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PROBLEMATIC ISSUES OF METAL PLASMONICS OF BULK POLARITONS IN THE MAGNETOSTATIC FIELD

A method has been proposed to formalize the solution of the problems in the electrodynamics of bulk plasmon-polaritons in which there arises a difficulty associated with the choice of additional boundary conditions independent of the number of waves in the electronic component of plasmon-polaritons. This method is based on the application of Green's operator for the wave equation describing bulk plasmon-polaritons and the residue theory of the complex-variable analysis. In the framework of the general formulation of the problem and using the methods of tensor algebra, the matrix coefficients of reflection and refraction of electromagnetic waves at the metal surface have been determined under conditions when bulk plasmon-polaritons exist. Green's operator for the wave equation of bulk plasmon-polaritons in the magnetostatic field $\mathbf{H}_0$ *has been constructed, and their dispersion "surfaces"* $\omega = f(\mathbf{k}, \mathbf{H}_0)$ *have been analyzed.*

Keywords: Green's operator, plasmons, plasmon-polaritons, spatial dispersion, additional boundary conditions, magnetostatic field.

1. Introduction

In our previous work [1], we generalized the Drude–Lorentz model to the case of bulk and surface plasmons (by making allowance for the spatial dispersion effects but without taking the retardation effects into account, $c \to \infty$) in a non-magnetic metal specimen located in the external static magnetic, $\mathbf{H}_0$, and electric, $\mathbf{E}_0$, fields. In this work, we extend the analysis of metal-plasmonic phenomena to the case of plasmon-polaritons (for which the retardation effects are significant, $c \neq \infty$) in a non-magnetic metal specimen, provided the same external conditions that were adopted earlier [1]. Similarly to what was done earlier, illustrative calculations were performed for indium antimonide (InSb, the n-type semiconductor with a narrow bandgap of about 0.18 eV) taken as an example, which is widely used in electronics and instrument engineering due to its unique physical properties [2].

It is quite reasonable that taking spatial dispersion effects into account revives the old problem of additional boundary conditions [3–7] because ordinary electrodynamic boundary conditions are not enough in this case. There are no fundamental difficulties here. In particular, boundary conditions for the electronic component (plasmons) of plasmon-polaritons can be used. Instead, there arises the problem of argumentation in favor of that or another choice of additional boundary conditions.

In this paper, we propose a method that allows the solution of electrodynamic problems dealing with bulk plasmon-polaritons in the magnetostatic field $\mathbf{H}_0$ to be formalized. Furthermore, the choice of additional boundary conditions turns out independent of the number of waves in the electronic component of plasmon-polaritons. This method is based on the application of Green's operator [8] for the wave equation of bulk plasmon-polaritons and the residue theory of the complex-variable analysis [9].

In the framework of the general problem formulation, as well as using Green's operator and the methods of tensor algebra, the reflection, $\hat{N}$, and refraction, $\hat{R}$, matrix coefficients have been constructed for an electromagnetic wave incident on the surface of a metal specimen located in the magnetostatic field $\mathbf{H}_0$, with the wave frequency being within the existence domain of bulk plasmon-polaritons. The challenging character of this problem is associated with the wide implementation of stealth technologies in the modern aerospace industry.

A general dispersion equation for bulk plasmon-polaritons in the magnetostatic field $\mathbf{H}_0$ has been found. The influence of spatial dispersion and magnetostatic field $\mathbf{H}_0$ on the physical properties of plasmon-polaritons has been analyzed. It has been shown that, in contrast to bulk plasmon-polaritons in the standard Drude–Lorentz model [10–12], there arise two additional types of low-frequency bulk plasmon-polaritons. Plasmon-polaritons of the first type (with the lower frequency) are formed owing to both the spatial dispersion and the magnetostatic field $\mathbf{H}_0$, whereas plasmon-polaritons of the other type are mainly generated by only the magnetostatic field $\mathbf{H}_0$ because the influence of spatial dispersion on their physical properties is negligibly weak.

As concerns high-frequency bulk plasmon-polaritons, the influence of spatial dispersion and magnetostatic field $\mathbf{H}_0$ on their physical properties is insignificant, so they are similar to those in the standard Drude–Lorentz model [10–12].

2. Green's Operator and the Problem of Additional Boundary Conditions in Metal Plasmonics with Spatial Dispersion

Consider a macroscopic monochromatic electromagnetic field

$$\begin{aligned} \mathbf{E} &= \mathbf{E}(\omega, \mathbf{r}) \exp(-i\omega t), \\ \mathbf{H} &= \mathbf{H}(\omega, \mathbf{r}) \exp(-i\omega t), \end{aligned} \tag{1}$$

which satisfies a wave equation in a linear crystalline medium with given material parameters [13, 14]. Its solution can be formally expressed via Green's integral operator $\hat{G}$ [8],

$$\mathbf{E}(\omega, \mathbf{r}) = \int \hat{G}(\omega, \mathbf{r} - \mathbf{r}')\, \mathbf{J}\,(\omega, \mathbf{r}') d\mathbf{r}', \tag{2}$$

where $\mathbf{J} = \mathbf{J}(\omega, \mathbf{r})$ is the vector of electric current density (see below).

The equation satisfied by the Fourier image of Green's operator

$$\hat{G}(\omega, \mathbf{k}) = \int \hat{G}(\omega, \mathbf{r}) \exp(-i\mathbf{k}\mathbf{r}) d\mathbf{r}$$

looks like

$$W_{\alpha\gamma}(\omega, \mathbf{k}) G^{\gamma\beta}(\omega, \mathbf{k}) = \frac{4\pi i\omega}{c^2} \delta_\alpha^\beta, \tag{3}$$

where $\hat{W}$ is the wave operator obtained proceeding from the Maxwell equations for the non-magnetic metal,

$$W_{\alpha\beta} = \mathbf{k}^2 \delta_{\alpha\beta} - k_\alpha k_\beta - q^2 \hat{\varepsilon}_{\alpha\beta}(\omega, \mathbf{k}), \quad q = \frac{\omega}{c},$$

and $\hat{\varepsilon}$ is the tensor of dielectric permittivity for the metal in the magnetostatic field $\mathbf{H}_0$ [1].

The Fourier image of Green's operator is expressed using the matrix

$$A_{\alpha\beta}(\hat{W}) = \frac{1}{2}\, e_{\alpha\nu\mu} e_{\beta\gamma\lambda} W^{\gamma\nu} W^{\lambda\mu} \tag{4}$$

and the determinant $\det(\hat{W})$ of the wave operator $\hat{W}$ as follows:

$$G_{\alpha\beta} = \frac{4\pi i\omega}{c^2} \frac{A_{\alpha\beta}(\hat{W})}{\det(\hat{W})}. \tag{5}$$

The poles of the Fourier image of Green's operator $\hat{G}$ determine the dispersion equation for bulk plasmon-polaritons in the usual way, i.e., $\det(W) = 0$.

The electric field of plasmon-polaritons, $\mathbf{E} = \mathbf{E}(\omega, \mathbf{r})$, is determined in the physical space as the integral

$$\mathbf{E} = \frac{1}{(2\pi)^3} \int \hat{G}(\omega, \mathbf{k})\, \mathbf{J}\,(\omega, \mathbf{k})\, \exp(i\mathbf{k}\mathbf{r})\, d\mathbf{k}, \tag{6}$$

which is calculated using the methods of the residue theory of the complex-variable analysis [9]. When calculating integral (6), the choice of the integration contour must be consistent with the radiation principle.

By calculating integral (6), we obtain a superposition of electromagnetic waves with the frequency ω. Their number is equal to the number of physically significant poles of the Fourier image of Green's operator. It is quite clear that the amplitudes of these waves (irrespective of their number) depend on the same vector $\mathbf{J}$. In particular, while considering the problem of electromagnetic wave reflection from a flat interface between two media, $z = 0$, the electric field of refracted waves is determined by the one-dimensional integral

$$\mathbf{E} = \frac{1}{2\pi} \int \hat{G}(\omega, \mathbf{k})\, \mathbf{J}\,(\omega, \mathbf{k})\, \exp(i\mathbf{k}\mathbf{r})\, dk_z. \tag{7}$$

Formally the result of integration in expression (7) has the following representation:

$$\mathbf{E} = \sum_{\mathbf{k}_j} \hat{U}(\omega, \mathbf{k}_j)\, \mathbf{J}\,(\omega, \mathbf{k}_j)\, \exp(i\mathbf{k}_j\mathbf{r} - \omega t). \tag{8}$$

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The explicit form of the matrix $\hat{U} = \hat{U}(\omega, \mathbf{k})$ is determined by the material parameters of the metal specimen. The vector $\mathbf{J}$ in Eq. (7) is an analog of the integration constant, and its expression can be determined using electrodynamic boundary conditions (see below).

Thus, it was shown that the application of Green's operator and the residue theory of the complex-variable analysis in metal plasmonics with spatial dispersion makes it possible to avoid the problem of choosing additional boundary conditions for the electronic component of plasmon-polaritons. This approach is reasonable because the influence of the state of the near-surface zone on the process of plasmon-polariton formation in the depth of metal specimen turns out insignificant.

As for surface plasmon-polaritons, the influence of the near-surface zone on their properties can be significant. However, it can be taken into account analogously to what was done in work [1] owing to the reduction of electrodynamic boundary conditions.

3. The Application of Green's Operator Method in the General Formulation of the Problem of Electromagnetic Wave Reflection and Refraction at the Metal Surface

Let us consider the problem of electromagnetic wave reflection and refraction at the plane insulator-metal interface $z = 0$ in the general formulation and using the Green's operator method. We are interested in the frequency intervals where plasmon-polaritons exist.

Let the metal fill the half-space $z \leq 0$. The electric field of the electromagnetic wave incident on the surface $z = 0$ is approximated by the plane monochromatic wave

$$\mathbf{E} = \mathbf{E}_0\,(\omega, \mathbf{k})\,\exp{(i\mathbf{kr} - i\omega t)}, \quad \text{at } z \geq 0. \tag{9}$$

In principle, expressions in the Fourier space for the reflection, $\hat{N} = \hat{N}(\omega, \mathbf{k})$, and refraction, $\hat{R} = \hat{R}(\omega, \mathbf{k})$, matrix coefficients, which determine the electric fields of the reflected, $\mathbf{E}_1 = \mathbf{E}_1(\omega, \mathbf{k})$, and refracted, $\mathbf{E}_2 = \mathbf{E}_2(\omega, \mathbf{k})$, electromagnetic waves are well-known for some special geometries of the problem [15,16]. However, in the general formulation of the problem of bulk polaritons in metal plasmonics, there arise a number of difficulties. For their solution, it is pertinent to apply the Green's operator method and express the electric fields

$$\begin{aligned} &\mathbf{E}_1 = (\hat{N}(\omega, \mathbf{k})\,\mathbf{E}_0(\omega, \mathbf{k})), \\ &\mathbf{E}_2 = (\hat{R}(\omega, \mathbf{k})\,\mathbf{E}_0(\omega, \mathbf{k})) \end{aligned} \tag{10}$$

and the matrix coefficients $\hat{N}$ and $\hat{R}$ in terms of expressions (6)–(8).

The logic of applying the Green's operator method requires an introduction of the auxiliary surface electric current density

$$\mathbf{J} = \mathbf{J}_0\,\delta(z)\,\exp{(i\mathbf{kr} - i\omega t)}, \tag{11}$$

into consideration, which would play the role of a source of the reflected and refracted electromagnetic waves. Its Fourier image looks like

$$\mathbf{J}(\omega', \mathbf{k}'_\tau) = \frac{\mathbf{J}_0}{2\pi}\delta(\omega - \omega')\delta(\mathbf{k}_\tau - \mathbf{k}'_\tau), \tag{12}$$

where $\mathbf{k}_\tau = (k_x, k_y, 0)$ is the wave vector tangent to the interface $z = 0$ between the contacting media.

First, let us determine the electric field of plasmon-polaritons. By combining formulas (5), (7), and (12), we obtain the field representation in the form of the integral

$$\mathbf{E} = \frac{2i\omega}{c^2}\int \frac{\hat{A}(\hat{W}(\omega, \mathbf{k}))}{\det(\hat{W}(\omega, \mathbf{k}))}\,\mathbf{J}_0\,\exp{(i\mathbf{kr}_i\omega t)}dk_z. \tag{13}$$

It can be calculated using the method of the residue theory of the complex-variable analysis. Formally the integration result can be written as follows:

$$\mathbf{E} = \sum_{\mathbf{k}_j}(\hat{U}(\omega, \mathbf{k}_j)\,\mathbf{J}_0)\,\exp{(i\mathbf{k}_j\mathbf{r} - i\omega t)}, \tag{14}$$

where $\mathbf{k}_j$ are wave vectors determined by the poles of the integrand in Eq. (13), i.e., the solutions k_z's of the equation $\det \hat{W}(\omega, \mathbf{k}) = 0$. The integration contour in Eq. (13) is chosen so that the result of integration leads to electromagnetic waves propagating from the interface $z = 0$ between two media into the metal depth.

The magnetic field of plasmon-polaritons is determined in a similar way,

$$\mathbf{H} = \sum_{\mathbf{k}_j}\frac{\omega}{c}(\hat{B}(\omega, \mathbf{k}_j)\cdot\mathbf{J}_0)\,\exp{(i\mathbf{k}_j\mathbf{r} - i\omega t)}, \tag{15}$$

where

$\hat{B}(\omega,\mathbf{k}) = (k^{\times} \cdot \hat{U}(\omega,\mathbf{k}))$,

and

$$k^{\times} = \begin{bmatrix} 0 & -k_z & k_y \\ k_z & 0 & -k_x \\ -k_y & k_x & 0 \end{bmatrix} \tag{16}$$

is the dual tensor of the wave vector $\mathbf{k}$.

Formulas (14) and (15) completely determine the structure of the electromagnetic field of plasmon-polaritons in a metal specimen embedded in a magnetostatic field. Concerning the quantity $\mathbf{J}_0$ in Eqs. (14) and (15), this vector is determined according to electrodynamic boundary conditions [13] via the amplitude of the electric field $\mathbf{E}_0$ of the electromagnetic wave incident on the metal surface.

In order to determine expressions for the matrix coefficients, let us use electrodynamic boundary conditions [13]. In so doing, we should take into account that a non-magnetic metal specimen is considered in this paper. Therefore, the magnetic field is continuous across the interface $z = 0$. Under such conditions, the system of electrodynamic boundary conditions for the electromagnetic fields takes the form

$$\begin{aligned} &((\mathbf{E}_0 + \mathbf{E}_1 - \mathbf{E}_2) \times \mathbf{n})\Big|_{z=0} = 0, \\ &(\mathbf{H}_0 + \mathbf{H}_1 - \mathbf{H}_2)\Big|_{z=0} = 0, \end{aligned} \tag{17}$$

where $\mathbf{E}_{0,1,2}$ are the electric fields, $\mathbf{H}_{0,1,2}$ the magnetic ones, and $\mathbf{n}$ is the external normal vector to the metal surface. Then, using the second Maxwell equation [13]

$$(\mathbf{k} \times \mathbf{H}) = -\frac{\omega}{c}\mathbf{D}, \tag{18}$$

where $\mathbf{D}$ is the vector of electric field induction, as well as formulas (14) and (15), we arrive at the following closed system of equations for the unknown quantities $\mathbf{E}_1$ and $\mathbf{J}_0$:

$$\begin{aligned} &((\mathbf{E}_0 + \mathbf{E}_1 - (\hat{U}_2 \cdot \mathbf{J}_0)) \times \mathbf{n})\Big|_{z=0} = 0, \\ &((\mathbf{k}_0 \times \mathbf{E}_0) + (\mathbf{k}_1 \times \mathbf{E}_1) - \varepsilon(\hat{B}_2 \cdot \mathbf{J}_0))\Big|_{z=0} = 0, \end{aligned} \tag{19}$$

where

$$\hat{U}_2 = \sum_{\mathbf{k}_j} \hat{U}(\omega,\mathbf{k}_j), \quad \hat{B}_2 = \sum_{\mathbf{k}_j} \hat{B}(\omega,\mathbf{k}_j). \tag{20}$$

In an isotropic medium with the dielectric permittivity $\varepsilon = \text{const}$, the polarization of the electromagnetic field is transverse, and its phase is determined by the dispersion equation

$$(\mathbf{k}_{0,1}\,\mathbf{E}_{0,1}) = 0, \quad {\mathbf{k}_{0,1}}^2 - q^2\varepsilon = 0. \tag{21}$$

Vectorially multiplying the first equation in (19) by $\mathbf{k}_1$ and taking into account that the electromagnetic waves are transverse in the region $z > 0$ (see Eq. (21)), we obtain the following expression for the electric field amplitude of the reflected wave:

$$\mathbf{E}_1 = \frac{1}{(\mathbf{k}_1 \cdot \mathbf{n})}(\mathbf{k}_1 \times (\mathbf{n} \times ((\hat{U}_2 \cdot \mathbf{J}_0) - \mathbf{E}_0))). \tag{22}$$

Substituting this expression into the second equation in (19), we obtain the equation

$$\begin{aligned} &(\mathbf{k}_1 \times (\mathbf{k}_1 \times (\mathbf{n} \times (\hat{U}_2 \cdot \mathbf{J}_0)))) - \varepsilon(\mathbf{k}_1 \cdot \mathbf{n})(\hat{B}_2 \cdot \mathbf{J}_0) - \\ &- (\mathbf{k}_1 \times (\mathbf{k}_1 \times (\mathbf{n} \times \mathbf{E}_0))) + (\mathbf{k}_1 \cdot \mathbf{n})(\mathbf{k}_0 \times \mathbf{E}_0) = 0 \end{aligned} \tag{23}$$

for the vector $\mathbf{J}_0$. To solve it, let us rewrite it in the matrix representation. For this purpose, besides (16), let us introduce the matrices

$$\begin{aligned} &n^{\times} = \begin{bmatrix} 0 & -n_z & n_y \\ n_z & 0 & -n_x \\ -n_y & n_x & 0 \end{bmatrix}, \\ &P_{1\alpha\beta} = {\mathbf{k}_1}^2\delta_{\alpha\beta} - k_{1\alpha}k_{1\beta}. \end{aligned} \tag{24}$$

Then, applying notations (24) to Eq. 23), we obtain the matrix equation

$$(\hat{X} \cdot \mathbf{J}_0) = -(\hat{F} \cdot \mathbf{E}_0), \tag{25}$$

for $\mathbf{J}_0$, where

$$\begin{aligned} &\hat{X} = (\hat{P}_1 \cdot n^{\times} \cdot \hat{U}_2) + \varepsilon(\mathbf{k}_1 \cdot n)\hat{B}_2, \\ &\hat{F} = (\hat{P}_1 \cdot n^{\times}) - (\mathbf{k}_1 \cdot n)n^{\times}. \end{aligned} \tag{26}$$

Its solution is

$$\mathbf{J}_0 = (\hat{Z} \cdot \mathbf{E}_0), \tag{27}$$

where

$$\hat{Z} = -(\hat{X}^{-1} \cdot \hat{F}).$$

Formulas (22), (14), and (27) completely determine the structure of the electric fields of the reflected and refracted electromagnetic waves at the interface $z = 0$

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between the isotropic insulator and the non-magnetic metal:

$$\mathbf{E}_1 = (\hat{N}\cdot\mathbf{E}_0),\quad \mathbf{E}_2 = (\hat{R}\cdot\mathbf{E}_0), \tag{28}$$

where

$$\hat{N} = \frac{1}{(\mathbf{k}_1\cdot\mathbf{n})}\,(k_1^{\times}\cdot n^{\times}\cdot((\hat{U}_2\cdot\hat{Z}) - I)),$$
$$\hat{R} = (\hat{U}_2\cdot\hat{Z}),$$

and $\hat{I}$ is the unit tensor.

A comparison between expressions (28) and (10) testifies that the quantities $\hat{N}$ and $\hat{R}$ in Eq. (28) are the matrix coefficients of electromagnetic wave reflection and refraction, respectively, at the interface $z = 0$ between the isotropic insulator and the non-magnetic metal, which were obtained in the framework of the general problem formulation.

The quantities $\hat{N}$ and $\hat{R}$ have a clear for understanding but rather cumbersome matrix structure. In essence, it is not difficult to program this structure in one of the high-level algorithmic languages for performing numerical calculations.

4. Green's Operator for Plasmon-Polaritons in Non-Magnetic Metals Embedded in the Magnetostatic Field $\mathbf{H}_0$

In what follows, we will confine ourselves to the consideration of plasmon-polaritons in the non-magnetic ($\hat{\mu} = 1$) metal located in the external magnetostatic field $\mathbf{H}_0$. The dielectric permittivity $\hat{\varepsilon}$ of the metal, which is described by the generalized Drude–Loretz model [1], can be written in the form

$$\varepsilon_{\alpha\beta} = \delta_{\alpha\beta} - \omega_p^2\left(\frac{D_0}{D_0^2 - \omega^2\boldsymbol{\omega}_H^2}\delta_{\alpha\beta} - \right.$$
$$\left. - \frac{\omega^2\omega_{H\alpha}\omega_{H\beta}}{D_0(D_0^2 - \omega^2\boldsymbol{\omega}_H^2)} + i\frac{\omega e_{\alpha\beta\gamma}{\omega_H}^{\gamma}}{D_0^2 - \omega\boldsymbol{\omega}_H^2}\right), \tag{29}$$

where

$$\omega_p^2 = \frac{4\pi n e^2}{m^*},\quad \boldsymbol{\omega}_H = \frac{e\mathbf{H}_0}{m^* c},$$
$$D_0 = \omega^2 - \omega_{\mathbf{k}}^2 + 2i\omega\gamma,\quad \omega_{\mathbf{k}} = \omega_p r_{\mathrm{D}}|\mathbf{k}|,$$

ω_p is the cyclic plasma frequency, ω_H the cyclotron frequency vector, r_{D} the Debye radius of electron screening in the metal, and γ the plasmon damping parameter.

Let us introduce the following parameters:

$$Q = \mathbf{k}^2 - q^2\varepsilon_0,\quad \varepsilon_0 = 1 - \frac{{\omega_p}^2 D_0}{{D_0}^2 - \omega^2{\boldsymbol{\omega}_H}^2}, \tag{30}$$

and

$$M_{\alpha\beta} = q^2 M m_\alpha m_\beta,\ M = \frac{{\omega_p}^2\omega^2{\boldsymbol{\omega}_H}^2}{D_0({D_0}^2 - \omega^2{\boldsymbol{\omega}_H}^2)},$$
$$G_{\alpha\beta} = q^2 G {e_{\alpha\beta}}^{\gamma} m_\gamma,\ G = -\frac{{\omega_p}^2\omega|\boldsymbol{\omega}_H|}{{D_0}^2 - \omega^2{\boldsymbol{\omega}_H}^2}, \tag{31}$$

where $\mathbf{m} = \boldsymbol{\omega}_H/|\boldsymbol{\omega}_H|$ is the unit vector directed along the magnetostatic field $\mathbf{H}_0$. Quantities (29)–(31) make it possible to express the dielectric permittivity of the metal and the wave operator in the following form

$$\varepsilon_{\alpha\beta} = \varepsilon_0\delta_{\alpha\beta} + M m_\alpha m_\beta + iG{e_{\alpha\beta}}^{\gamma} m_\gamma,$$
$$W_{\alpha\beta} = Q\delta_{\alpha\beta} - (k_\alpha k_\beta + M_{\alpha\beta}) - iG_{\alpha\beta}. \tag{32}$$

Then making use of the algebraic cofactors

$$A_{\alpha\beta}(\hat{W}) = Q(Q\delta_{\alpha\beta} - (\mathbf{k}^2\delta_{\alpha\beta} - k_\alpha k_\beta) -$$
$$- q^2 M(\mathbf{m}^2\delta_{\alpha\beta} - m_\alpha m_\beta)) +$$
$$+ q^2(M{e_\alpha}^{\nu\mu}k_\nu m_\mu {e_\beta}^{\nu\mu}k_\nu m_\mu - q^2G^2 m_\alpha m_\beta) -$$
$$- iq^2 G{e_{\alpha\beta}}^{\nu}((\mathbf{k}\cdot\mathbf{m})k_\nu - (Q - q^2M)m_\nu) \tag{33}$$

and the determinant

$$\det(\hat{W}) = Q(Q(Q - \mathbf{k}^2 - q^2M) +$$
$$+ q^2(M(\mathbf{k}\times\mathbf{m})^2 - q^2G^2)) + q^4G^2((\mathbf{k}\cdot\mathbf{m})^2 + q^2M) \tag{34}$$

of the wave operator (32), the Fourier images of Green's operator and the electric field of plasmon-polaritons can be written in the form

$$G_{\alpha\beta} = \frac{4\pi i\omega}{c^2}\frac{A(\hat{W})_{\alpha\beta}}{\det(\hat{W})},\quad \mathbf{E} = \hat{G}\mathbf{J}, \tag{35}$$

where $\mathbf{J} = \mathbf{J}(\omega, \mathbf{k})$ is the Fourier image of the electric current density.

5. Bulk Plasmon-Polaritons in Non-Magnetic Metals Embedded in the Magnetostatic Field $\mathbf{H_0}$

The key task of plasmonics of metals in the magnetostatic field $\mathbf{H}_0$ is finding the solutions to the dispersion equation $\det(\hat{W}) = 0$. These solutions determine the set of poles of the Fourier image of Green's

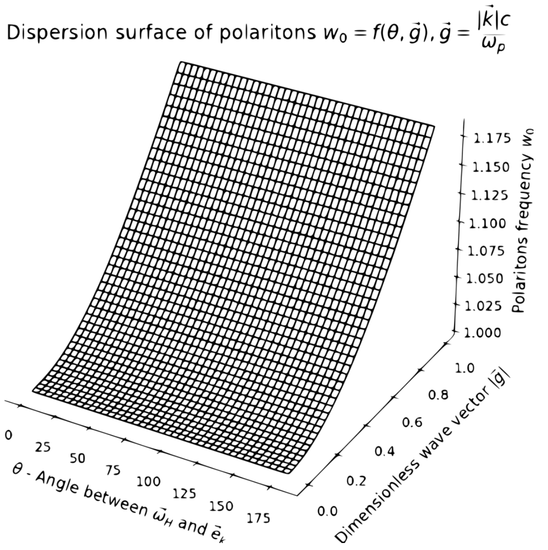

Fig. 1. Dispersion "surface" of high-frequency plasmon-polaritons in the negligibly weak magnetostatic field ($\mathbf{H}_0 \to 0$)

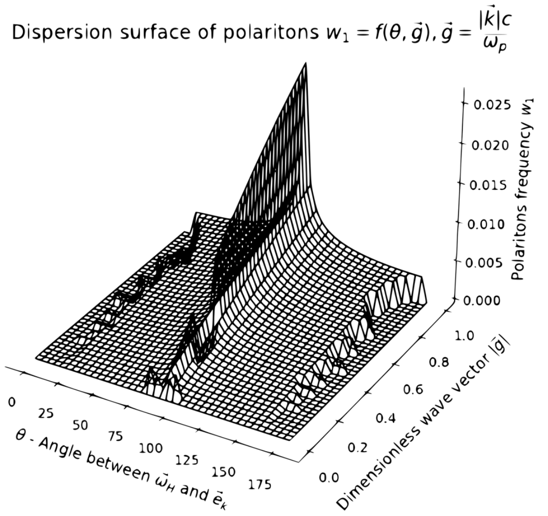

Fig. 2. Dispersion "surface" of low-frequency plasmon-polaritons taking the magnetostatic field $\mathbf{H}_0$ and the spatial dispersion into account

operator and, therefore, the structure of the electromagnetic field of bulk plasmon-polaritons. To find the solutions, it is convenient to introduce the following dimensionless variables:

$$w = \frac{\omega}{\omega_p}, \quad \mathbf{w}_H = \frac{\boldsymbol{\omega}_H}{\omega_p}, \quad \mathbf{g} = \frac{\mathbf{k}c}{\omega_p},$$
$$\mathbf{w}_g = \rho_\mathrm{D}\mathbf{g}, \quad \rho_\mathrm{D} = \frac{\omega_p}{c} r_\mathrm{D}, \quad \frac{\gamma}{\omega_p} \ll 1. \tag{36}$$

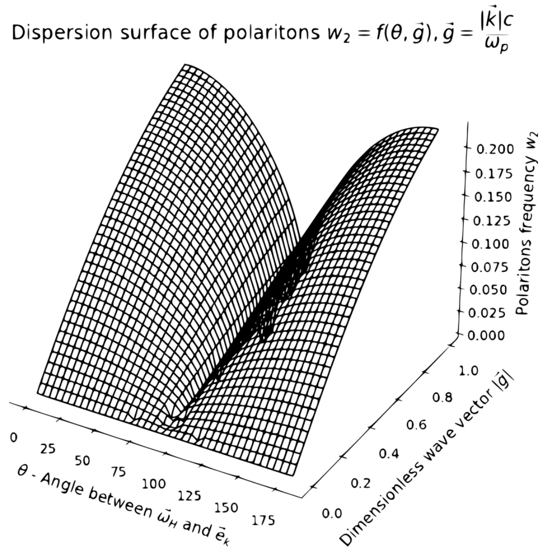

Fig. 3. Dispersion "surface" of low-frequency plasmon-polaritons taking the magnetostatic field $\mathbf{H}_0$ into account

Using them in Eqs. (33) and (34), we obtain the required dispersion equation for bulk plasmon-polaritons in the following form:

$$Q(Q(Q - \mathbf{g}^2 - w^2 M) + w^2(M(\mathbf{g}\times\mathbf{m})^2 - w^2 G^2)) +$$
$$+ w^4 G^2((\mathbf{g}\cdot\mathbf{m})^2 + w^2 M) = 0, \tag{37}$$

where

$$Q = \mathbf{g}^2 - w^2\varepsilon_0, \quad \varepsilon_0 = 1 - \frac{D_g}{{D_g}^2 - w^2{w_H}^2},$$
$$M = \frac{w^2{w_H}^2}{D_g({D_g}^2 - w^2{w_H}^2)}, \tag{38}$$
$$G = -\frac{w w_H}{{D_g}^2 - w^2{w_H}^2}, \quad D_g = w^2 - {w_g}^2.$$

First, let us find a solution for the dispersion equation of plasmon-polaritons of a non-magnetic metal in the absence of the external magnetostatic field ($\mathbf{H}_0 \to 0$),

$$Q = \mathbf{g}^2 - w^2\Big(1 - \frac{1}{w^2 - w_\mathbf{g}^2}\Big) = 0 \tag{39}$$

The solution is unique and looks like

$${w_0}^2 = \frac{1}{2}(\mathbf{g}^2 + 1 - {w_\mathbf{g}}^2) +$$
$$+ \frac{1}{2}\sqrt{(\mathbf{g}^2 + 1 - {w_\mathbf{g}}^2)^2 + 4{w_\mathbf{g}}^4}. \tag{40}$$

The graphical representation of solution (40) of Eq. (39) is shown in Fig. 1. The latter will be used further to compare the dispersion "surfaces" of plasmon-polaritons in metal in the external magnetostatic field $\mathbf{H}_0$. Since $w_{\mathbf{g}} \ll 1$, the influence of spatial dispersion on the physical properties of plasmon-polaritons is negligibly small here, and the solution of Eq. (39) practically coincides with the dispersion relationship for plasmon-polaritons in the standard Drude–Lorentz model [10–12].

In our previous paper [1], it was shown that the arrangement of a metal specimen in an external magnetostatic field $\mathbf{H}_0$ together with the assumption $c \to \infty$ (the retardation effects are neglected) results in the appearance of two additional types of low-frequency bulk plasmons. It is quite clear that taking the retardation effects into account ($c < \infty$) leads to the appearance of corresponding plasmon-polaritons. Their dispersion "surfaces" are determined by the solutions of Eq. (37), which can be calculated only numerically. The very structure of Eq. (37) testifies to a considerable influence of the magnetostatic field $\mathbf{H}_0$ on the dispersion of plasmon-polaritons.

Numerical solutions of the dispersion equation (37) corresponding to low-frequency plasmon-polaritons form the dispersion "surfaces" depicted in Figs. 2 and 3. In particular, Fig. 2 illustrates the dispersion "surface" of low-frequency plasmon-polaritons formed due to the complex effect of spatial dispersion and the magnetostatic field $\mathbf{H}_0$ (the microwave frequency interval, $\omega \approx \omega_{\mathbf{k}}$). One can see that in this case plasmon-polaritons are formed in fact if the vectors $\mathbf{k}$ and $\mathbf{H}_0$ are mutually orthogonal. We also draw attention to the circumstance that the dependence $\omega_1 = f_1(\mathbf{k})$ is nonmonotonic.

On the other hand, Fig. 3 demonstrates the dispersion "surface" for low-frequency plasmon-polaritons induced by the magnetostatic field $\mathbf{H}_0$ (the UHF interval, $\omega \approx \omega_H$). In this case, the influence of spatial dispersion is negligibly weak ($\omega_{\mathbf{k}} \ll \omega_H$).

By comparing Figs. 3 and 1, we can make a conclusion that the magnetostatic field $\mathbf{H}_0$ suppresses the formation of plasmon-polaritons if the relative orientation of the vectors $\mathbf{k}$ and $\mathbf{H}_0$ is close to orthogonal.

The remaining (third and last) available solution of the dispersion equation (37) corresponds (see Fig. 4) to high-frequency plasmon-polaritons ($\omega \approx \omega_p$), which are analogous to those in the standard Drude–Lorentz model [10–12]. From comparing Fig. 4 with Fig. 1, a conclusion follows that in the former case the spatial dispersion and the magnetostatic field $\mathbf{H}_0$ weakly affect the physical properties of plasmon-polaritons because $\omega_{\mathbf{k}} \ll \omega_p$ and $\omega_H \ll \omega_p$.

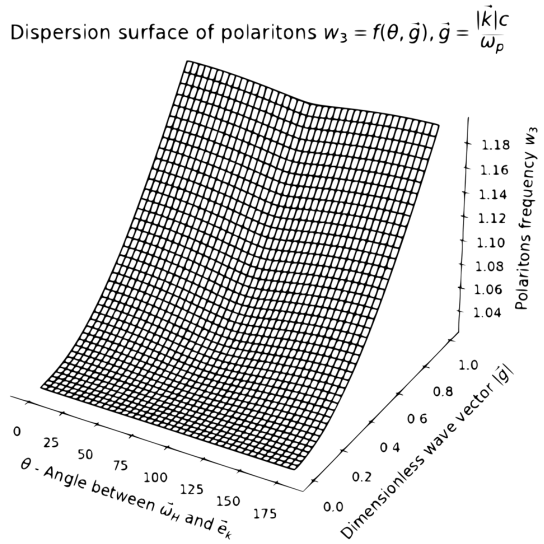

Fig. 4. Dispersion "surface" of high-frequency plasmon-polaritons in the magnetostatic field $\mathbf{H}_0$

6. Conclusions

In this paper, a method has been proposed that formalizes the solution of the problems dealing with electrodynamics of bulk plasmon-polaritons, where there arises a difficulty associated with the selection of additional boundary conditions. The method is independent of the number of waves in the electronic component of bulk plasmon-polaritons. It is based on the application of Green's operator for the wave equation of bulk plasmon-polaritons and the residue theory of the complex-variable analysis.

Using the properties of Green's operator for the wave equation of bulk plasmon-polaritons and tensor algebra methods, the matrix coefficients of reflection and refraction of electromagnetic waves at the metal surface in the frequency interval of bulk plasmon-polariton existence have been found in the framework of the general formulation of the problem. Green's operator of the wave equation of bulk plasmon-polaritons in the magnetostatic field $\mathbf{H}_0$ has been constructed. The poles of the Fourier image of Green's operator determine the spectrum of bulk plasmon-polaritons of the metal specimen embedded in a magnetostatic field. It was found that the spatial dispersion and the magnetostatic field $\mathbf{H}_0$ lead to the appearance of additional types (as compared to the standard Drude–Lorentz model) of bulk plasmon-polaritons with the dispersion that substantially depends on the mutual orientation of their propagation direction $\mathbf{e}_k$ and the magnetostatic field $\mathbf{H}_0$. The dependence of the physical properties of plasmon-polaritons on the magnetostatic field $\mathbf{H}_0$ can be used to implement control in applied problems of metal plasmonics.

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ПРОБЛЕМНІ ПИТАННЯ
МЕТАЛОПЛАЗМОНІКИ ОБ'ЄМНИХ
ПОЛЯРИТОНІВ У МАГНІТОСТАТИЧНОМУ ПОЛІ

Запропоновано метод, який формалізує розв'язок задач електродинаміки об'ємних плазмон-поляритонів, де виникає проблема з вибором додаткових граничних умов, незалежний від кількості хвиль у їх електронній компоненті. Цей метод базується на використанні оператора Гріна для хвильового рівняння об'ємних плазмон-поляритонів та теорії лишків функцій комплексної змінної. У загальній постановці задачі, використовуючи методи тензорної алгебри, знайдено матричні коефіцієнти відбиття та заломлення електромагнітних хвиль на поверхні металу у області існування об'ємних плазмон-поляритонів. Побудовано оператор Гріна хвильового рівняння об'ємних плазмон-поляритонів у магнітостатичному полі $\mathbf{H}_0$ та проаналізовано їх дисперсійні "поверхні" $\omega = f(\mathbf{k}, \mathbf{H}_0)$.

Ключові слова: оператор Гріна, плазмони, плазмон-поляритони, просторова дисперсія, додаткові граничні умови, магнітостатичне поле.