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DARCY–BRINKMAN BIO-THERMAL CONVECTION IN A POROUS ROTATING LAYER SATURATED BY A NEWTONIAN FLUID CONTAINING GYROTACTIC MICROORGANISMS

The bio-thermal convection in a rotating layer of a porous medium saturated with a Newtonian fluid with gyrotactic microorganisms is studied on the basis of the Darcy–Brinkman model. A linear analysis of the bio-thermal convection is carried out using the Galerkin method for rigid-rigid boundary conditions. In a stationary regime, we obtained a dispersion equation with a relation between the thermal Rayleigh–Darcy number and the Rayleigh–Darcy number of bioconvection. The influence of the Peclet number, gyrotaxis, Darcy number, Rayleigh–Darcy number, cell eccentricity, and rotation parameter on bioconvective processes is analyzed and shown graphically. The results indicate that an increase in the rotation parameter (Taylor number) delays the onset of the bioconvection, whereas an increase in the cell eccentricity can stimulate the onset of the bioconvection.

Keywords: Darcy–Brinkman model, bio-thermal convection, Coriolis force, porous medium, gyrotactic microorganism.

1. Introduction

The study of the physics of the flow of fluids through a porous medium is of great practical importance in such fields as soil mechanics, groundwater hydrology, oil production, industrial filtration, *etc.* The well-known books by Ingham and Pop [1], Nield and Bejan [2] are devoted to the problems of thermal instability of a fluid layer in a porous medium. The issues of the fluid flow and heat transfer in rotating porous media are studied in detail in the recent review by Vadasz [3].

Recently, a new area of research has attracted the increasing attention: bioconvection in porous media. The problems of bacterial movement and biofilm growth are of current importance in the technologies of microbiological oil production. In this regard, theoretical studies are needed in the field of the interaction between bio- and natural convections.

Bioconvection accounts for the movement of bacteria and algae, which have a greater density than water. Gravity forces (gyrotactic microorganisms), oxygen concentration gradients (oxytactic microorganisms), light radiation (phototaxis microorganisms), nutrition gradients (chemotaxis microorganisms), and other variables can cause bacteria to migrate. The number of self-propelled microorganisms can be rather large, ranging from 10^7 cm^{-3} in a low concentration regime to 10^{11} cm^{-3} in a turbulent regime containing nearly densely packed microorganisms. Pedley *et al.* [4–6] constructed a linear theory of the stability of bioconvection of gyrotactic microorganisms in a limited-depth layer of an ordinary fluid. The conditions for the start of a bioconvective flow were identified in these studies.

There are a lot of publications on the effect of gyrotactic microorganisms on fluid flows in bounded porous media. Kuznetsov, Nield, and Avramenko [7–11] made a significant contribution to the dynamics of biological processes in porous media. In [7], it is

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Цитування: Копп М.Й., Яновській В.В. Біотеплова конвекція Дарсі–Брінкмана в пористому шарі, який обертається, насиченому ньютонівською рідиною, що містить гіротактичні мікроорганізми. *Укр. фіз. журн.* **68**, №1, 30 (2023).

established that if the permeability is smaller than critical, the system is stable and the bioconvection does not develop. On the contrary, if the permeability is greater than critical, the bioconvection may develop. In [8], they studied the problem of the occurrence of bioconvection in a horizontal layer occupied by a saturated porous medium. The critical Rayleigh numbers are obtained for various values of the Peclet number of bioconvection, the gyrotaxis number, and the cell eccentricity. In [9], the influence of a vertical flow on the onset of bioconvection in a suspension of gyrostatic microorganisms in a porous medium was studied. On the basis of a linear analysis, an equation for the critical Rayleigh number was obtained. In [9], it was shown that the vertical through flow stabilizes the considered system. In [10], a continuum model of thermobioconvection of oxytate bacteria in a porous medium is presented to study the effect of the heating of microorganisms from below on the stability of the medium in a horizontal layer filled with a fluid saturated with a porous medium. Using the Galerkin method to solve a problem of linear stability made it possible to obtain the relation between the critical value of the Rayleigh number and the thermal Rayleigh number. In [11] within the Lorenz approach [12], a nonlinear theory of bioconvection for gyrotactic microorganisms in a layer of an ordinary liquid was developed. In [11], the boundaries of various hydrodynamic regimes of the two-dimensional bioinvection were determined.

Hwang and Pedley [13] investigated the role of uniform shear at the bioconvective instability of a shallow suspension of swimming gyrotactic cells. The shear was introduced by applying a flat Couette flow, which significantly violated the gravity of the cell. It has been shown that the bioconvective instability in a fine suspension arises as a result of three different physical processes: gravitational overturning, cell gyrotaxis, and negative cross-diffusion flow. Shear with sufficiently high velocities also plays a stabilizing role, as in the Rayleigh–Benard convection. However, at low shear rates, it destabilizes these perturbations through the overstability mechanism discussed by Hill, Pedley, and Kessler [5]. A detailed review of the main aspects of the bioconvection process in nanofluids and porous media is presented by Dmitrenko [14]. In [14], a mathematical model of bioconvection based on Darcy's law for porous media is given.

Sharma and Kumar [15] investigated the influence of high-frequency vertical vibrations on the onset of bioconvection in a dilute solution of gyrotactic microorganisms analytically and numerically. It was observed [15] that high-frequency low-amplitude vertical vibrations and the bioconvection Peclet number have a stabilizing effect on the system. A more detailed analysis of the stability of a vibrational system of shallow layers filled with randomly swimming gyrotactic microorganisms was carried out by Kushwaha *et al.* [16].

Zhao *et al.* [17] expanded the study of bio-thermal convection [18] to the case of a highly porous medium saturated with a suspension of gyrotactic microorganisms. Darcy's law states that the drag is proportional to the velocity in a low-porosity porous medium. The Darcy–Brinkman equation is commonly used to investigate the flows in porous media with a high porosity. The classical Darcy equation is extended with an additional Laplacian (viscous) element (Brinkman term) [19]. Based on the Darcy–Brinkman model, work [17] presents a stability analysis for the bio-thermal convection in a suspension of gyrotactic microorganisms in a highly porous medium heated from below.

It follows from the above review of the literature that no work has yet been reported on the influence of the rotation effect (Coriolis force) on the bio-thermal convection in a layer of a porous medium saturated with a suspension containing gyrotactic microorganisms. Thus, the aim of this paper is to investigate the abovementioned problem. This study can be used in laboratory and geophysical researches on the bioconvection in highly porous media.

2. Statement of the Problem and Basic Equations

Let us consider the infinite horizontal layer of a porous medium saturated by a Newtonian fluid containing gyrotactic microorganisms. The porous layer with a thickness of h is kept rotating about the vertical axis at a constant angular velocity Ω_0 and heated from below, as shown in Fig. 1. The heating from below of the layer causes a disturbance, where T_0, T_1 are the temperatures at the lower and upper boundaries, respectively. Figure 1 depicts problem's geometry. We use the Cartesian coordinates (x, y, z) with the z-axis pointing vertically upward. The gravitational force $\mathbf{g} = (0, 0, -g)$ acts vertically downwards. For a di-

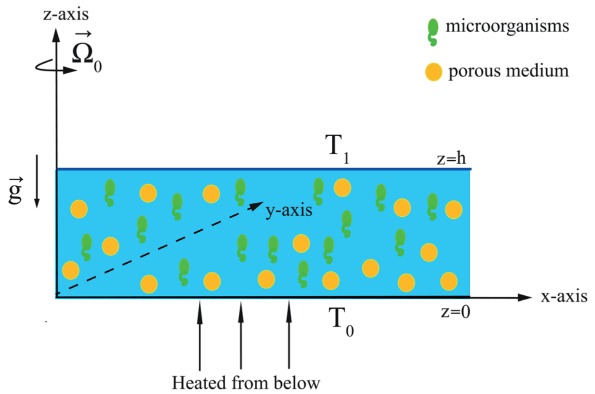

Fig. 1. Geometry and coordinate system of the problem

lute suspension of swimming microorganisms, we assume that it is incompressible, and that the porous matrix does not absorb microorganisms. In addition, the Boussinesq approximation is used in the Darcy–Brinkman model, and the influence of the rotation effect is taken into account by the Coriolis force. According to the above assumptions, the continuity, momentum, and energy equations and conservation equation for cells are as follows [2, 3, 5, 7]:

$$\nabla\cdot\mathbf{V}_D=0, \tag{1}$$

$$\frac{\rho_0}{\varepsilon}\frac{\partial\mathbf{V}_D}{\partial t}=-\nabla P+\widetilde{\mu}\nabla^2\mathbf{V}_D-\frac{\mu}{K}\mathbf{V}_D+\frac{2\rho_0}{\varepsilon}[\mathbf{V}_D\times\mathbf{\Omega_0}]+$$
$$-\mathbf{e}g\rho_0(1-\beta(T-T_0))-\mathbf{e}g(\delta\rho)\mathcal{V}n, \tag{2}$$

$$(\rho c)_m\frac{\partial T}{\partial t}+(\rho c)_f\mathbf{V}_D\cdot\nabla T=k_m\nabla^2 T, \tag{3}$$

$$\frac{\partial n}{\partial t}=-\mathrm{div}\left(n\mathbf{V}_D+nW_c\hat{\mathbf{l}}-D_m\nabla n\right). \tag{4}$$

Here, $\mathbf{V}_D=(u,v,w)$ is the Darcy velocity, which is related to the fluid velocity $\mathbf{V}$ as $\mathbf{V}_D=\varepsilon\mathbf{V}$, ε is the porosity of the porous medium, K is the permeability of the porous medium, ρ_{f0} is fluid's density at the reference temperature, P is the pressure, β is the thermal expansion coefficient, g is the gravitational acceleration, $\mathbf{e}=(0,0,1)$ is a unit vector in the direction of the axis z, $\widetilde{\mu}$ is the Brinkman effective viscosity, μ is the viscosity of the fluid, $(\rho c)_f$ is the heat capacity of the fluid, $(\rho c)_m$ is the effective heat capacity, k_m is the effective thermal conductivity, n is the concentration of microorganisms, $\delta\rho$ is the density difference between microorganisms and the base fluid: $\rho_m-\rho_f$, $\mathcal{V}$ is the average volume of a microorganism, and D_m is the diffusivity of microorganisms. We assumed that random motions of microorganisms are simulated by a diffusion process, $W_c\hat{\mathbf{l}}$ is the average microorganism swimming velocity (W_c is constant, $\hat{\mathbf{l}}$ is a unit vector of the movement of microorganisms).

The temperature is considered to remain constant at the borders. Thus, the boundary conditions are as follows [8, 17]:

$$w=0,\quad T=T_0,\quad \mathbf{J}\cdot\mathbf{e}=0,\quad \text{at } z=0, \tag{5}$$

$$w=0,\quad T=T_1,\quad \mathbf{J}\cdot\mathbf{e}=0,\quad \text{at } z=h, \tag{6}$$

where $\mathbf{J}=n\frac{\mathbf{V}_D}{\varepsilon}+nW_c\hat{\mathbf{l}}-D_m\nabla n$ is the flux of microorganisms.

Let us introduce the following non-dimensional parameters

$$(x^*,y^*,z^*)=\frac{(x,y,z)}{h},\quad \mathbf{V}_D^*=\mathbf{V}_D\frac{h}{\alpha_m},$$
$$t^*=\frac{t\alpha_m}{h^2\widetilde{\sigma}},\quad T^*=\frac{T-T_1}{T_0-T_1}, \tag{7}$$
$$P^*=\frac{PK}{\mu\alpha_m},\ \alpha_m=\frac{k_m}{(\rho c)_f},\ \widetilde{\sigma}=\frac{(\rho c)_m}{(\rho c)_f},\ n^*=n\mathcal{V}.$$

Using expressions (7) and omitting the asterisks, we get the following system of dimensionless equations:

$$\nabla\cdot\mathbf{V}_D=0, \tag{8}$$

$$\frac{1}{\mathcal{V}_a}\frac{\partial\mathbf{V}_D}{\partial t}=-\nabla P+D_a\nabla^2\mathbf{V}_D-\mathbf{V}_D-\mathbf{e}\frac{\mathrm{R}_b}{L_b}n+$$
$$+\mathbf{e}\,\mathrm{Ra}\,T+\sqrt{\mathrm{Ta}}\,[\mathbf{V}_D\times\mathbf{e}], \tag{9}$$

$$\frac{\partial T}{\partial t}+(\mathbf{V}_D\nabla)T=\nabla^2 T, \tag{10}$$

$$\frac{1}{\widetilde{\sigma}}\frac{\partial n}{\partial t}=-\nabla\left(n\mathbf{V}_D+\frac{\mathrm{Pe}}{L_b}n\hat{\mathbf{l}}-\frac{1}{L_b}\nabla n\right). \tag{11}$$

In Eqs. (8)–(11), we introduced the following dimensionless parameters:

$$\mathcal{V}_a=\frac{\varepsilon(\rho c)_m\widetilde{\mu}}{\rho_0 k_m D_a}$$

is the modified Vadasz number,

$$D_a=\frac{\widetilde{\mu}K}{\mu h^2}$$

is the Darcy number,

$$\mathrm{Ta}=\frac{4\Omega_0^2K^2}{\varepsilon^2\mu^2}\rho_0^2$$

is the Taylor–Darcy number,

$$\mathrm{R}_b=\frac{g(\delta\rho)hK}{\mu D_m}$$

is the bioconvection Rayleigh–Darcy number,

$$L_b = \frac{\alpha_m}{D_m}$$

is the bioconvection Lewis number,

$$\mathrm{Ra} = \frac{\rho_0 g h K \beta (T_0 - T_1)}{\mu \alpha_m}$$

is the Rayleigh–Darcy number,

$$\mathrm{Pe} = \frac{W_c h}{D_m}$$

is the bioconvection Peclet number. Equations (8)–(11) are supplemented with boundary conditions in non-dimensional form:

$$w = 0, \quad T = 1, \quad n\mathrm{Pe} = \frac{dn}{dz}, \quad \text{at } z = 0, \tag{12}$$

$$w = 0, \quad T = 0, \quad n\mathrm{Pe} = \frac{dn}{dz}, \quad \text{at } z = 1. \tag{13}$$

3. Basic State

Assume that the basic state is time-independent and is given by

$$\begin{aligned} &\mathbf{V}_D = \mathbf{V}_b = 0, \quad P = P_b(z), \\ &T = T_b(z), \quad n = n_b(z). \end{aligned} \tag{14}$$

The steady profiles of the temperature $T_b(z)$, concentration of microorganisms $n_b(z)$, and the pressure distribution $P_b(z)$ in the basic state are found from the solutions of the equations:

$$\frac{d^2 T_b}{dz^2} = 0, \tag{15}$$

$$\frac{dn_b}{dz} = n_b(z)\,\mathrm{Pe}, \tag{16}$$

$$\frac{dP_b}{dz} = -\frac{\mathrm{R}_b}{L_b} n_b(z) + \mathrm{Ra}\, T_b(z). \tag{17}$$

After integrating Eq. (15) and taking the boundary conditions (12)–(13) into account, we obtain the temperature distribution $T_b(z)$ as a linear dependence on z:

$$T_b(z) = 1 - z. \tag{18}$$

Futher, we obtain a solution for n_b, which coincides with the result of paper [8]

$$n_b(z) = n_b(0) \exp(\mathrm{Pe}\, z), \tag{19}$$

where $n_b(0) = n_0$ is the value of the number density at the bottom of the layer. The constant $n_b(0)$ is found as

$$n_b(0) = \frac{\langle n \rangle \mathrm{Pe}}{\exp(\mathrm{Pe}) - 1}, \quad \langle n \rangle = \int_0^1 n_b(z) dz. \tag{20}$$

By assuming that $P = P_0$ at $z = 1$, we find the pressure distribution in the basic state as

$$\begin{aligned} P_b(z) = P_0 + \frac{\mathrm{R}_b}{L_b \mathrm{Pe}} n_b(0)(e^{\mathrm{Pe}} - e^{z\mathrm{Pe}}) + \\ + \mathrm{Ra}\left(z - \frac{z^2 + 1}{2}\right). \end{aligned} \tag{21}$$

4. Stationary Perturbation Equations

The heating from below of the mixed nanofluid layer causes disturbances in the main flow, and these disturbances are assumed to be small:

$$\begin{aligned} &\mathbf{V}_D = \mathbf{V}', \; T = T_b + T', \; n = n_b + n', \\ &P = P_b + P', \; \hat{\mathbf{l}} = \mathbf{e} + \widehat{\mathbf{m}}', \end{aligned} \tag{22}$$

According to publications [4, 7], the equation for the perturbation of a unit vector indicating the direction of the swimming of microorganisms has the following form:

$$\widehat{\mathbf{m}}' = \mathcal{B}\zeta \mathbf{i} - \mathcal{B}\xi \mathbf{j} + 0 \cdot \mathbf{e}, \tag{23}$$

where $\mathbf{i}$ and $\mathbf{j}$ are the unit vectors in the x- and y-directions, respectively; $\mathcal{B} = (3\widetilde{\mu}/\rho_m g d)(\alpha_m/h^2)$ is a dimensionless parameter characterizing the reorientation of microorganisms under the action of a gravitational moment against the viscous resistance, d is a displacement of the center of mass of the cell from the center of buoyancy. In Eq. (23), the parameters ζ and ξ in the x- and y-components of the vector $\widehat{\mathbf{m}}'$ are

$$\begin{aligned} &\zeta = -(1 - \alpha_0)\frac{\partial w'}{\partial x} + (1 + \alpha_0)\frac{\partial u'}{\partial z}, \\ &\xi = (1 - \alpha_0)\frac{\partial w'}{\partial y} - (1 + \alpha_0)\frac{\partial v'}{\partial z}, \end{aligned} \tag{24}$$

α_0 is the cell eccentricity which is given by the following equation [4, 7]:

$$\alpha_0 = \frac{r_{\max}^2 - r_{\min}^2}{r_{\max}^2 + r_{\min}^2}, \tag{25}$$

where $r_{\max}$ and $r_{\min}$ are the semimajor and semiminor axes of the spheroidal cell.

We substitute expressions (22) in Eqs. (8)–(11), then the resulting equations for variables u', v', w', T', n' are linearized. As a result, we get

$$\nabla \cdot \mathbf{V}' = 0, \tag{26}$$

$$\frac{1}{\mathcal{V}_a}\frac{\partial \mathbf{V}'}{\partial t} = -\nabla P' + D_a \nabla^2 \mathbf{V}' - \mathbf{V}' - \mathbf{e}\frac{\mathrm{R}_b}{L_b} n' + + \mathbf{e}\,\mathrm{Ra}\, T' + \sqrt{\mathrm{Ta}}\,[\mathbf{V}' \times \mathbf{e}], \tag{27}$$

$$\frac{\partial T'}{\partial t} + w'\frac{dT_b}{dz} = \nabla^2 T', \tag{28}$$

$$\frac{1}{\widetilde{\sigma}}\frac{\partial n'}{\partial t} = -w'\frac{dn_b}{dz} - \frac{\mathrm{Pe}}{L_b}\frac{\partial n'}{\partial z} - \frac{1}{L_b}\nabla^2 n' + \mathrm{PeG} n_b \times \times \left((1+\alpha_0)\frac{d^2 w'}{dz^2} + (1-\alpha_0)\left(\frac{\partial^2 w'}{\partial x^2} + \frac{\partial^2 w'}{\partial y^2}\right)\right), \tag{29}$$

where $\mathrm{G} = D_m \mathcal{B}/h^2$ is a dimensionless orientation parameter [4].

In order to eliminate the pressure, we operate with ($\mathbf{curl} \equiv \nabla\times$) on Eq. (27) and obtain

$$\left(D_a\nabla^2 - \frac{1}{\mathcal{V}_a}\frac{\partial}{\partial t} - 1\right)(\nabla \times \mathbf{V}') = \frac{\mathrm{R}_b}{L_b}(\nabla n' \times \mathbf{e}) - - \mathrm{Ra}\,(\nabla T' \times \mathbf{e}) - \sqrt{\mathrm{Ta}}\nabla \times (\mathbf{V}' \times \mathbf{e}). \tag{30}$$

Operating on Eq. (30) with **curl** and with the identity **curl curl** = **grad div** $-\nabla^2$, we find equations for the z-component

$$\left(D_a\nabla^2 - \frac{1}{\mathcal{V}_a}\frac{\partial}{\partial t} - 1\right)\omega_z' = -\sqrt{\mathrm{Ta}}\,\frac{\partial w'}{\partial z}, \quad \omega_z' = (\nabla \times \mathbf{V}')_z, \tag{31}$$

$$\left(D_a\nabla^2 - \frac{1}{\mathcal{V}_a}\frac{\partial}{\partial t} - 1\right)\nabla^2 w' = \frac{\mathrm{R}_b}{L_b}\nabla_\perp^2 n' - \mathrm{Ra}\nabla_\perp^2 T' + + \sqrt{\mathrm{Ta}}\frac{\partial \omega_z'}{\partial z}, \quad \nabla_\perp^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}. \tag{32}$$

Next, the method of normal modes is applied to analyze the equations for perturbations (28)–(31). In the absence of the oscillatory convection, we can take that the perturbation quantities are of the form

$$[w', \omega_z', T', n'] = [W(z), Z(z), \Theta(z), N(z)]e^{i(k_x x + k_y y)}, \tag{33}$$

where $a = \sqrt{k_x^2 + k_y^2}$ is the horizontal wave number of the disturbances. Substituting (33) into Eqs. (28), (29) and (31), (32), we get

$$D_a(D^4 - 2a^2D^2 + a^4)\widetilde{W} - (D^2 - a^2)\widetilde{W} + a^2\widehat{\mathrm{R}}_b\widetilde{N} - - a^2\mathrm{Ra}\,\widetilde{\Theta} - \sqrt{\mathrm{Ta}}\,D\widetilde{Z} = 0, \quad D \equiv \frac{d}{dz}, \tag{34}$$

$$(D_a(D^2 - a^2) - 1)\widetilde{Z} + \sqrt{\mathrm{Ta}}\,D\widetilde{W} = 0, \tag{35}$$

$$(D^2 - a^2)\,\widetilde{\Theta} + \widetilde{W} = 0, \tag{36}$$

$$\mathrm{Pe}^{-1}(D^2 - a^2)\widetilde{N} - D\widetilde{N} - e^{z\mathrm{Pe}}(1 + \mathrm{G}(a^2(1-\alpha_0) - - (1+\alpha_0)D^2))\,\widetilde{W} = 0. \tag{37}$$

Here, $\widetilde{W} = L_b W$, $\widetilde{Z} = L_b Z$, $\widetilde{\Theta} = L_b\Theta$, $\widetilde{N} = N/n_0$ are the rescaled variables, $\widehat{\mathrm{R}}_b = n_0\mathrm{R}_b$.

The boundary conditions considered for solving the given system of equations (34)–(37) are

$$\widetilde{W} = 0, \quad \widetilde{\Theta} = 0, \quad D\widetilde{W} = \widetilde{Z} = 1, \quad \mathrm{Pe}\,\widetilde{N} = \frac{d\widetilde{N}}{dz}, \quad \text{at } z = 0, \tag{38}$$

$$\widetilde{W} = 0, \quad \widetilde{\Theta} = 0, \quad D\widetilde{W} = \widetilde{Z} = -1, \quad \mathrm{Pe}\,\widetilde{N} = \frac{d\widetilde{N}}{dz}, \quad \text{at } z = 1. \tag{39}$$

For the solution of Eqs. (34)–(37), we employ the simple Galerkin method [20]. The functions satisfying the the boundary conditions (38)–(39) are as follows:

$$\widetilde{W} = A_1 W_1 = A_1(z - z^2), \ \widetilde{\Theta} = B_1\Theta_1 = B_1(z - z^2), \quad \widetilde{Z} = C_1 Z_1 = C_1(1 - 2z), \quad \widetilde{N} = F_1 N_1 = F_1(2 - \mathrm{Pe}(1-2z) - \mathrm{Pe}^2(z - z^2)), \tag{40}$$

where A_1, B_1, C_1, F_1 are constants. Using the standard procedure [20], we obtain the stationary bioconvective and thermal Rayleigh–Darcy numbers which can be written as

$$\widehat{\mathrm{R}}_b = \frac{\mathrm{Pe}^2 r_0 r_1}{a^2(10 - \mathrm{Pe}^2)(a^2 + 10)r_2}, \tag{41}$$

$$\mathrm{Ra} = \frac{(a^2+10)^2 + D_a(a^4 + 20a^2)(a^2+10)}{a^2} - -\widehat{\mathrm{R}}_b(10 - \mathrm{Pe}^2)(a^2+10)\frac{r_2}{r_0} + \frac{10\mathrm{Ta}(a^2+10)}{a^2(1 + a^2 D_a)}, \tag{42}$$

where

$r_0 = 10\mathrm{Pe}^4 + a^2(120 - 10\mathrm{Pe}^2 + \mathrm{Pe}^4)$,

$r_1 = (a^2+10)^2 + D_a(a^4 + 20a^2)(a^2+10) - a^2\mathrm{Ra} +$

$$+\frac{10\text{Ta}(a^2+10)}{1+a^2D_a},$$
$$r_2 = 30(1+a^2\text{G}(1-\alpha_0))[-e^{\text{Pe}}(\text{Pe}-4)^2+(\text{Pe}+4)^2]+$$
$$+120\text{Pe}^2\text{G}(1+\alpha_0)(e^{\text{Pe}}-1).$$

It should be noted that, in the case where there is no rotation Ta = 0, expressions (41), (42) coincide with the results of work [17]. For the case where there is no heating (Ra = 0), and the dynamic viscosity is negligible ($D_a = 0$), we get the result of work [7]. In the next section, we will examine, in detail, the effect of rotation on the bio-thermal convection in the Darsy–Brinkman model.

5. Results and Discussion

In this section, we investigate the influence of the bioconvective Peclet number, gyrotaxis, Darcy number, Rayleigh thermal number, and microorganism geometric forms on the bio-thermal convection in rotating porous media. If we assume that microorganisms (or cells) are heavier than water (or other fluid), which means that the bioconvective Rayleigh number is always positive.

In Fig. 2, the bioconvective Rayleigh–Darcy number $\widehat{\text{R}}_b$ is plotted against the dimensionless wavenumber a for different values of the bioconvective Peclet number Pe for the case of a rotating (Ta = 5) and non-rotating (Ta = 0) media. It is seen that, as the Taylor number increases, the bioconvective Rayleigh–Darcy number also increases for fixed values $D_a = 0.1$, $\alpha_0 = 0.2$, G = 1, Ra = 25. Thus, the rotation delays the development of the bio-thermal convection. As seen in Fig. 2, raising the Pe makes the suspension less stable. The physical meaning is as follows: a higher Pe value is related to a larger W_c of the cells. Hence, a suspension with rapid cells is less stable than a suspension with slower cells, and the bioconvection grows more easily in it both with and without the medium rotation.

The bioconvective Rayleigh–Darcy number is plotted against the dimensionless wavenumber in Fig. 3 for fixed values Pe = 1, $D_a = 0.1$, $\alpha_0 = 0.2$, Ra = 25. This demonstrates that if the gyrotaxis number G rises, the bioconvective Rayleigh–Darcy number decreases. This effect is enhanced in the presence of a rotation. The gyrotaxis number G characterizes a deviation of the cell movement direction from a strict vertical. If G = 0, then there is no gyrotaxis, and microorganisms float vertically upwards (show negative

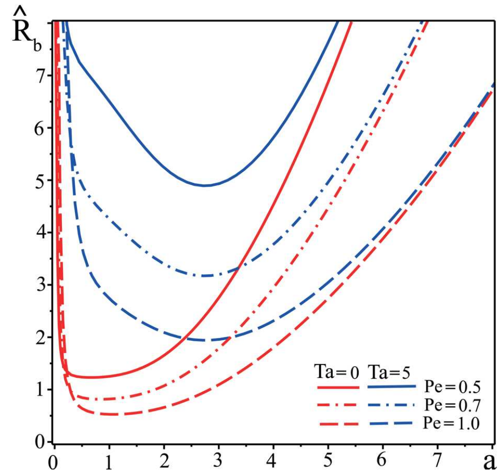

Fig. 2. Dependence of the bioconvective Rayleigh–Darcy number $\widehat{\text{R}}_b$ on the wavenumber a for different bioconvection Peclet numbers Pe = 0.5, 0.7, 1.0

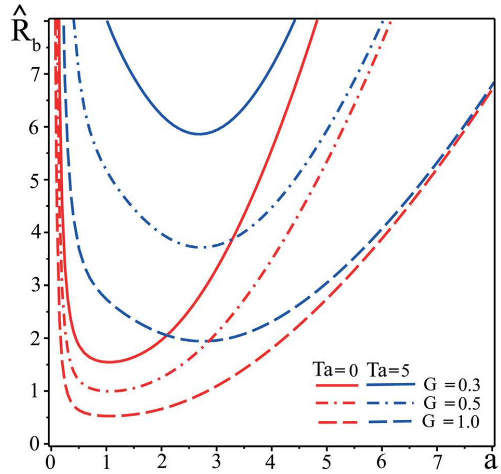

Fig. 3. Dependence of the bioconvective Rayleigh–Darcy number $\widehat{\text{R}}_b$ on the wavenumber a for different gyrotaxis numbers G = 0.3, 0.5, 1.0

geotaxis). Pedley *et al.* [4] showed that a suspension of gyrotactic microorganisms (G > 0) is unstable. As a result, gyrotaxis plays an important role in the occurrence of the bioconvective instability.

Figure 4 shows the effect of the Darcy number on the onset of the bio-thermal instability for fixed values Pe = 1, G = 1, $\alpha_0 = 0.2$, Ra = 25. It can be seen that, with an increase in the Darcy number, the critical Rayleigh number of bioconvection $\widehat{\text{R}}_b$ also increases in both rotating and non-rotating porous media. An enhancement in the Darcy number is caused by an

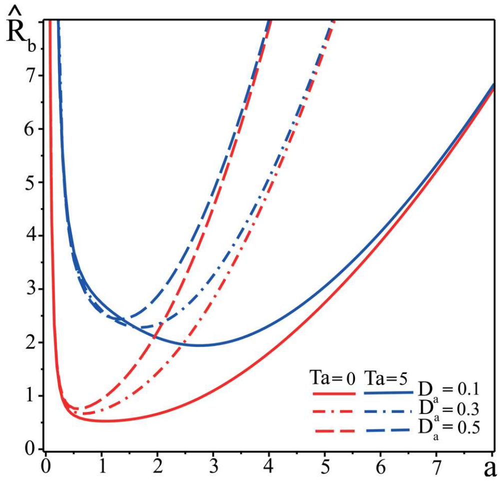

Fig. 4. Dependence of the bioconvective Rayleigh–Darcy number $\widehat{R}_b$ on the wavenumber a for different Darcy numbers.3, 0.5

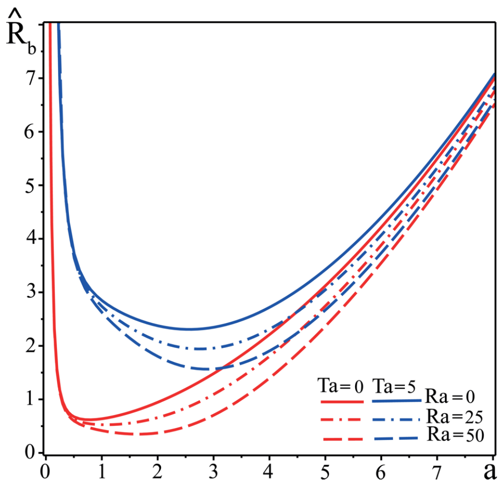

Fig. 5. Dependence of the bioconvective Rayleigh–Darcy number $\widehat{R}_b$ on the wavenumber a for different Rayleigh–Darcy numbers Ra = 0.25, 50

increase in the effective viscosity $\widetilde{\mu}$, which stabilizes the suspension and inhibits the development of the bioconvection.

Figure 5 depicts the effect of the heating on bioconvective processes for a rotating and non-rotating media. Figure 5 is plotted for fixed values $D_a = 0.1, \mathrm{Pe} = 1, \mathrm{G} = 1, \alpha_0 = 0.2$. From Fig. 5, we see that increasing the value of Ra means increasing the temperature difference between two layers, which destabilizes the system and improves the bioconvection process. The physical explanation is that a temperature increase leads to the natural convection. As a result, the bioconvection occurs, when the cell transport is caused by a convective fluid flow.

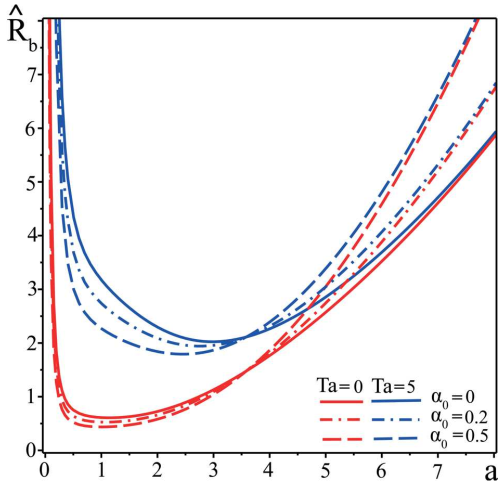

Fig. 6. Dependence of the bioconvective Rayleigh–Darcy number $\widehat{R}_b$ on the wavenumber a for different cell eccentricities $\alpha_0 = 0,\ 0.2,\ 0.5$

The results shown in Figs. 2–5 for the case $\Omega_0 = 0$ are in good agreement with the conclusions of work [17].

In Fig. 6, the bioconvective Rayleigh–Darcy number $\widehat{R}_b$ is plotted against the dimensionless wave number a for different values of the cell eccentricity α_0 for fixed values $D_a = 0.1$, Pe = 1, G = 1, Ra = 25. As is seen from Fig. 6, the spherical shape of microorganisms $\alpha_0 = 0$ slows down the development of the bioconvection in both rotating and non-rotating porous media. Therefore, the spherical shape of microorganisms contributes to the stabilization of the bioconvection. A similar conclusion was made in [8, 11] for the case of a non-rotating medium.

6. Conclusions

We have investigated the linear stage of the bio-thermal convection in a rotating layer of a highly porous medium saturated with a Newtonian fluid with gyrotactic microorganisms. In the stationary mode, the critical bioconvective and thermal Rayleigh–Darcy numbers are obtained. The effects of the Peclet number, gyrotaxis, Darcy number, Rayleigh–Darcy number, cell eccentricity, and rotation parameter on the occurrence of the bioconvection under conditions of the heating of the lower boundary of the porous layer are established. The results obtained are presented graphically. The main conclusions of the research are as follows:

1. The rotation inhibits the onset of a bioconvection.

2. An increase in the Peclet number, gyrotaxis, and Rayleigh–Darcy number reduces the threshold for the occurrence of the bioconvective instability. On the other hand, an increase in the Darcy number increases the threshold of the bioconvective instability.

3. The spherical shape of microorganisms delays the development of the bioconvection.

In conclusion, we note that, in order to study the issue of the structure formation at bioconvection Rayleigh–Darcy numbers near the critical value, a weakly nonlinear analysis is required.

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БІОТЕПЛОВА КОНВЕКЦІЯ ДАРСІ–БРІНКМАНА В ПОРИСТОМУ ШАРІ, ЯКИЙ ОБЕРТАЄТЬСЯ, НАСИЧЕНОМУ НЬЮТОНІВСЬКОЮ РІДИНОЮ, ЩО МІСТИТЬ ГІРОТАКТИЧНІ МІКРООРГАНІЗМИ

В роботі на основі моделі Дарсі–Брінкмана досліджується біотеплова конвекція в шарі пористого середовища, що обертається, насиченого ньютонівською рідиною з гіротактичними мікроорганізмами. Лінійний аналіз біотермічної конвекції проводився методом Галеркіна для жорстко-жорстких граничних умов. У стаціонарному режимі отримано дисперсійне рівняння зі зв'язком між тепловим числом Релея–Дарсі та числом біоконвекції Релея–Дарсі. Проаналізовано та показано графічно вплив числа Пекле, гіротаксису, числа Дарсі, числа Релея–Дарсі, ексцентриситету клітини та параметра обертання на біоконвективні процеси. Результати показують, що збільшення параметра обертання (числа Тейлора) затримує початок біоконвекції, тоді як збільшення ексцентриситету клітини може посилити початок конвекції.

Ключові слова: модель Дарсі–Брінкмана, біотеплова конвекція, сила Коріоліса, пористе середовище, гіротактичний мікроорганізм.