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EXISTENCE OF SMALL-AMPLITUDE DOUBLE LAYERS IN TWO-TEMPERATURE NON-ISOTHERMAL PLASMA

In the presence of warm negative ions, ion-acoustic small-amplitude monotonic double layers are theoretically investigated in a plasma consisting of warm positive ions, warm positrons, and two-temperature non-isothermal electrons under the variation of the trapping parameters of electrons, concentration of positrons, and mass ratios of heavier negative ions to lighter positive ions by the Sagdeev pseudopotential method. The corresponding double layer solutions are also discussed for the same variation. Consequently, ion-acoustic solitary waves and double layers have been observed in auroral and magnetospheric plasmas with two-temperature electron distributions found in a laboratory, as well as in the space. This paper shows the effects of trapping parameters of electrons, positron concentration, and mass ratios of heavier negative ions to lighter positive ions on the Sagdeev potential function $\psi(\phi)$ and double layer solutions ϕ_{DL} for small-amplitude monotonic double layers. The results are presented graphically in Figs. 1 to 6.

Keywords: two-temperature non-isothermal electrons, Sagdeev potential method, heavier negative ions, double layers, double-layer solutions.

1. Introduction

Double layers, non-linear wave structures [1, 2], have been regarded as an interesting and significant topic. They occur frequently in astrophysics and space physics. In the past few years, the investigation of ion-acoustic double layers in electron-positron-ion plasmas attracted much attention theoretically, as well as experimentally in both non-relativistic and relativistic regimes. The study of electron-positron-ion plasmas is growing extensively due to their presence in the early universe, in active galactic nuclei, in pulser magnetosphere, in ionosphere, in the solar atmosphere, *etc.* It is well known that if positrons are introduced into an electron-ion plasma, the response of the electrostatic waves changes significantly in comparison to the usual two-component plasmas. Further, an electron-positron-ion plasmas can also appear in the pair production in a plasma due to the propagation of intense laser pulses. As a consequence, a number of authors studied and analyzed the ion-acoustic double layers in a plasma consisting of warm positive and negative ions, warm positrons, and two-temperature electrons. In an electron-ion plasma, a double layer is defined as a monotonic transition of the electric potential connecting smoothly two differently biased plasmas giving rise to a net poten-

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Цитування: Чаттопадхяй С. Існування низькоамплітудних подвійних шарів малої амплітуди у неізотермічній плазмі з двома температурами. *Укр. фіз. журн.* **68**, №12, 797 (2023).

tial difference, but in electron-positron-ion plasmas in addition to electrons, positrons also contribute to the double layer potential. Thus, the properties of wave motion in such plasmas are very much different from those in electron-positron plasmas. Again, the large- and weak-amplitude ion-acoustic double layers [3–7] in different plasma systems have been observed by several authors using the reductive perturbation method. Merlino and Loomis [8] reported on the experimental observation of a strong double layer in a plasma composed of positive ions, negative ions, and electrons. Goswami *et al.* [9] studied obliquely propagating double layers in a magnetic field for the two-electron Boltzmann model. In most cases, electron distributions were then considered as Maxwellian and were found in thermal plasmas. Further, satellite-based observations confirm that the plasmas usually deviate from the Maxwellian particle distributions. As a result, non-Maxwellian particle distributions for electrons such as non-thermal, kappa, vortex-type, and Tsallis distributions have been proposed. In general, it is worth mentioning that the plasma distributions near a double layer are strongly non-Maxwellian, and this double layer with non-Maxwellian electron distributions has been the focus of recent studies. Moreover, in a two- or single-temperature non-isothermal electron plasma, the concept and role of trapping (reflecting) parameters for electrons are attracted much attention. In [1, 2, 6, 7], the trapping and reflecting parameters were self-consistently determined from the phase velocity of a perturbation and its amplitude in a kinetic description. Using the reductive perturbation method, Gill *et al.* [10] investigated ion-acoustic solitons and double layers in non-thermal electrons with isothermal positive and negative ions. Tagare *et al.* [11] and Mishra *et al.* [12] took the plasma composition (H^+, O_2^-) and (H^+, H^-) which are expected to occur in the D region of the ionosphere for finding solitary waves and double layers in the plasma. This double layer has been a topic of great interest because of its relevance in cosmic applications [13–15], confinement of plasmas in tandem mirror devices [16], *etc.* Further, Popel *et al.* [17] established the existence of finite-amplitude ion-acoustic solitons of positive electrostatic potential in electron-positron-ion plasmas using the hydrodynamic model for the ion fluid and the kinetic description for hot electron and positron fluids, the masses of which are equal. The present author [18–19] also found out first- and second-order compressive solitary wave solutions in single- and two-temperature non-isothermal electron plasmas for cold positive and negative ions and for warm positive and negative ions under the variation of different plasma parameters by a "simple non-linear wave solution method" and a new technique called as "tanh-method". Schamel [20] studied the effect of the interaction of ion-acoustic waves and electrons, where some electrons can be trapped in the ion-acoustic waves, while another ones still move freely in the space. This implies that the plasma system consists of two possible types of electrons. However, Schamel considered the Maxwellian distribution for both free and trapped electrons in the investigation of non-linear ion-acoustic wave phenomena. In addition to that, Chattopadhyay [21] studied ion-acoustic solitary waves and small-amplitude double layers in a plasma containing warm positive ions, warm negative ions, warm positrons, and two-temperature non-isothermal electrons by the same well-known Sagdeev pseudopotential method.

The main ourpose of this paper is to study the ion-acoustic small-amplitude monotonic double layers for understanding the effect of trapping or reflecting parameters of electrons, concentration of positrons, and mass ratios of heavier negative ions to lighter positive ions on the profiles of the Sagdeev potential function $\psi(\phi)$ and on the profiles of double-layer solutions ϕ_{DL}.

The plan of the paper is arranged in the following ways:

In Sec. 2, the basic set of normalized equations for warm positive ions, warm negative ions, warm positrons along with Poisson's equations, concentrations of electrons and positrons, charge neutrality, and boundary conditions are given. The expression for the Sagdeev potential function $\psi(\varphi)$ is obtained in this section as well. Using the required double-layer conditions, the profiles of the Sagdeev potential function $\psi(\varphi)$ and the double-layer solution (ϕ_{DL}) for small-amplitude compressive double layers are stated in this part. In Sec. 3, this problem is discussed comprehensively under the variation of some plasma parameters. Concluding remarks are given in Sec. 4.

2. Formulations

We will consider a collisionless, unmagnetized, non-relativistic plasma consisting of warm positive ions with density n_i, velocity u_i, pressure p_i, mass m_i, and temperature T_i; warm negative ions with density n_j, velocity u_j, pressure p_j, mass m_j, and temperature T_j; warm positrons with density χ, temperature T_p, and two-temperature non-isothermal electrons with density n_e comprising of the densities of low-temperature electrons n_{el} and high-temperature electrons n_{eh}, and effective temperature $T_{\rm eff}$, consisting of the hot component with temperature T_{eh} and the cold component with temperature T_{el} which are followed by $T_{eh,f}$, $T_{eh,t}$ and $T_{el,f}$, $T_{el,t}$, where $T_{eh,f}$, $T_{eh,t}$ are, respectively, the temperatures of free and trapped electrons at high temperatures, and $T_{el,f}$, $T_{el,t}$ are, respectively, the temperatures of free and trapped electrons at low temperatures. Further, we assumed that low-frequency electrostatic waves propagate in plasmas. The non-linear behavior of ion-acoustic waves may be described by the following set of normalized basic equations:

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i u_i) = 0, \tag{1}$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} + \frac{\sigma_i}{n_i}\frac{\partial p_i}{\partial x} + \frac{\partial \phi}{\partial x} = 0, \tag{2}$$

$$\frac{\partial p_i}{\partial t} + u_i \frac{\partial p_i}{\partial x} + 3p_i \frac{\partial u_i}{\partial x} = 0, \tag{3}$$

$$\frac{\partial n_j}{\partial t} + \frac{\partial}{\partial x}(n_j u_j) = 0, \tag{4}$$

$$\frac{\partial u_j}{\partial t} + u_j \frac{\partial u_j}{\partial x} + \frac{\sigma_j}{Q n_j}\frac{\partial p_j}{\partial x} - \frac{Z}{Q}\frac{\partial \phi}{\partial x} = 0, \tag{5}$$

$$\frac{\partial p_j}{\partial t} + u_j \frac{\partial p_j}{\partial x} + 3\,p_j \frac{\partial u_j}{\partial x} = 0, \tag{6}$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_e - n_i + Z n_j - n_p. \tag{7}$$

Here, the electrons are assumed to be in a quasi-equilibrium state with low-frequency ion-acoustic wave. We take the following general expression for the electron density [22–23]:

$$n_e(\phi) = \left(\int\limits_{-\infty}^{-\sqrt{2\phi}} dv + \int\limits_{\sqrt{2\phi}}^{\infty} dv\right) f + \frac{g_+}{2}\left[1+\operatorname{sgn}(\xi-\xi_m)\right] \times$$
$$\times \int\limits_{-\sqrt{2\phi}}^{\sqrt{2\phi}} dv\, f_+ + \frac{g_-}{2}\left[1-\operatorname{sgn}(\xi-\xi_m)\right] \int\limits_{-\sqrt{2\phi}}^{\sqrt{2\phi}} dv\, f_-.$$

Here, the symbols f and $f_\pm$ represent the free and reflected electron distribution functions. The above-mentioned density normalization constants g_+ and g_- are positive. In this case, electrons can have different densities depending on the sign of $(\xi - \xi_m)$, where ξ_m is the position of the minimum of ion-acoustic double-layer potential $(\phi = 0)$, and $\operatorname{sgn}(\xi - \xi_m)$ is the constant of motion for all the reflected particles.

By using the quasi-neutrality condition and the drift approximation for electrons in small-amplitude ion-acoustic double layers, we may expand the electron density $n_e(\phi)$ [22–24] as follows:

$$n_e(\phi) = 1 + A_1\phi + \delta^{\frac{1}{2}} A_2 \operatorname{sgn}(\xi - \xi_m)\, \phi^{\frac{3}{2}} + A_3\phi^2 + \dots.$$

The above coefficients A_1, A_2, A_3, ... are given as:

$$\int\limits_{-\infty}^{+\infty} dv\, f(v) = 1, \quad A_1 = -\mathrm{P}\int dv\, \frac{1}{v}\frac{\partial f}{\partial v},$$

$$A_2 = \left(\frac{4\sqrt{2}}{3}\right)\left\{g_\pm f''_\pm(0) + f''_\pm(0)\right\},$$

$$A_3 = -\frac{1}{2}\mathrm{P}\int dv\left(\frac{1}{v}\frac{\partial}{\partial v}\right) f,$$

where “P” represents the principal value of the integral.

Again for the general formulation of monotonic double layers, we now consider the following modified Schamel-type [22–26] electron distribution:

$$f = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\operatorname{sgn}(v)\sqrt{\varepsilon} - v_d\right)^2\right] \cdot \theta(\varepsilon),\ \varepsilon > 0,$$

$$f_\pm = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}\, g_\pm v_d^2\right) \exp\left(-\frac{1}{2}\,\delta_\pm\, \varepsilon\right) \cdot \theta(-\varepsilon),\ \varepsilon \le 0.$$

Here $\varepsilon^2 = v^2 - 2\phi$, $\phi(x) \ge 0$, and θ is the Heaviside step function.

The electron velocity and the potential are, respectively, normalized to the electron thermal velocity $\sqrt{\frac{K\,T_{\rm eff}}{m_e}}$ and the electron temperature $\frac{K\,T_{\rm eff}}{e}$, and v_d represents the electron drift velocity. The thermal distribution scaling $(\delta_\pm)$ is positive.

From the electron distribution functions given above, the corresponding density for electrons can be found by simple velocity-space integrations as follows:

$$n_e(\phi) = \exp\left(-\frac{v_d^2}{2}\right)\left[I\left(\frac{v_d^2}{2}, \phi\right) + T_\pm(\beta, \phi)\right].$$

Here, I and T are defined as follows:

$$I\left(\frac{v_d^2}{2},\phi\right)=\sqrt{\frac{2}{\pi}}\int\limits_0^\infty dV\times$$
$$\times\left[\frac{V}{\sqrt{V^2+2\,\phi}}\exp\left(-\frac{V^2}{2}\right)\cosh\left(V,\,v_d\right)\right],$$
$$T_+\left(\beta,\,\phi\right)=\frac{1}{\sqrt{\beta}}\exp\left(\beta\phi\right)\operatorname{erf}\left(\sqrt{\beta\phi}\right),\quad\beta>0,$$
$$T_-\left(\beta,\,\phi\right)=\frac{2}{\sqrt{\pi\left|\beta\right|}}\exp\left(-\left|\beta\right|\phi\right)\int\limits_0^{\sqrt{|\beta|\phi}}dy\,\exp\left(y^2\right),$$
$$\beta<0.$$

Here, the "erf" represents the error function.

In this paper, the electron density is defined from the Vlasov equations involving the free and trapped electrons as

$$n_e\left(\phi\right)=\int\limits_{-\propto}^{\propto}f_e\left(x,v\right)dv=$$
$$=K_0\left[\exp\left(\phi\right)\operatorname{erfc}\left(\sqrt{\phi}\right)+\frac{1}{\sqrt{\beta_1}}\times\right.$$
$$\left.\times\begin{Bmatrix}\exp(\beta_1\phi)\operatorname{erf}\left[\sqrt{|\beta_1\phi|}\right] & \text{for }\beta_1\geq 0,\\ \frac{2}{\sqrt{\pi}}\exp\left[-\left\{\sqrt{|\left(-\beta_1\phi\right)|}\right\}^2\times\right. & \\ \left.\times\int\limits_0^{\sqrt{|(-\beta_1\phi)|}}\exp(X^2)dX\right] & \text{for }\beta_1<0\end{Bmatrix}\right].$$

Here, $\operatorname{erf}(x)=\frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt$, $\operatorname{erfc}(x)=1-\operatorname{erf}(x)$, K_0 is some constant, $f_e(x,v)$ is the electron distribution function, $\beta_1=\frac{T_{el,f}}{T_{eh,t}}$ is the temperature ratio of free ($T_{el,f}$) and trapped ($T_{eh,t}$) electrons at low and high temperatures.

Here, we consider the case $\beta_1>0$ and take

$$n_e\left(\phi\right)=\exp\left(\phi\right)\operatorname{erfc}\left(\sqrt{\phi}\right)+\frac{1}{\sqrt{\beta_1}}\times$$
$$\times\exp\left(\beta_1\phi\right)\operatorname{erf}\left[\sqrt{|\beta_1\phi|}\right]=\exp\left(\phi\right)\left[1-\operatorname{erf}\left(\sqrt{\phi}\right)\right]+$$
$$+\frac{1}{\sqrt{\beta_1}}\exp\left(\beta_1\phi\right)\operatorname{erf}\left[\sqrt{|\beta_1\phi|}\right]=$$
$$=\exp\left(\phi\right)\left[1-\frac{2}{\sqrt{\pi}}\int\limits_0^{\sqrt{\phi}}\exp\left(-t^2\right)dt\right]+$$
$$+\frac{1}{\sqrt{\beta_1}}\exp\left(\beta_1\phi\right)\left[1-\frac{2}{\sqrt{\pi}}\int\limits_0^{\sqrt{\beta_1\,\phi}}\exp(-t^2)\,dt\right].$$

The normalized electron density $n_e\left(\phi\right)$ for a two-temperature non-isothermal electron plasma is obtained by the Taylor series expansion from above under the condition $\phi\ll 1$ as

$$n_e=n_{el}+n_{eh}=$$
$$=\left[1+\left(\frac{\phi}{\mu+\nu\beta_1}\right)-\frac{4}{3}b_l\left(\frac{\phi}{\mu+\nu\beta_1}\right)^{\frac{3}{2}}+\frac{1}{2}\left(\frac{\phi}{\mu+\nu\beta_1}\right)^2-\right.$$
$$\left.-\frac{8}{15}b_l^{(1)}\left(\frac{\phi}{\mu+\nu\beta_1}\right)^{\frac{5}{2}}+\frac{1}{6}\left(\frac{\phi}{\mu+\nu\beta_1}\right)^3+...\right]+$$
$$+\nu\left[1+\left(\frac{\beta_1\phi}{\mu+\nu\beta_1}\right)-\frac{4}{3}b_h\left(\frac{\beta_1\phi}{\mu+\nu\beta_1}\right)^{\frac{3}{2}}+\frac{1}{2}\left(\frac{\beta_1\phi}{\mu+\nu\beta_1}\right)^2-\right.$$
$$\left.-\frac{8}{15}b_h^{(1)}\left(\frac{\beta_1\phi}{\mu+\nu\beta_1}\right)^{\frac{5}{2}}+\frac{1}{6}\left(\frac{\beta_1\phi}{\mu+\nu\beta_1}\right)^3+...\right]=$$
$$=1+\phi-\frac{4}{3}\frac{\left(\mu b_l+\nu b_h\beta_1^{\frac{3}{2}}\right)}{\left(\mu+\nu\beta_1\right)^{\frac{3}{2}}}\phi^{\frac{3}{2}}+\frac{1}{2}\frac{\left(\mu+\nu\beta_1^2\right)}{\left(\mu+\nu\beta_1\right)^2}\phi^2-$$
$$-\frac{8}{15}\frac{\left(\mu b_l^{(1)}+\nu b_h^{(1)}\beta_1^{\frac{5}{2}}\right)}{\left(\mu+\nu\beta_1\right)^{\frac{5}{2}}}\phi^{\frac{5}{2}}+\frac{1}{6}\frac{\left(\mu+\nu\beta_1^3\right)}{\left(\mu+\nu\beta_1\right)^3}\phi^3-...,\tag{8}$$
$$n_p=\chi e^{-\sigma_p\phi},\tag{9}$$
$$b_l=\frac{1-\beta_l}{\sqrt{\pi}},\quad b_h=\frac{1-\beta_h}{\sqrt{\pi}},\quad b_l^{(1)}=\frac{1-\beta_l^2}{\sqrt{\pi}},$$
$$b_h^{(1)}=\frac{1-\beta_h^2}{\sqrt{\pi}},\quad\beta_1=\frac{T_{el,f}}{T_{eh,t}},\quad\beta_l=\frac{T_{el,f}}{T_{el,t}},$$
$$\beta_h=\frac{T_{eh,f}}{T_{eh,t}},\quad\mu+\nu=1,$$
$$\sigma_p=\frac{T_{\text{eff}}}{T_p},\quad\sigma_i=\frac{T_i}{T_{\text{eff}}},\quad\sigma_j=\frac{T_j}{T_{\text{eff}}},\quad Q=\left(\frac{m_j}{m_i}\right)^{\frac{1}{2}},$$

i is for positive ion, j is for negative ion, $T_{\text{eff}}==\frac{T_{el}\,T_{eh}}{\mu\,T_{eh}+\nu\,T_{el}}$; for the non-isothermal plasma, $0<b_l$ or $b_h<\frac{1}{\sqrt{\pi}}$ and $0<b_l^{(1)}$ or $b_h^{(1)}<\frac{1}{\sqrt{\pi}}$.

In this case, μ and ν are, respectively, the unperturbed number density of low-temperature and high-temperature electrons; $T_{el,f}$, $T_{eh,f}$ are, respectively, the temperatures of free electrons at low and high temperatures, whereas $T_{el,t}$, $T_{eh,t}$ are the temperatures of trapped electrons at low and high temperatures. Moreover, β_1, β_l and β_h are the temperature ratio for free and trapped electrons at low and high temperatures; temperature ratio of free and trapped electrons at low temperature and temperature ratio of free and trapped electrons at high temperature. Here, the mass ratio Q is taken with respect to Refs. [10, 28, 29].

The charge neutrality condition is

$$1 + Zn_{jo} = n_{io} + \chi. \tag{10}$$

The boundary conditions are $n_i \to n_{io}$, $n_j \to n_{jo}$, $u_i \to u_{io}$, $u_j \to u_{jo}$, $p_i \to p_{i0}$, $p_j \to p_{j0}$, $n_e \to 1$, $n_p \to \chi$ and $\phi \to 0$ at $|x| \to \infty$.

The above equations (1) to (9) are normalized in the following ways: densities n_i, n_j, n_e, and n_p are normalized by their equilibrium values n_0; velocities u_i, u_j by the ion-acoustic wave speed in the mixture $C_s = \sqrt{\frac{KT_{\text{eff}}}{m_\alpha}}$, where m_α is the mass of ions, T_{eff} is the effective temperature of electrons, and K is the Boltzmann constant; pressures p_i, p_j by $p_0 = Kn_0T_\alpha$, where T_α is the temperature of ions; potential (ϕ) by $\frac{K\,T_{\text{eff}}}{e}$, where e is the electron charge; time (t) by the inverse of the ion-plasma frequency in the mixture $\omega_{p_\alpha}^{-1} = \sqrt{\frac{m_\alpha}{4\pi n_0 e^2}}$ and the space coordinate (x) by the Debye length $\lambda_{\text{D}} = \sqrt{\frac{K\,T_{\text{eff}}}{4\pi n_0 e^2}}$, where $\alpha = i$ stands for positive ions, and $\alpha = j$ stands for negative ions.

Using the boundary conditions and the Galilean transformation $\eta = x - Vt$, where V is the velocity of the solitary waves, we observe that Eqs. (3) and (6) are consistent with the equation of state $p_i = p_{i0}\left(\frac{n_i}{n_{i0}}\right)^3$ and $p_j = p_{j0}\left(\frac{n_j}{n_{j0}}\right)^3$ in the one-dimensional motion; i.e., we consider the adiabatic case, and, hence, $p_{i0} = 1$ and $p_{j0} = 1$. After some calculations (taking terms up to ϕ^4), we get finally from the above equations, by using the Sagdeev pseudopotential method:

$$\begin{aligned}\psi(\phi) = \Bigg[&-\phi - \frac{1}{2}\phi^2 + \\ &+ \frac{8}{15}\frac{\mu b_l + \nu b_h \beta_1^{\frac{3}{2}}}{(\mu + \nu\beta_1)^{\frac{3}{2}}}\phi^{\frac{5}{2}} - \frac{1}{6}\frac{\mu + \nu\beta_1^2}{(\mu + \nu\beta_1)^2}\phi^3 + \\ &+ \frac{16}{105}\frac{\mu b_l^{(1)} + \nu b_h^{(1)}\beta_1^{\frac{5}{2}}}{(\mu + \nu\beta_1)^{\frac{5}{2}}}\phi^{\frac{7}{2}} - \frac{1}{24}\frac{\mu + \nu\beta_1^3}{(\mu + \nu\beta_1)^3}\phi^4\Bigg] + \\ &+ \frac{1}{6}\sqrt{\frac{n_{io}^3}{3\sigma_i}}\Bigg[\left\{\left(V - u_{io} - \sqrt{\frac{3\sigma_i}{n_{io}}}\right)^2 - 2\phi\right\}^{\frac{3}{2}} - \\ &- \left\{\left(V - u_{io} + \sqrt{\frac{3\sigma_i}{n_{io}}}\right)^2 - 2\phi\right\}^{\frac{3}{2}} + \\ &+ \left(V - u_{io} + \sqrt{\frac{3\sigma_i}{n_{io}}}\right)^3 - \left(V - u_{io} - \sqrt{\frac{3\sigma_i}{n_{io}}}\right)^3\Bigg] + \\ &+ \frac{1}{6}\sqrt{\frac{Q^3 n_{io}^3}{3\sigma_j}}\Bigg[\left\{\left(V - u_{jo} - \sqrt{\frac{3\sigma_j}{Qn_{jo}}}\right)^2 + \frac{2Z\phi}{Q}\right\}^{\frac{3}{2}} - \\ &- \left\{\left(V - u_{jo} + \sqrt{\frac{3\sigma_j}{Qn_{jo}}}\right)^2 + \frac{2Z\phi}{Q}\right\}^{\frac{3}{2}} + \\ &+ \left(V - u_{jo} + \sqrt{\frac{3\sigma_j}{Qn_{jo}}}\right)^3 - \left(V - u_{jo} - \sqrt{\frac{3\sigma_j}{Qn_{jo}}}\right)^3\Bigg] + \\ &+ \frac{\chi}{\sigma_p}(1 - e^{-\sigma_p\phi}).\end{aligned} \tag{11}$$

Here,

$$\begin{aligned}n_i = \frac{1}{2}\sqrt{\frac{n_{io}^3}{3\sigma_i}}\Bigg[&\sqrt{\left(V - u_{io} + \sqrt{\frac{3\sigma_i}{n_{io}}}\right)^2 - 2\phi} - \\ &- \sqrt{\left(V - u_{io} - \sqrt{\frac{3\sigma_i}{n_{io}}}\right)^2 - 2\phi}\Bigg],\end{aligned}$$

$$\begin{aligned}n_j = \frac{1}{2}\sqrt{\frac{Qn_{jo}^3}{3\sigma_j}}\Bigg[&\sqrt{\left(V - u_{jo} + \sqrt{\frac{3\sigma_j}{Qn_{jo}}}\right)^2 + \frac{2Z\phi}{Q}} - \\ &- \sqrt{\left(V - u_{jo} - \sqrt{\frac{3\sigma_j}{Qn_{jo}}}\right)^2 + \frac{2Z\phi}{Q}}\Bigg].\end{aligned}$$

Expanding $\psi(\phi)$ in a power series in ϕ by the Taylor formula, one may obtain

$$\frac{d^2\phi}{d\eta^2} = H_1\phi - H_2\phi^{\frac{3}{2}} + H_3\phi^2 - H_4\phi^{\frac{5}{2}} + H_5\phi^3 - ... = -\frac{\partial\psi}{\partial\phi} \tag{12}$$

and

$$\psi(\phi) = -\frac{1}{2}H_1\phi^2 + \frac{2}{5}H_2\phi^{\frac{5}{2}} - \frac{1}{3}H_3\phi^3 + \frac{2}{7}H_4\phi^{\frac{7}{2}} - \frac{1}{4}H_5\phi^4. \tag{13}$$

Here,

$$\begin{aligned}H_1 = \Bigg[&1 - n_{i0}\left\{(V - u_{i0})^2 - \frac{3\sigma_i}{n_{i0}}\right\}^{-1} - \\ &- Z^2 n_{j0}\left\{Q(V - u_{j0})^2 - \frac{3\sigma_j}{n_{j0}}\right\}^{-1} + \chi\sigma_p\Bigg],\end{aligned}$$

$$H_2 = \frac{4}{3}\frac{\left(\mu b_l + \nu b_h\beta_1^{\frac{3}{2}}\right)}{(\mu + \nu\beta_1)^{\frac{3}{2}}},$$

$$H_3 = \frac{1}{2}\left[\frac{\mu+\nu\beta_1^2}{(\mu+\nu\beta_1)^2} - \frac{n_{i0}^{\frac{3}{2}}}{2\sqrt{3\sigma_i}}\left\{\left(V-u_{i0}-\sqrt{\frac{3\sigma_i}{n_{i0}}}\right)^{-3} - \right.\right.$$
$$\left. - \left(V-u_{i0}+\sqrt{\frac{3\sigma_i}{n_{i0}}}\right)^{-3}\right\} +$$
$$+ \frac{Z^3 n_{j0}^{\frac{3}{2}}}{2Q\sqrt{3Q\sigma_j}}\left\{\left(V-u_{j0}-\sqrt{\frac{3\sigma_j}{Qn_{j0}}}\right)^{-3} - \right.$$
$$\left.\left. - \left(V-u_{j0}+\sqrt{\frac{3\sigma_j}{Qn_{j0}}}\right)^{-3}\right\} - \chi\sigma_p^2\right],$$
$$H_4 = \frac{8}{15}\frac{\left(\mu b_l^{(1)}+\nu b_h^{(1)}\beta_1^{\frac{5}{2}}\right)}{(\mu+\nu\beta_1)^{\frac{5}{2}}},$$
$$H_5 = \frac{1}{2}\left[\frac{1}{3}\frac{\mu+\nu\beta_1^3}{(\mu+\nu\beta_1)^3} - \frac{n_{i0}^{\frac{3}{2}}}{2\sqrt{3\sigma_i}}\left\{\left(V-u_{i0}-\sqrt{\frac{3\sigma_i}{n_{i0}}}\right)^{-5} - \right.\right.$$
$$\left. - \left(V-u_{i0}+\sqrt{\frac{3\sigma_i}{n_{i0}}}\right)^{-5}\right\} +$$
$$+ \frac{Z^4 n_{j0}^{\frac{3}{2}}}{2Q^2\sqrt{3\sigma_{jQ}}}\left\{\left(V-u_{j0}+\sqrt{\frac{3\sigma_j}{Qn_{j0}}}\right)^{-5} - \right.$$
$$\left.\left. - \left(V-u_{j0}-\sqrt{\frac{3\sigma_j}{Qn_{j0}}}\right)^{-5}\right\} + \frac{\chi\sigma_p^3}{3}\right]. \quad (14)$$

Now, the conditions for the formation of double layers are:

$$\begin{aligned} &\text{(i)} \quad \psi(\phi)=0 \ \text{ at } \phi=0 \text{ and } \phi=\phi_m \\ &\text{(ii)} \quad \frac{\partial\psi}{\partial\phi}=0 \ \text{ at } \phi=0 \ \text{ and } \phi=\phi_m \\ &\text{(iii)} \quad \frac{\partial^2\psi}{\partial\phi^2}<0 \ \text{ at } \phi=0 \text{ and } \phi=\phi_m \\ &\text{(iv)} \quad \psi(\phi)<0 \text{ for } 0<\phi<\phi_m \text{ and } \phi>\phi_m \end{aligned} \quad (15)$$

Using these conditions, the following relations are obtained finally for small-amplitude ion-acoustic monotonic double layers:

$$H_1 = \frac{2}{3}H_3\phi_m \ \text{ and } H_2 = \frac{5}{3}H_3\phi_m^{\frac{1}{2}}, \quad (16)$$

$$\psi(\phi) = -\frac{1}{3}H_3\phi^2\left(\sqrt{\phi}-\sqrt{\phi_m}\right)^2, \quad (17)$$

$$\frac{\partial\psi(\phi)}{\partial\phi} = -\frac{1}{3}H_3\phi\left(\sqrt{\phi}-\sqrt{\phi_m}\right)\left(3\sqrt{\phi}-2\sqrt{\phi_m}\right), \quad (18)$$

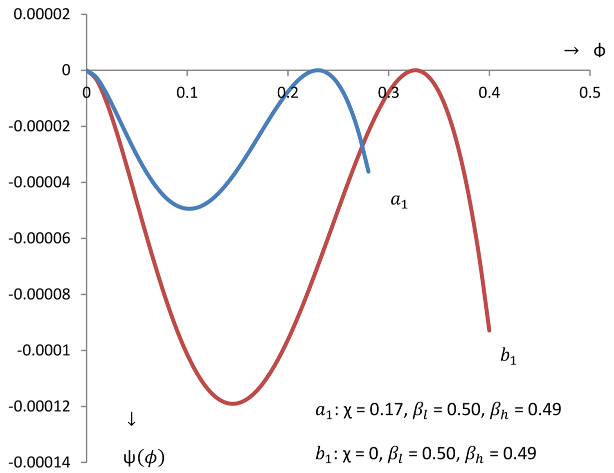
$$\phi_{\rm DL} = \frac{1}{4}\phi_m\left[1-\tanh\left(\sqrt{\frac{H_3\phi_m}{24}}\,\eta\right)\right]^2, \quad (19)$$

where $H_3 > 0$.

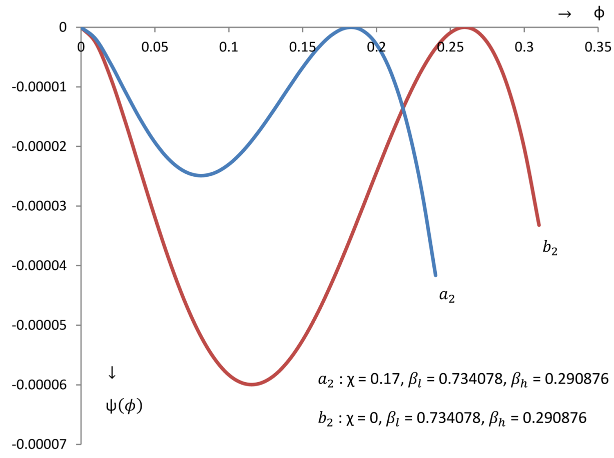
The double layers are localized asymmetric potential structures with a net potential drop. It can accelerate or decelerate or reflect plasma particles. The existence of ion-acoustic double layers has also been observed in auroral and magnetospheric plasmas. In this paper, the trapping and reflecting parameters for electrons are two important and useful parameters determined self-consistently [1, 2, 6, 7] from the phase velocity of the perturbation and its amplitude in a kinetic description. The monotonic double layer described in this paper cannot trap particles, but it reflects them. Thus, the role of reflecting parameters for electrons is mainly responsible for the formation of ion-acoustic monotonic double layers in the small-amplitude case. It is further observed from Ref. [27] that the distribution state of reflected particle is controlled by the trapping parameters β_l and β_h for the two-temperature non-isothermal electron plasma at low and high temperatures. So, we discuss the profiles of the Sagdeev potential function $\psi(\phi)$ against ϕ and double-layer solutions $\phi_{\rm DL}$ against η for small-amplitude double layers under the variation of the trapping parameters for electrons instead of the reflecting parameters for electrons. In the presence of warm negative ions and positrons, we will now show the profiles of the Sagdeev potential function $\psi(\phi)$ and the double-layer solution $\phi_{\rm DL}$ for small-amplitude (weak) monotonic double layers in a two-temperature non-isothermal electron plasma with warm positive ions under the variation of the trapping parameters for electrons $(\beta_l\beta_h)$, concentration of positrons (χ), and mass ratios (Q) of heavier negative ions (m_j) to lighter positive ions (m_i).

3. Results and Discussions

In this paper, the author has studied the profiles of the Sagdeev potential function $\psi(\phi)$ against ϕ and the double-layer solution $\phi_{\rm DL}$ against η for small-amplitude compressive monotonic double layers in a two-temperature non-isothermal electron plasma under the variation of the trapping parameters for electrons (β_l, β_h), concentration of positrons (χ), and mass ratios (Q) of heavier negative ions (m_j) to lighter positive ions (m_i) given in Figs. 1–6. As the distribution state of reflected particles is entirely controlled by the trapping parameters β_l and β_h (Ref. [27]), we are studying the profiles of the Sagdeev potential function $\psi(\phi)$ against ϕ and the double-layer solution $\phi_{\rm DL}$ against η for small-amplitude compressive



a



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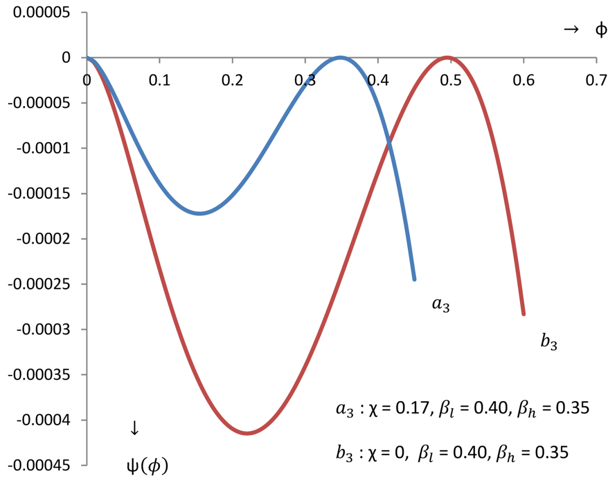

c

Fig. 1. Profiles of the Sagdeev potential function $\psi(\phi)$ against ϕ under the variation of the trapping parameters $(\beta_l\, \beta_h)$ for electrons in the presence $(\chi = 0.17)$ and in the absence $(\chi = 0)$ of positrons for $V = 1.65$, $u_{io} = 0.4$, $u_{jo} = 0.2$, $\sigma_i = \frac{1}{100}$, $\sigma_j = \frac{1}{20}$, $n_{jo} = 0.05$, $n_{io} = 0.88$, 1.05, $Q = 1.9$, $Z = 1$, $\sigma_p = 0.41$, 0; $\beta_1 = 0.25$, $\mu = 0.15$, $\nu = 0.85$: $\beta_l = 0.50$, $\beta_h = 0.49$ (a); $\beta_l = 0.734078$, $\beta_h = 0.290876$ (b); $\beta_l = 0.40$, $\beta_h = 0.35$ (c)

monotonic double layers in a two-temperature non-isothermal electron plasma under the variation of the trapping parameters for electrons (β_l, β_h) instead of the reflecting parameters for electrons.

In Fig. 1, the profiles of the Sagdeev potential function $\psi(\phi)$ against ϕ for small-amplitude double layers in the presence $(\chi = 0.17)$ and in the absence $(\chi = 0)$ of positrons are shown under the variation of the trapping parameters for electrons $(\beta_l,\ \beta_h)$.

In Fig. 1, *a*, the profiles of the Sagdeev potential function $\psi(\phi)$ against ϕ for small-amplitude double layers denoted by the curve a_1 for $\beta_l = 0.50$, $\beta_h = 0.49$ with $\chi = 0.17$, cuts the ϕ – axis at $\phi_m = 0.229726$, while the other curve b_1 for $\beta_l = 0.50$, $\beta_h = 0.49$ with $\chi = 0$ cuts the ϕ axis at $\phi_m = 0.326524$. It is seen from these two curves that, in the absence of positrons, the amplitude (ϕ_m) is larger than the amplitude (ϕ_m) in the presence of positrons for the trapping parameters for electrons $\beta_l = 0.50$ and $\beta_h = 0.49$ satisfying the inequality $0 < b_l$ or $b_h < \frac{1}{\sqrt{\pi}}$.

Figure 1, *b* shows the profiles of the Sagdeev potential function $\psi(\phi)$ against ϕ for small-amplitude double layers in the presence $(\chi = 0.17)$ and in the absence $(\chi = 0)$ of positrons denoted by the curves a_2 for any chosen values of the trapping parameters $\beta_l = 0.734078$, $\beta_h = 0.290876$ satisfying the inequality $0 < b_l$ or $b_h < \frac{1}{\sqrt{\pi}}$ with $\chi = 0.17$ and b_2 for the same chosen values of the trapping parameters $\beta_l = 0.734078$, $\beta_h = 0.290876$ satisfying the inequal-

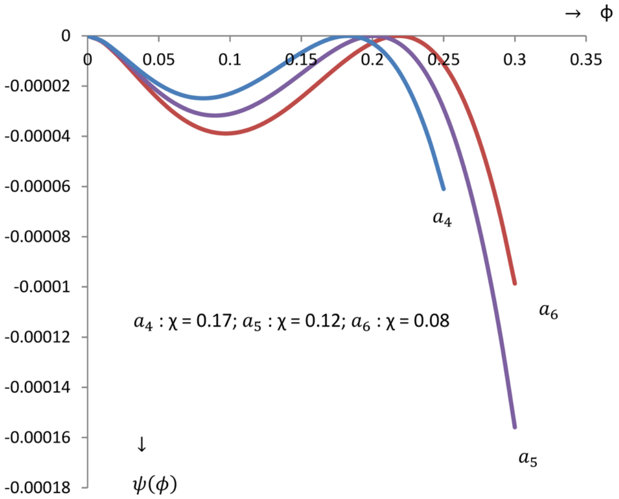

Fig. 2. Profiles of the Sagdeev potential function $\psi(\phi)$ against ϕ for a small-amplitude double layer under the variation of the concentration of positrons (χ) in a two-temperature non-isothermal electron plasma, when $V = 1.65$, $u_{io} = 0.4$, $u_{jo} = 0.2$, $\sigma_i = \frac{1}{100}$, $\sigma_j = \frac{1}{20}$, $n_{jo} = 0.05$, $n_{io} = 0.97, 0.93, 0.88$; $Q = 1.9$, $Z = 1$, $\sigma_p = 0.41$, $\beta_1 = 0.25$, $\mu = 0.15$, $\nu = 0.85$, $\beta_l = 0.734078$, $\beta_h = 0.290876$, $\chi = 0.08, 0.12, 0.17$

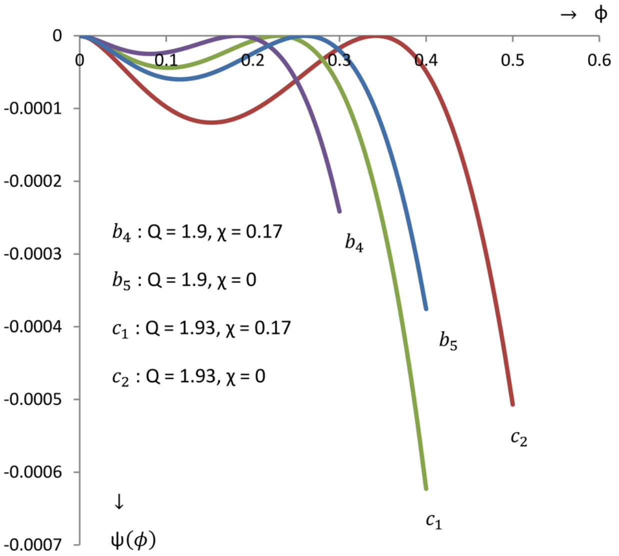

Fig. 3. Profiles of the Sagdeev potential function $\psi(\phi)$ against ϕ for small-amplitude double layers under the variation of the mass ratios (Q) of heavier negative (m_j) to lighter positive (m_i) ions in the presence $(\chi = 0.17)$ and in the absence $(\chi = 0)$ of positrons for a two-temperature non-isothermal electron plasma, when $V = 1.65$, $u_{io} = 0.4$, $u_{jo} = 0.2$, $\sigma_i = \frac{1}{100}$, $\sigma_j = \frac{1}{20}$, $n_{jo} = 0.05$, $n_{io} = 0.88, 1.05$; $Q = 1.9, 1.93$; $Z = 1$, $\sigma_p = 0.41, 0$; $\beta_1 = 0.25$, $\mu = 0.15$, $\nu = 0.85$, $\beta_l = 0.734078$. $b_h = = 0.290876$, $\chi = 0.17, 0$

ity $0 < b_l$ or $b_h < \frac{1}{\sqrt{\pi}}$ with $\chi = 0$. The curve a_2 cuts the ϕ – axis at $\phi_m = 0.182775$, and b_2 cuts the ϕ – axis at $\phi_m = 0.259789$ which are lower values than in Fig. 1, *a*, when $\beta_l = 0.50$ and $\beta_h = 0.49$ satisfying the inequality $0 < b_l$ or $b_h < \frac{1}{\sqrt{\pi}}$.

Similarly, Fig. 1, *c* shows the profiles of the Sagdeev potential function $\psi(\phi)$ against ϕ in the presence $(\chi = 0.17)$ and in the absence $(\chi = 0)$ of positrons for small-amplitude double layers under the variation of the trapping parameter for electrons denoted by the curves a_3 for any chosen values of the trapping parameters $\beta_l = 0.40$, $\beta_h = 0.35$ satisfying the inequality $0 < b_l$ or $b_h < \frac{1}{\sqrt{\pi}}$ with $\chi = 0.17$ and b_3 for $\beta_l = 0.40$, $\beta_h = 0.35$ satisfying the inequality $0 < b_l$ or $b_h < \frac{1}{\sqrt{\pi}}$ with $\chi = 0$. The curve a_3 cuts the ϕ – axis at $\phi_m = 0.348255$, while the other curve b_3 cuts the ϕ – axis at $\phi_m = 0.494997$.

It is clearly seen from Figs. 1, *a, b* and *c* that if the chosen value of the trapping parameter β_l increases, the maximum value (ϕ_m) of the electrostatic potential (ϕ) decreases, and ϕ_m will be larger for smaller values of β_l in the presence $(\chi = 0.17)$ and in the absence $(\chi = 0)$ of positrons. A similar result is observed for increasing values of another chosen values of the trapping parameter β_h. The effect of electron trapping parameters (β_l, β_h) satisfying the inequality $0 < b_l$ or $b_h < \frac{1}{\sqrt{\pi}}$ is shown in the Figs. 1, *a, b* and *c*, respectively.

It follows from Figs. 1, *a, b* and *c* that the values of the trapping parameters used in this paper for electrons are any chosen values, no specific reasons exist for choosing those values of the concerned parameters, provided the trapping parameters (β_l, β_h) must satisfy the inequality $0 < b_l$ or $b_h, < \frac{1}{\sqrt{\pi}}$. Now, we show only the changes of the profiles of the Sagdeev potential function $\psi(\phi)$ against ϕ and double layer solutions ϕ_{DL} against η for small-amplitude double layers in the presence $(\chi \neq 0)$ and in the absence $(\chi = 0)$ of positrons (χ) under the variation of the above-mentioned chosen values of the electron trapping parameters (β_l, β_h) satisfying the inequality $0 < b_l$ or $b_h < \frac{1}{\sqrt{\pi}}$.

In Fig. 2, the profiles of the Sagdeev potential function $\psi(\phi)$ against ϕ for small-amplitude double layers are shown under the variation of the concentration of positrons $(\chi = 0.08, 0.12, 0.17)$ in a two-temperature non-isothermal electron plasma for a particular value of the electron trapping parameter (β_l, β_h) satisfying the inequality $0 < b_l$ or $b_h < \frac{1}{\sqrt{\pi}}$. The double-layer curves formed for $\chi = 0.08$, 0.12 and 0.17 in this figure are represented by a_6, a_5 and a_4. The maximum value (ϕ_m) of the electrostatic potential (ϕ) will be large for a restricted higher value of the density of positrons(χ).

Figure 3 shows the profiles of the Sagdeev potential function $\psi(\phi)$ against ϕ for small-amplitude double layers in the presence ($\chi = 0.17$) and in the absence of ($\chi = 0$) positrons under the variation of the mass ratios (Q) of heavier negative ions (m_j) to lighter positive ions (m_i) for any chosen values of the trapping parameters for electrons (β_l, β_h) satisfying the inequality $0 < b_l$ or $b_h < \frac{1}{\sqrt{\pi}}$. The Sagdeev potential function $\psi(\phi)$ against ϕ for small-amplitude double layers in the presence ($\chi = 0.17$) and in the absence of ($\chi = 0$) positrons are denoted by c_1, c_2 for (K^+, SF_6^-) a plasma with $Q = 1.93$ and by b_4, b_5 for (Ar^+, SF_6^-) a plasma with $Q = 1.9$ [Refs. 10, 28, 29]. It is observed from these curves that, as the mass ratio (Q) increases, the above-mentioned curves cut the $\phi -$axis at larger values than the previous one, found both in the presence and in the absence of positrons for this two-temperature non-isothermal electron plasmas.

In Fig. 4, the profiles of small-amplitude double-layer solution (ϕ_{DL}) against η are shown in the presence ($\chi = 0.17$) and in the absence ($\chi = 0$) of positrons under the variation of any chosen values of the trapping parameters (β_l, β_h) for electrons satisfying the inequality $0 < b_l$ or $b_h < \frac{1}{\sqrt{\pi}}$. The curves so generated for double-layer solutions ϕ_{DL} against η are represented by w_1, w_2 and w_3, when the chosen set of trapping parameters are $\beta_l = 0.40$, $\beta_h = 0.35$; $\beta_l = 0.50$, $\beta_h = 0.49$ and $\beta_l = 0.734078$, $\beta_h = 0.290876$ satisfying the inequality $0 < b_l$ or $b_h < \frac{1}{\sqrt{\pi}}$ with $\chi = 0.17$ shown in Fig. 4, *a*, while those by r_1, r_2 and r_3 with $\chi = 0$ shown in Fig. 4, *b*. In the presence ($\chi = 0.17$) and in the absence ($\chi = 0$) of positrons, it is found that the values of ϕ_{DL} will be smaller, when β_l and β_h are increasing, and this shows the effect of trapping parameters for electrons on the double-layer solution ϕ_{DL} against η, where the electron trapping parameters (β_l, β_h) satisfy the inequality $0 < b_l$ or $b_h < \frac{1}{\sqrt{\pi}}$.

Figure 5 shows the profiles of the small-amplitude double-layer solution (ϕ_{DL}) against η under the variation of the concentration of positrons (χ) for any chosen values of the electron trapping parameters (β_l, β_h)

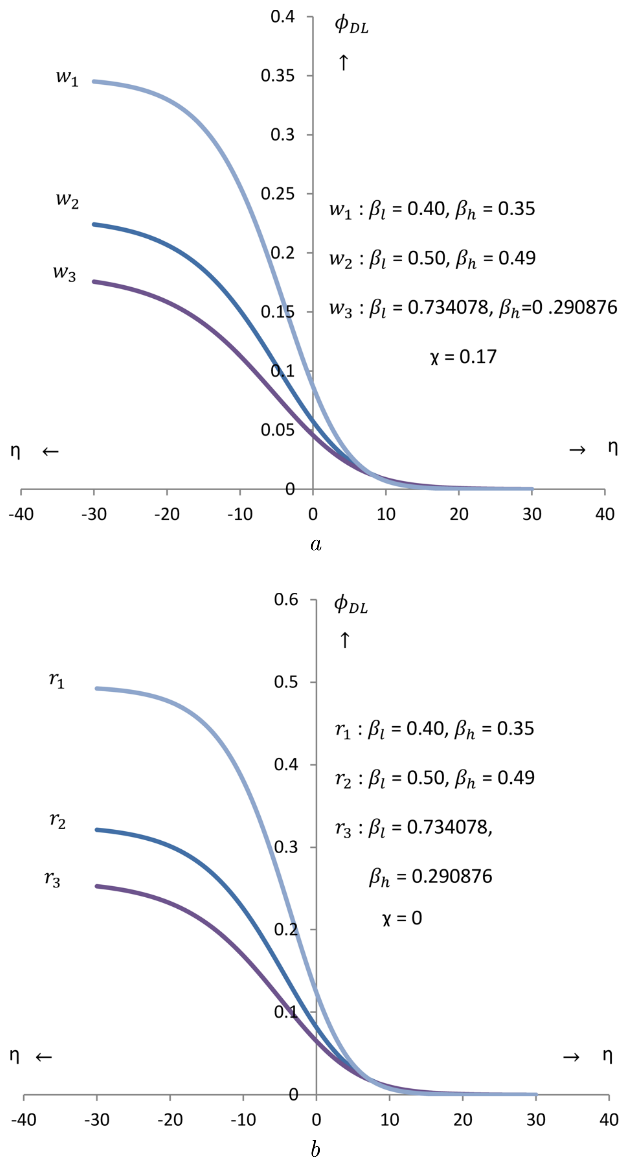

Fig. 4. Profiles of the small-amplitude double-layer solution (ϕ_{DL}) against η in the presence of positrons ($\chi = 0.17$) under the variation of any chosen values of the trapping parameters for electrons (β_l, β_h) for $V = 1.65$, $u_{io} = 0.4$, $u_{jo} = 0.2$, $\sigma_i = \frac{1}{100}$, $\sigma_j = \frac{1}{20}$, $n_{jo} = 0.05$, $n_{io} = 0.88$, $Q = 1.9$, $Z = 1$, $\sigma_p = 0.41$, $\beta_1 = 0.25$, $\mu = 0.15$, $\nu = 0.85$, $\beta_l = 0.40$, $\beta_h = 0.35$; $\beta_l 0.50$, $\beta_h = 0.49$; $\beta_l = 0.734078$, $\beta_h 0.290876$ (*a*). Profiles of the small-amplitude double-layer solution (ϕ_{DL}) against η in the absence of positrons ($\chi = 0$) under the variation of any chosen values of the trapping parameters for electrons (β_l, β_h) for $V = 1.65$, $u_{io} = 0.4$, $u_{jo} = 0.2$, $\sigma_i = \frac{1}{100}$, $\sigma_j = \frac{1}{20}$, $n_{jo} = 0.05$, $n_{io} = 1.05$, $Q = 1.9$, $Z = 1$, $\sigma_p = 0$, $\beta_1 = 0.25$, $\mu = 0.15$, $\nu = 0.85$, $\beta_l = 0.40$, $\beta_h = 0.35$; $\beta_l = 0.50$, $\beta_h = 0.49$; $\beta_l = 0.734078$, $\beta_h = 0.290876$ (*b*)

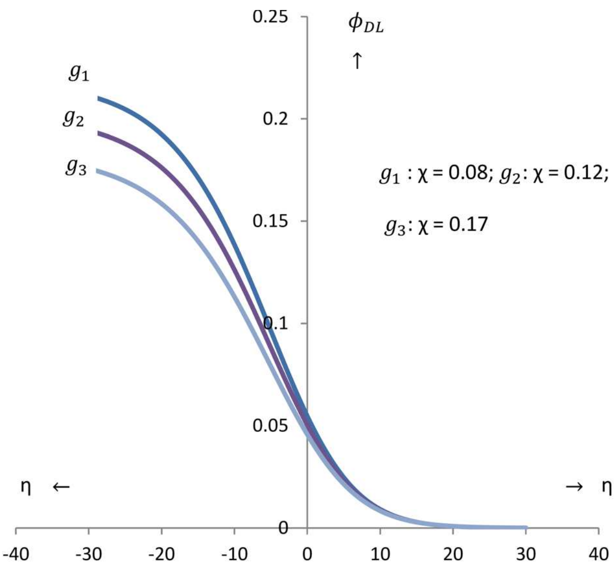

Fig. 5. Profiles of the small-amplitude double-layer solution ($\phi_{\rm DL}$) against η under the variation of the concentration of positrons (χ) in a two-temperature non-isothermal electron plasma, when $V = 1.65$, $u_{io} = 0.4$, $u_{jo} = 0.2$, $\sigma_i = \frac{1}{100}$, $\sigma_j = \frac{1}{20}$, $n_{jo} = 0.05$, $n_{io} = 0.97$, 0.93, 0.88; $Q = 1.9$, $Z = 1$, $\sigma_p = 0.41$, $\beta_1 = = 0.734078$, $\beta_h = 0.290876$, $\chi = 0.08$, 0.12, 0.17

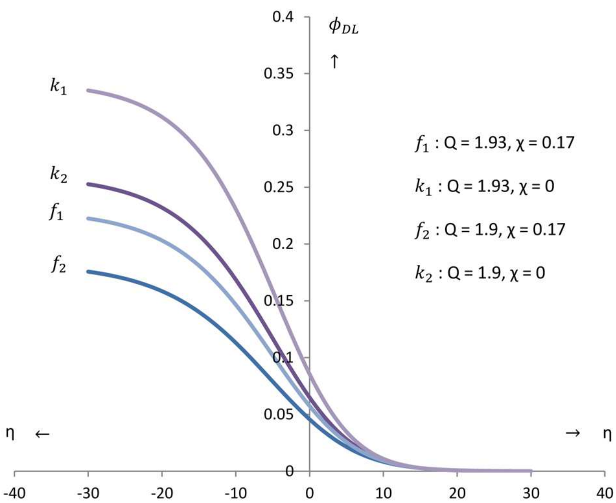

Fig. 6. Profiles of the double-layer solutions ($\phi_{\rm DL}$) against η for small-amplitude double layers under the variation of the mass ratios (Q) of negative (m_j) to positive (m_i) ions in the presence ($\chi = 0.17$) and in the absence ($\chi = 0$) of positrons for a two-temperature non-isothermal electron plasma, when $V = 1.65$, $u_{io} = 0.4$, $u_{jo} = 0.2$, $\sigma_i = \frac{1}{100}$, $\sigma_j = \frac{1}{20}$, $n_{jo} = 0.05$, $n_{io} = 0.88$, 1.05; $Q = 1.93$, 1.9; $Z = 1$, $\sigma_p = 0.41$, 0; $\beta_1 = 0.25$, $\mu = 0.15$, $\nu = 0.85$, $\beta_l = 0.734078$. $\beta_h = 0.290876$, $\chi = 0.17$, 0

satisfying the inequality $0 < b_l$ or $b_h < \frac{1}{\sqrt{\pi}}$. The small-amplitude double-layer solution ($\phi_{\rm DL}$) curves against η are represented by g_1, g_2 and g_3 for $\chi =$ = 0.08, 0.12 and 0.17. The curve g_1 for $\chi = 0.08$, g_2 for $\chi = 0.12$ and g_3 for $\chi = 0.17$ cut the $\phi_{\rm DL}$- axis at 0.054617, 0.050354 and 0.045694, respectively. The curve g_1 for $\chi = 0.08$ cuts the $\phi_{\rm DL}$ axis at larger values than those of the curves g_2 for $\chi = 0.12$ and g_3 for $\chi = 0.17$. It is thus concluded that the value of $\phi_{\rm DL}$ will be higher for smaller values of the concentration of positrons (χ).

In Fig. 6, the profiles of the samall-mplitude double-layer solution ($\phi_{\rm DL}$) against η in the presence ($\chi = 0.17$) and in the absence of ($\chi = 0$) positrons under the variation of the mass ratios (Q) of heavier negative ions (m_j) to lighter positive ions (m_i) for any chosen values of the trapping parameters for electrons (β_l, β_h) satisfying the inequality $0 < b_l$ or $b_h < \frac{1}{\sqrt{\pi}}$. In the presence ($\chi = 0.17$) and in the absence ($\chi = 0$) of positrons for ($K^+$, SF_6^-) a plasma with $Q = 1.93$, the profiles of small-amplitude double-layer solutions ($\phi_{\rm DL}$) against η are denoted by $f_1\,(\chi = 0.17)$ and $k_1\,(\chi = 0)$, while those for (Ar^+, SF_6^-) a plasma with $Q = 1.9$ [Refs. 10, 28, 29], the profiles of small-amplitude double-layer solutions ($\phi_{\rm DL}$) against η are denoted by $f_2\,(\chi = 0.17)$ and $k_2\,(\chi = 0)$. It is observed from these figures that, in the absence of positrons ($\chi = 0$), the respective curves cut the $\phi_{\rm DL}$ – axis at larger values than in the presence of positrons ($\chi = 0.17$) for the two values of the mass ratio (Q).

4. Conclusions

The profiles of the Sagdeev potential function $\psi(\phi)$ against ϕ and the double-layer solution $\phi_{\rm DL}$ against η for small-amplitude monotonic double layers have been investigated theoretically under the variation of the trapping parameters (β_l, β_h) for electrons satisfying the inequality $0 < b_l$ or $b_h < \frac{1}{\sqrt{\pi}}$, concentration of positrons (χ), and mass ratios (Q) of heavier negative ions (m_j) to lighter positive ions (m_i). This present theoretical attempt shows that the distribution state of reflected particles is controlled by the trapping parameters of the two-temperature non-isothermal electrons at low and high temperatures (observed by Tae Han Kim, 2006, Ref. 27) for the formation of small-amplitude monotonic double layers. So, we have discussed the Sagdeev potential function $\psi(\phi)$ against ϕ and the double-layer solutions $\phi_{\rm DL}$ against η in favour of the trapping parameters for electrons instead of reflecting parameters for electrons. The effects of trapping parameters for electrons (β_l, β_h) are shown in Fig. 1, *a* to Fig. 1, *c* for the presence $(\chi = 0.17)$ and absence $(\chi = 0)$ of positrons, while Fig. 2 shows the effects of the concentration (χ) of positrons, and Fig. 3 represents the effects of the mass ratios (Q) of heavier negative ions (m_j) to lighter positive ions (m_i). On the other hand, Figs. 4 to 6 show the effects of the trapping parameters for electrons (β_l, β_h) where the trapping parameters (β_l, β_h) satisfy the inequality $0 < b_l$ or $b_h < \frac{1}{\sqrt{\pi}}$, the concentration of positrons (χ) and mass ratios (Q) of heavier negative ions (m_j) to lighter positive ions (m_i) on the profiles of small-amplitude monotonic double-layer solutions (ϕ_{DL}) against η. It is also found that the present system supports small-amplitude compressive monotonic double layers, when the ratio (σ_j) of the temperatures of negative ions (T_j) and electrons (T_{eff}) is greater than such ratio (σ_i) for positive ions (T_i) and electrons (T_{eff}). Ion-acoustic double layers have been observed in the magnetospheric and auroral plasmas and also exist in those plasmas, where negative ions are present. In the presence $(\chi = 0.17)$ and in the absence $(\chi = 0)$ of positrons, the profiles of the Sagdeev potential function $\psi(\phi)$ against ϕ and the double-layer solution ϕ_{DL} against η for small-amplitude ion-acoustic monotonic double layers are presented along with the variation of the amplitudes for (K^+, SF_6^-) and (Ar^+, SF_6^-) plasmas. The most important observation is the effect of the trapping parameters for non-isothermal electrons on the Sagdeev potential function $\psi(\phi)$ and the double-layer solution ϕ_{DL}. It is seen that if the trapping parameters (β_l, β_h) for electrons are increasing, the amplitudes of the double layers are decreasing which is an important situation. The amplitudes are also increasing for the increasing values of the concentration of positrons (χ) and mass ratios (Q) of heavier negative ions (m_j) to lighter positive ions (m_i) for ion-acoustic small-amplitude monotonic double layers. The higher order trapping of electrons in the potential well yields new findings of spiky and explosive solitary waves along with double layers which are not yielded from "Freja Scientific Satellite observations". Moreover, this indicates that the order of non-linearity causes by the degree of trapped electrons which might show several acoustic modes in the space. The degree of non-isothermality can be adjusted by controlling the experimental plasma parameters from outside and the desired features of different double-layer structures can be observed. We note that double layers can be created in discharge tubes, where the sustained energy is provided within the layer for the electron acceleration. Finally, we would like to point out that the results presented in this paper may be helpful in understanding the acceleration of charged particles to high energies in space and astrophysical plasmas. The future plan of the present author is to find large-amplitude ion-acoustic double layers in magnetized plasmas consisting of warm negative ions, warm positive ions, and two-temperature isothermal electrons by the Sagdeev pseudopotential method.

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ІСНУВАННЯ НИЗЬКОАМПЛІТУДНИХ ПОДВІЙНИХ ШАРІВ МАЛОЇ АМПЛІТУДИ У НЕІЗОТЕРМІЧНІЙ ПЛАЗМІ З ДВОМА ТЕМПЕРАТУРАМИ

Методом псевдопотенціалу Сагдєєва теоретично досліджено іонно-акустичні монотонні подвійні шари малої амплітуди у плазмі, яка складається з розігрітих позитивних іонів, позитронів і неізотермічних електронів з двома температурами, при зміні параметрів захоплення електронів, концентрації позитронів і співвідношення мас важчих негативних іонів до легших позитивних іонів. Іонно-акустичні поодинокі хвилі та подвійні шари спостерігалися в авроральній та магнітосферній плазмі з двотемпературним розподілом електронів, як у лабораторних умовах, так і в космосі. У даній роботі показано вплив параметрів захоплення електронів, концентрації позитронів і відношення мас важчих негативних іонів до легших позитивних іонів на потенціальну функцію Сагдєєва $\psi(\phi)$ і розв'язки подвійного шару $\phi_{\rm DL}$ для монотонних подвійних шарів малої амплітуди. Результати подано графічно.

Ключові слова: неізотермічні електрони з двома температурами, метод потенціалу Сагдєєва, більш важкі негативні іони, подвійні шари, розв'язки з подвійним шаром.