019].
22. V.L. Ginzburg, L.D. Landau. On the theory of superconductivity. *J. Exp. Theor. Phys.* **20**, 1064 (1950).

23. K.G. Wilson, J. Kogut. Renormalization group and ε-expansion. *Phys. Rep. C* **12**, 75 (1974).
24. M.E. Fisher. The renormalization group in the theory of critical behavior. *Rev. Mod. Phys.* **46**, 597 (1974).
25. B. Lev, A. Zagorodny. *Applications of Field Theory Methods in Statistical Physics of Nonequilibrium Systems* (World Scientific, 2021) [ISBN: 978-9811229978].
26. *Bogolyubov Institute for Theoretical Physics, NAS of Ukraine1966-2016.* Edited by A.G. Zagorodniy (Akademperiodika, 2015) [ISBN: 978-966-360-301-8].
27. I.M. Mryglod, V.V. Ignatyuk, Yu.V. Golovach. *Mykola Bogolyubov and Ukraine, Series: Library "World of Physics"* (Eurosvit, 2009).
28. I.R.Yukhnovsky. *Phase Transitions of the Second Order. Method of Collective Variables* (Naukova Dumka, 1984).
29. I.R.Yukhnovsky, M.P. Kozlovsky, I.V. Pilyuk. *Microscopic Theory of Phase Transiyions in Three-Dimensional Systems* (Eurosvit, 2001).
30. M.P. Kozlovsky. *Influence of an External Field into the Critical Behavior of Three-Dimensional Systems* (Galician Druker, 2012).
31. *Order, Disorder and Criticality. Vol. 6. Adanced Problems of Phase Transitions Theory.* Edited by Yu.V. Holovatch (World Scientific, 2020) [ISBN 978-981-121-621-3].
32. L.P. Kadanoff. Critical behavior, universality and scaling, *Proc. Int. School Phys.* (Acad. Press, 1971).
33. M.E. Fisher. The theory of critical point singularities. In: *Critical Phenomena, Proceedings of the International School of Physics "Enrico Fermi.* Edited by M.S. Green (Academic Press, 1971)
34. A.Z. Patashinskii, V.L. Pokrovskii. *Fluctuation Theory of Phase Transition* (Pergamon Press, 1979).
35. H.E. Stanley. *Introduction to Phase Transition and Critical Phenomena* (Oxford University Press, 1987).
36. M.A. Anisimov. *Critical Phenomena in Liquids and Liquid Crystals* (Gordon & Breach, 1991).
37. M.A. Anisimov, E.E. Gorodetskii, V.D. Kulikov, A.A. Povodyrev, J.A. Sengers. A general isomorphism approach to thermodynamic and transport properties of binary liquid mixtures near critical points. *Physics A* **220**, 277 (1995).
38. L.D. Landau, E.M. Lifshits. *Theoretical Physics, Statistical Physics* (Pergamon Press, 2013), part 1, vol. 5.
39. K. Binder. Phase transitions in reduced geometry. *Annu. Rev. Phys. Chem.* **43**, 33 (1992).
40. A.V. Chalyi, L.A. Bulavin, V.F. Chekhun, K.A. Chalyy, L.M. Chernenko, A.N. Vasilev, E.V. Zaitseva, G.V. Khrapijchuk, A.V. Severin, M.V. Kovalenko. Universality classes and critical phenomena in confined liquid systems, *Cond. Matt. Phys.* **16**, 23008 (2013).
41. A.M. Polyakov. Conformal symmetry of critical fluctuations. *J. Exp. Theor. Phys. Lett.* **12,** 381 (1970).
42. V.L. Pokrovskii. Feasibility of experimental verification of conformal invariance hypothesis. *J. Exp. Theor. Phys. Lett.* **17**, 156 (1973).
43. H. Haken. *Synergetics. An Introduction. Nonequilibrium Phase Transitions and Self-Organization in Physics, Chemistry and Biology* (Springer-Verlag, 1978).
44. H. Haken. Advanced Synergetics. *Instability Hierarchies of Self-Organizating Systems and Devices.* (Springer-Verlag, 1983).
45. R.P. Feynman, R.B. Leighton, M. Sands. *The Feynman Lectures on Physics*, vol. 1, Chapter 26. (California Institute of Technology, 2013); [URL: https://www.feynmanlectures.caltech.edu].
46. E. Noether. Invariant variation problems. *Transport Theory and Statistical Physics* **1** 186 (1971).
47. L.D. Landau, E.M. Lifshits. *Theoretical Physics. V. 1. Mechanics.* (Pergamon Press, 2000).
48. I.R. Yukhnjvsky, M.F. Golovko. *Statistical Theory of Classical Systems* (Naukova Dumka, 1980).
49. A.V. Chalyi. Dynamic anomalies in confined supercooled water and bulk fluids. In: *Modern Problems of Molecular Physics.* Edited by L. Bulavin, A. Chalyi (Springer International Publishing, 2018) [ISBN: 978-3-319-61108-2].
50. K.A. Chalyy, K. Hamano, A.V. Chalyi. Correlating properties of a simple liquid at criticality in a reduced geometry. *J. Mol. Liq.* **92**, 153 (2001).
51. L.A. Bulavin, K.O. Chalyy. *Neutron Optics of Mesoscopic Liquids* (Naukova Dumka, 2006).
52. R.C. Reid, J.V. Prausnitz, T.K. Sherwood. *The Properties of Gases and Liquids* (McGraw-Hill, 1977).
53. A.V. Chalyi. Surface tension in bulk and bounded liquids. *J. Mol. Liq.* **288**, 110873 (2019).
54. A.V. Chalyi. Dimensional crossover in liquids in reduced geometry. In: *Physics of Liquid Matter: Modern Problem.* Edited by L. Bulavin, N. Lebovka (Springer International Publishing, 2015) [ISBN: 978-3319372235].
55. D.A. Fuentevilla, M.A. Anisimov. Scaled equation of state for supercooled water near the liquid-liquid critical point. *Phys. Rev. Lett.* **97**, 195702 (2006).
56. C.E. Bertrand, M.A. Anisimov. Peculiar thermodynamics of the second critical point in supercooled water. *J. Phys. Chem. B* **115**, 14099 (2011).
57. V. Holten, C.E. Bertrand, M.A. Anisimov, J.V. Sengers. Thermodynamics of supercooled water. *J. Chem. Phys.* **136**, 094507 (2012).
58. V.M. Sysoev, A.V. Chalyi. Correlation functions and dynamic structure factor of non-isotopic systems. *Theor. Math. Phys.* **26**, 126 (1976).
59. S.V. Fomichev, S.B. Khohlachev. On the matter equation of state near the critical point liquid-vapor. *J. Exp. Theor. Phys.* **66**, 3 (1974).
60. D. Hubel, C. Stevens, H. Lee *et al.* The Brain. *Scientific American* **241**, 45 (1979).
61. V.A. Berezovsky, N.N. Kolotilov. *Biophysical Characteristics of Human Tissues. Reference Book* (Naukova Dumka, 1982).
62. P.G. Kostyuk, O.A. Kryshtal. *Mechanisms of Electrical Excitability of Nervous System* (Nauka, 1981).
63. *Medical and Biological Physics.* Textbook for students of higher medical institutions. Edited by A.V. Chalyi (Nova Knyga, 2020) [ISBN 978-966-382-804-6].
64. V. Tsymbalyuk, V. Medvedev. *Man and His Brain* (Nova Knyga, 2022) [ISBN 978-966-382-925-8].

65. A.V. Chalyi. Inverse correlation radius expansions and the "Rectilinear diameter" singularity. *Ukr. J. Phys.* **21**, 474 (1976).
66. J. Wang, M.A. Anisimov. Nature of vapor-liquid asymmetry in fluid criticality. *Phys. Rev. E* **75**, 051107 (2007).
67. O. Bakai, M. Bratchenko, S. Dyuldya. On the singularity of the liquid-gas curve dimeter. *Ukr. J. Phys.* **65**, 802 (2020).
68. L.M. Aptyukhoskaya, E.T. Shimanskaya, Yu.I. Shimanskii. Heptane coexistence curve near the critical point. *J. Exp. Theor. Phys.* **63**, 2153 (1972).
69. E.T. Shimanskaya, I.V. Bezrychko, B.I. Basok, Yu.I. Shimanskii. Experimental definition of the critical exponents, asymmetric and non-asymptotic corrections in the coexistence-curve equation of freon-113. *J. Exp. Theor. Phys.* **80**, 139 (1981).
70. M. Nakata, H. Kawahara. Coexistence curve for polystyrene-cyclohexane near the critical point. *J. Chem. Phys.* **62**, 430349 (1975).
71. B.S. Maccabee, J. White. A non-symmetrical parametric equation of state for the extended critical region of carbon dioxide. *Phys Lett. A* **60**, 179 (1977).
72. L.A. Bulavin, Yu.I. Shimanskii. The singularity of the coexistence-curve diameter of etheane. *J. Exp. Theor. Phys. Lett.* **29**, 482 (1979).
73. A.V. Chalyi, L.M. Chernenko. Phase transitions in finite-size systems and synaptic transmission In: *Dynamic Phenomena at Interfaces, Surfaces and Membranes.* Edited by D. Beysens, N. Boccara, G. Forgacs (Nova Sci. Publ., 1993) [ISBN: 978-1560720898].
74. A.V. Chalyi, A.N. Vasil'ev. Correlation properties, critical parameters and critical light scattering in finite-size systems. *J. Mol. Liq.* **84**, 203 (2000).
75. K.A. Chalyy, L.A. Bulavin, A.V. Chalyi. Dynamic scaling and central component width of critical opalescence spectrum in liquids with restricted geometry. *J. Phys. Stud.* **9**, 66 (2005).
76. A.V. Chalyi, E.V. Zaitseva. Strange attractor in kinetic model of synaptic transmission. *J. Phys. Stud.* **11**, 322 (2007).
77. A.V. Chalyi, E.V. Zaitseva. A kinetic model of synaptic transmission on intercell interaction. *Ukr. J. Phys.* **54**, 366 (2009).
78. A.V. Chalyi, A.N. Vasilev, E.V. Zaitseva. Synaptic transmission as a cooperative phenomenon in confined systems. *Cond. Matter Phys.* **20**, 13804 (2017).
79. I.S. Braude, N.N. Gal'tsov, V.I. Dotsenko, E.N. Chaika. Phase compositions of cast dental Ni–Cr–Mo and Co–Cr–Mo alloys. *Phys. Metals and Metallography* **100**, 165 (2005).
80. V.I. Dotsenko, E.N. Chaika. Influence of progressive remelts on structure and physical-mechanical properties of alloy germanium GM-700. *Metallophysics and Advanced Technologies* **25**, 297 (2003).
81. D.A. Gavryushenko, K.V. Cherevko, V.M. Sysoev. The influence of the chemical reactions on the diffusion phenomena in the cylindrical systems bounded with the membranes. *J. Mol. Liq.* **127**, 71 (2006).
82. K.A. Chalyy, L.A. Bulavin, V.F. Chekhun, A.V. Chalyi, Y.V. Tsekhmister, L.M. Chernenko. Fundamentals and medical applications of neutron and light spectroscopy of confined liquids. *IFMBE Proceedings* **25**, 197 (2009).
83. K.V. Cherevko, D.A. Gavryushenko, O.V. Korobko, V.M. Sysoev. Entropy production in the diffusion of a Margules solution in a flat-parallel pore. *Ukr. J. Phys.* **58**, 988 (2018).
84. L.A. Bulavin, V. F. Chekhun, A.A. Vasilkevich, V.I. Kovalchuk, V.T. Krotenko, V.I. Slisenko, V.P. Trindyak, K.A. Chalyy, S.D. Galyant. Neutron investigations of self-diffusion of water molecules in plasmatic membranes. *J. Phys. Stud.* **8**, 334 (2004).
85. K.V. Cherevko, D.A. Gavryushenko, L.A. Bulavin. Entropy production in a model biological system with facilitated diffusion. *Ukr. J. Phys.* **66**, 714 (2021).

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ФІЗИЧНІ АСПЕКТИ НОБЕЛІВСЬКОЇ ПРЕМІЇ З ФІЗІОЛОГІЇ АБО МЕДИЦИНИ 2014 РОКУ: 2. ПЕРШИЙ ПРИНЦИП ТА КЛАС УНІВЕРСАЛЬНОСТІ ДЛЯ КЛІТИН СІТКИ У МОЗКУ

Основною метою даної оглядової статті є використання флуктуаційної теорії фазових переходів для вивчення клітин гексагональної сітки як головної складової орієнтаційної системи в мозку людини і тварин (Нобелівська премія з фізіології та медицини 2014 р.). Особливу увагу приділено застосуванню фейнманівської класифікації трьох етапів вивчення природних явищ для: 1) короткого опису експериментального етапу відкриття гексагональних структур клітин сітки в мозку людини і тварин; 2) теоретичного етапу дослідження формування гексагонів у фізичній системі клітин Бенара, а також у нейрофізіологічній системі клітин сітки, відкритої Едвардом Мозером і Мей-Брітт Мозер; 3) найважливішого етапу, який дозволяє сформулювати перший принцип виникнення клітин сітки в мозку і, взагалі кажучи, першооснову формування шестикутників в об'єктах неживої і живої природи. Наші оригінальні теоретичні результати пов'язані з наступними твердженнями: (*a*) гіпотеза конформної інваріантності Полякова порушується для системи клітин сітки в мозку людини і тварин; (*b*) система клітин сітки належить до класу універсальності, котрий включає 3D-модель Ізінга в магнітному полі, а також реальну класичну систему рідина–пара; (*c*) для надійного теоретичного обґрунтування першого принципу, який пояснює виникнення клітин гексагональної сітки в мозку, необхідно використати флуктуаційну частину термодинамічного потенціалу Гіббса (гамільтоніан Гінзбурга–Ландау) для системи з хімічними (біохімічними) реакціями.

К л ю ч о в і с л о в а: перший принцип, клас універсальності, клітини сітки, шестикутники в мозку людини, гіпотеза конформної інваріантності, гамільтоніан Гінзбурга–Ландау.