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KINEMATIC DYNAMO MODEL OF A SOLAR MAGNETIC CYCLE

The paper deals with the problem of explaining the origin and nature of the space-time variations in the magnetic activity of the Sun. It presents a new hydrodynamic model of the solar magnetic cycle, which uses helioseismological data on the differential rotation of the solar convective zone. The model is based on the hypothesis of the emergence of global flows as a result of the loss of stability of a differentially rotating plasma layer in the convective zone. First, the hydrodynamic global plasma flows are calculated without accounting for the effect of a magnetic field on them. Under this condition, it is shown that the solutions found describe all global flows observed on the surface of the Sun: permanent meridional circulation from the equator to the poles, torsional oscillations and space-time variations of the meridional flow. We conclude that the last two flows are azimuthal and meridional components of a single three-dimensional global hydrodynamic flow. Second, to simulate the dynamics of the magnetic field, the found velocities of global migrating flows and the spatial profile of the angular velocity of the internal differential rotation of the solar convective zone obtained from helioseismic measurements were used. Good coincidences have been obtained between the characteristics of the calculated dynamics of global migrating flows and the variable global magnetic fields generated by them with the observed values on the solar surface. An explanation is given for some phenomena on the surface of the Sun, which could not be explained within the framework of the available models.

K e y w o r d s: plasma, plasma confinement, magnetohydrodynamics, solar magnetic cycle, solar dynamo.

1. Introduction

Cyclic regeneration of the global magnetic field of the Sun underlies all the phenomena known collectively as the “solar activity” (SA). Sunspots, flares, floccules, protrusions, *etc.* are the external observed manifestations of SA. Sunspots were the simplest and historically the first observable manifestation of the Sun’s magnetic activity. R. Wolf from Bern in 1848 determined the length of the average period of changes in the number of sunspots of about 11.1 years [1]. This is how the basic law of SA, according to which a change in the frequency of the appearance of sunspots occurs cyclically with an average period of about 11 years. After the discovery of the magnetic nature of the spots, it was established that the spatio-temporal pattern of the magnetic polarity of groups of sunspots is restored after 22 years in average [2], thus determining the magnetic cycle of the Sun, which was called the Hale cycle. Recently, as a result of studying the distributions of the number of groups of sunspots according to their equivalent diameters, the evidence of a double Hale cycle (about 44 years) was found [3]. As was correctly noted in [3], it seems that the 22-year Hale cycle and the 44-year cycle of the distribution of diameters of sunspot groups constitute a physically unified formation.

We consider it most obvious that the well-studied magnetic Hale cycle serves as the observed surface manifestation of the global MHD rearrangement of the solar magnetism in depths of the convective zone, with which all other observed magnetic fluctuations (especially, age cycles) are associated. Therefore, when studying the magnetic cyclicity of the Sun, researchers' efforts were focused on the development of models of the Hale magnetic cycle.

In connection with global changes in the Earth's climate, the study of the so-called great minima and great maxima of the Sun's magnetic activity during the Holocene (the last 12,000 years) has gained importance in recent years within the framework of astrophysical models of the influence of SA on the climate. One of the main features of long-term changes in SA is the irregular behavior of the latter, which cannot be described by a combination of quasperiodic processes, since it contains an essentially random component. Therefore, most researchers assume that the anomalous behavior of SA, in particular, the absence of sunspots during the Maunder minimum (1645–1715), is determined by self-organized stochastic processes related to the accumulation and release of energy (see review [4]).

At present, it is believed that the solar magnetic cycle is due to the action of the movements of an electrically conductive plasma fluid inside the Sun. The most widespread idea among researchers is that the driving mechanism of the solar cycle is the process of hydromagnetic dynamo [5–7]. An important milestone in the construction of dynamo models was the idea of E. Parker [8] that the Coriolis force could impart a systematic cyclonic twist to rising turbulent fluid elements in the solar convection zone (SCZ). Due to the action of the cyclonic convection on the toroidal magnetic field, it was possible to reproduce the generation of the poloidal component of the magnetic field, which is necessary to close the solar magnetic cycle. This groundbreaking idea was rigorously quantified in mean-field magnetohydrodynamics (MHD) (the so-called macroscopic MHD), which soon became the main theory for the solar dynamo modeling [9].

The objects of research in macroscopic MHD are not the fields of true values of physical quantities, but the averaged (on scales, time, or ensemble) values of particle velocities, electric currents, magnetic and electric fields [10].

The model of the generation of the global magnetic field of the Sun, built on the basis of mean-field theory, is called the $\alpha\Omega$ dynamo in the literature. This model takes two effects into account. The differential rotation of the Sun stretches the magnetic lines of force of the poloidal field and generates a toroidal magnetic field (Ω-effect). From the toroidal field, the mean helical turbulence regenerates a new poloidal component directed in antiparallel to its original orientation (α-effect), thereby closing the solar cycle. Over the past several decades, a number of $\alpha\Omega$ dynamo models have been proposed to explain the main observed regularities of the solar cycle [6, 7, 9, 11–14].

However, over time, several problems have been identified for the $\alpha\Omega$ dynamo models of the solar cycle [7]. The physical essence of the difficulty is as follows. Numerical modeling of the turbulent convection in a rotating spherical convective shell led to a magnetic field migration that was not similar to the migration pattern observed on the Sun. Further, more detailed theoretical calculations and numerical simulations have questioned the ability of the α-effect and turbulent magnetic diffusion to work as expected in the mean-field theory. Finally, the most decisive difficulty is associated with the results of helioseismological studies on the internal differential rotation of the Sun. The structure of internal rotation, calculated on the basis of helioseismology data, turned out to be significantly different from that required to obtain dynamo solutions within the mean-field theory. Therefore, it can be stated that at present there is no consensus on the detailed nature and relative importance of various possible contributions of inductive currents and related different operating modes of the solar dynamo [7]. This situation not only indicates the need to radically modify the existing dynamo-models of mean magnetic field, but also the need to search for new physical mechanisms of magnetic field generation.

Earlier, we proposed a fairly simple model of the space-time structure of the global flows of the Sun with inductive action [15–20]. This model uses data from helioseismological studies on the internal structure of a rotating SCZ. It is based on the hypothesis of the emergence of global currents due to the loss of stability of a differentially rotating layer of solar plasma. Our calculations indicated the existence, in the SCZ, of a region in which the latitudinal flow becomes unstable. Further studies showed that this instability is responsible for the generation of all known global hydrodynamic flows on the Sun: constant meridional circulation, zonal changes in the angular velocity (torsional oscillations) [17], and space-time variations of the poloidal circulation [15, 16]. Our results are in good agreement with observational data and confirm our assumption that global hydrodynamic flows on the Sun arise due to the instability of differential rotation. In contrast to the models known to us, in which, as a rule, torsional oscillations and variations of the meridional circulation are considered independent flows, we have established that these two flows are components of a single three-dimensional hydrodynamic global flow. We called the latter the *global migratory flow* (GMF). In [18–20], we have demonstrated the dominant role of the global migratory flow in the generation of a variable magnetic field of the Sun. In this paper, we briefly outline the kinematic model of the solar magnetic cycle developed by us and give an explanation, with its help, of some observational phenomena on the solar surface that could not be explained on the base of mean-field theory.

In Section 2, a scheme of the processes of generation of hydrodynamic currents and global magnetic fields in the convective zone of the Sun is constructed. Section 3 presents a mathematical model of the formation of a whole 3-dimensional global hydrodynamic flow due to the loss of stability of the differential rotation of the Sun (Ω formation). Section 4 is devoted to the study of the evolution of the global magnetic field. First, the generation of the constant component of the toroidal magnetic field as a result of the effect of differential rotation on the relict constant poloidal magnetic field (Ω effect) is considered (Subsection 4.1). Then the mathematical concept of the generation of an alternating magnetic field under the influence of the global migrating current (hydrodynamic Ω^2 model of the magnetic cycle) is formulated (Subsection 4.2) and solutions are obtained. Section 5 describes some features of the distribution of polarities and tilt angles of magnetic bipoles on the surface of the Sun, which have not yet received a reliable theoretical explanation. Section 6 compares our results of model calculations of the evolution of global migrating currents and the alternating magnetic fields generated by them with the pattern of changes in the structure of magnetism and hydrodynamic currents observed on the solar surface. It was found that the obtained numerical solutions allow us to explain the correlation between the cyclic rearrangement of the surface magnetism and the spatio-temporal evolution of torsional oscillations. In Conclusions, the main results of the work are formulated.

2. Kinematic Model of Generation of the Global Variable Magnetic Field of the Sun

Solar non-stationary processes are mainly associated with local fine-structured (discrete) magnetic fields of active regions (which include magnetic fields both in sunspots and outside sunspots, [21]). However, regardless of the degree of concentration of the magnetic field in local beams, when averaging over scales that exceed the size of the active regions, it is always possible to distinguish a smooth large-scale (global) magnetic field [9] by averaging true "kilogauss" fields in sub-telescopic power tubes [21] and in areas between them, where there are no fields. The study of the global field makes it possible, while diverting from complex local phenomena, to identify the main processes characteristic of the Sun as a whole, which cause the cyclic magnetic activity.

According to modern concepts (e.g., [22, 23]), the global magnetic field of the Sun has two components. The first component is a weak poloidal (oriented along the meridians) magnetic field ($\mathbf{H}_P$). The lines of force of this field cross the Sun's surface near the poles and form polar magnetic fields. The second component is a strong deep toroidal (oriented in the azimuthal direction) magnetic field ($\mathbf{H}_T$), which, when crossing the solar surface, determines the intensity of sunspot formation in the photosphere.

Observations show [13] that the manifestations of surface magnetism caused by global components oscillate in time and space with an average period of about 22 years in antiphase. In particular, the 11-year cycle of the sunspot activity is simultaneously a cycle of changing the sign of the polar fields. That is, Hale's magnetic cycle consists of two 11-year cycles according to Wolf numbers. Therefore, it is obvious that both global magnetic components are connected to each other (regenerate each other), and are probably excited by one process, which has an oscillating cyclical character. As already noted, the belief that the triggering mechanism of the magnetic cycle is the process of a turbulent dynamo is the most widespread among researchers of the global magnetism of the Sun [5–7].

It is generally accepted that the poloidal magnetic field has a relict origin (e.g., [23]) and has two components: a primary constant component ($H_P^{(1)}$) and a secondary evolutionary component, variable in time ($H_P^{(2)}$). As for the toroidal field, it is believed that it was fully formed during the evolution of the Sun. In modern models of the global variable magnetic field, it is generally accepted that the toroidal magnetic field ($\mathbf{H}_T$) is generated from the poloidal field ($\mathbf{H}_P$) as a result of the differential rotation with the frequency of the solar convective zone (the so-called Ω-effect [8,22,23]). Due to the Ω-effect, the toroidal field ($\mathbf{H}_T$) also has two components. The first is the constant toroidal component ($H_T^{(1)}$) associated with the effect of differential rotation on the primary constant poloidal field ($H_P^{(1)}$). The second variable component ($H_T^{(2)}$) is due to the interaction of the differential rotation and the secondary variable poloidal field ($H_P^{(1)}$), with respect to the original in the previous cycle.

Most researchers associate the generation of an variable poloidal magnetic field ($H_P{}^{(2)}$) with helical *turbulent* motions in the SCZ. It is assumed that the magnetic field lines of the toroidal field ($H_T^{(2)}$) under the influence of the helical turbulent convection are drawn into magnetic loops with a nonzero component in the meridional plane (the so-called α-effect [9]). Merging due to the diffusion, magnetic loops generate a poloidal field of opposite polarity ($H_P^{(2)}$) [9].

In contrast to this concept, we believe that the poloidal field ($H_P^{(2)}$) is formed (like both components of the toroidal field – $H_T^{(1)}$ and $H_T^{(2)}$) under the influence of *global* hydrodynamic flows and differential rotation. Our proposed model is an alternative to the generally accepted hydromagnetic (turbulent) dynamo theory. At the same time, one can point to the diffusion model of the magnetic cycle of the Sun [23], in which small-scale pulsations of the plasma in the SCZ play a decisive role, and the spiral nature of the turbulence is not taken into account. In this model, it is assumed that the magnetic cycle is formed by a separate portion of the magnetic flux, which comes from the radiant zone into the SCZ. Interacting here (in the SCZ) with the turbulent environment, it is organized into a large-scale magnetic structure. This magnetic structure in its development gives an observable picture of the magnetic cycle on the surface of the Sun. However, it is not entirely clear how new portions of the magnetic flux arise/are generated. In contrast to the diffusion model with the defining role of small-scale pulsations, in our scenario of the magnetic cycle, the dominant role is played by large-scale (global) hydrodynamic flows excited in the SCZ.

When studying the observed regularities and fluctuations of the magnetic cycle [7], an important role is traditionally assigned to global hydrodynamic flows: torsional oscillations [24] and meridional (poloidal) flows [25]. The relevance of studying the torsional oscillations is due to the fact that active magnetic regions arise mainly at the boundary between the zones of fast and slow rotation of surface layers [25]. Deep meridional circulation, which is indicated by the results of helioseismology, can cause the waves of the toroidal field to propagate toward the equator. This explains the occurrence of sunspots in the low-latitude belt [26, 27].

When constructing our model, we proceeded from the assumption that the solar magnetic cycle is a steady process. Therefore, we can leave aside the question of its origin. This allowed us to single out, from the whole variety of phenomena occurring on the Sun, those that are directly related to the polarity reversal, and to establish a connection between them. Comparing our calculations with the phenomena accompanying the magnetic cycle, on the one hand, we saw the correctness of our approach in their qualitative agreement, and, on the other hand, gave explanations for these phenomena.

The model of the kinematic dynamo proposed by us assumes the leading role of the hydrodynamic flows of the conducting plasma in relation to the magnetic processes. This assumption makes it possible to construct a mathematical model for the generation of variable global flows and variable magnetic fields of the solar cycle. For this, we separate the hydrodynamic equations of a conducting plasma fluid from the complete system of magnetohydrodynamic equations and solve them separately from the equations of

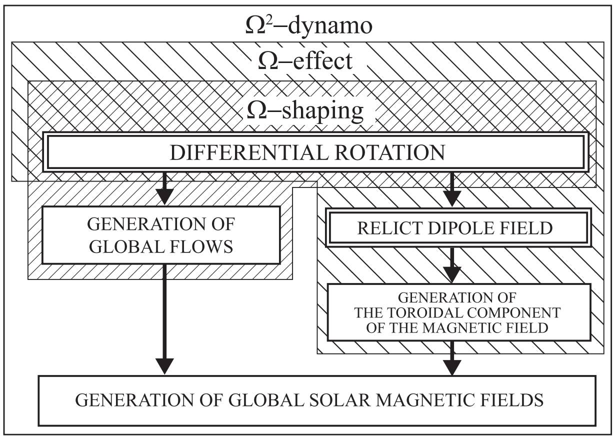

Fig. 1. Diagram of the processes of generation of hydrodynamic flows and global magnetic fields in the convective zone of the Sun (model of the kinematic Ω^2 dynamo). The block 'Ω-shaping' (Ω-formation) describes the process of hydrodynamic generation of global flows as a result of the loss of stability of the differential rotation of the Sun. The block 'Ω-effect' describes the process of generating the toroidal component of the magnetic field by the differential rotation from the dipole relict field. Block 'Ω^2-dynamo' – describes the process of generating the global magnetic fields of the Sun. The double bold lines mark the input parameters of the model

magnetic field generation. In this case, we neglect the influence of the magnetic field on the hydrodynamic plasma flow. At the same time, the spatial structure and dynamics of the magnetic field are completely determined by the plasma flow. The block diagram of our model is shown in Fig. 1. A detailed description of the model and research results can be found in works [15–20].

The following sections present the simulation results and their comparison with observational data.

3. Ω Formation

We consider the solar plasma to be an ideally conducting medium. To calculate the global flows, we use the system of equations:

$$\rho\left[\frac{\partial \mathbf{V}}{\partial t}+(\mathbf{V}\cdot\nabla)\,\mathbf{V}\right]=-\nabla P-\rho\nabla U, \quad (1)$$

$$\operatorname{div}(\rho\mathbf{V})=0. \quad (2)$$

Here P is the pressure, U is the gravitational potential, $\mathbf{V}$ is the flow velocity, and ρ is the plasma density. We assume that the plasma density does not depend on time (the assumption of anelasticity [28]) and is a function of only the spatial coordinates. The dependence of the plasma density on the radius is taken from the standard model of the Sun [29].

Equations (1) and (2) are analyzed in a spherical coordinate system (R,θ,φ), the polar axis of which coincides with the axis of rotation of the Sun. The solution of these equations under the assumption of axial symmetry $(\partial/\partial\varphi=0)$ is sought in the form:

$$\mathbf{V}=\mathbf{V}_\varphi+\mathbf{v}, \quad |\mathbf{V}_\varphi|\gg|\mathbf{v}|, \quad (3)$$

$$P=P_0+p, \quad P_0\gg p, \quad (4)$$

$$U=U_0+u, \quad U_0\gg u, \quad (5)$$

$$\rho=\rho_0. \quad (6)$$

Here, $\mathbf{V}_\varphi$ is the stationary azimuthal velocity of the Sun's rotation in the convective zone, determined by the methods of helioseismology, and $\mathbf{v}$ is an unknown small correction that accounts for the flow arising from the instability of the differential rotation.

Equations (1) and (2) are supplemented by two boundary conditions consisting in the vanishing of the velocity component v_n normal to the boundary surfaces: at the bottom of the convective zone, S_i, and on the outer surface of the photosphere, S_e:

$$v_n|_{S_i}=0, \quad v_n|_{S_e}=0. \quad (7)$$

After substituting (3)–(6) into Eqs. (1) and (2), we obtain the equations of the zero order

$$\rho_0\left[(\mathbf{V}_\varphi\cdot\nabla)\,\mathbf{V}_\varphi\right]=-\nabla P_0-\rho_0\nabla U_0, \quad (8)$$

$$\operatorname{div}(\rho_0\mathbf{V}_\varphi)=0, \quad (9)$$

and the equations of the first order of smallness

$$\rho_0\left[\frac{\partial \mathbf{v}}{\partial t}+(\mathbf{V}_\varphi\cdot\nabla)\,\mathbf{v}+(\mathbf{v}\cdot\nabla)\,\mathbf{V}_\varphi\right]= =-\nabla p-\rho_0\nabla u, \quad (10)$$

$$\operatorname{div}(\rho_0\mathbf{v})=0. \quad (11)$$

Equation (9) is satisfied automatically, since both ρ_0 and $\mathbf{V}_\varphi$ do not depend on φ. The solutions of Eq. (8) describe the equilibrium of the Sun rotating with the speed $\mathbf{V}_\varphi$, and the equilibrium distribution of density, pressure, and gravitational potential inside it. Since the oblateness of the Sun is equal to 9×10^{-6} [30], we will further consider the Sun as a sphere, and the equilibrium values of ρ_0, P_0, and U_0 – depending only on the radius.

Applying the operation **curl** to Eq. (10), we obtain the following equations for considering the vortex motion:

$$\mathrm{curl}\left(\rho_0\left[\frac{\partial\mathbf{v}}{\partial t}+(\mathbf{V}_\varphi\cdot\nabla)\,\mathbf{v}+(\mathbf{v}\cdot\nabla)\,\mathbf{V}_\varphi\right]\right)=0,\quad(12)$$

$$\mathrm{div}\,(\rho_0\mathbf{v})=0.\quad(13)$$

The solution to Eq. (13), as expected, describes a vortex motion, where $\mathbf{A}=A(R,\theta,t)\mathbf{e}_\varphi$. Whence follows

$$\mathbf{v}=(v_R,v_\theta,v_\varphi)=$$
$$=\left(\frac{1}{\rho_0 R\sin\theta}\frac{\partial}{\partial\theta}\,(A\sin\theta),\,-\frac{1}{\rho_0 R}\frac{\partial}{\partial R}\,(RA),\,v_\varphi\right).\quad(14)$$

Substituting (14) into Eq. (12) and writing out the resulting equation in terms of the basis vectors of the spherical coordinate system, we arrive at two linear partial differential equations for determining A and v_φ:

$$\frac{\partial^2(\Delta A)}{\partial t^2}-\frac{1}{R}\left\{\ln\left(R^2\sin^2\theta\right),\,\frac{V_\varphi}{R^3\sin^2\theta}\mathit{\Pi}\right\},\quad(15)$$

$$\frac{\partial v_\varphi}{\partial t}+\frac{1}{\rho_0R^3\sin^2\theta}\,(-\mathit{\Pi})=0.\quad(16)$$

Here,

$$\mathit{\Pi}=\{AR\sin\theta,V_\varphi R\sin\theta\},$$

$$\Delta A=\frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial A}{\partial R}\right)+\frac{1}{R^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial A}{\partial\theta}\right)-$$
$$-\frac{A}{R^2\sin^2\theta},$$

and curly brackets denote the Poisson brackets

$$\{f,g\}=\left\{\frac{\partial f}{\partial R}\frac{\partial g}{\partial\theta}-\frac{\partial f}{\partial\theta}\frac{\partial g}{\partial R}\right\}.$$

Thus, in order to determine all three components of the velocity $\mathbf{v}$, it is necessary to solve the differential equation (15) with variable coefficients relative to the function $A(R,\theta,t)$. From the structure of Eq. (15), it follows that its solution is completely determined by the function $V_\varphi(R,\theta)$ and its derivatives. The solutions to Eq. (15) by the Galerkin method [31] give flows of two types [15]: 1) flows that are symmetric about the equator and 2) flows that are mirror-symmetric about the equator. In other words, flows whose streamlines do not cross the equator, and flows whose streamlines cross the equator. The presence of both types of flows on the Sun simultaneously can be the reason for a slight asymmetry in the dynamics of currents in the southern and northern hemispheres, usually explained by inaccuracies in measurements [25].

An essential feature of the found solutions is that flows that periodically change their direction to the opposite one are added to the flows that do not change their direction over time. The maximum value of the speed of these flows, having arisen at the pole, over time moved to the equator. We called these flows *global migratory flows* (GMF). For the components of the velocity of this flow, we will use the notation

$$\mathbf{v}_{\mathrm{gmf}}(R,\theta,t)=$$
$$=\left(v^R_{\mathrm{gmf}}(R,\theta,t),v^\theta_{\mathrm{gmf}}(R,\theta,t),v^\varphi_{\mathrm{gmf}}(R,\theta,t)\right),$$

where $v^R_{\mathrm{gmf}}, v^\theta_{\mathrm{gmf}}$, and v^φ_{gmf} are the radial, meridional, and latitudinal components of the velocity, respectively. Typical meridional v^θ_{gmf} and azimuthal v^φ_{gmf} components of the GMT near the surface in coordinates (t,θ) for one period are shown in Figs. 2, *a* and 2, *b*, respectively. Let us compare the obtained result with the observational data on the temporal variation of the meridional flow and torsional oscillations shown in Figs. 3, *a* and 3, *b*, respectively.

In view of the inversion of the sign of the velocity in the southern hemisphere (Fig. 3, *b*, specially made by Basu&Antia [25] to emphasize the connection between Fig. 3, *a* and Fig. 3, *b*, the similarity of the calculated spatio-temporal variations of the poloidal flow and torsional oscillations (Figs. 2, *a* and 2, *b*) with the observational data looks quite obvious.

It is worth to note that both flows depicted in Figs. 3, *a*, 3, *b*, were discovered as independent from each other [19, 20]. Their connection with each other is assumed, but has not yet been proven. Within the framework of our model, this relationship is obvious, since the values of the velocity shown in Figs. 2, *a* and 2, *b* are projections of the same flow.

4. Solar Magnetic Cycle Model

4.1. Generation of a toroidal field (Ω-effect)

Using the results of the previous section, we present the results of modeling the dynamics of global mag-

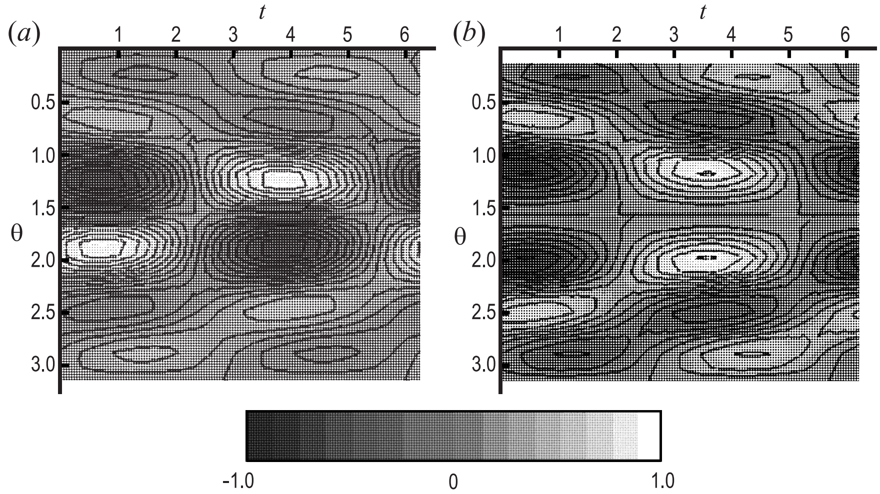

Fig. 2. Distributions over the polar angle, θ (in radians, along ordinates), for the meridional, $v^{\theta}_{\rm gmf}$ (a), and azimuthal, $v^{\varphi}_{\rm gmf}$ (b), components of the GMF velocity (in arb. u.) on the solar surface, which are presented as a function of time (abscissa axis). The angle $\pi/2$ corresponds to the equator. The values of the flow velocities are marked in shades of gray, with the maximum positive value corresponding to white (1.0), and the maximum negative value to dark gray (–1.0). The abscissa shows the phase of the oscillations t ($t = 2\pi T/T^*$, where $0 \le T \le T^*$, T^* is the oscillation period). The radial velocity component is absent in the figure, since it vanishes at the surface

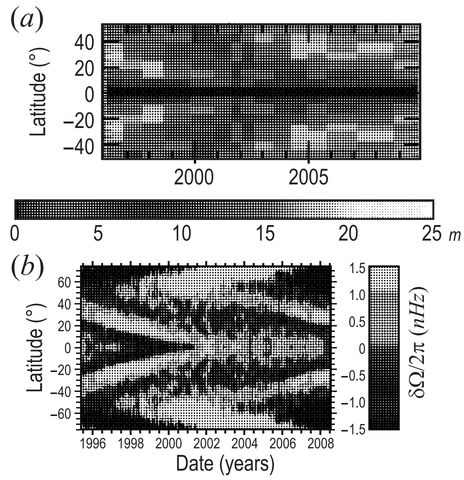

Fig. 3. Dependence of the poloidal flow velocity on the time (abscissa axis) and solar latitude (ordinate axis) for $r = R/R_0 = 0.998$. The direction of speed in the southern hemisphere is reversed by the authors [32] of the figure (a). Torsional oscillations – variations in the frequency of rotation of the Sun $\delta\Omega/2\pi$ depending on the time (abscissa) and solar latitude (ordinate) for the relative radius of the Sun $r = R/R_0 = 0.99$ (R_0 is the radius of the Sun) [31] (b)

netic fields using helioseismological data on the differential rotation. We proceed from the equations

$$\frac{\partial \mathbf{H}}{\partial t} = \mathrm{curl}\,[\mathbf{V} \times \mathbf{H}], \tag{17}$$

$$\mathrm{div}\mathbf{H} = 0, \tag{18}$$

where $\mathbf{H}$ – magnetic field strength.

We represent the speeds of global flows in the form as

$$\mathbf{V}\,(R,\theta) = \mathbf{V}_{\varphi}\,(R,\theta) + \epsilon \mathbf{v}_{\rm gmf}\,(R,\theta) + ..., \tag{19}$$

where $\mathbf{V}_{\varphi} = \Omega\,(R,\theta)\,R \sin\theta\,\mathbf{e}_{\varphi}$ is the linear speed of the differential rotation of the Sun, and ϵ is a small parameter, $0 < \epsilon \ll 1$. The angular velocity profile was taken from helioseismological calculations [32]. Since the induction equation (17) is linear in $\mathbf{H}$, the time-dependent magnetic field will be sought in the form

$$\mathbf{H}\,(R,\theta,t) = \epsilon \mathbf{H}_1\,(R,\theta,t) + \epsilon^2 \mathbf{H}_2\,(R,\theta,t) + \dots . \tag{20}$$

Magnetic field

$$\mathbf{H}_1(R,\theta,t) = (H^{(1)}_R(R,\theta), H^{(1)}_{\theta}(R,\theta), H^{(1)}_{\varphi}(R,\theta,t))$$

has three components. The first two components, $H^{(1)}_R(R,\theta)$ and $H^{(1)}_{\theta}(R,\theta)$, are the radial and meridional components of the relict dipole magnetic field [33], respectively. The third component, $H^{(1)}_{\varphi}(R,\theta,t)$, is the azimuthal magnetic field resulting from the interaction of the differential rotation with the relict dipole field (Ω-effect). This component is determined from the system of equations of the first order in ϵ. After the substitution of (18) in (17), we get the expressions for the velocity and magnetic field:

$$\begin{aligned}
&\frac{\partial H_R^{(1)}}{\partial t}+\frac{V_\varphi}{R\sin\theta}\frac{\partial H_R^{(1)}}{\partial\varphi}=0,\\
&\frac{\partial H_\theta^{(1)}}{\partial t}+\frac{V_\varphi}{R\sin\theta}\frac{\partial H_\theta^{(1)}}{\partial\varphi}=0,\\
&\frac{\partial H_\varphi^{(1)}}{\partial t}+\frac{V_\varphi}{R\sin\theta}\frac{H_\varphi^{(1)}}{\partial\varphi}+H_R^{(1)}\left(\frac{V_\varphi}{R}-\frac{\partial V_\varphi}{\partial R}\right)+\\
&+H_\theta^{(1)}\left(\frac{V_\varphi}{R}\cot\theta-\frac{1}{R}\frac{\partial V_\varphi}{\partial\theta}\right)=0,
\end{aligned}\tag{21}$$

where

$$\mathrm{div}\mathbf{H}_1=\frac{1}{R^2}\frac{\partial\left(R^2H_R^{(1)}\right)}{\partial R}+\frac{1}{R\sin\theta}\frac{\partial\left(\sin\theta H_\theta^{(1)}\right)}{\partial\theta}+$$
$$+\frac{1}{R\sin\theta}\frac{\partial\left(H_\varphi^{(1)}\right)}{\partial\varphi}=0.$$

It follows from the first and second equations of system (21) that the components $H_R^{(1)}$ and $H_\theta^{(1)}$ of any form satisfy them, as long as they satisfy the equation $\mathrm{div}\mathbf{H}_1=0$. We have chosen the dipole form of these components [20], since the relict dipole magnetic field is well observed on the Sun [25]. Hence,

$$H_R^{(1)}(R,\theta)=-\frac{2\cos\theta}{R^3},\tag{22}$$

$$H_\theta^{1)}(R,\theta)=-\frac{\sin\theta}{R^3}\tag{23}$$

From the third equation of system (21), we obtain the following expression for the azimuthal increase in time field:

$$H_\varphi^{(1)}=\left(-\frac{\sin2\theta}{R^2}\frac{\partial\Omega}{\partial R}-\frac{(\sin\theta)^2}{R^3}\frac{\partial\Omega}{\partial\theta}\right)t=H_\varphi^*t.\tag{24}$$

The structures of the rate of increase of the magnetic field H_φ^* in the SCZ and on the solar surface are shown in Figs. 4, *a* and 4, *b*, respectively.

Equation (24) gives a linear increase in $H_\varphi^{(1)}$ with respect to time and thereby confirms our initial assumption about the presence of a strong latitudinal magnetic field under the surface of the Sun [17].

4.2. Ω^2 dynamo

The terms quadratic in ϵ in Eq. (17) give the equation

$$\frac{\partial\mathbf{H}_2}{\partial t}-\mathrm{curl}\left[\mathbf{V}_\varphi\times\mathbf{H}_2\right]=\mathrm{curl}\left[\mathbf{v}_{\mathrm{gmf}}\times\mathbf{H}_1\right].\tag{25}$$

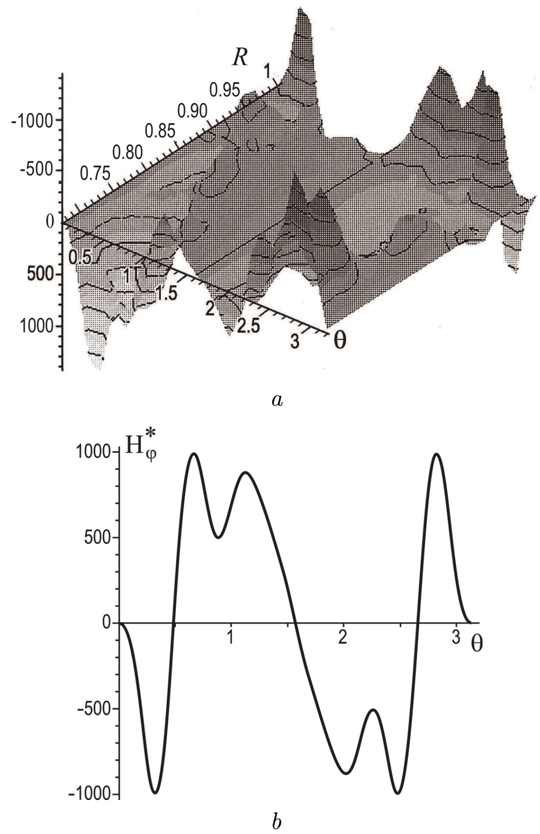

Fig. 4. Distributions of the relative amplitude of the azimuthal component of the rate of increase of the magnetic field H_φ^* in the SCZ [17]. H_φ^* dependences in the coordinate system $(R,\ \theta)$ in the SCZ, where R is the relative radius of the Sun $(0.68\le R\le1, 0\le\theta\le\pi)$ (*a*). The equator corresponds to a straight line parallel to the R axis passing through the $\pi/2$ point of the θ axis. Dependence of H_φ^* on the polar angle θ at the surface of the Sun (*b*)

Here,

$$\mathbf{H}_2(R,\theta,t)=$$
$$=\left(H_R^{(2)}(R,\theta,t),\,H_\vartheta^{(2)}(R,\theta,t),\,H_\varphi^{(2)}(R,\theta,t)\right)$$

is a variable magnetic field arising from the interaction of torsional oscillations, spatio-temporal variations of the poloidal flow and the differential rotation of the Sun.

Equation (18) of the second order in ϵ has the form:

$$\mathrm{div}\,\mathbf{H}_2=\frac{1}{R^2}\frac{\partial\left(R^2H_R^{(2)}\right)}{\partial R}+\frac{1}{R\sin\theta}\frac{\partial\left(\sin\theta H_\theta^{(2)}\right)}{\partial\theta}+$$
$$+\frac{1}{R\sin\theta}\frac{\partial H_\varphi^{(2)}}{\partial\varphi}=0.\tag{26}$$

From Eqs. (25) and (26), we have

$$\frac{\partial H_R^{(2)}}{\partial t} = \frac{1}{R\sin\theta}\frac{\partial\left[\left(v_{\rm gmf}^R H_\theta^{(1)} - v_{\rm gmf}^\theta H_R^{(1)}\right)\sin\theta\right]}{\partial\theta},$$
$$\frac{\partial H_\theta^{(2)}}{\partial t} = -\frac{1}{R}\frac{\partial\left[\left(v_{\rm gmf}^R H_\theta^{(1)} - v_{\rm gmf}^\theta H_R^{(1)}\right)R\right]}{\partial R},$$
$$\frac{\partial H_\varphi^{(2)}}{\partial t} = H_R^{(2)} R\sin\theta\frac{\partial\Omega}{\partial R} + H_\theta^{(2)}\sin\theta\frac{\partial\Omega}{\partial\theta} + $$
$$+\frac{1}{R}\frac{\partial\left[\left(v_{\rm gmf}^\varphi H_R^{(1)} - v_{\rm gmf}^R H_\varphi^{(1)}\right)R\right]}{\partial R} -$$
$$-\frac{1}{R}\frac{\partial\left(v_{\rm gmf}^\theta H_\varphi^{(1)} - v_{\rm gmf}^\varphi H_\theta^{(1)}\right)}{\partial\theta}. \quad (27)$$

Let us substitute the expression for

$$\mathbf{H}_1(R,\theta,t) = \left[H_R^{(1)}(R,\theta), H_\vartheta^{(1)}(R,\theta), H_\varphi^{(1)}(R,\theta,t)\right]$$

into the system of equations (27) and integrate the first and second equations over the time. After substituting the obtained result into the third equation of the system, we find:

$$H_R^{(2)} = \frac{1}{R\sin\theta}\int\frac{\partial\left[(-\varXi_1)\sin\theta\right]}{\partial\theta}dt,$$
$$H_\theta^{(2)} = \frac{1}{R}\int\frac{\partial\left[(\varXi_1)R\right]}{\partial R}dt,$$
$$H_\varphi^{(2)} = \frac{\partial\Omega}{\partial R}\int\left(\int\frac{\partial\left[(-\varXi_1)\sin\theta\right]}{\partial\theta}dt\right)dt -$$
$$-\frac{\sin\theta}{R}\frac{\partial\Omega}{\partial\theta}\int\left(\int\frac{\partial\left[(-\varXi_1)R\right]}{\partial R}dt\right)dt +$$
$$+\frac{1}{R}\int\frac{\partial\left(\left[-v_{\rm gmf}^\varphi\frac{2\cos\theta}{R^3} + v_{\rm gmf}^R(\varXi_2)t\right]R\right)}{\partial R}dt +$$
$$+\frac{1}{R}\int\frac{\partial\left[v_{\rm gmf}^\theta(\varXi_2)t - v_{\rm gmf}^\varphi\frac{\sin\theta}{R^3}\right]}{\partial\theta}dt, \quad (28)$$

where

$$\varXi_1 = v_{\rm gmf}^R\frac{\sin\theta}{R^3} - v_{\rm gmf}^\theta\frac{2\cos\theta}{R^3},$$

$$\varXi_2 = \frac{\sin 2\theta}{R^3}R\frac{\partial\Omega}{\partial R} + \frac{\sin^2\theta}{R^3}\frac{\partial\Omega}{\partial\theta}.$$

The equations for the components of the magnetic field $H_R^{(2)}(R,\theta,t)$ and $H_\theta^{(2)}(R,\theta,t)$ describe the superposition of standing oscillations with a complex distribution of amplitudes constant in time over the space of the SCZ. The equation for the azimuthal field $H_\varphi^{(2)}(R,\theta,t)$ contains terms whose structure is similar to the structure described above, as well as terms describing standing oscillations with an amplitude increasing in time. Over time, terms with constant amplitude (independent of time t) become insignificant and can be disregarded. Accounting for the above considerations, the expressions for the components $\mathbf{H}_2(R,\theta,t)$ take the following form:

$$H_R^{(2)} = \frac{1}{R\sin\theta}\int\frac{\partial\left[(-\varXi_1)\sin\theta\right]}{\partial\theta}dt,$$
$$H_\theta^{(2)} = \frac{1}{R}\int\frac{\partial\left[(\varXi_1)R\right]}{\partial R}dt,$$
$$H_\varphi^{(2)} = \frac{1}{R}\int\left(\frac{\partial\left[v_{\rm gmf}^R(\varXi_2)R\right]}{\partial R} + \frac{\partial\left[v_{\rm gmf}^\theta(\varXi_2)\right]}{\partial\theta}\right)tdt,$$
$$H_\varphi^{(2)} = H_\varphi^{*(2)}t. \quad (29)$$

The time behavior of all three components of the magnetic field of the second order of smallness is shown in Figs. 5, *a–c*.

Formulas (28)–(29) clearly show the dominant role of the differential rotation of the Sun in our proposed model of the laminar dynamo of the solar magnetic cycle (see Figs. 2, *a, b*). Indeed, firstly, the differential rotation (terms with $\Omega(R,\theta)$) enter into the equations through the components of the velocity of the global migratory flow $\mathbf{v}_{\rm gmf} = (v_{\rm gmf}^R, v_{\rm gmf}^\theta, v_{\rm gmf}^\varphi)$. Secondly, the differential rotation $\Omega(R,\theta)$ is explicitly included in the expression for the magnetic field $H_\varphi^{(2)}$ in combination with the components of the GMF velocity, i.e., ultimately, "quadratically". Therefore, the proposed model can be called "Hydrodynamic Ω^2 model of the solar magnetic cycle". Thus, it is the differential rotation that is the main trigger that sets the Hale magnetic cycle.

Before proceeding to the comparison of simulation results with observational data, we briefly recall some information about the observed magnetic structures on the solar surface.

5. Magnetic Bipoles and Ephemeral Active Regions

In [34], the concept of "magnetic bipole" was introduced, which, along with the observed bipolar sunspot

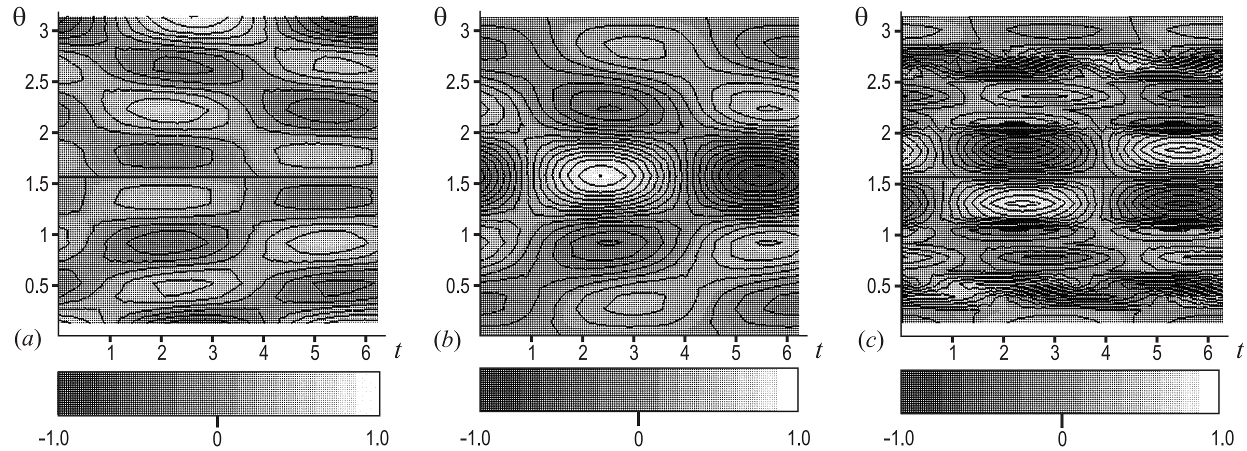

Fig. 5. The distributions over the polar angle θ ($0 \le \theta \le \pi$) and time t ($t = 2\pi T/T^*$, $0 \le T \le T^*$, T^* is the oscillation period) of the relative amplitudes of the three components of the variable magnetic field $\mathbf{H}_2(\theta, t)$: $H_R^{(2)}(\theta, t)$ (a), $H_\theta^{(2)}(\theta, t)$ (b), and $H_\varphi^{*(2)}$: $H_\varphi^{(2)}(\theta, t) = H_\varphi^{*(2)}(\theta, t) \cdot t$ (c). The angle $\pi/2$ corresponds to the equator. The values of the magnetic field are marked in shades of gray, with the maximum positive value corresponding to white (1.0), and the maximum negative value to dark gray (–1.0)

groups, is usually applied to short-lived small-scale bipolar structures with a weak magnetic field. According to Harvey [34], the total magnetic flux through the solar surface is formed from the fluxes of bipolar magnetic bipoles with different sizes. Large magnetic bipoles form sunspots of the "royal zone" and are identified as active regions [35, 36]. At the same time, when calculating the total magnetic flux, one should also consider small "bipoles", called "ephemeral regions" (ER) [37]. Subsequent observations on the solar surface (see, e.g., [38, 39]) revealed a large number of regularly present ephemeral regions. The distribution of ERs covers a wider range of latitudes than the distribution of sunspots and most normal active centers. In particular, ephemeral regions are found at high latitudes, where sunspots are absent.

Further, Stenflo [40] showed that observations make it possible to find the tilt angles of bipolar magnetic configurations. This made it possible to establish that the axes of the bipolar sunspot groups are oriented at a slight angle to the east-west latitudinal direction. As a rule, the western head sunspots of the groups are usually located closer to the equator than the eastern tail sunspots (Joy's law) [41]. In [42], from the analysis of observational data for active regions in 21–24 solar cycles, the latitudinal-temporal distribution of polarities and averaged tilt angle of bipoles (both of sunspot groups and ephemeral regions) was obtained. It was found that the magnetic polarities of large bipoles in each solar cycle corresponded to Hale's law of polarity [41, 43]. The spatially averaged tilt angles of large bipoles, in accordance with Joy's law, was positive in the northern hemisphere of the Sun and negative in the southern one. At the same time, a deviation from these regularities was found in smaller high-latitude bipoles. Shallow bipoles often "violated" Hale's polarity law and had the inverse Joy's law [44]. A similar picture of the distribution of magnetic bipoles on the surface of the Sun was obtained as a result of our numerical simulations.

6. Comparison of Simulation Results with Observational Data

By now, the main regularities of the solar magnetic activity have been established. However, a number of questions remain about the cause-and-effect relationship between the observed changes in magnetism and the hydrodynamic motion of the plasma. The discovery of *torsional oscillations* (TO) was of great importance for the establishment of these connections [45]. It turned out that the processes of rearrangement of surface magnetism during the Hale cycle correlate fairly well with the observed space-time evolution of TO.

Before proceeding to the analysis of these correlations, let us emphasize the main result of our modeling for global hydrodynamic flows. In our model of the solar cycle, there are radial $v^R_{\rm gmf}$, meridional $v^\theta_{\rm gmf}$, and azimuthal $v^R_{\rm gmf}$ components of the velocity of a single three-dimensional global migratory flow, i.e., all three velocity components take part in the generation of a variable magnetic field $\mathbf{H}_2(R, \theta, t)$ (see Eqs. 28). Therefore, the behavior of the velocity components $\mathbf{v}_{\rm gmf}$ simulated by us can be used to explain the rearrangement of surface magnetism.

When observing the surface hydrodynamic movements, it is usually possible to analyze only torsional oscillations and meridional circulation. Therefore, when studying the phenomena associated with the magnetic restructuring, observers primarily pay attention to torsional oscillations and, to a less extent, to the meridional circulation. To date, the following correlations have been established between surface magnetic phenomena and torsional oscillations:

• The beginning of the magnetic cycle (the appearance of the first high-latitude spots) coincides with the appearance of TOs at high latitudes of the Sun, which slowly drift toward the equator. After two 11–year cycles, TOs disappear at the equator, and the magnetic cycle ends [13].

• The attainment of the maximum magnetic field at the poles [46] coincides in time with the onset of TO.

• The meridional drift of the maximum of the magnetic field over the surface of the Sun coincides in time with the line of zero velocity of the TO. The latter lies between its increased and decreased values relative to the background velocity at a given latitude [47, 48].

• The onset of polarity reversal of the polar magnetic field in time is shifted by a quarter of the period from the moment the TO appears [13, 46].

• The zone of formation of sunspots coincides with the zone of zero drift of the TO velocity [13].

• The behavior of the spatio-temporal variations of the meridional flow (see Figs. 3, *a, b*) is consistent with the dynamics of the TO [13, 48].

The listed observational results are in good agreement with the results of our numerical simulation. Figures 6, *a–c* show the relative amplitudes of the components of the variable magnetic field $\mathbf{H}_2 = (H_R^{(2)}, H_\theta^{(2)}, H_\varphi^{(2)})$ (see Figs. 5, *a–c*) with superimposed lines of zero azimuthal velocity $v_{\rm gmf}^{\varphi}$ on the solar surface (see Figs. 2, *a, b*). From Figs. 6, *a–c*, it follows that all components of the magnetic field reach a maximum on the lines of zero velocity $v_{\rm gmf}^{\varphi}$. It can also be seen that the beginning of the emergence of the line of zero velocity $v_{\rm gmf}^{\varphi}$ coincides in time with the attainment of the maximum of the radial magnetic field at the poles, and the magnetic field itself at the poles becomes maximum a little later, when the zero line $v_{\rm gmf}^{\varphi}$ reaches the equator. Note also that the latitudinal and meridional components of magnetic fields increase with the distance from the poles and reach a maximum at the equator.

At the same time, for a number of well-known observational regularities, it *was not possible to establish* reliable connections with torsional oscillations ($v_{\rm gmf}^{\varphi}$) and with time variations of the meridional flow ($v_{\rm gmf}^{\theta}$). In particular, these patterns include the following:

1. Cyclic change of magnetic signs in bipoles [40, 42, 43, 48, 49] at a fixed latitude in accordance with the Hale–Nicholson law [2, 41, 50].

2. Orientation of the axes of bipolar groups of sunspots at a small angle to the latitudinal direction (Joy's law, [41]).

3. Difficulty trying to reconcile the toroidal field values calculated by the $\alpha\Omega$ dynamo model with observational data. The essence of the problem is as follows. Cyclic reversal of the direction of a strong toroidal magnetic field, which is excited from a weak poloidal one by the differential rotation (Ω - effect), requires a significant efficiency of the generation process. However, the coefficient of transformation of the poloidal field into the toroidal one during the cycle (lasting 11 years), estimated from the helioseismological data on the gradients of the angular velocity of rotation, does not exceed 800 [51]. This value is insufficient to obtain large observed values of toroidal fields (of the order of 3000 G and higher).

In order to find the answers to these questions, let us compare the results of our modeling with the observational data of magnetic bipoles of various scales. When comparing, we will consider both the results of early observations of magnetic fields [2, 41, 50] and information about bipolar structures obtained recently as a result of processing the observational data using ground-based telescopes, as well as using instruments on spacecrafts [40, 42, 43, 49, 50]. Consider Fig. 7 obtained in our work [20]. This figure shows a picture of the distribution of orientations of the vector $\mathbf{H}^* = (H_\theta^{(2)}, H_\varphi^{*(2)})$ introduced by us into consideration. Here, $H_\theta^{(2)}$ and $H_\varphi^{*(2)}$ are the meridional and azimuthal components of the variable surface magnetic field during the full magnetic cycle. The vector $\mathbf{H}^*$ obtained in [20] as a result of the modeling was called, by us, the "model bipolar sunspot

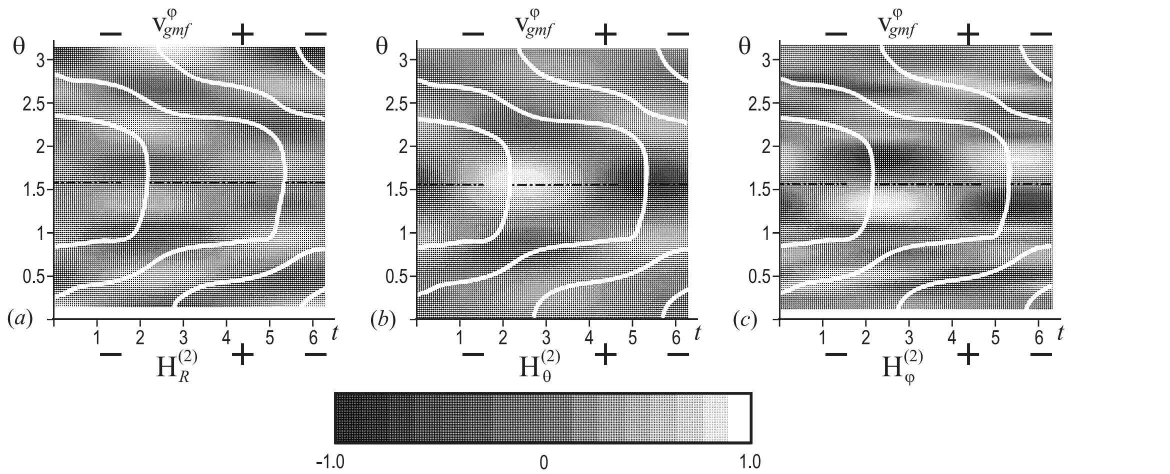

Fig. 6. The distributions over the polar angle θ ($0 \le \theta \le \pi$) and time t ($t = 2\pi T/T^*$, $0 \le T \le T^*$, T^* is the oscillation period) of the relative amplitudes of the three components of the variable magnetic field $\mathbf{H}_2(\theta, t)$: $H_R^{(2)}(\theta, t)$ (a), $H_\theta^{(2)}(\theta, t)$ (b), and $H_\varphi^{*(2)}$: $H_\varphi^{(2)}(\theta, t) = H_\varphi^{*(2)}(\theta, t)\, t$ (c), taken from Fig. 5, combined with a plot of lines of zero velocity TC (v^φ_{gmf}) on the surface of the Sun from Fig. 2. The North Pole corresponds to the polar angle $\theta = 0$ (bottom of the figures), the South Pole – to $\theta = \pi$, and the equator of the Sun – to $\theta = \pi/2$. The pluses and minuses above and below the figures indicate the zones of increased and decreased velocities of zonal currents (relative to the background speed of the Sun's rotation) between zero lines (in white). The magnetic field values are marked in shades of gray, with the maximum positive value corresponding to white (+1) and the maximum negative value to dark gray (–1)

group". Since the literature uses the term "magnetic bipole", we will further call the vector $\mathbf{H}^*$ *"surface model magnetic bipole"*. Therefore, the small arrows in Fig. 7 depict the distribution of the directions of the magnetic bipole $\mathbf{H}^*$ on the solar surface. It can be seen that the region of allocation of the modeled bipole covers the range from the equator to high heliolatitudes, reaching almost the poles. This result fully agrees with the picture of the allocation of bipoles on the solar surface obtained in [42] as a result of processing the observational data on more than 10^5 surface magnetic bipoles of various scales (see Fig. 1 in [42]).

It also follows from Fig. 7 that the distribution of polarities of the calculated bipoles in the "royal zone" obey the Hale–Nicholson law [2, 41, 50], according to which the head and tail sunspots have opposite signs in the northern and southern hemispheres. Note also that the signs of the radial magnetic field in the near-pole regions coincide with the signs of the head spots of bipoles [48, 49]. Thus, the presence of the velocity v^θ_{gmf} in the system of equations (21) makes it possible to explain the connection between the observed regularities of magnetism and the temporal variations of the meridional flow (see point 1 above).

Note also that the axes of the calculated magnetic bipoles in the "royal zone" are oriented at a small an-

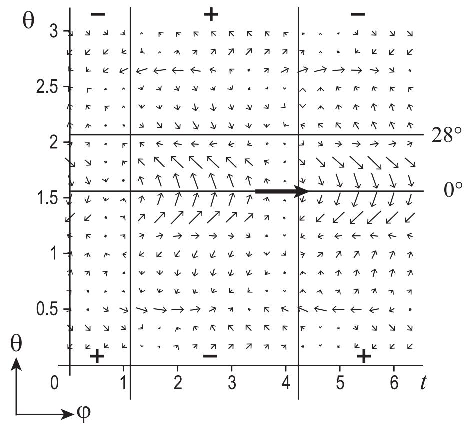

Fig. 7. Schematic representation of directions and magnitudes of $\mathbf{H}^* = [H_\theta^{(2)}, H_\varphi^{*(2)}]$. Short arrows show the distribution of directions (from positive to negative pole) of the "magnetic bipole" $\mathbf{H}^*$ on the Sun's surface at different times ($0 \le t \le 2\pi$) of the 22-year magnetic cycle, depending on the polar angle θ ($0 \le \theta \le 2\pi$) [20]. The North Pole corresponds to the polar angle $\theta = 0$ (bottom of the figure), and the South Pole – to $\theta = \pi$

gle to the latitudinal direction, which is in full agreement with Joy's law [41]. As a result, one more problematic issue, noted in point 2, is removed. At the same time, according to our calculations, in the region of middle and high latitudes, the Hale–Nicholson law and Joy's rule are violated. In this regard, we note that a small number of magnetic bipoles violating the Hale–Nicholson law and Joy's rule were recorded at all heliolatitudes during observations (see, e.g., [40, 42, 48, 49, 52]).

As for another problematic issue (point 3) about the difficulty of achieving a large conversion factor of the poloidal field into a toroidal field during the magnetic cycle [51], the velocities $v^R_{\rm gmf}$ and $v^\theta_{\rm gmf}$ found by us allow us to explain the increased efficiency of the excitation of a variable azimuthal field (see the third equation of the system of equations (31) for the latitudinal component of the magnetic field $H^{(2)}_\varphi$). This makes it possible to explain the large observed values of magnetic fields in sunspots.

7. Conclusions

It is now generally accepted that differential rotation (DR) plays a decisive role in the global magnetism of the Sun, since the action of the DR on a weak relict poloidal field provides the excitation of a strong toroidal field (Ω-effect). We have established another important role of the DR, namely, its ability to excite global hydrodynamic flows in the SCZ. For this, we investigated the stability of the DR in the SCZ. We found an area, where the DR loses its stability and leads to the generation of a secondary hydrodynamic flow, which outwardly resembles the global flows observed on the Sun (meridional circulation, torsional oscillations, variations of the meridional circulation).

The hydrodynamic model of the magnetic cycle proposed by us is based on the hypothesis of the hydrodynamic nature of the emergence of global flows as a result of the loss of stability of a differentially rotating fluid layer. The model uses a differential rotation spatial profile found from helioseismological data. We believe that the differential rotation plays a dominant role both in the generation of the magnetic field and in the excitation of global migrating flows. Therefore, we could call our model "Kinematic Ω^2 dynamo model of the solar magnetic cycle".

The model makes it possible to calculate hydrodynamic global plasma flows without regard for the effect of a magnetic field on them. The obtained numerical solutions qualitatively describe the behavior of all types of global flows observed on the surface of the Sun: The permanent meridional circulation from the equator to the poles, zonal changes in the angular velocity (torsional oscillations), and space-time variations in the meridional flow. Based on calculations and the analysis of processes on the solar surface, we made the assumption that the last two flows are the azimuthal and meridional components of a single 3-dimensional global migratory hydrodynamic flow.

We used the obtained velocities of global migratory flows and the spatial profile of the angular velocity of the internal differential rotation of the SCZ obtained from helioseismic measurements for numerical calculations of time-varying magnetic fields. These fields are generated by the interaction of these flows with a constant dipole magnetic field of the Sun of relict origin.

We compared the results of modeling the hydrodynamic flows and time-varying magnetic fields with observational data on the behavior of torsional oscillations and variations in the meridional flow. A good temporal coincidence of the calculated dynamics of global migratory flows and the alternating magnetic fields generated by them with the observed cyclical changes in magnetism and flows on the solar surface is obtained.

Thus, our studies allow us to conclude that the differential rotation of the SCZ apparently determines all the main parameters of the solar magnetic cycle.

The question of the influence of the dependence of the plasma density on the depth of the convective zone on the processes under study remained outside the scope of our model. We plan to take this dependence into account in the further development of the model, which involves i the entire region of the convective zone.

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1. R. Wolf. Extract of a letter to Mr. Carrington. *Mon. Not. R. Astron. Soc.* **19**, 85 (1859).
2. G.E. Hale, S.B. Nicholson. The low of Sun-spot polarity. *Astrophys. Journ.* **62**, 270 (1925).
3. V.M. Efimenko, V.G. Lozitsky. Essential features of long-term changes of areas and diameters of sunspot groups

in solar activity cycles 12–24. *Adv. Space Res.* **61**, 2820 (2018).

4. I.G. Usoskin, S.K. Solanki, G.A. Kovaltsov. Grand minima and maxima of solar activity: New observational constraints. *Astron. Astrophys.* **471**, 301 (2007).
5. Ya.B. Zeldovich, A.A. Ruzmaikin, D.D. Sokoloff. *Magnetic Fields in Astrophysics* (Gordon and Breach, 1983).
6. M. Ossendrijver. The solar dynamo. *Astron. Astrophys. Review* **11** (4), 287 (2003).
7. P. Charbonneau. Dynamo models of the solar cycle. *Living Rev. Solar Phys.* **17** (4), 1 (2020).
8. E.N. Parker. The formation of sunspots from the solar toroidal field. *Astrophys. J.* **121**, 491 (1955).
9. F. Krause, K.-H. Raedler. *Mean Field Magnetohydrodynamics and Dynamo Theory* (Pergamon Press, Ltd., 1980).
10. A.G. Zagorodniy, O.K. Cheremnykh. *Introduction to Plasma Physics* (Naukova Dumka, 2014).
11. V.N. Krivodubskij. Turbulent dynamo near tachocline and reconstruction of azimuthal magnetic field in the solar convection zone. *Astronomische Nachrichten* **326** (1), 61 (2005).
12. V.N. Kryvodubskyj. Dynamo parameters of the solar convection zone. *Kinematics and Physics of Celestial Bodies* **22** (1), 1 (2006).
13. D.H. Hathaway. The solar cycle. *Living Rev. Solar Phys.* **12** (1), 1 (2015).
14. V.N. Krivodubskij. Role of rotational radial magnetic advection in possible explaining a cycle with two peaks. *Advances in Space Research* **68** (9), 3943 (2021).
15. A.A. Loginov, N.N. Salnikov, O.K. Cheremnykh, V.N. Krivodubskij, N.V. Maslova. Hydrodynamic model for generation of global poloidal flow of the Sun. *Kosm. nauka tehnol.* **17** (1), 29 (2011).
16. A.A. Loginov, N.N. Sal'nikov, O.K. Cheremnykh, Y.I. Zyelyk, N.V. Maslova. On the hydrodynamic mechanism of the generation of the global poloidal flux on the Sun. *Kinematics and Physics of Celestial Bodies* **27** (5), 217 (2011).
17. A.A. Loginov, O.K. Cheremnykh, V.N. Krivodubskij, N.N. Sal'nikov. Hydrodynamic model of torsional oscillations of the Sun. *Kosm. nauka tehnol.* **18** (1), 74 (2012).
18. A.A. Loginov, V.N. Krivodubskij, N.N. Salnikov, Y.V. Prutsko. Simulating the generation of the solar toroidal magnetic field by differential rotation. *Kinematics and Physics of Celestial Bodies.* **33** (6), 265 (2017).
19. A.A. Loginov, V.N. Krivodubskij, O.K. Cheremnykh. Generation of the radial magnetic field of the Sun by global hydrodynamic flows. *Kinematics and Physics of Celestial Bodies.* **36** (2), 56 (2020).
20. A.A. Loginov, V.N. Krivodubskij, O.K. Cheremnykh. Generation by global migratory flow of variable magnetic field of the Sun. *Kinematics and Physics of Celestial Bodies.* **37** (1), 16 (2021).
21. O.O. Botygina, M.Yu. Gordovskyy, V.G. Lozitsky. Estimation of the flux tube diameters outside sunspots using Hinode observations. Preliminary results. *Advanc. Astron. Space Phys.* **6**, 20 (2016).
22. H.W. Babcock. The topology of the Sun's magnetic field and the 22 year cycle. *Astrophys. J.* **133**, 572 (1961).
23. E.N. Parker. *Cosmical Magnetic Fields: Their Origin and Their Activity* (Clarendon Press, 1979).
24. A.A. Solov'ev, E.A. Kiritchek. *The Diffusion Theory of Solar Magnetic Cycle* (Elista-Saint-Petersburg, Kalmyk State University, 2004).
25. S. Basu, H.M. Antia. Characteristics of solar meridional flows during solar cycle 23. *Astrophys. J.* **717**, 488 (2010).
26. V.V. Pipin, A.G. Kosovichev. The asymmetry of sunspot cycles and Waldmeier relations as a result of nonlinear surface-shear shaped dynamo. *Astrophys. J.* **741** (1), 1 (2011).
27. V.V. Pipin, A.G. Kosovichev. The mean-field solar dynamo with a double cell meridional circulation pattern. *Astrophys. J.* **776** (1), 34 (2013).
28. D.R. Durran. Improving the anelactic approximation. *J. Atmos. Sci.* **46** (11), 1453 (1989).
29. NASA. Solar Interior. Web page. Accessed: 2021-09-30. http://solarscience.msfc.nasa.gov/interior.shtml (2021).
30. C.W. Allen. *Astrophysical Quantities*, 3rd Edition (London University: Athlon, 1976).
31. E.B. Gladzer, F.V. Dolzhansky, A.M. Obukhov. *Systems of Hydrodynamic Type and Their Application* (Nauka, 1982).
32. M.J. Thompson, J. Christensen-Dalsgaard, M.S. Miesch, J. Toomre. The Internal rotation of the Sun. *Annu. Rev. Astron. Astrophys.* **41** (1), 599 (2003).
33. J. Braithwaite, H.C. Spruit. Magnetic fields in non-convective regions of stars. *R. Soc. Open Sci.* **4**, 160271 (2017).
34. K.L. Harvey. *Magnetic Bipoles on the Sun* (Ph.D. thesis, Univ. Utrecht 1993).
35. Y. Fan. Magnetic Fields in the Solar Convection Zone. *Living Rev. Solar Phys.* **6** (4), 1 (2009).
36. J.M. Borrero, K. Ichimoto. Magnetic Structure of Sunspots. *Living Rev. Solar Phys.* **8** (4), 1 (2011).
37. H.W. Dodson. High latitude sunspot, August 13, 1953. *Publ. Astron. Soc. Pacific* **65** (386), 256 (1953).
38. B. Gillespie, J. Harvey, W. Livingston, K. Harvey. Polar magnetic fields and the new solar cycle. *Astrophys. J.* **186**, L85 (1973).
39. K.L. Harvey, J.W. Harvey, S.F. Martin. Ephemeral active regions in 1970 and 1973. *Solar. Phys.* **40** (1), 87 (1975).
40. J.O. Stenflo, A.G. Kosovichev. Bipolar magnetic regions on the Sun: global analysis of the SOHO//MDI data set. *Astrophys. J.* **745** (2), 129 (2012).
41. G.E. Hale, F. Ellerman, S.B. Nicholson, A.H. Joy. The magnetic polarity of sun-spots. *Astrophys. J.* **49**, 153 (1919).
42. A. Tlatov, E. Illarionov, D. Sokoloff, V. Pipin. A new dynamo pattern revealed by the tilt angle of bipolar sunspot groups. *Monthly Notices of the Royal Astronomical Society* **432** (4), 2975 (2013).
43. A.G. Tlatov, V.V. Vasil'eva, A.A. Pevtsov. Distribution of magnetic bipoles on the Sun over three solar cycles. *Astrophys. J.* **717** (1), 357 (2010).
44. R. Howard, B.J. LaBonte. The Sun is observed to be a torsional oscillator with a period of 11 years. *Astrophys. J.* **239**, L33 (1980).

45. S.S. Mahajan. *Observational Constraints on the Solar Dynamo and the Hunt for Precursors to Solar Flares* (PhD thesis, Georgia State University, 2019).
46. J. Zhao, A.G. Kosovichev. Torsional oscillation, meridional flows, and vorticity inferred in the upper convection zone of the Sun by time-distance helioseismology. *Astrophys. J.* **603** (2), 776 (2004).
47. R. Howe. Solar interior rotation and its variation. *Living Rev. Solar Phys.* **6** (1), (2009).
48. Y.-M. Wang, N.R.J. Sheeley. Average properties of bipolar magnetic regions during sunspot cycle 21. *Solar Phys.* **124** (1), 81 (1989).
49. A.I. Khlystova, D.D. Sokoloff. Toroidal magnetic field of the sun from data on hale-rule-violating sunspot groups. *Astronomy Reports* **53** (3), 281 (2009).
50. G.E. Hale, S.B. Nicolson. Magnetic Observations of Sunspots, 1917–1924. Part I. In: *Publ. Carnegie Inst. Washington D.C.* **438** (1938).
51. L.L. Kitchatinov, A.A. Nepomnyashchikh. Diamagnetic pumping in a rotating convection zone. *Advances in Space Research* **58** (8), 1554 (2016).
52. E. Illarionov, A. Tlatov, D. Sokoloff. The properties of the tilts of bipolar solar regions. *Solar Phys.* **290**, 351 (2015).

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МОДЕЛЬ КІНЕМАТИЧНОГО ДИНАМО ДЛЯ СОНЯЧНОГО МАГНІТНОГО ЦИКЛУ

У статті розглядається проблема пояснення походження та природи просторово-часових варіацій магнітної активності Сонця. У роботі представлена нова гідродинамічна модель сонячного магнітного циклу, яка використовує геліосейсмічні дані про диференціальне обертання сонячної конвективної зони. В основі моделі лежить гіпотеза виникнення глобальних потоків у результаті втрати стійкості диференційно ротаційного шару плазми в конвективній зоні. Спочатку розраховуються гідродинамічні глобальні потоки плазми без урахування впливу на неї магнітного поля. За цієї умови показано, що знайдені розв'язки описують усі глобальні потоки, що спостерігаються на поверхні Сонця: постійну меридіональну циркуляцію від екватора до полюсів, крутильні коливання та просторово-часові варіації меридіонального потоку. У роботі зроблено висновок про те, що останні два потоки є азимутальною та меридіональною складовими єдиного тривимірного глобального гідродинамічного потоку. По-друге, для моделювання динаміки магнітного поля були використані знайдені швидкості глобальних мігруючих потоків та просторовий профіль кутової швидкості внутрішнього диференціального обертання сонячної конвективної зони, отриманий за результатами геліосейсмічних вимірювань. Отримано хороше узгодження характеристик розрахункової динаміки глобальних міграційних потоків і створюваних ними змінних глобальних магнітних полів із спостережуваними значеннями на поверхні Сонця. Дано пояснення деяким явищам на поверхні Сонця, які неможливо пояснити в рамках існуючих моделей.

Ключові слова: плазма, утримання плазми, магнітогідродинаміка, сонячний магнітний цикл, сонячне динамо.